

HW 3: Optimal Economic Dispatch in Distribution Feeders with Renewables

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Problem 1: Network Parameters

(a)

The plot of active and reactive power consumption is shown in Figure 1.

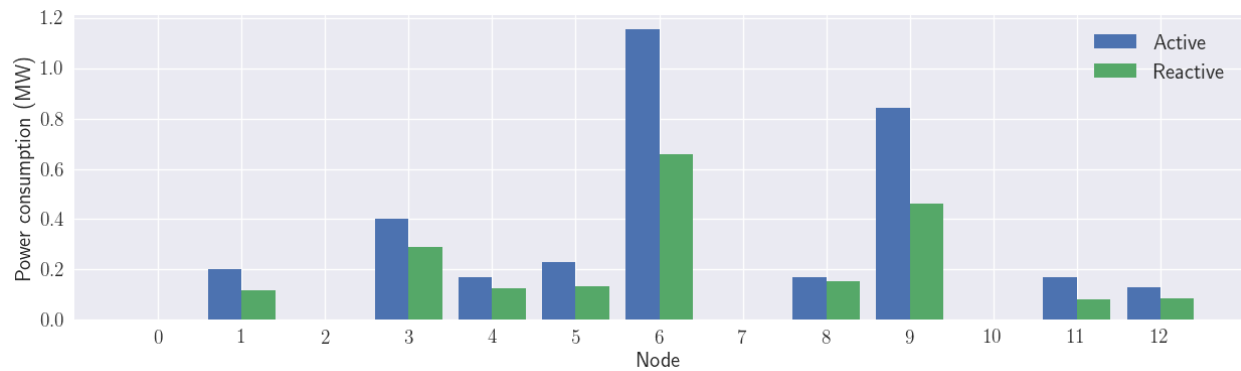


Fig. 1: Active and reactive power consumption

(b)

