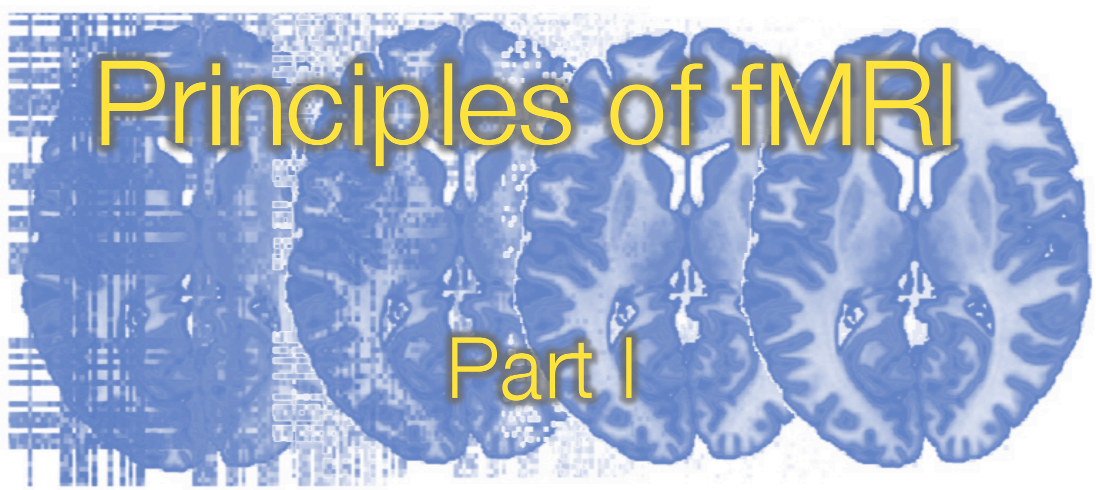
Principles of fMRI

Part I



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@fMRIstats

GLM Model Building 1:
Multiple conditions and contrasts

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REVIEW: KEY CONCEPTS

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Lindquist & Wager

The structural model for the GLM

Simple or multiple regression, t-tests, ANOVA, ANCOVA

$$\mathbf{Y} = \mathbf{X}\beta + \varepsilon$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & \cdots & X_{1p} \\ 1 & X_{21} & \cdots & X_{2p} \\ \vdots & \vdots & & \vdots \\ 1 & X_{np} & \cdots & X_{np} \end{bmatrix} \times \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Observed Data

