Principles of fMRI

Part II

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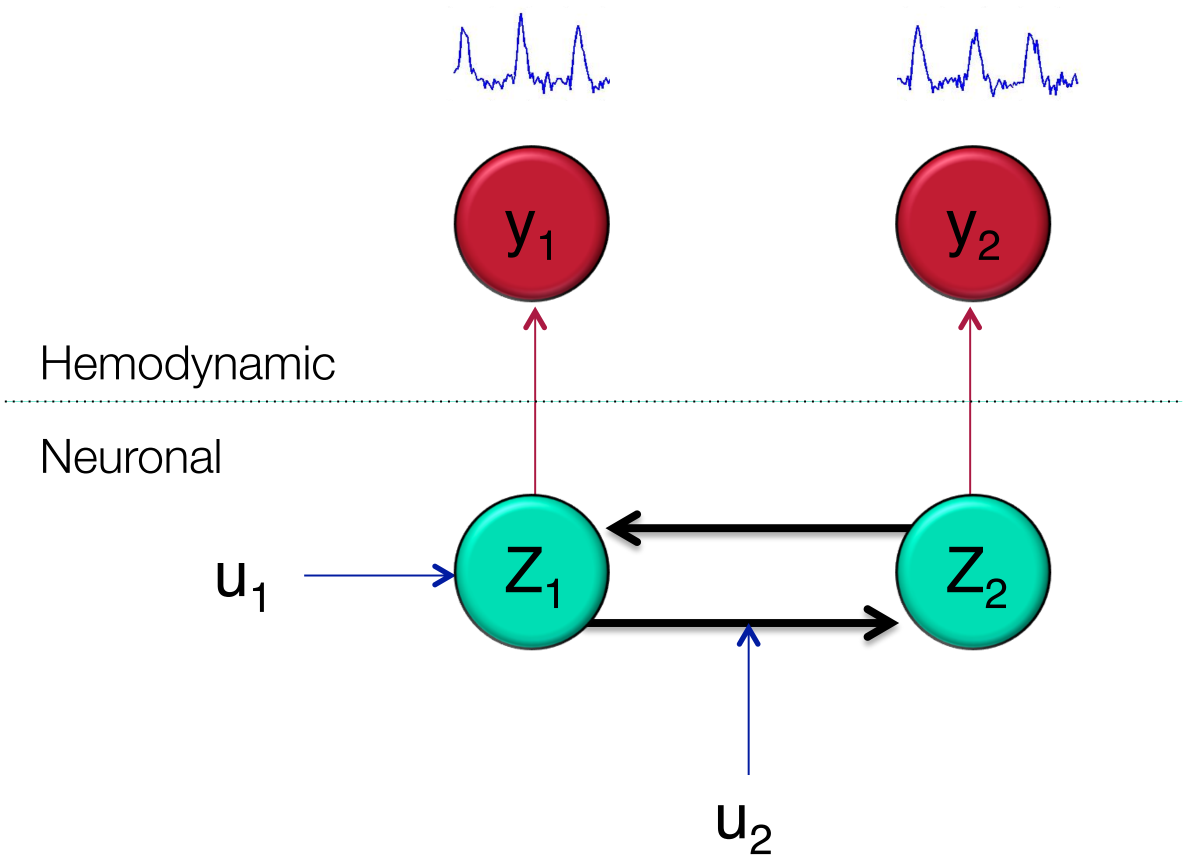
Dynamic Causal Modeling

Dynamic Causal Modeling

- DCM attempts to model latent neuronal interactions using hemodynamic time series.
  - Based on a neuronal model of interacting regions, supplemented with a forward model of how neuronal activity is transformed into the observed response.

- Effective connectivity is parameterized in terms of the coupling among unobserved neuronal activity in different regions.
  - We can estimate these parameters by perturbing the system and measuring the response.

