

The algorithm is stated as follow:

Algorithm 1 Algorithm for problem 2

Sort the m students by their days a_i in chronological order $S_1, S_2 \dots S_m$

Sort the n employees by their interval starting days s_i in chronological order $E_1, E_2, \dots E_n$

While select student S_i in reverse chronological order (From S_m to S_1)

While select employee E_j from E_n to E_1

If $s_i \leq a_j \leq f_i$

 Match S_i and E_j

Break

End if

End while

End while

Return the set M of matched pairs.

Proof of Correctness (referred to class note): For employees E_1 to E_m corresponding to students S_1 to S_m , we want to prove each s_i is as late as possible. Considering another solution e'_1 to e'_k , we need to prove $s_i \geq s'_i$ by induction on i .

Base case: $i = 1$: $s_i \geq s'_i$ since it is our rule

Induction: For $i \geq 2$, E' has no more options before s_i , and the algorithm select the latest s_{i+1} . Hence, this algorithm is correct.

Time Complexity Analysis:

The complexity of selecting each student is $O(m)$.

For each student, the complexity of selecting each employee is $O(n)$.

m students \times n employees = $O(mn)$