Design matrix

Model parameters

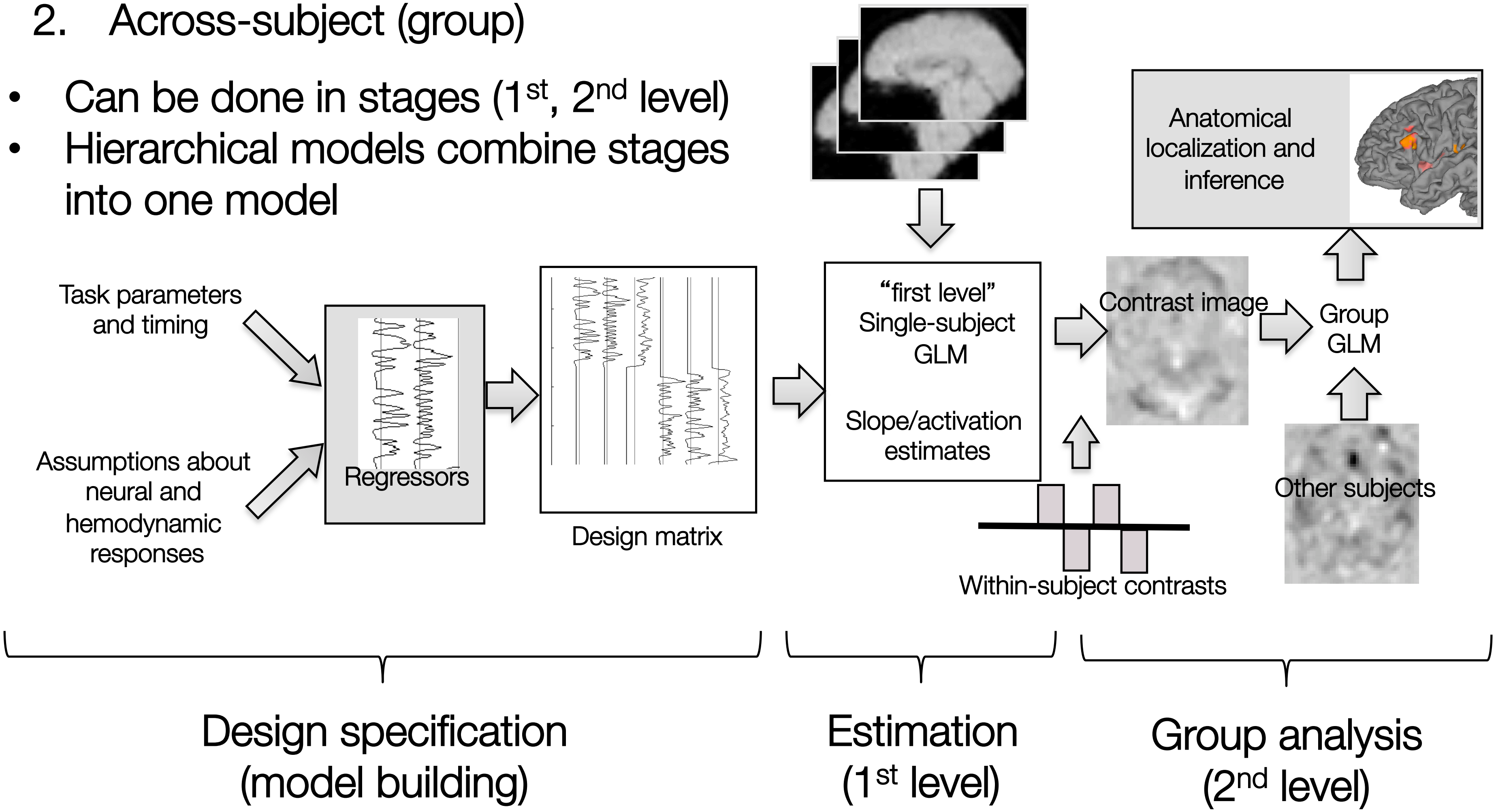
Residuals

Overview of the GLM analysis process

Typically a two-level hierarchical analysis

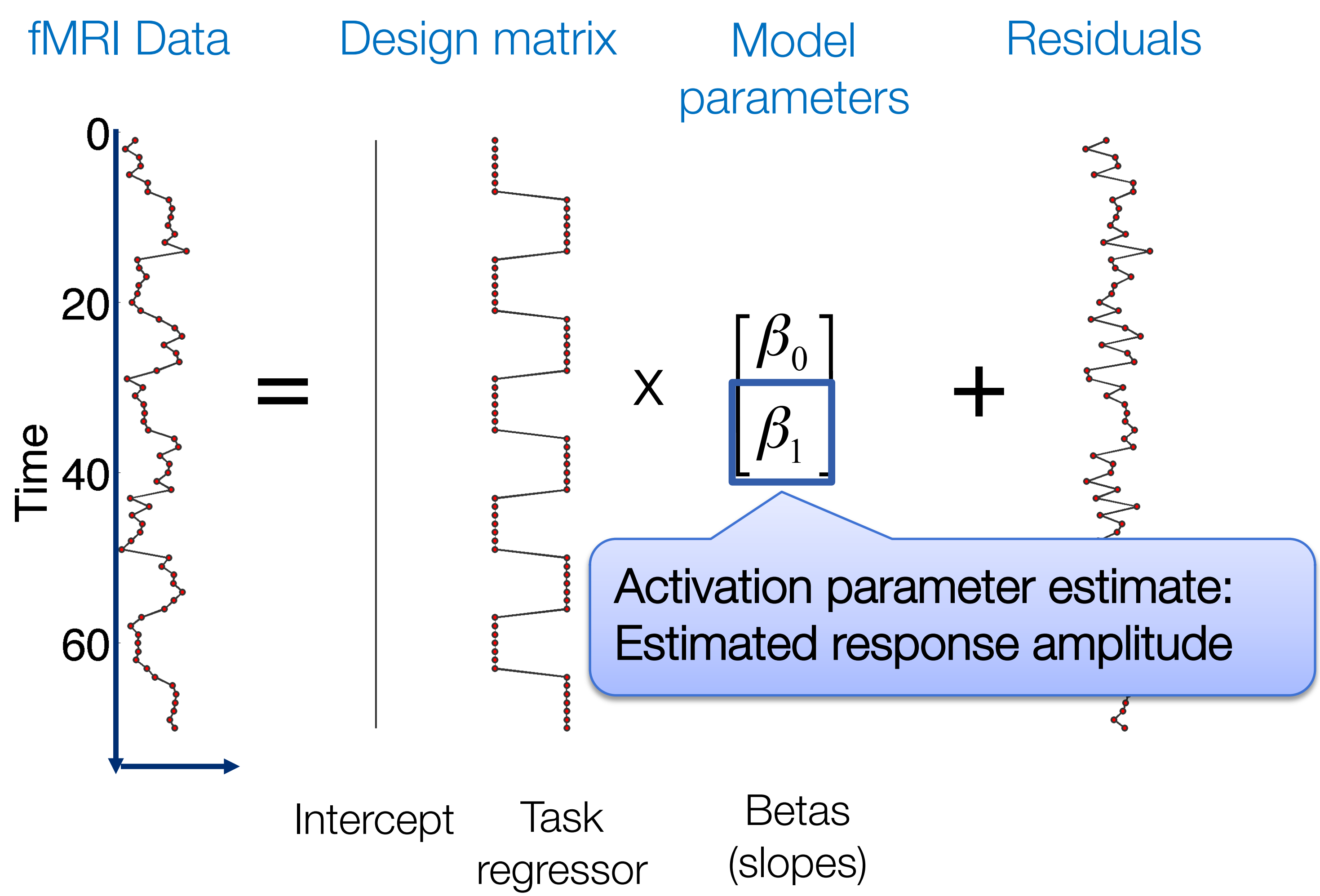
1. Within-subject (individual)
2. Across-subject (group)