The adjacency matrix is shown as follow:

```
0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 1 0 1 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 1 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 1 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

Problem 2: Balancing Supply & Demand without a Network

(a)

Optimization variables:

$\mathbf{p} \in \mathbb{R}^{13}$ (where p_i is the active power generated at node i)

$\mathbf{q} \in \mathbb{R}^{13}$ (where q_i is the reactive power generated at node i)

$\mathbf{s} \in \mathbb{R}^{13}$ (where s_i is the apparent power generated at node i)

(b)

Objective function:

$$\sum_{i=0}^{12} c_i s_i \quad (1)$$

(c)

Constraint 1 (apparent power limits):

$$s_i \leq s_{i,max} \quad \forall i \in [0, 12] \quad (2)$$

Constraint 2 (balance active power generation with consumption):

$$\sum_{i=0}^{12} p_i = \sum_{i=0}^{12} l_i^P \quad (3)$$

Constraint 3 (balance reactive power generation with consumption):

$$\sum_{i=0}^{12} q_i = \sum_{i=0}^{12} l_i^Q \quad (4)$$

Constraint 4 (power generation should be non-negative):

$$p_i \geq 0, \quad q_i \geq 0 \quad \forall i \in [0, 12] \quad (5)$$

Constraint 5 (compute apparent power from active and reactive power):

$$\sqrt{p_i^2 + q_i^2} = s_i \quad \forall i \in [0, 12] \quad (6)$$

(d)

This problem is **not** a linear program (LP), **not** a quadratic program (QP), **not** a convex program (CP), since the Constraint 5 (an equality constraint) is not affine.

However, if we relax it to an inequality constraint, the problem will become a **CP** problem, since this inequality constraint is convex.

(e)

The result is shown as follow:

Minimum Generating Cost : 255.36 USD

Node 0 [Grid] Gen Power : p_0 = 0.901 MW | q_0 = 0.546 MW | s_0 = 1.054 MW

Node 3 [Gas] Gen Power : p_3 = 0.000 MW | q_3 = 0.000 MW | s_3 = 0.000 MW

Node 9 [Solar] Gen Power : p_9 = 2.565 MW | q_9 = 1.556 MW | s_9 = 3.000 MW

Total active power : 3.466 MW consumed | 3.466 MW generated

Total reactive power : 2.102 MVar consumed | 2.102 MVar generated

Total apparent power : 4.063 MVA consumed | 4.054 MVA generated

Problem 3: Add Line Power Flows

(a)

Optimization variables:

$\mathbf{p} \in \mathbb{R}^{13}$ (where p_i is the active power generated at node i)

$\mathbf{q} \in \mathbb{R}^{13}$ (where q_i is the reactive power generated at node i)

$\mathbf{s} \in \mathbb{R}^{13}$ (where s_i is the apparent power generated at node i)

$\mathbf{P} \in \mathbb{R}^{13 \times 13}$ (where P_{ij} is the active power flowing on line (i, j))

$\mathbf{Q} \in \mathbb{R}^{13 \times 13}$ (where Q_{ij} is the reactive power flowing on line (i, j))

(b)

Constraint 1 (apparent power limits):

$$s_i \leq s_{i,max} \quad \forall i \in [0, 12] \quad (7)$$

Constraint 2 (balance active power generation with consumption):

$$\sum_{i=0}^{12} p_i = \sum_{i=0}^{12} l_i^P \quad (8)$$

Constraint 3 (balance reactive power generation with consumption):

$$\sum_{i=0}^{12} q_i = \sum_{i=0}^{12} l_i^Q \quad (9)$$

Constraint 4 (power generation should be non-negative):

$$p_i \geq 0, \quad q_i \geq 0 \quad \forall i \in [0, 12] \quad (10)$$

Constraint 5 (compute apparent power from active and reactive power (relaxed version)):

$$\sqrt{p_i^2 + q_i^2} \leq s_i \quad \forall i \in [0, 12] \quad (11)$$

Constraint 6 (power flow should align with power generation and consumption at each node):

$$P_{ij} = (l_j^P - p_j) + \sum_{k=0}^{12} A_{jk} P_{jk} \quad \forall j \in [0, 12], \quad i = \rho(j) \quad (12)$$

$$Q_{ij} = (l_j^Q - q_j) + \sum_{k=0}^{12} A_{jk} Q_{jk} \quad \forall j \in [0, 12], \quad i = \rho(j) \quad (13)$$

(c)

The result is shown as follow:

Minimum Generating Cost : 255.36 USD

Node 0 [Grid] Gen Power : p_0 = 0.901 MW | q_0 = 0.546 MW | s_0 = 1.054 MW ||
mu_s0 = 0 USD/MW

Node 3 [Gas] Gen Power : p_3 = 0.000 MW | q_3 = 0.000 MW | s_3 = 0.000 MW ||
mu_s4 = 0 USD/MW

Node 9 [Solar] Gen Power : p_9 = 2.565 MW | q_9 = 1.556 MW | s_9 = 3.000 MW ||
mu_s9 = 50 USD/MW

Total active power : 3.466 MW consumed | 3.466 MW generated

Total reactive power : 2.102 MVar consumed | 2.102 MVar generated

Total apparent power : 4.063 MVA consumed | 4.054 MVA generated

From the result we can notice that the minimum and minimizers remain the **same** as Problem 2. The reason is that when adding constraint 6, as we tune P_{ij} and Q_{ij} , such a constraint will always be true no matter what p_j or q_j is, which means such a constraint have no connection to other constraints or the objective function. Hence, we actually did not “add” a constraint compared to Problem 2 (This is an inactive constraint).

(d)

Actually in CVXPY we do not need to re-compute the problem The result is shown as follow:

Minimum Generating Cost : 255.36 USD

