Principles of fMRI

Part II

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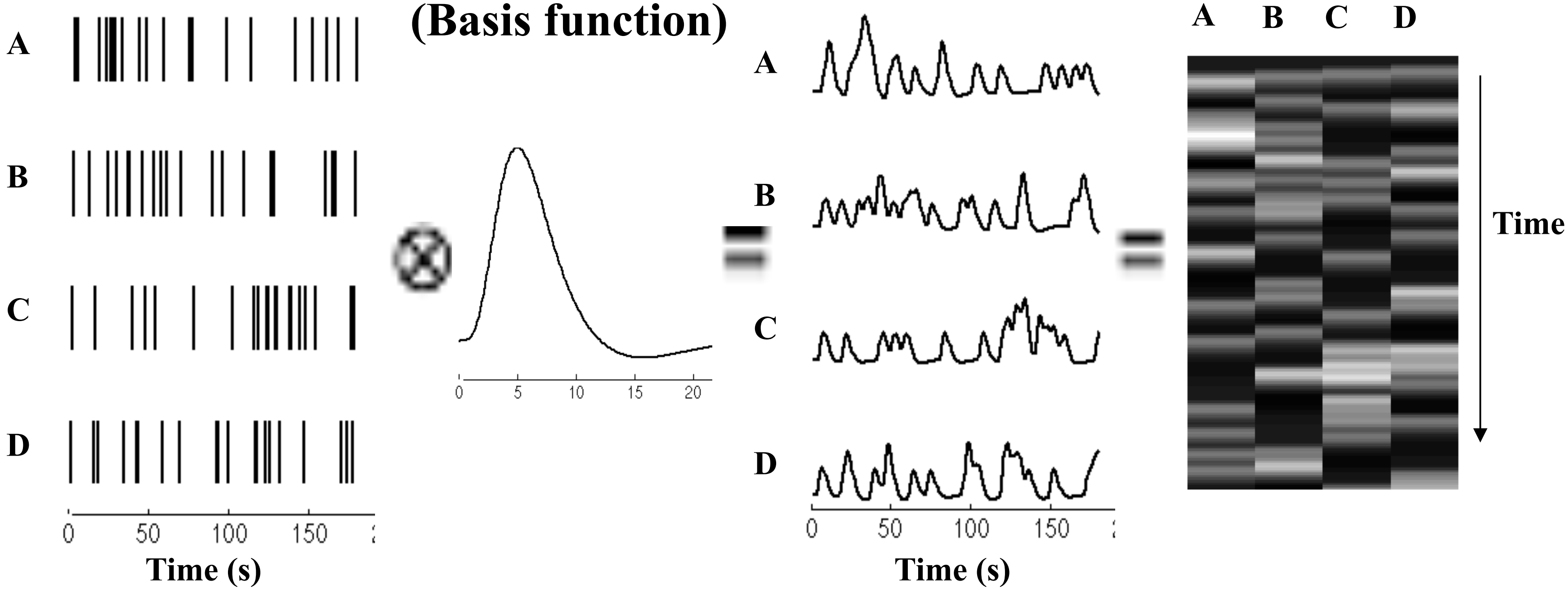
The General Linear Model: Assumptions and Multicolinearity

Model Building for Multiple Predictors

Indicator functions (onsets) | **Assumed HRF (Basis function)** | **Design Matrix (X^T)** | **Design Matrix (X)**



Assumptions!

Assume neural activity function is correct

Assume HRF is correct

Assume LTI system

Assumptions Required for Valid p-values

What to do?

- Check assumptions
  - Look for outliers, skewed variables
  - Pay particular attention to behavioral predictors in brain-behavioral correlations

- Fixes: Variable transformation
  - e.g., log transform behavioral data for positive skew

- Fixes: Nonparametric and robust statistics
  - SnPM, Statistical nonParametric Mapping (Nichols & Holmes, 2001)
  - Rank statistics (e.g., Spearman's rho)
  - Robust statistics (e.g., IRLS, Wager et al., 2005)

A Note on Multicolinearity

- Correlated predictors increase the variance (uncertainty) in parameter estimates

- Because of fundamental uncertainty in which predictor should be “assigned credit” for variation in the data.