- Can be done in stages (1st, 2nd level)
- Hierarchical models combine stages into one model

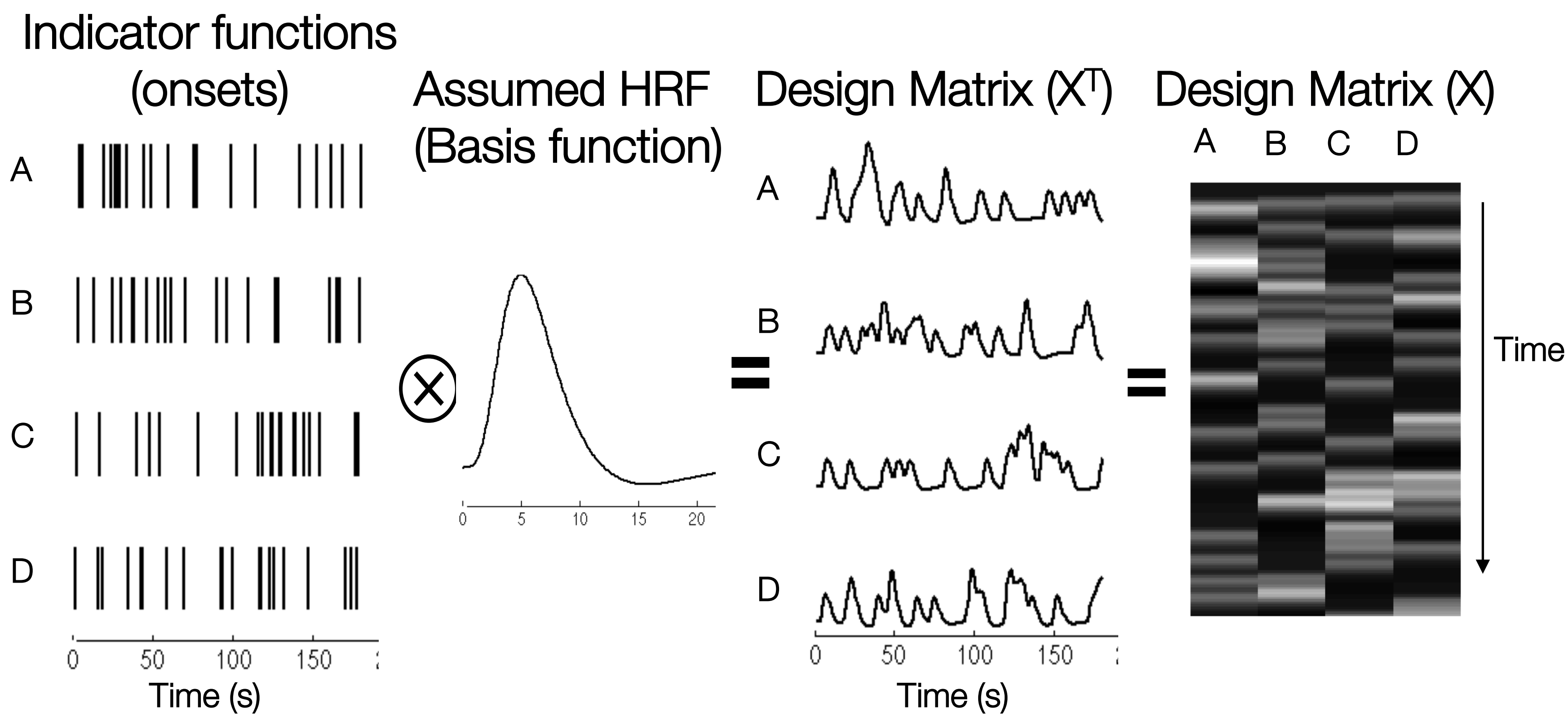


First-level GLM: Single-voxel, single-subject

- A basic design matrix:



Model building for multiple predictors: Single-subject, single voxel



MODEL BUILDING: PREDICTORS AND CONTRASTS

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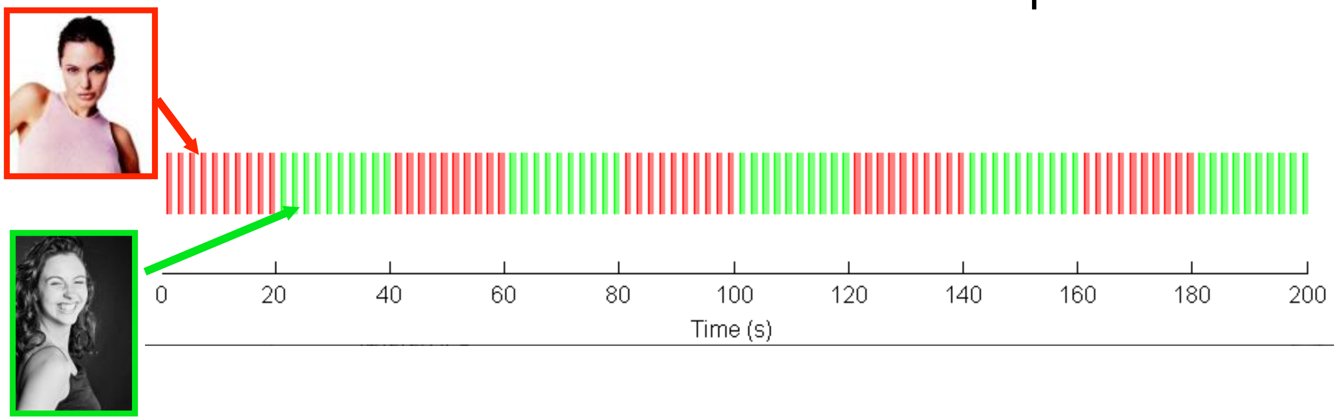
Lindquist & Wager

Multiple-predictor designs and contrasts

- Famous vs. non-famous face example:



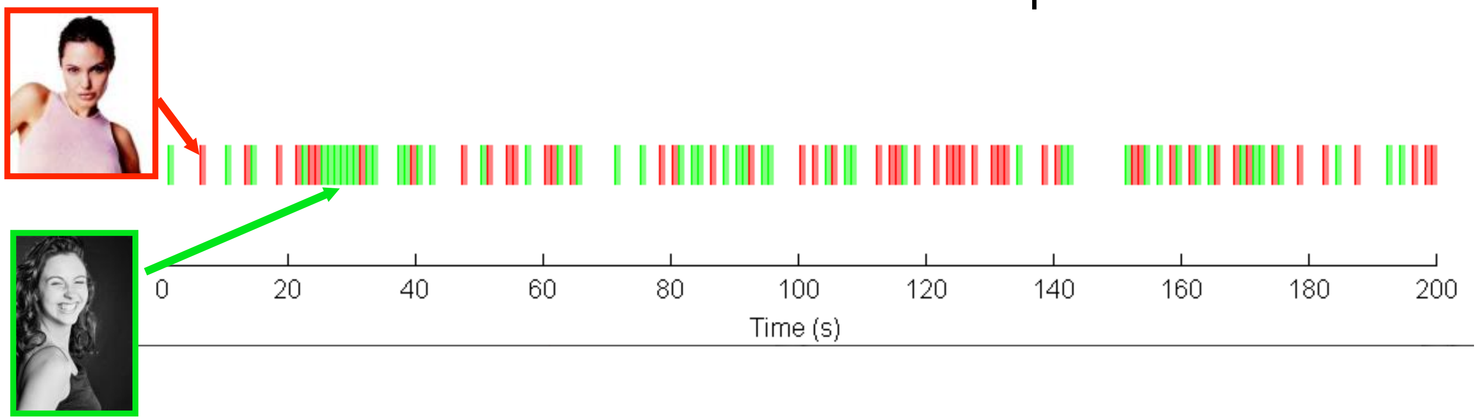
- You care about the difference between famous and non-famous faces. This is a contrast across two conditions.

- With block design, one can use a single regressor that captures the difference

Multiple-predictor designs and contrasts

Principles of fMRI
Part I

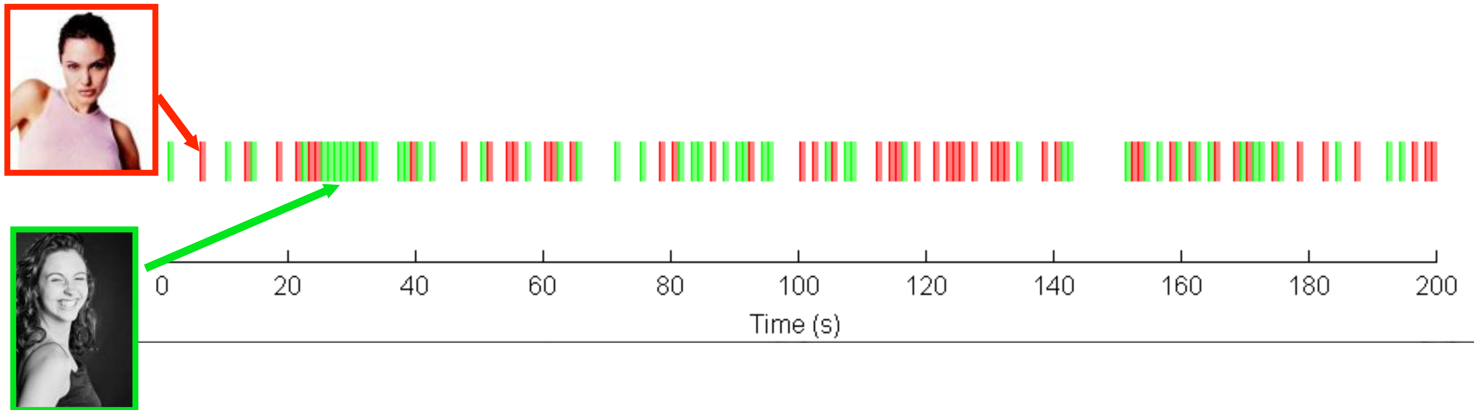
- Famous vs. non-famous face example:



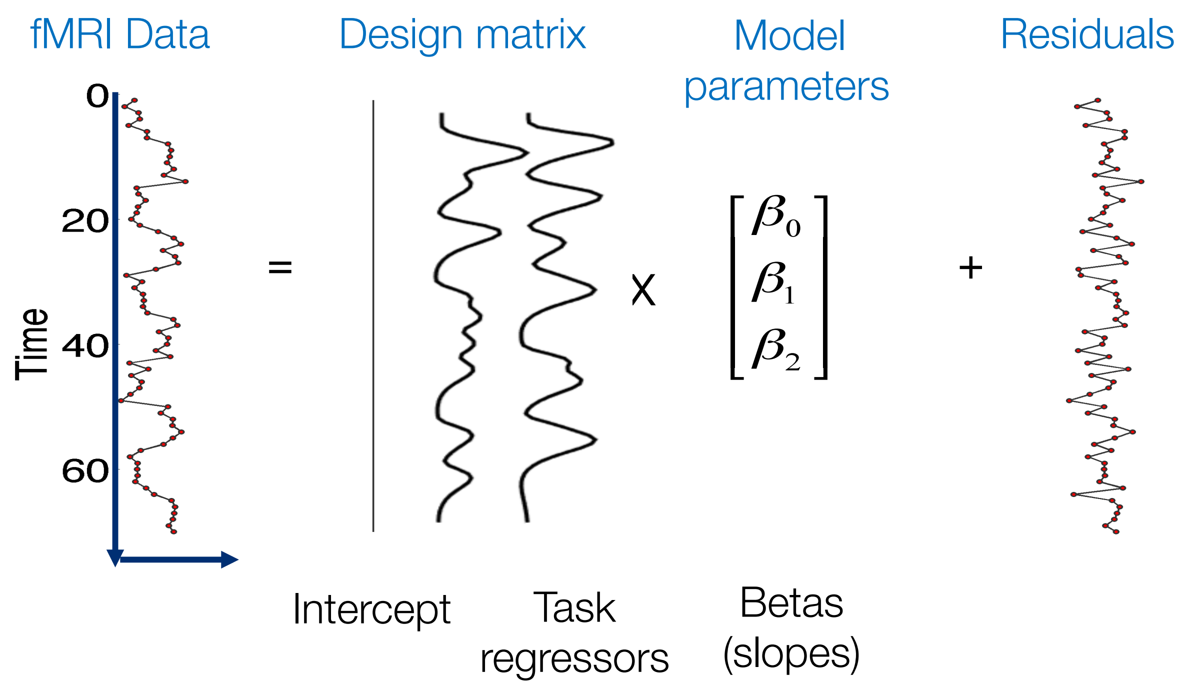
- With event-related designs, model each event type separately

Multiple-predictor designs and contrasts

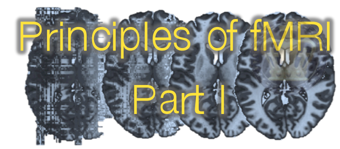
- With event-related designs, model each event type separately



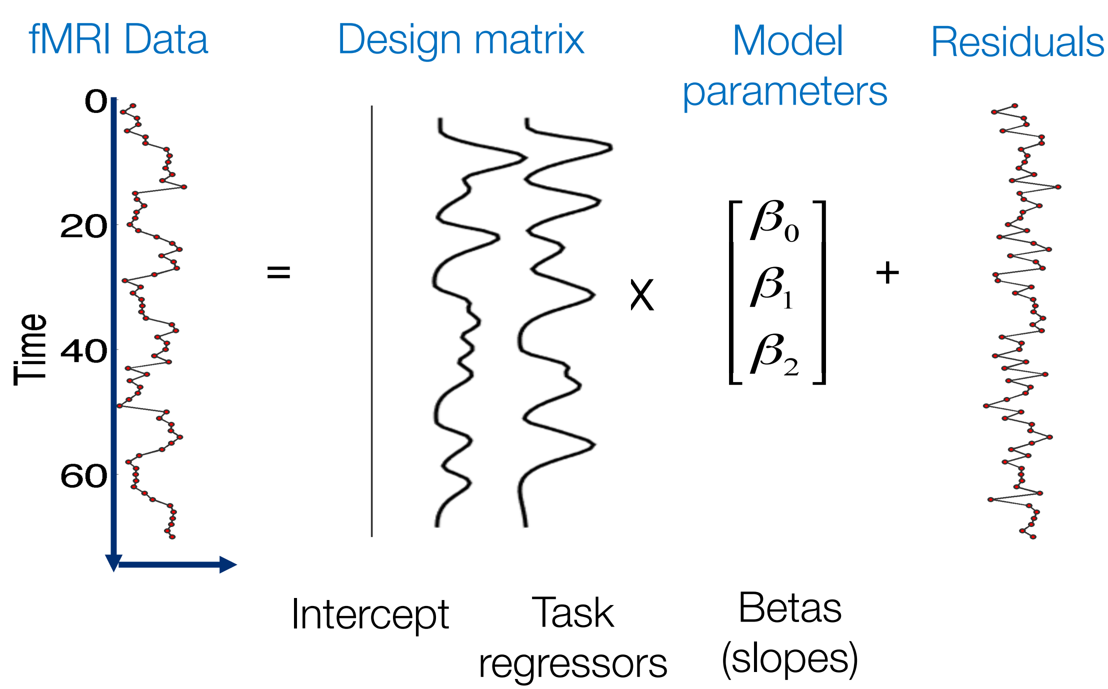
- You can now assess the *difference*, each one *separately*, or their *average*
- These functions are specified by different linear contrasts



