Nonlinear and Discrete Optimization—Homework Sets 4-5

1. Consider the optimization problem

$$\min_{x_1,x_2} \quad e^{x_1-x_2-0.4} + e^{x_1+x_2-0.4} + (x_1-1)^2 + (x_2+1)^2$$

Write a code to solve this problem using the Gradient method with the backtracking parameters $\alpha = 1$ and $\beta = 0.6$. Draw $f(x^{(k)})$ verses k for $k = 0, 1, 2, \ldots, 50$. Show the trajectory of points $x^{(0)}, x^{(1)}, ..., x^{(50)}$ in the 2-dimensional (x_1, x_2) plane.

2. Redo the previous problem with Newton's method.

3. Consider the optimization problem

$$\min_{x\in\mathbb{R}^n} \quad -\sum_{i=1}^{n} \log(2 - x_i^2) - \sum_{i=1}^{n} \log(1 - 2a_i^T x)$$

where $n = 500$ and $a_i \in \mathbb{R}^n$ are randomly generated vectors. Solve this problem using Newton's method with the backtracking line search ($\alpha = 0.4$ and $\beta = 0.4$). Draw $f(x^{(k)})$ versus k for $k = 0, 1, \ldots, 300$.