

We add a new node v' to the graph G and connect each $v \in T$ to v' and denote this new graph as G' . Then we run Ford-Fulkerson Algorithm in graph G' using h as source and v' as sink. We assign the capacity of edges connected to sink in the Graph as the values w_v and all other capacities as 1. Denote the residual graph of G' with max flow f as G'_f . Then starting at h , we run BFS to find out the set of nodes that are reachable from h and denote such a set as R . We let E' be the edges in G' that are from a node in R to a node in $V - R$. We remove edges in E' that are from any $v \in T$ to v' and denote the remaining edges as F . Then F is the objective set that maximizing $q(F) - |F|$

Proof of Correctness:

After we have found the min cut in G' , we can divide E' into F and E'' . Given that nodes in $V - R$ are not reachable from h in G'_f , $q(f)$ will be the set of nodes in $V - R$ after we find the min cut. Then there are $|T| - q(f)$ terminal nodes in R . Since we know there is no augmenting path in G'_f (min cut), edges from $|T| - q(f)$ nodes in R to v' have to be saturated. Hence, $|E''| = |T| - q(f)$. **Because the min cut in G' is the cut that minimizes the number of edges, it aims to minimize $|F| - q(f)$, which is exactly the same as maximizing $q(f) - |F|$.** Since edges in E'' are not actually existed in E , they must be removed from E' . However, removing E'' from E' will not change $q(f) - |F|$ since every edge in E'' corresponds a v which is reachable from h .

Time Complexity Analysis::

Denote number of nodes as n and number of edges as m . Using Ford-Fulkerson would cost $O(n^3)$ to find the max flow. Running BFS would use $O(m) = O(n^2)$. Removing edges in F takes $O(n)$ since there as at most $n - 1$ terminal nodes. Hence, the total time complexity is $O(n^3) + O(n^2) + O(n) = O(n^3)$