Contrasts: A flexible and powerful tool



- **Contrast**: A linear combination of GLM parameters
  - T-contrast: Single, planned contrast -> t-test
  - Specified by a vector of weights (*c*), so that $\mathbf{c}^T\hat{\boldsymbol{\beta}}$ = a scalar value
  - Signed: Can have negative or positive values



Difference

$$\begin{bmatrix} 0 & 1 & -1 \end{bmatrix}$$

Sum (average)

$$\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$$

Single event

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$[\ \beta$

Model building and contrasts: multiple predictors

	Factor 2	
Factor 1	**A**	**B**
	C	**D**

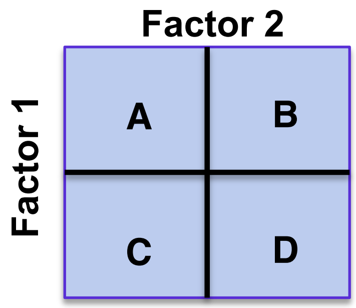
Example: Memory experiment
Four word types, grouped into two factors:

Factor 1: Visual vs. Auditory presentation (2 levels)
Factor 2: High vs. low imageability (2 levels)

Dependent Variable	**Predictors**	**Analysis**
4+ repeated measures	**0 between** ***2+ categorical* within**	**Factorial RM-ANOVA**

"Factorial repeated-measures ANOVA design"
Very typical in fMRI experiments

Model building and contrasts: multiple predictors



Example: Memory experiment
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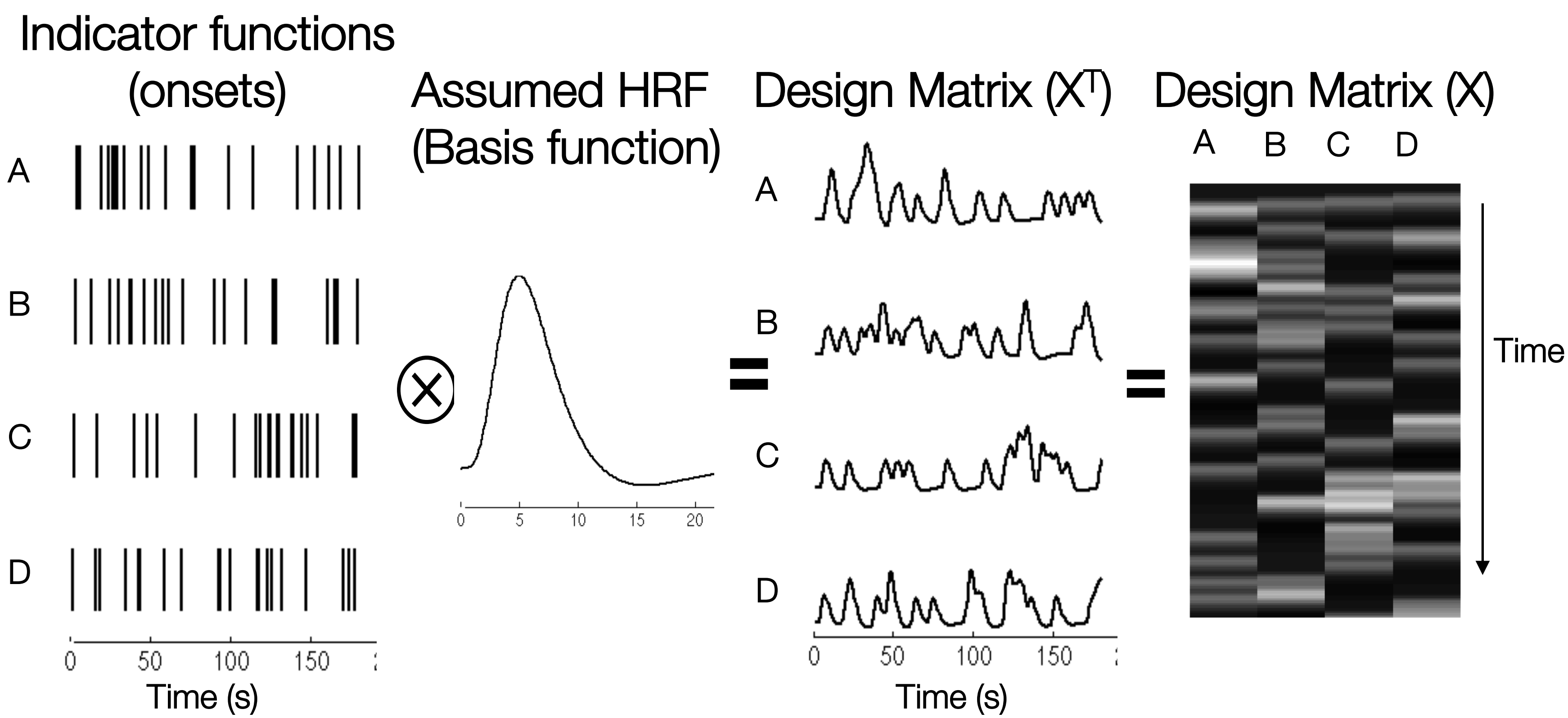
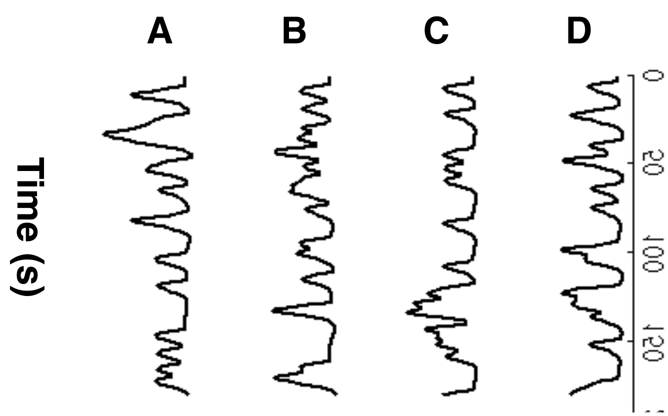