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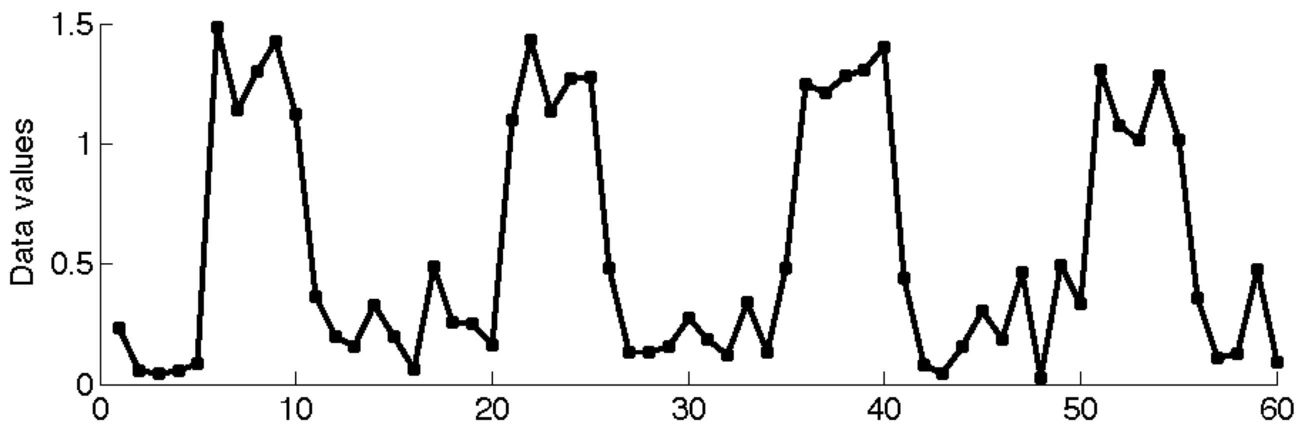
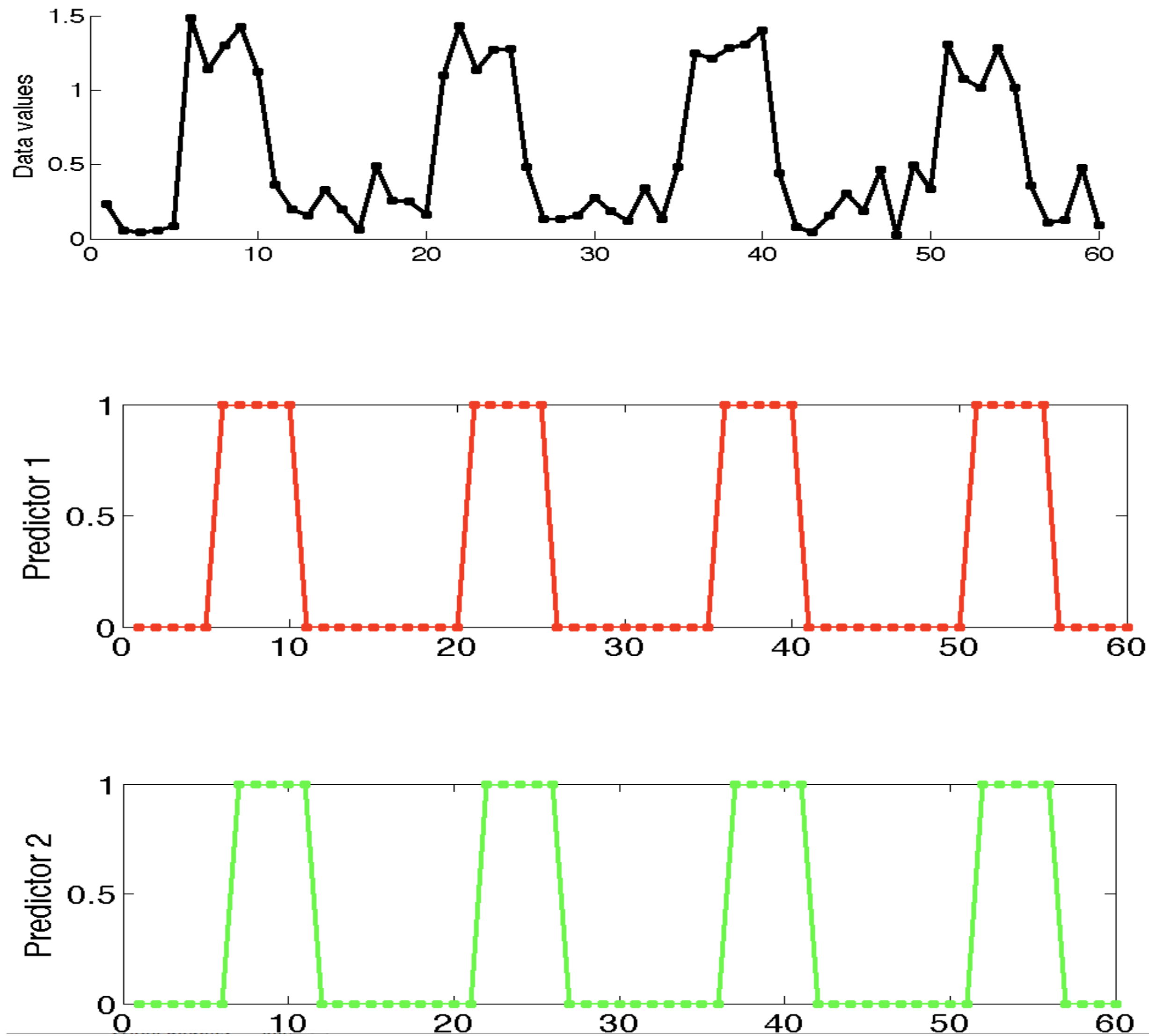


Some data

A Note on Multicolinearity



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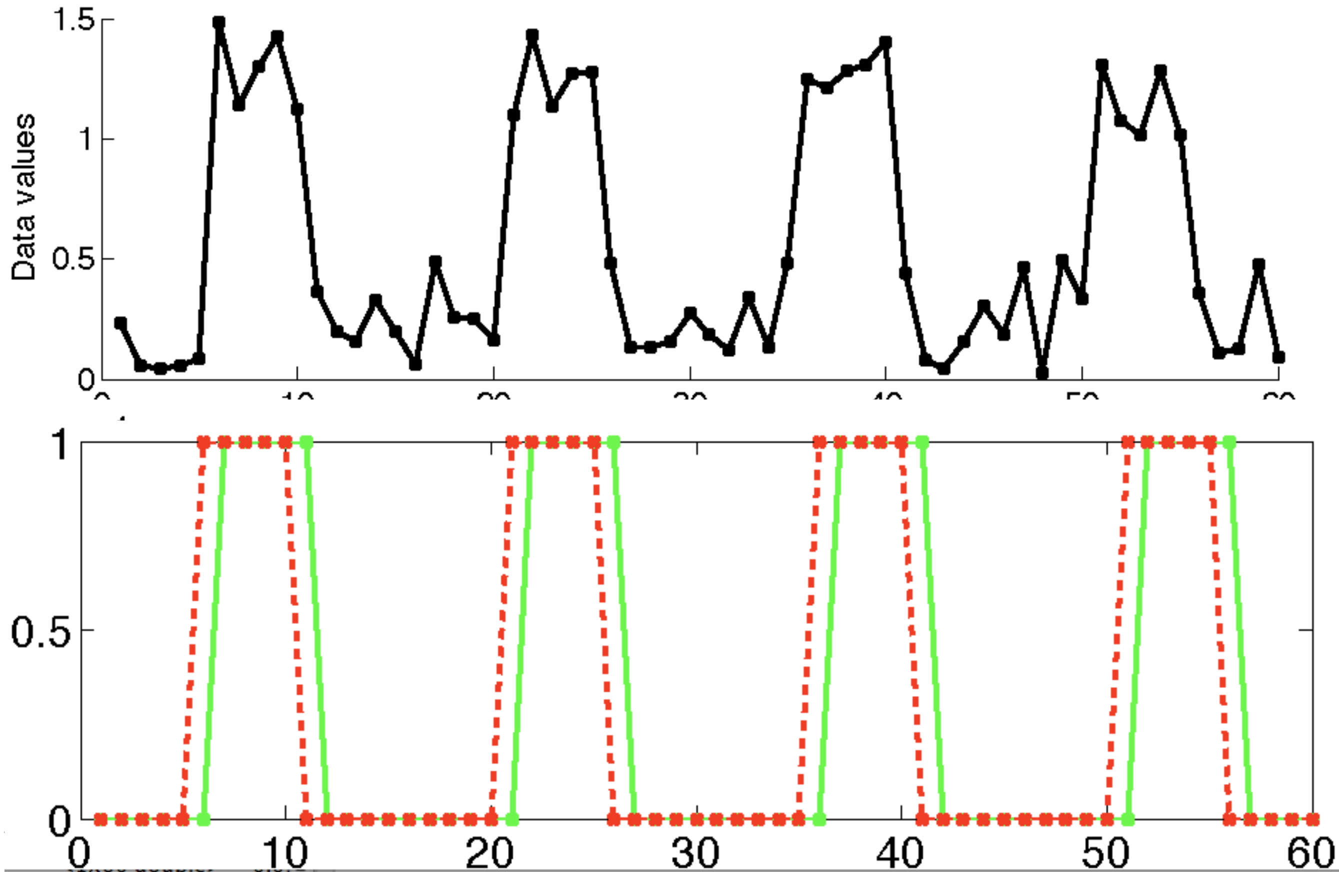
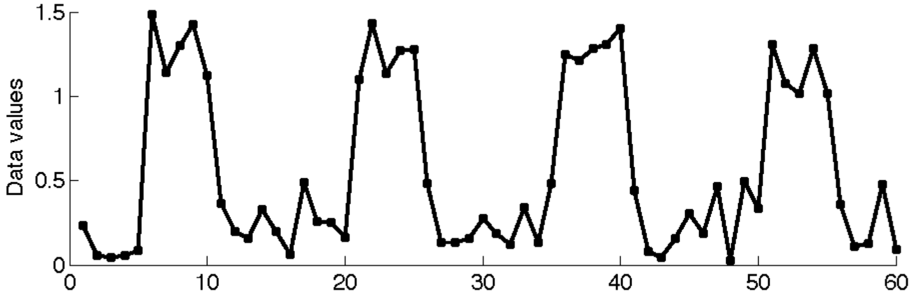
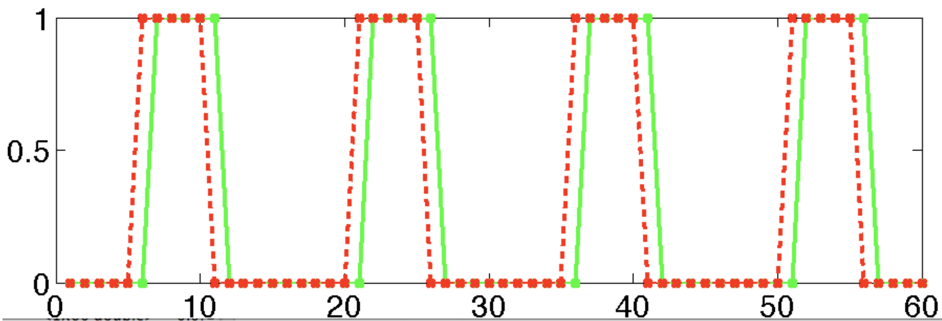


