The problem is in NP

MONOTONE ALMOST-ALL-TRUE SAT is in NP, since given an assignment where k variables are set to false (**certificate**), we can check the number of false variables for each clause and the result of the formula Φ in polynomial time (**certifier**) since the number of variables and clauses is polynomial.

The problem is NP-hard

We choose INDEPENDENT SET to show that INDEPENDENT SET $\leq_P$ MONOTONE ALMOST-ALL-TRUE SAT.

Given a graph $G = (V, E)$ and integer k, we would like to now whether there exists an independent set of size k in G. we can solve it using a black box for MONOTONE ALMOST-ALL-TRUE SAT. The construction looks like the follows: for each edge in E, the nodes at the end of the edge correspond to the variables $x_1, x_2, x_3, \cdots$, and the edge correspond to a clause in formula Φ. Then we constructed a MONOTONE ALMOST-ALL-TRUE SAT from INDEPENDENT SET.

The reduction can be done in polynomial time since the only thing we implemented in the construction of a MONOTONE ALMOST-ALL-TRUE SAT is to assign the edges and nodes to variables and clauses, which costs $O(E)$ time.

Now we would like to prove the correctness by showing it is an if and only if statement:

If we have found an independent set I of size k on G, the variables correspond to the nodes in I can be set to false, while other variables are set to true. Since we know that such nodes are not connected (which is the definition of independent set), there will be at most 1 variable set to false for each clause. Given that for each clause there are at least 2 variables and there is at most 1 variable set to false, the formula Φ will be true.

If we have found an assignment of MONOTONE ALMOST-ALL-TRUE SAT, the corresponding graph G can be constructed. We simply add n nodes where n is the numbers of the variables and connect the nodes based on the clauses where the variables correspond to the nodes that are at the end of edges. We would like to know if the set of nodes correspond to variables with false values is a independent set. It is true since for each clause, there is only one variables set to false, indicating that is no edge that connect two or more nodes in the set (there is no clauses contain 2 or more false variable).