Contrasts for ANOVA design: Planned comparisons to identify effects of interest

Design Matrix (X)



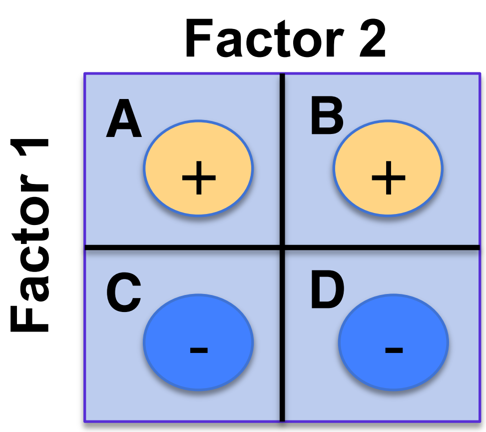
Contrast weights

$$* \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} =$$

Effect

(A+B) - (C+D)

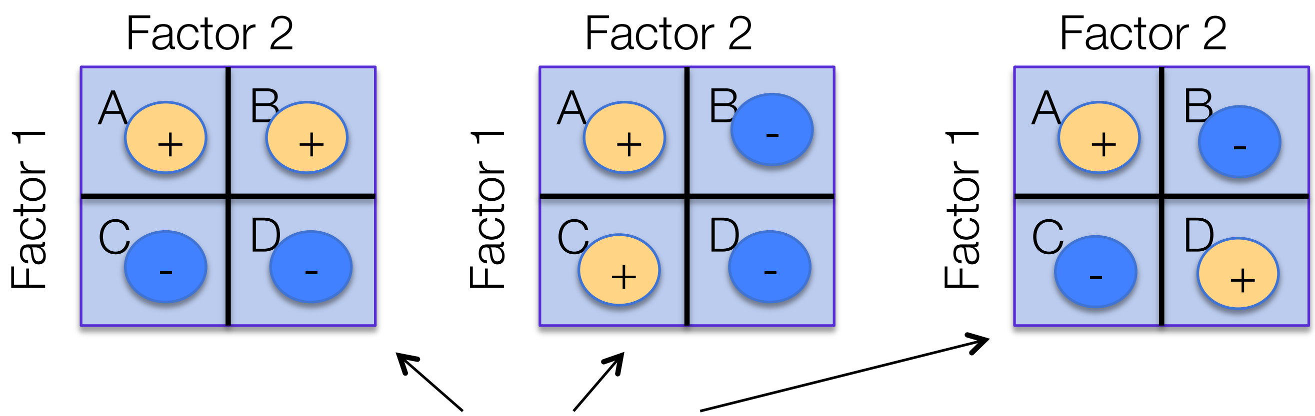
Main effect of Factor 1



Factor 1: Visual vs. Auditory presentation (2 levels)
Factor 2: High vs. low imageability (2 levels)

Rules for T-contrasts

- C can be a matrix
  - Columns are applied independently and do not affect one another, so each is a separate test



$$C = \begin{bmatrix} .5 & .5 & .5 \\ .5 & -.5 & -.5 \\ -.5 & .5 & -.5 \\ -.5 & -.5 & .5 \end{bmatrix}$$

$c^T\hat{\beta}$ has three columns, corresponding to main effects and interaction: standard ANOVA contrasts