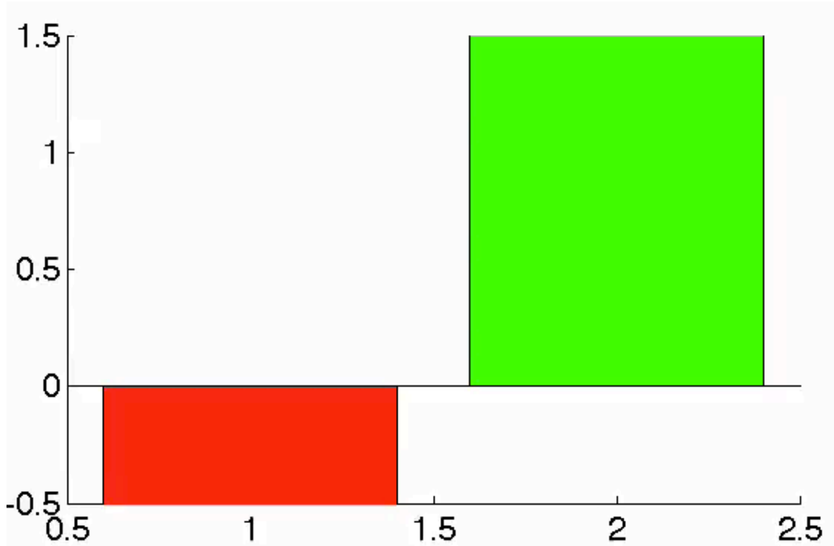
Where are the predictor values different?
Which predictor fits best depends only the data at these points!

A Note on Multicolinearity





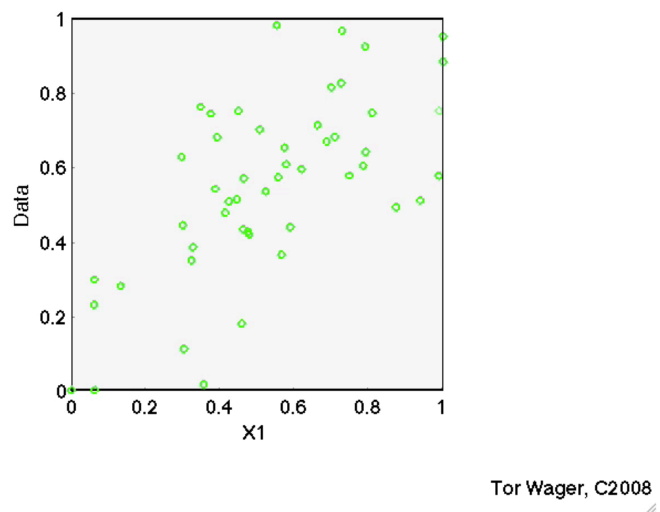
Amplitudes (betas)



Regression with Two Predictors

No multicolinearity

Multicolinearity



- With multicolinearity, P-values can be misleading: Easy to 'flip-flop' from significant positive to negative

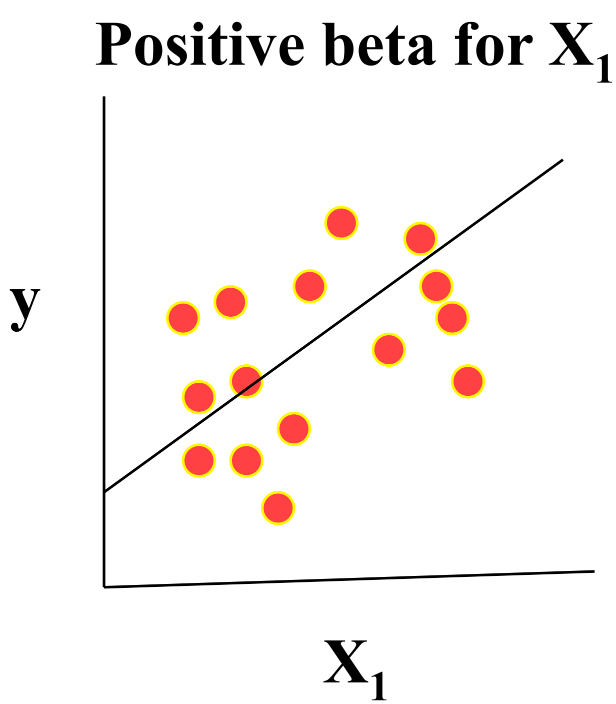
Ground truth in simulations is the same:

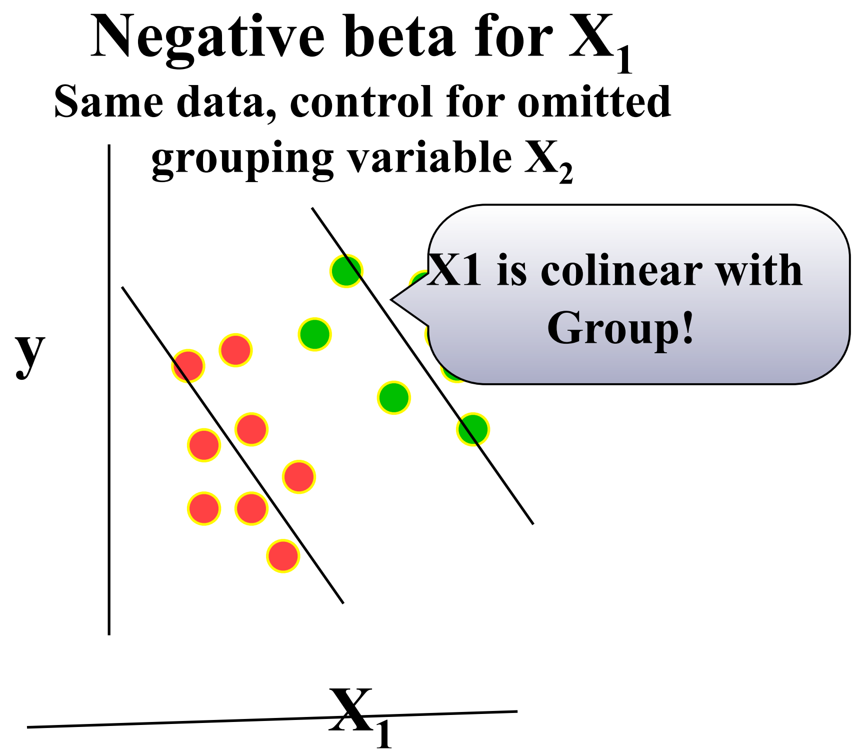
- Strong, positive effects of both predictors X_1 and X_2
- Same noise variance

Multiple Regression is Tricky Business!

Principles of fMRI

- Inferences on parameters (betas) depend on getting the model for *other* parameters right
- Including parameters you didn't know should be included in the model!
- Interpretation of parameters is model-dependent



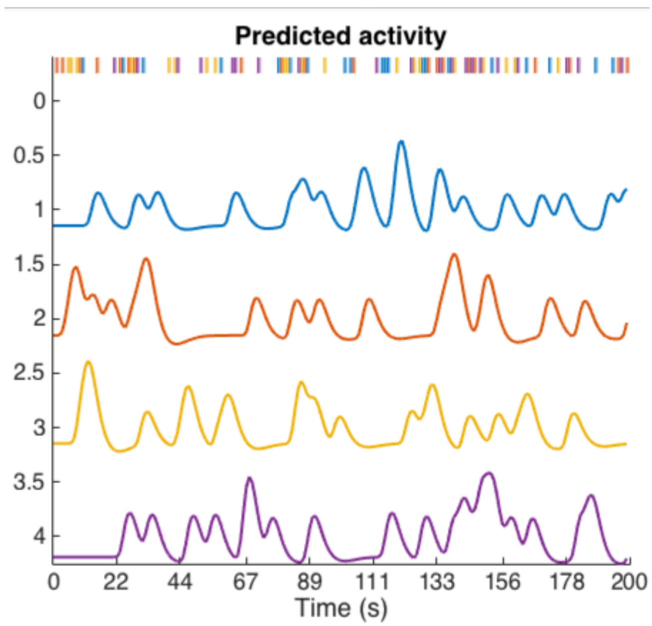


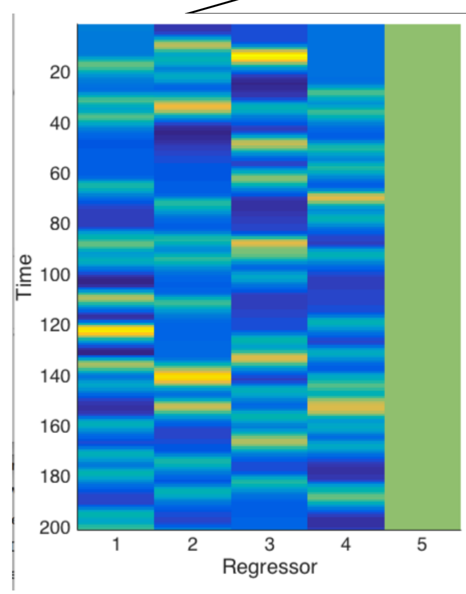
Diagnosis: Variance inflation factors

- Variance Inflation Factors (VIFs) of each regressor in your design matrix
- Increase in error variance due to design multicolinearity.
- e.g., VIF = 2 means your error variance will be *doubled*.

- For a design matrix X with columns i = 1…I

$$vif_i = \frac{1}{1 - R_i^2}$$





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e.g., Kutner et al. 2004

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Principles of fMRI

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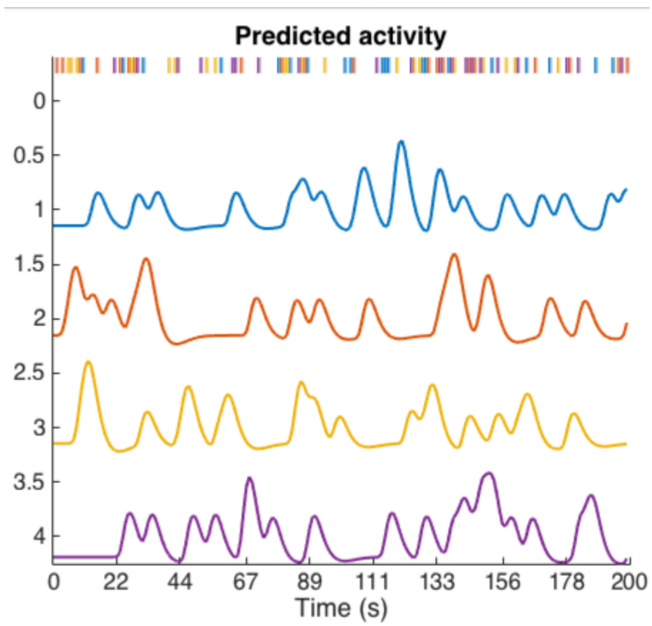
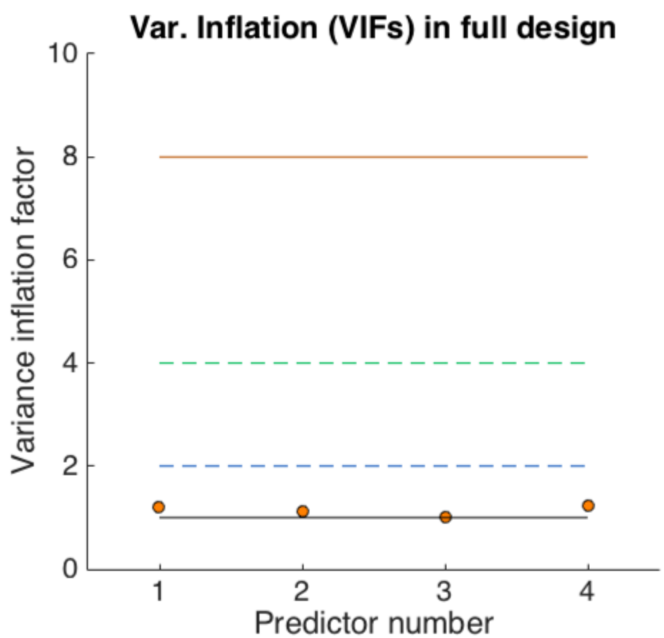


