COMP 478/6771 Image Processing Assignment

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1

The 3 by 3 spatial filter is:

$$h(x,y) = \frac{1}{4}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

And the corresponding filter in frequency domain:

$$\begin{aligned} H(u,v) &= \sum_{x=-1}^{1}\sum_{y=-1}^{1} h(x,y)e^{-j2\pi(xu+yv)} \\ &= \frac{1}{4}(e^{-j2\pi u} + e^{j2\pi u} + e^{j2\pi v} + e^{-j2\pi v}) \\ &= \frac{1}{4}(2\cos(2\pi u) + 2\cos(2\pi v)) \\ &= \frac{1}{2}\cos(2\pi u) + \frac{1}{2}\cos(2\pi v) \end{aligned}$$

Here I use the trigonometric identity $\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$.
In conclusion, the equivalent filter $H(u,v)$ in the frequency domain is $\frac{1}{2}\cos(2\pi u) + \frac{1}{2}\cos(2\pi v)$.

2

The first derivatives in 2D in spatial domain:

$$g(x,y) = f(x,y) - f(x-1,y) + f(x,y) - f(x,y-1)$$

Fourier transform gives:

$$\begin{aligned} \mathfrak{F}(g(x,y)) &= F(u,v) - F(u,v)e^{-j2\pi u} + F(u,v) - F(u,v)e^{-j2\pi v} \\ &= (1 - e^{-j2\pi u})F(u,v) + (1 - e^{-j2\pi v})F(u,v) \\ &= [2 - (e^{-j2\pi u} + e^{-j2\pi v})]F(u,v) \\ &= H(u,v)F(u,v) \end{aligned}$$

Hence, the equivalent filter $H(u,v)$ in the frequency domain is $2 - (e^{-j2\pi u} + e^{-j2\pi v})$.
The figure of this filter is showing in Figure 1. It is clear can be seen that this filter is a high-pass filter.

3

3.1 a

First, for simplification, I will express filtering as convolution, the filtered image is:

$$g(x,y) = h(x,y)\bigstar f(x,y)$$

where h is the corresponding spatial filter and f is the input image. Then, do histogram equalization, obtain:

$$s(x,y) = T[g(x,y)] = T[h(x,y)\bigstar f(x,y)]$$

where T is the histogram equalization.

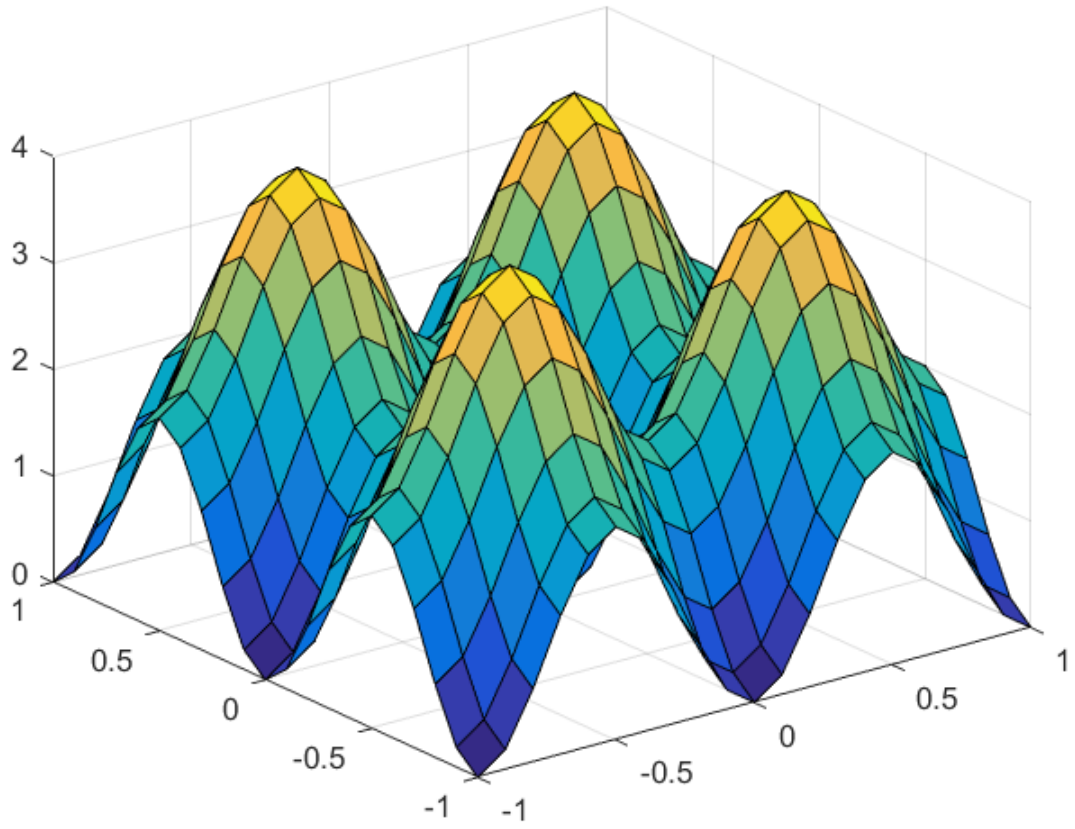

Figure 1: filter figure

On the other side, if we do histogram equalization first, then

$$g(x, y) = T[f(x, y)]$$

then convolution:

$$s(x, y) = h(x, y)\bigstar T[f(x, y)]$$

and since T is a nonlinear transform determined by the nature of the pixels in the image from which it is computed. Thus, in general:

$$T[h(x, y)\bigstar f(x, y)] \neq h(x, y)\bigstar T[f(x, y)]$$

In conclusion, it does matters which process is applied first.

3.2 b

As highpass filtering severely diminishes the contrast of an image. Thus, if an image is histogram equalized first, the gain in contrast improvement will essentially be lost in the filtering process. Therefore, the procedure in general is to filter first and histogram equalize the image after that.

4

Figure 2 shows the result images of sobel and cannny edge detector.
From the result images, it can be seen that Sobel detector mainly find the main contours in the image, whereas Canny detector find more tiny edges in the image, but also it introduces some noise. The Sobel and Canny detectors are both based on gradient magnitude.

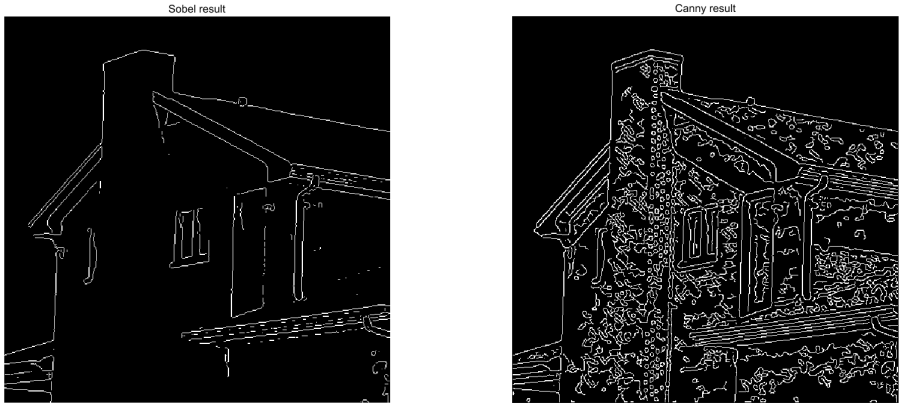

Figure 2: Result images