The problem is in NP

BLOONSTOWERDEFENSE is in NP, since given a set of M monkeys and there location assignment (**certificate**), we can check whether the monkeys can pop all if the balloons in polynomial time (**certifier**) since we just have to compute the sum of h_i step by step after each monkey has output all its damage, which takes $O(MB)$.

The problem is NP-hard

We choose SET COVER to show that SET COVER $\leq_P$ BLOONSTOWERDEFENSE.

Given a set B of N elements, a collection $S_1, S_2, \cdots, S_m$ of subsets and a number L, we would like to know whether there exist a collection of at most L of these sets whose union is equal to all of B. We can solve it using a black box for BLOONSTOWERDEFENSE. The construction looks like the follows: for each monkey at location l, we assign the total damage it can produce $d_m = L$. The health of each balloon $h_i = 1$.

The reduction can be done in polynomial time since the only thing we implemented in the construction of a BLOONSTOWERDEFENSE is to assign the sets and nodes to monkeys and balloons, which costs $O(N + B)$ time.

Now we would like to prove the correctness by showing it is an if and only if statement:

If we have found a set cover S, then a collection $A_1, A_2, \cdots A_L \subseteq B$ and a number M has been found so that the union of at most M of these sets is equal to all of B. Hence we have found the solution of the BLOONSTOWERDEFENSE.

If we have found an assignment of BLOONSTOWERDEFENSE, then a valid assignment of at most M monkeys in M locations where $A_l \subseteq B$ such that $U(M, A_l) = B$, which means that every ballon has been shot at least one time. Since we already known that $h_i = 1$, all balloons are popped. Hence, this is a valid assignment.