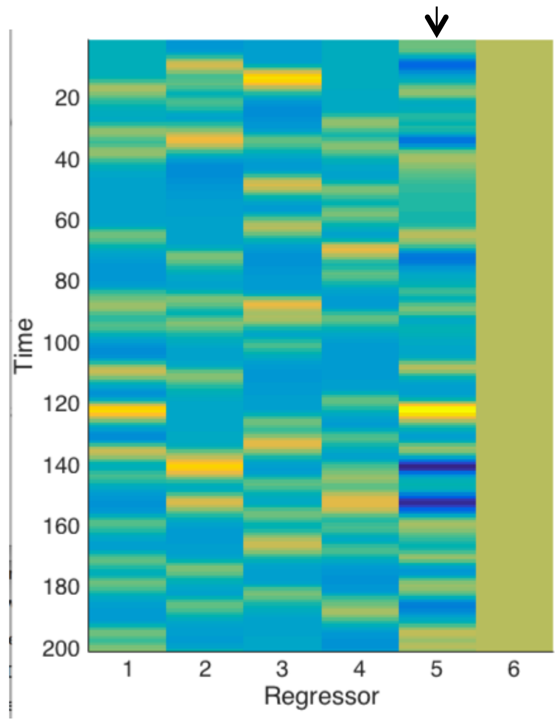


e.g., Kutner et al. 2004

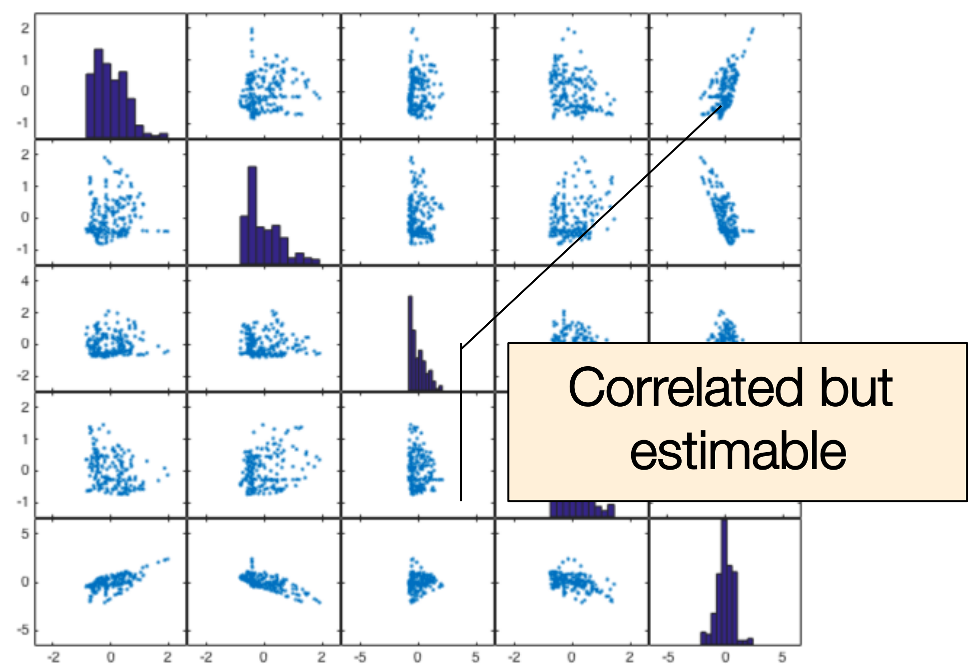
Properties of VIFs

- \- Estimated for each column in design matrix; some may have high VIFs, others low VIFs
- \- Adding nuisance regressors may increase VIFs for some regressors more than others
- \- Pairwise correlations are not enough to assess multicolinearity

