(a)

Since there is only 3 different colors, so the number of cases that two ends of edge e get the same color is 3. The total number of possible cases is 3×3. Hence, the answer is:

$$Pr = \frac{3}{3 \times 3} = \frac{1}{3} \tag{1}$$

(b)

For each edge e, we have computed in part (a) that the probability that it is badly colored is $1/3$, hence the probability that it is colored properly is $1 - 1/3 = 2/3$. For the whole graph, since the color is sampled independently for each node, then the Expectations will be:

$$E[\text{properlyColored}] = m \times \frac{2}{3} = \frac{2m}{3} \tag{2}$$

$$E[\text{badlyColored}] = m \times \frac{1}{3} = \frac{m}{3} \tag{3}$$

(c)

According to **Markov's inequality** ($Pr(X \geq a) \leq E[X]/a$ if $a > 0$), the probability that more than half of the edges are badly colored should be (we know $m > 0$):

$$Pr(\text{badlyColored} \geq \frac{m}{2}) \leq \frac{E[\text{badlyColored}]}{m/2} = \frac{m/3}{m/2} = \frac{2}{3} \approx 67\% \tag{4}$$