```
Node 0 [Grid]  Gen Power : p_0 = 0.901 MW | q_0 = 0.546 MW | s_0 = 1.054 MW ||
mu_s0 =      0 USD/MW
Node 3 [Gas]   Gen Power : p_3 = 0.000 MW | q_3 = 0.000 MW | s_3 = 0.000 MW ||
mu_s4 =      0 USD/MW
Node 9 [Solar] Gen Power : p_9 = 2.565 MW | q_9 = 1.556 MW | s_9 = 3.000 MW ||
mu_s9 =     50 USD/MW
```

```
Total active power   : 3.466 MW   consumed | 3.466 MW   generated
Total reactive power : 2.102 MVar consumed | 2.102 MVar generated
Total apparent power : 4.063 MVA  consumed | 4.054 MVA  generated
```

Dual variable mu_s:

```
[[ 1.19192177e-09]
 [ 1.36704047e+02]
 [ 1.37527656e+02]
 [ 1.26366831e-09]
 [ 1.37994308e+02]
 [ 1.38501208e+02]
 [ 1.40611644e+02]
 [ 1.40743436e+02]
 [ 1.40787583e+02]
 [ 5.32612423e-09]
 [ 1.41842060e+02]
 [ 1.42320563e+02]
 [ 1.42401951e+02]]
```

If we increase the solar power capacity by 1MW, the result is shown as follow:

Minimum Generating Cost : 205.36 USD

Node 0 [Grid] Gen Power : p_0 = 0.046 MW | q_0 = 0.028 MW | s_0 = 0.054 MW ||
mu_s0 = 0 USD/MW
Node 3 [Gas] Gen Power : p_3 = 0.000 MW | q_3 = 0.000 MW | s_3 = 0.000 MW ||
mu_s4 = 0 USD/MW
Node 9 [Solar] Gen Power : p_9 = 3.420 MW | q_9 = 2.074 MW | s_9 = 4.000 MW ||
mu_s9 = 50 USD/MW

Total active power : 3.466 MW consumed | 3.466 MW generated
Total reactive power : 2.102 MVar consumed | 2.102 MVar generated
Total apparent power : 4.063 MVA consumed | 4.054 MVA generated

As we can see from the results above, we would save **50 USD** (nearly 20 percent!) if we could increase the solar power capacity by 1MW.

Problem 4: The Complete Optimal Economic Dispatch with Distflow Equations

(a)

Optimization variables:

$\mathbf{p} \in \mathbb{R}^{13}$ (where p_i is the active power generated at node i)

$\mathbf{q} \in \mathbb{R}^{13}$ (where q_i is the reactive power generated at node i)

$\mathbf{s} \in \mathbb{R}^{13}$ (where s_i is the apparent power generated at node i)

$\mathbf{P} \in \mathbb{R}^{13 \times 13}$ (where P_{ij} is the active power flowing on line (i, j))

$\mathbf{Q} \in \mathbb{R}^{13 \times 13}$ (where Q_{ij} is the reactive power flowing on line (i, j))

$\mathbf{V} \in \mathbb{R}^{13}$ (where V_i is the squared magnitude of voltage at node i)

$\mathbf{L} \in \mathbb{R}^{13 \times 13}$ (where L_{ij} is the squared magnitude of complex current on line (i, j))

(b)

Constraint 1 (apparent power limits):

$$s_i \leq s_{i,max} \quad \forall i \in [0, 12] \quad (14)$$

Constraint 2 (balance active power generation with consumption):

$$\sum_{i=0}^{12} p_i = \sum_{i=0}^{12} l_i^P \quad (15)$$

Constraint 3 (balance reactive power generation with consumption):

$$\sum_{i=0}^{12} q_i = \sum_{i=0}^{12} l_i^Q \quad (16)$$

Constraint 4 (power generation should be non-negative):

$$p_i \geq 0, \quad q_i \geq 0 \quad \forall i \in [0, 12] \quad (17)$$

Constraint 5 (compute apparent power from active and reactive power (relaxed version)):

$$\sqrt{p_i^2 + q_i^2} \leq s_i \quad \forall i \in [0, 12] \quad (18)$$

Constraint 6 (power flow should align with power generation and consumption at each node):

$$P_{ij} = (l_j^P - p_j) + r_{ij}L_{ij} + \sum_{k=0}^{12} A_{jk}P_{jk} \quad \forall j \in [0, 12], \quad i = \rho(j) \quad (19)$$

$$Q_{ij} = (l_j^Q - q_j) + x_{ij}L_{ij} + \sum_{k=0}^{12} A_{jk}Q_{jk} \quad \forall j \in [0, 12], \quad i = \rho(j) \quad (20)$$

Constraint 7 (relationship between current flow and power flow):

$$L_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{V_j} \quad \forall j \in [0, 12], \quad i = \rho(j) \quad (21)$$

Constraint 8 (relationship between nodal voltage and resistance & reactance):

$$V_j = V_i + (r_{ij}^2 + x_{ij}^2)L_{ij} - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) \quad \forall j \in [0, 12], \quad i = \rho(j) \quad (22)$$

Constraint 9 (nodal voltage limits):

$$v_{min}^2 \leq V_j \leq v_{max}^2 \quad \forall i \in [0, 12] \quad (23)$$

Constraint 10 (squared line current limits):

$$L_{ij} \leq I_{ij,max}^2 \quad \forall j \in [0, 12], i = \rho(j) \quad (24)$$

(c)

This problem is **not** a CP since Constraint 7 (a equality constraint) is not affine. However, if we relax it to an inequality constraint, the problem will become a **CP** problem, since this inequality constraint is convex.

(d)

The result is shown as follow:

Minimum Generating Cost : 299.69 USD