New regressor

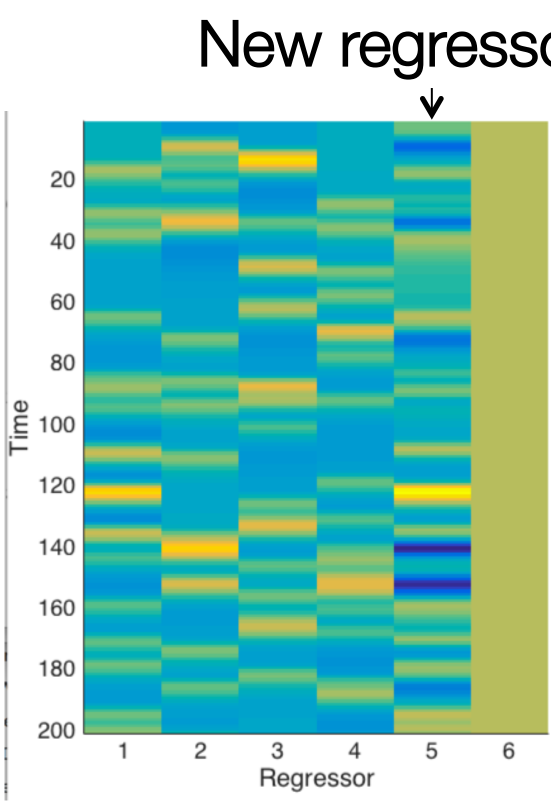


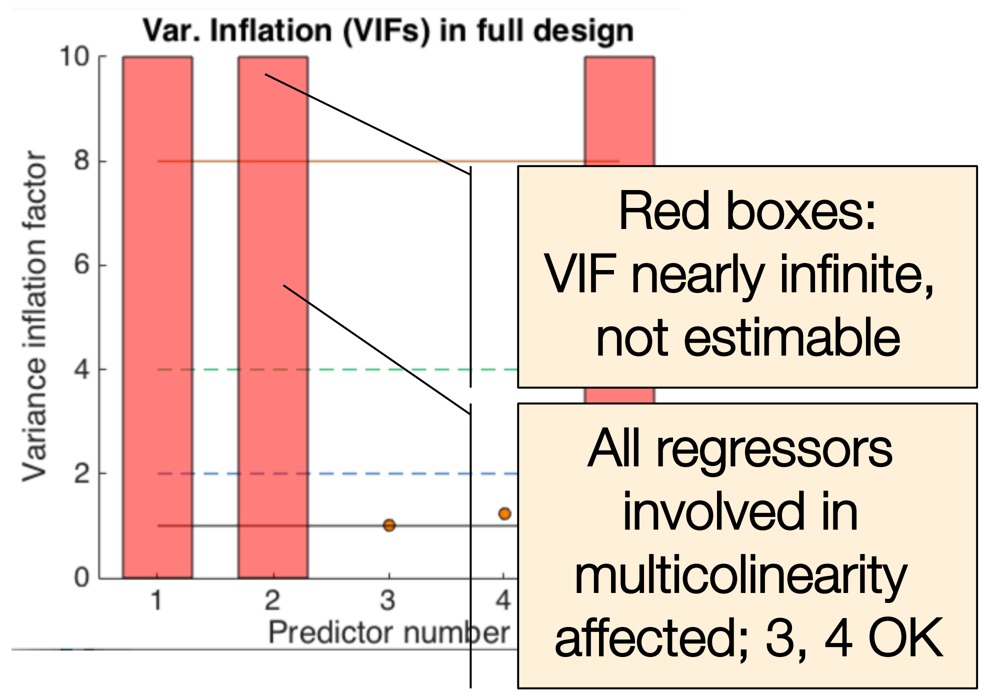
Pairwise correlations



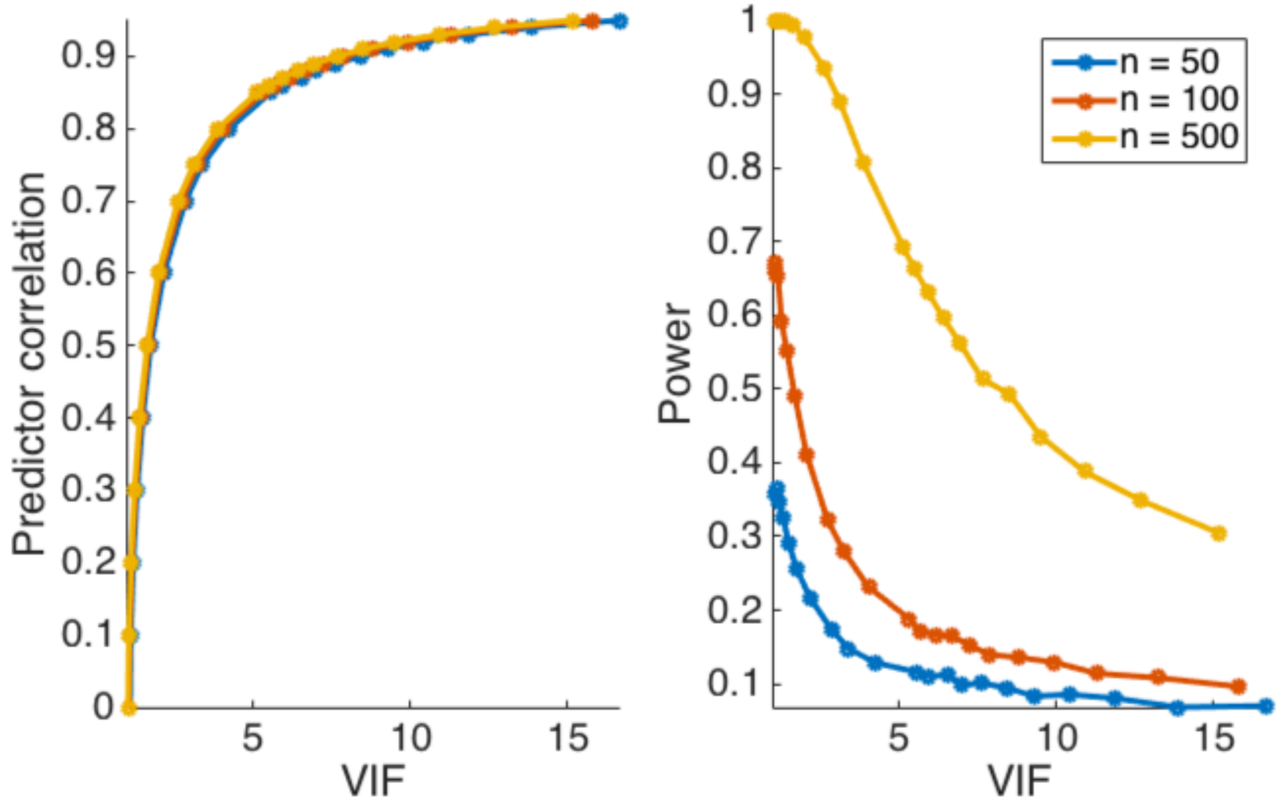
Properties of VIFs

- - Looks ok?
- - BUT: New regressor is perfect linear combination of original regressors: $X_1 - X_2$!
- - Which VIFs will be affected, and which model parameters not uniquely estimable?





How are VIFs related to correlations and power?



- VIFs above 5 signal strong multicolinearity (r = 0.8)
- Power depends on sample size too
- No hard and fast rule for “too high”
- P-values can be misleading: Can ‘flip-flop’ from significant positive to negative effects

Take-home: Regression

- Take-home: Multicolinearity
  - It is important to check for multicolinearity
  - Look at your design matrix visually
  - Pairwise correlations are not enough
  - Variance inflation factors are a good way to check

- Take-home: Interpretation
  - P-values, t-values, Z-values are only valid if the GLM assumptions hold
  - A predictor with a significant fit does not mean that the predictor is the "right" model
  - Variables that you have not modeled may actually be causing effects in your data