Illustration



Neuronal Model

- Define the neuronal states as:

$$z = (z_1, \ldots z_N)^T$$

- The effective connectivity model is described by:

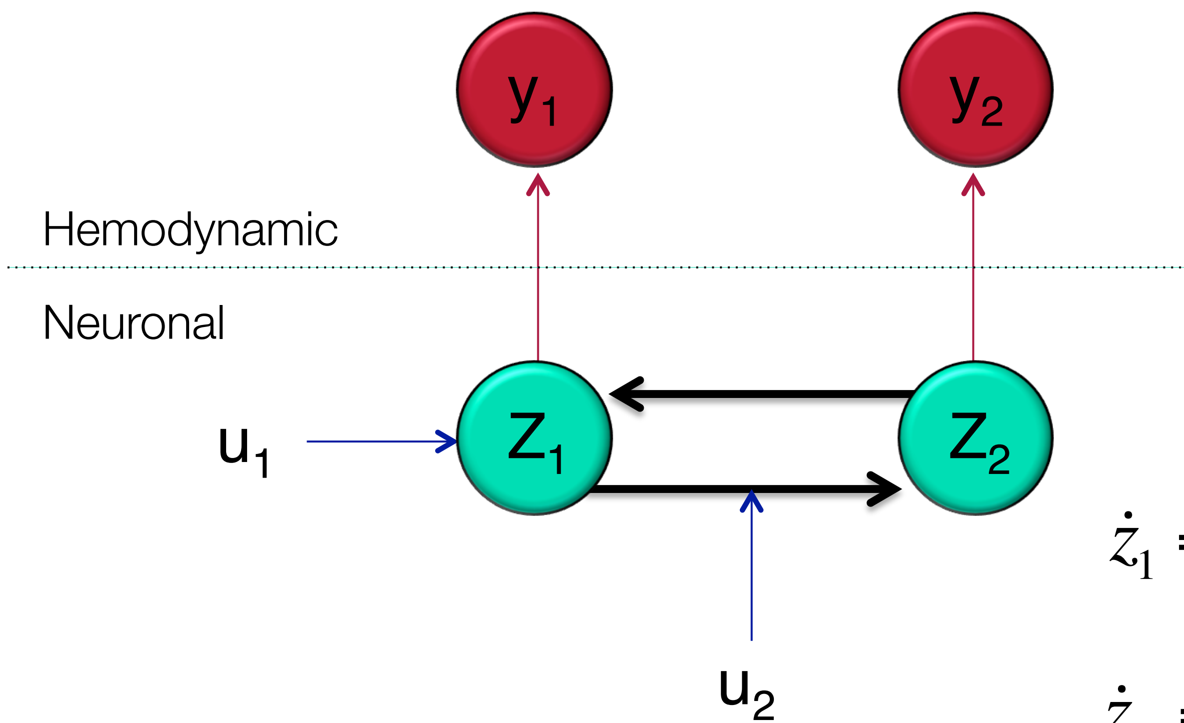
$$\dot{z}_t = \left(A + \sum_{j=1}^{J} u_t(j) B^j \right) z_t + C u_t$$

where z_t is the neuronal activity at time t (latent) and $u_t(j)$ is the j^{th} of J inputs at time t (known).

Interpretation

- The matrix A represents the first order connectivity among regions in the absence of input.
  - Specifies how regions are connected and whether these connections are uni- or bidirectional.
- The matrix C represents the extrinsic influence of inputs on neuronal activity.
  - Specifies how inputs are connected to regions.
- The matrices B_j represent the change in coupling induced by the jth input.
  - Specifies how connections are changed by inputs.

$$\dot{z}_t = \left(A + \sum_{j=1}^{J} u_t(j) B^j \right) z_t + C u_t$$



$$\dot{z}_1 = a_{11} z_1 + a_{12} z_2 + c_{11} u_1$$

$$\dot{z}_2 = a_{21} z_1 + a_{22} z_2 + b_{21}^2 u_2 z_1$$

Hemodynamic Model

- Neuronal activity causes changes in blood volume and deoxyhmoglobin that cause changes in the observed BOLD response.

- The hemodynamics are described using an extended Balloon model, which involves a set of hemodynamic state variables, state equations and hemodynamic parameters θ^h.