```
Node 0 [Grid]  Gen Power : p_0 = 1.568 MW | q_0 = 0.985 MW | s_0 = 1.852 MW ||
mu_s0 =      0 USD/MW
Node 3 [Gas]   Gen Power : p_3 = 0.000 MW | q_3 = -0.000 MW | s_3 = -0.000 MW ||
mu_s4 =      0 USD/MW
Node 9 [Solar] Gen Power : p_9 = 1.941 MW | q_9 = 1.216 MW | s_9 = 2.290 MW ||
mu_s9 =      0 USD/MW
```

```
Total active power   : 3.466 MW   consumed | 3.509 MW   generated
Total reactive power : 2.102 MVar consumed | 2.201 MVar generated
Total apparent power : 4.063 MVA  consumed | 4.142 MVA  generated
```

This time the solution is **different**. The reason is that new variables V and L are computed based on p and q , and they also have boundaries. Hence, the new added constraints are active, so the solution is different from previous ones.

(e)

Constraints are active if the corresponding dual variables are not equal to 0. So for voltage constraint and line current constraint, from the dual variables we may notice that those constraints are inactive for all nodes.

(f)

The result is shown as follow:

Minimum Generating Cost : 348.67 USD

Node 0 [Grid] Gen Power : $p_0 = 0.940$ MW | $q_0 = 0.174$ MW | $s_0 = 0.956$ MW ||
 $\mu_{s0} = 0$ USD/MW
Node 3 [Gas] Gen Power : $p_3 = 0.760$ MW | $q_3 = 0.518$ MW | $s_3 = 0.920$ MW ||
 $\mu_{s4} = 0$ USD/MW
Node 9 [Solar] Gen Power : $p_9 = 1.789$ MW | $q_9 = 1.450$ MW | $s_9 = 2.303$ MW ||
 $\mu_{s9} = 0$ USD/MW

Total active power : 3.466 MW consumed | 3.489 MW generated
Total reactive power : 2.102 MVar consumed | 2.142 MVar generated
Total apparent power : 4.063 MVA consumed | 4.178 MVA generated

Why did the solution change? Let's check the dual variables again. We can see that this time dual variables for those constraints are not all equal to zero, which means that they become active constraints. Hence, the solution changes.