End of Module

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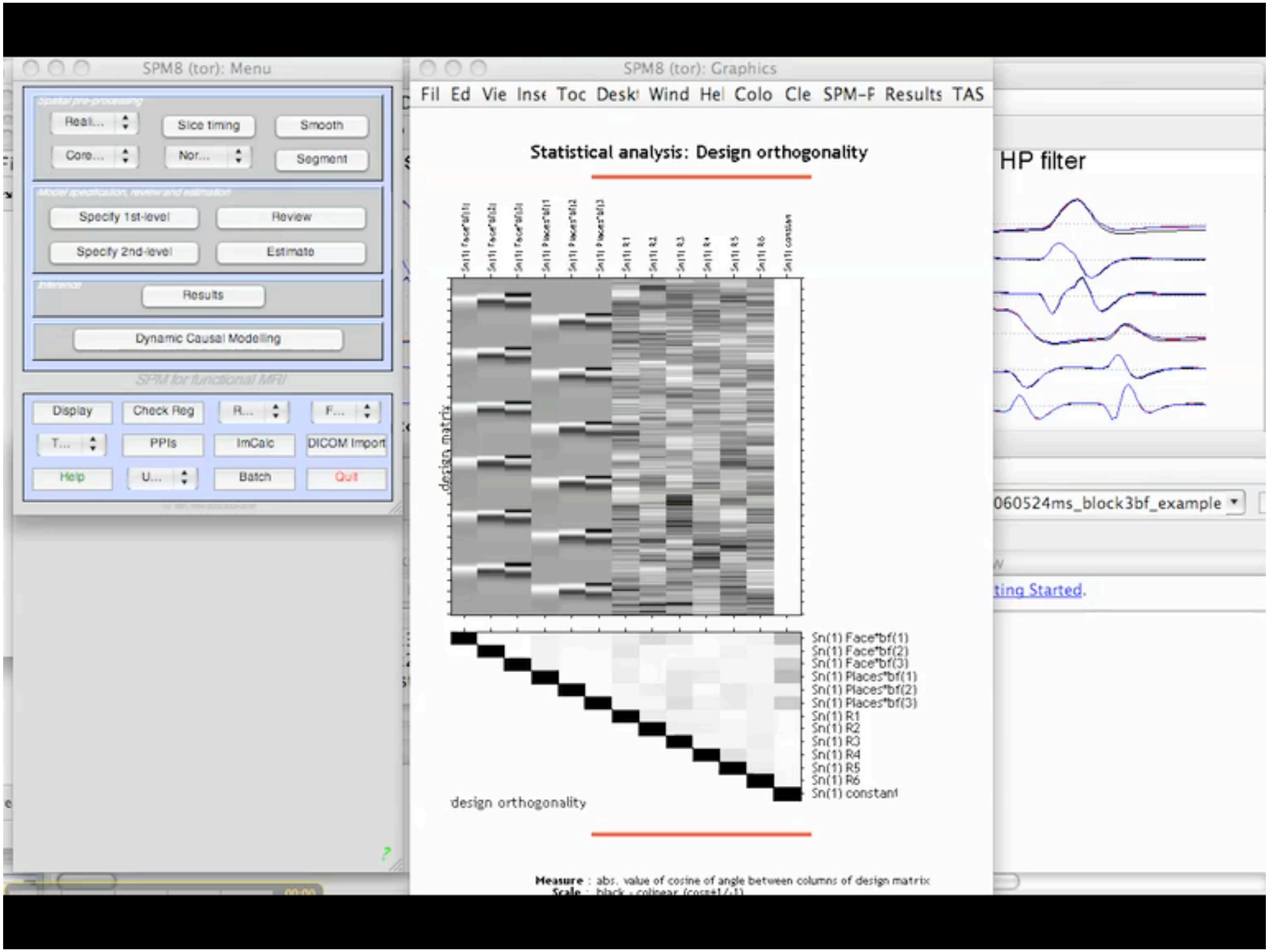
Practical: Checking an SPM design


SPM8 (tor): Menu
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Slice timing
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Specify 2nd-level
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SPM8 (tor): Graphics
Fil Ed Vie Inse Toc Desk Wind Hel Colo Cle SPM-F Results TAS
Statistical analysis: Design orthogonality
design matrix
Sn(1) Face*bf(1)
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Sn(1) R3
Sn(1) R4
Sn(1) R5
Sn(1) R6
Sn(1) constant
design orthogonality
Measure : abs. value of cosine of angle between columns of design matrix
HP filter
060524ms_block3bf_example
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Contrasts, filtering, and estimability

Having regressors that satisfy the above does not imply that contrasts do.

...run scn_spm_design_check after specifying contrasts.

