COMP 478/6771 Image Processing Assignment

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a.) Final answers:

1. The minimum size of the blurring mask is 7×7 (since the size of the mask must be odd).
2. The minimum value of the threshold to accomplish the task is 0.0204 ($\frac{1}{49}$).

Reasons:

1. Consider the extreme case that the filter is positioned on the center pixel of a 5-pixel gap, which is the maximum gap of broken strings. the 5×5 mask would encompass a completely black background field, so the next bigger size (7×7 size) would encompass some of the white pixels. Thus, the minimum size of the blurring mask is 7×7.

2. The extreme case is in the region of the 7×7 filter, there are only two pixels belongs to the white strings, but we want to connect the strings with no gaps in the strings, so the center pixel produced by this case must be white after thresholding, so the minimum value of the threshold to accomplish the task is 0.0204 ($\frac{1}{49}$).

b.) I can use a 7×7 max filter to repair the gaps. More specifically, a 7×7 max filter which is $R = max\{z_k | k = 1, 2, 3, ..., 49\}$, this filter is aim to find the brightest pixels in the region, so it can connect the strings and therefore repair the gaps.

2

Code implement in files Solution_2.m and method2.m. The output T for H03.bmp is 154, for H04.bmp is 91.

Figure 1 shows the original grayscale image and its binary result of H03.bmp.

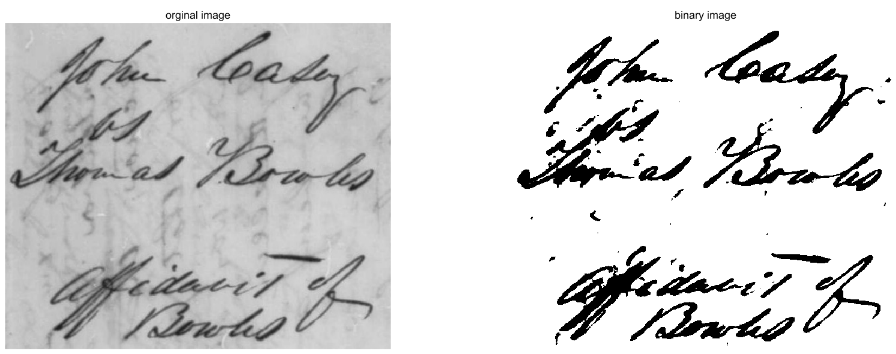

Figure 1: Image H03 and its binary result

Figure 2 shows the original grayscale image and its binary result of H04.bmp.

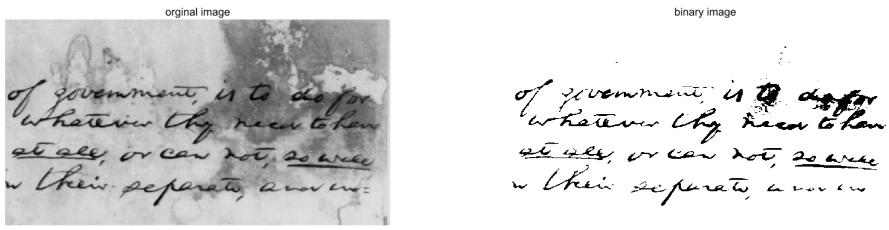

Figure 2: Image H04 and its binary result

3

a.)

$$
\begin{aligned}
F(\mu) &= \int_{-\infty}^{+\infty} f(t)e^{-j2\pi\mu t}dt = \int_{0}^{W} Ae^{-j2\pi\mu t}dt \\
&= \frac{-A}{j2\pi\mu}[e^{-j2\pi\mu t}]_0^W = \frac{-A}{j2\pi\mu}[e^{-j2\pi\mu W} - 1] \\
&= \frac{A}{j2\pi\mu}[1 - e^{-j2\pi\mu W}] = \frac{A}{j2\pi\mu}e^{-j\pi\mu W}(e^{j\pi\mu W} - e^{-j\pi\mu W}) \\
&= AW\frac{\sin(\pi\mu W)}{(\pi\mu W)}e^{-j\pi\mu W}
\end{aligned}
$$

The only difference between my result and the result in Example 4.1 is the exponential term, i.e. my result have an exponential term. Because the function has a shift in it, so the result has a phase term. Except this, the magnitude of the Fourier Transform results is the same.
When $A = W = 1$, the Fourier Transform of $f(t)$ in this case is: $\frac{\sin(\pi\mu)}{(\pi\mu)}e^{-j\pi\mu}$

b.)

Let's define the box function as:

$$
\text{rect}(t) = \prod(t) = \begin{cases} 0 & \text{if } |t| > \frac{1}{2} \\ \frac{1}{2} & \text{if } |t| = \frac{1}{2} \\ 1 & \text{if } |t| < \frac{1}{2} \end{cases}
$$

Using the definition of Fourier Transform:

$$
\begin{aligned}
F\{\text{rect}(t)\} &= \int_{-\infty}^{+\infty} \text{rect}(t)e^{-j2\pi\mu t}dt = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-j2\pi\mu t}dt \\
&= \frac{1}{-j2\pi\mu}[e^{-j2\pi\mu t}]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{1}{-j2\pi\mu}(e^{-j\pi\mu} - e^{j\pi\mu}) \\
&= \frac{1}{\pi\mu}(\frac{e^{j\pi\mu} - e^{-j\pi\mu}}{2j}) \\
&= \frac{1}{\pi\mu}\sin(\pi\mu) \\
&= \text{sinc}(\mu)
\end{aligned}
$$

where I use the trigonometric identity $\sin(\theta) = \frac{(e^{j\theta} - e^{-j\theta})}{2j}$ and the definition of $sinc$ function $\text{sinc}(m) = \frac{\sin \pi m}{(\pi m)}$.

By the convolution theorem, the Fourier Transform of the convolution of two functions in the spatial domain is equal to the product in the frequency domain of the Fourier transforms of the two functions. Since that the tent function is the convolution of two box functions, and the Fourier Transform of a box function is a $sinc$ function, that is:

$$\begin{aligned} F\{\text{tent}(t)\} &= F\{\text{rect}(t) \star \text{rect}(t)\} \\ &= F\{\text{rect}(t)\} \times F\{\text{rect}(t)\} \\ &= \text{sinc}(\mu) \times \text{sinc}(\mu) \\ &= \text{sinc}^2(\mu) \end{aligned}$$

In conclusion, the Fourier Transform of the tent function is a $sinc$ function squared.