Extended Balloon Model

Activity-dependent signal: $\dot{s} = z - \kappa s - \gamma (f - 1)$

Flow induction: $\dot{f} = s$

Changes in volume: $\tau \dot{v} = f - v^{1/\alpha}$

Changes in dHb: $\tau \dot{q} = f\, E(f, \rho)/\rho - v^{1/\alpha}\, q/v$

Hemodynamic response $y = \lambda(v, q)$

State Equations

Neuronal state:

Neuronal activity - z_t with parameters θ^c.

Hemodynamic states:

Vasodilatory signal - s_t

Inflow - f_t

Blood volume - v_t

Deoxygenation content - q_t

The observed data: $y_t=\lambda(q_t,v_t)$ with parameters θ^h.

Bayesian Analysis

- Combining the neuronal and hemodynamic states $x=\{z, s, f, v, q\}$ gives us the following state-space model:

$$\dot{x} = f(x, u, \theta)$$

$$y = \lambda(x, \theta)$$

- Analysis performed using Bayesian methods.
  - Normal priors are placed on θ.
  - The posterior density is used to make inferences about the connections.
  - Model comparison can be performed to determine whether the data favors one model over another.

Model Comparison

- The model evidence is defined as

$$p(y \mid m) = \int p(y \mid \theta, m)\, p(\theta \mid m)\, d\theta$$

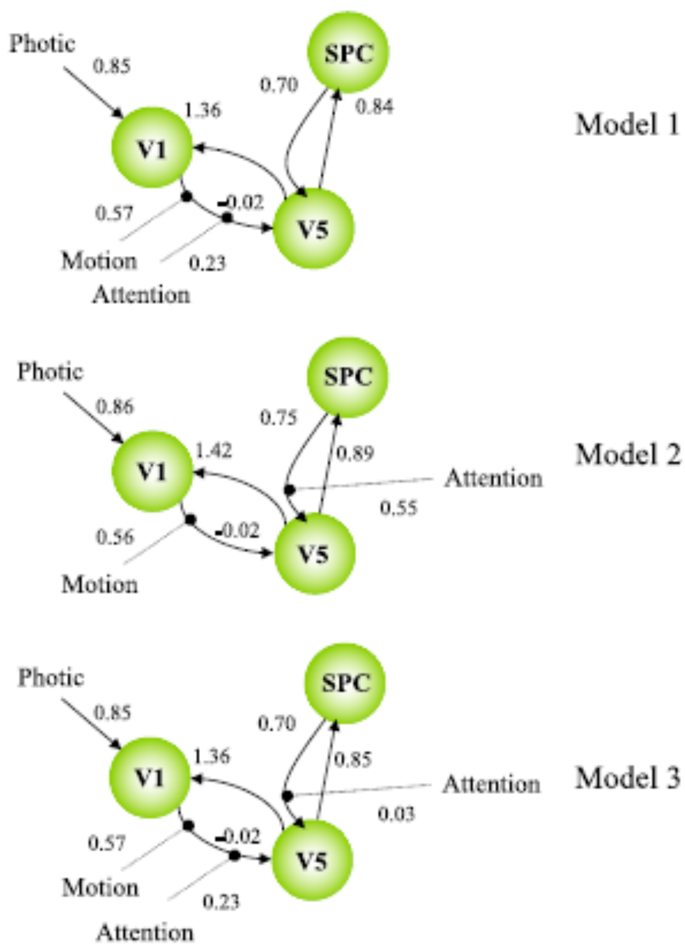
- The Bayes factor for comparing model i to j:

$$B_{ij} = \frac{p(y \mid m = i)}{p(y \mid m = j)}$$

If B_{ij} is large than i more likely than j.

- Various approximations (e.g., negative free energy, AIC or BIC) exist.

Example



Use Bayes factors to compare three different candidate DCMs.

Table 6
Attention data—comparing modulatory connectivities

	B_{12}	B_{13}	B_{32}
AIC	3.56	2.81	1.27
BIC	3.56	19.62	0.18

Bayes factors provide consistent evidence in favor of the hypothesis embodied in model 1, that attention modulates (solely) the bottom-up connection from V1 to V5. Model 1 is preferred to models 2 and 3.

Penny at al.

End of Module