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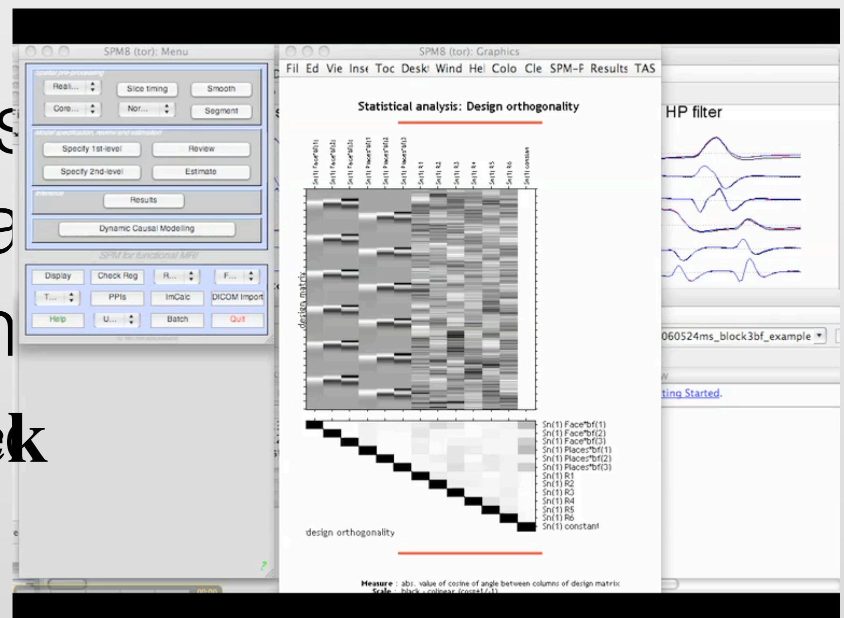
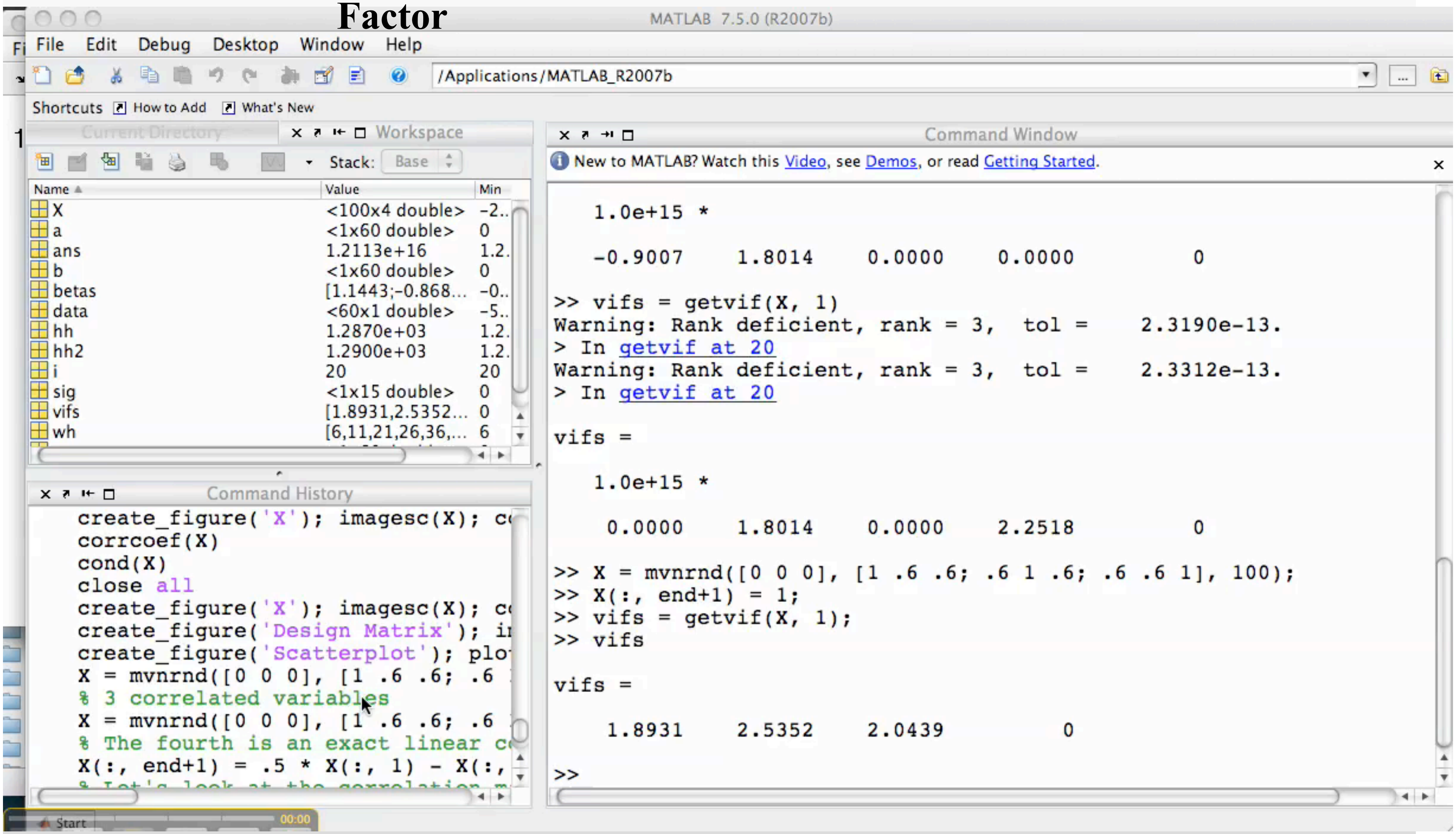
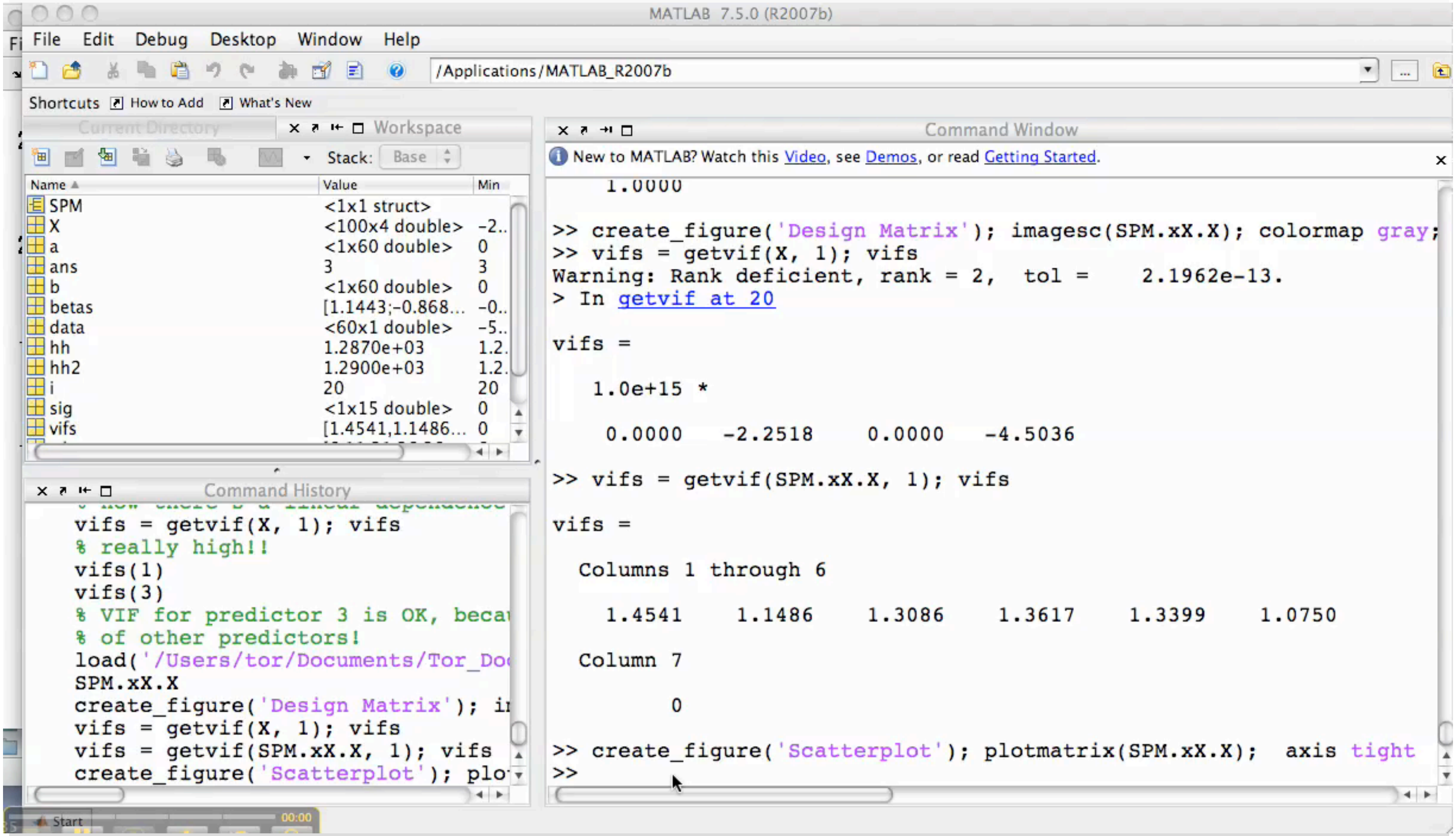


Contrasts are uniquely estimable

Contrast values are estimated with

Condition number and Variance Inflation Factor




MATLAB 7.5.0 (R2007b)
File Edit Debug Desktop Window Help
/Applications/MATLAB_R2007b
Shortcuts How to Add What's New
Current Directory
Workspace
Stack: Base
Name Value Min
SPM <1x1 struct>
X <100x4 double> -2..
a <1x60 double> 0
ans 3 3
b <1x60 double> 0
betas [1.1443;-0.868... -0..
data <60x1 double> -5..
hh 1.2870e+03 1.2..
hh2 1.2900e+03 1.2..
i 20 20
sig <1x15 double> 0
vifs [1.4541,1.1486... 0
Command History
vifs = getvif(X, 1); vifs
% really high!!
vifs(1)
vifs(3)
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vifs = getvif(SPM.xX.X, 1); vifs
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Command Window
New to MATLAB? Watch this Video, see Demos, or read Getting Started.
1.0000
>> create_figure('Design Matrix'); imagesc(SPM.xX.X); colormap gray;
>> vifs = getvif(X, 1); vifs
Warning: Rank deficient, rank = 2, tol = 2.1962e-13.
> In getvif at 20
vifs =
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0.0000 -2.2518 0.0000 -4.5036
>> vifs = getvif(SPM.xX.X, 1); vifs
vifs =
Columns 1 through 6
1.4541 1.1486 1.3086 1.3617 1.3399 1.0750
Column 7
0
>> create_figure('Scatterplot'); plotmatrix(SPM.xX.X); axis tight
>>
Start
00:00