

## HW 1: Battery Modeling, Analysis, and Simulation

Due: Friday February 3 at 5:00pm PT

This assignment will provide hands-on practice for modeling, linearization, stability analysis, and simulation.

### Problem 1: Review Submission Procedure from HW0 (ungraded)

Go to bCourses and examine HW0 and the solution: [bCourses/CE295/Files/HW Assignments/HW0](#). This HW explains the submission procedures and provides useful Matlab exercises. Note, you may use MS WORD or the  $\text{\LaTeX}$  template to generate your HW report. You may complete HW0 for practice, but it will not be collected, nor graded.

### Problem 2: Reading

Read Chapter 3 “Drivers for Change” in the following report available on bCourses

- A. N. Annaswamy, M. Amin, C. L. DeMarco, and T. Samad, “IEEE Vision for Smart Grid Controls: 2030 and Beyond,” IEEE, Tech. Rep., 2013. [bCourses/CE295/Files/Reading/Vision\\_SG\\_Controls.pdf](#)

Please summarize in one paragraph or two what recent issues have spurred growth and interest in developing a smart grid. How can modeling, controls, and optimization address these issues?

### Problem 3: Black-box vs. White-box Modeling

Please complete the table below, which summarizes the advantages and disadvantages of black-box and white-box modeling. Provide at least three items in each cell.

	Black-Box Models	White-Box Models
Advantages		
Disadvantages		

### Problem 4: Mathematical Modeling Uses

What are the five potential uses of a mathematical model? List all five uses, and explain.

### Problem 5: Mathematical Modeling

Suppose you work at an energy consulting firm contracted by Tesla Motors. Tesla is interested in utilizing their vehicles as a distributed energy storage source that can provide services to the smart grid. This provides their customers with an additional revenue stream to offset their EV investment. To this end, you wish to

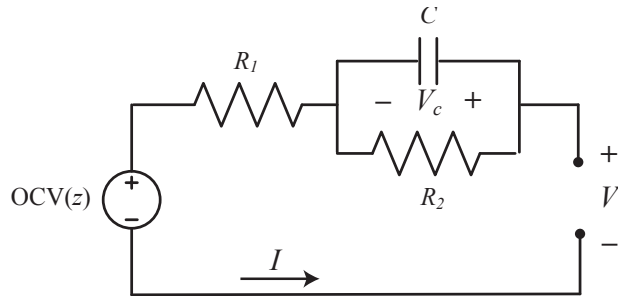


Figure 1: OCV-R-RC Equivalent circuit model of a battery. Includes an open circuit voltage in series with a resistance, in series with a resistor-capacitor pair.

develop a mathematical model of a battery that predicts its state-of-charge evolution, for analysis and control purposes.

Batteries can be modeled by a so-called “equivalent circuit model”, shown in Fig. 1. This model idealizes a battery cell as a voltage source  $OCV$  in series with a resistor  $R_1$  and resistor-capacitor ( $R_2, C$ ) pair. The battery terminal voltage is  $V(t)$  and current is  $I(t)$ . The voltage source  $OCV$  is not a constant voltage source. In fact, it is a nonlinear function of the battery state-of-charge (SOC), denoted by  $z$ . Moreover, it has the following integrator dynamics

$$\dot{z}(t) = \frac{1}{Q} I(t) \quad (1)$$

where  $Q$  is the charge capacity of the battery.

- List the “reservoirs” in this model. That is, what are the states?
- Use Kirchoff’s voltage and current laws to derive the system equations. Include the resulting equations and integrator dynamics in your report.
- List all the parameters,  $\theta$ , of the model.
- Consider current  $I$  as the controllable input, and voltage  $V$  as the measurable output. Reformulate the model into a state-space form, as discussed in class. You should have multiple state equations and one output equation.
- Does the model have a linear state equation? If so, then what are the  $A, B$  matrices when written in vector-matrix form? Does the model have a linear output equation? If so, then what are the  $C, D$  matrices when written in vector-matrix form? If not, then what term produces the nonlinearity?

### Problem 6: Stability and Linearization

In the following, consider the following parameter values:  $R_1 = 0.05\Omega, R_2 = 0.005\Omega, C = 500F, Q = 3600$ . In addition suppose that the nonlinear function  $OCV(z)$  can be expressed by the polynomial

$$OCV(z) = p_0 + p_1 z + p_2 z^2 + p_3 z^3 \quad (2)$$

- (a). For zero input current, is the model marginally stable, asymptotically stable, or unstable? You may use the `eig` command in Matlab or the `numpy.linalg.eig()` method in Python to assist you (although you should be able to identify the eigenvalues easily by inspection).
- (b). Linearize the output equation from Problem 4(d), using (2) to describe  $OCV(z)$ , around  $z^{eq} = 0.5$ ,  $V_c^{eq} = 0$ , and  $I^{eq} = 0$ . Show all steps for full credit.

### Problem 7: Simulation and Analysis

This problem provides practical experience implementing and analyzing your two models. You will be provided skeleton code, `HW1_Skeleton.m` and `HW1_Skeleton.ipynb`, on bCourses to assist you.

- (a). Enter the missing model parameters into your code, and copy “part(a)” of your code into your report. Also provide a plot of the nonlinear OCV function versus state-of-charge.
- (b). Simulate the model with the true nonlinear output function, and the linearized output function. Provide one figure with three subplots: current vs. time, SOC vs. time, and voltage vs. time. On the third subplot, plot both the nonlinear and linearized voltage response. Use legends.
- (c). Comment on why the linearized model incurs errors when SOC drops below 25%. Explain using the OCV versus SOC plot. Over time, does the system move away from the linearization point? What are the implications for doing any optimization or control designs with a linearized model?

## Deliverables

Submit the following on bCourses. Be sure that all files are named exactly as specified (including spelling and case), and make sure the function declaration is exactly as specified.

`LASTNAME_FIRSTNAME_HW1.PDF`

`LASTNAME_FIRSTNAME_HW1.m` or `LASTNAME_FIRSTNAME_HW1.ipynb`