Rules for T-contrasts

Testing of custom hypotheses

- Not limited to ANOVA contrasts

- Can specify planned tests that make sense based on your hypotheses:

$$C = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$c^T\beta$ is 'simple effect', difference [A – B]

High – low imageability effect for visual items only

$$c = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

$c^T\hat{\beta}$ is effect magnitude estimate for 2*A - B – C

Rules for T-contrasts

- Scaling of weights affects magnitude, but not inference (t-values, p-values).
  - [1 -1] and [.5 -.5] give same statistical result.

$$c = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$ $c^T\hat{\beta}$ is effect magnitude estimate for 2*A - B – C

$$c = \begin{bmatrix} 1 \\ -0.5 \\ -0.5 \\ 0 \end{bmatrix}$$ $c^T\hat{\beta}$ is effect magnitude estimate for A – mean(B, C)
Statistically identical to above.

Tip: Contrast weights must be the same for all participants.

- Beware if you have missing sessions or runs!

Rules for T-contrasts

- Expected value of $c^T\beta$ under the null hypothesis should be 0
  - Ho (null): $c^T\beta = 0$. Alternative Ha: $c^T\beta \neq 0$, permits t-test

- If you are testing a difference between conditions, contrast weights should sum to zero
- Consider a contrast *C* across 4 conditions:

Valid: Expected value is zero with no true difference in β

$$C = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$ $c^T\hat{\beta}$ is effect magnitude estimate for 2*A – B

Invalid: Expected value is not zero

$$C = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$ $c^T\hat{\beta}$ is effect magnitude estimate for 2*A – B - C

Rules for T-contrasts

- One case in which weights do not need to sum to zero: Testing the average of one or more conditions against the implicit baseline

Ho: $\beta_A = 0$

- Under null hypothesis of no activation in A, $E(\beta_A) = 0$

$$C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

C' b is beta for A only
i.e., whether response to A > 0

Ho: $(\beta_A + \beta_B) / 2 = 0$

$$C = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

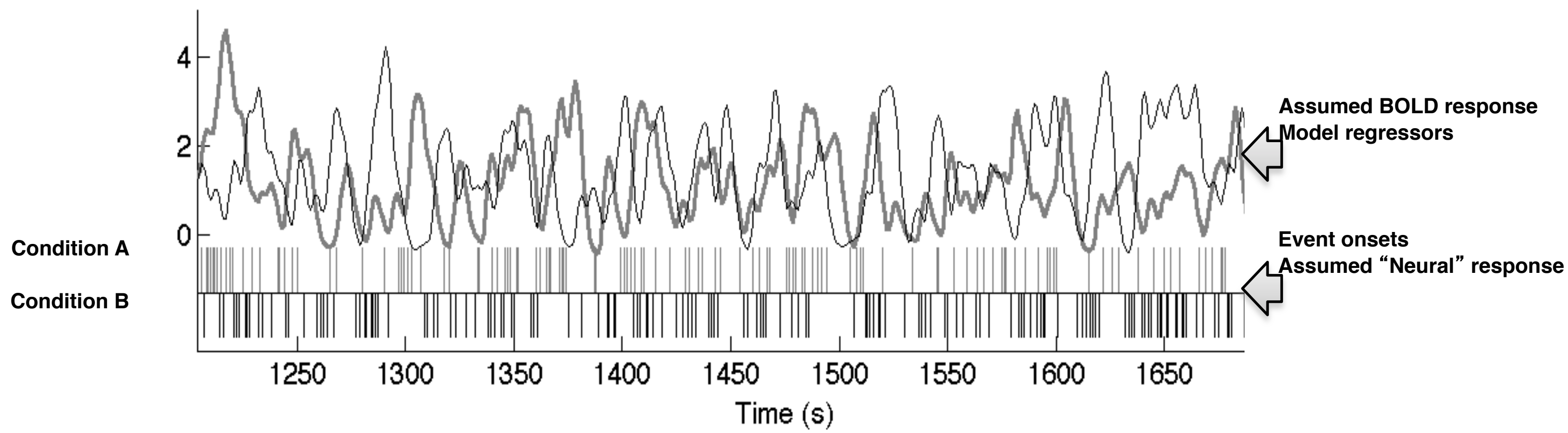
C' b is sum (or average) beta for A and B

Summary

- Specifying contrasts for hypotheses of interest
  - Simple differences and averages
  - ANOVA contrasts
  - Custom hypotheses

- Rules for contrasts
  - Scaling
  - Expected value must be zero under null

End of Module


Assumed BOLD response
Model regressors
Event onsets
Assumed "Neural" response
Condition A
Condition B
4
2
0
1250
1300
1350
1400
1450
1500
1550
1600
1650
Time (s)

Rules for T-contrasts

- I have 4 conditions, but am interested in testing specific differences among them
- Define a matrix C whose columns define linear combinations of 4 predictors

$$C = \begin{bmatrix} 0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{bmatrix}$$

Then C' X is mean(A,B) - mean(C,D)
e.g., a main effect across 4 conditions

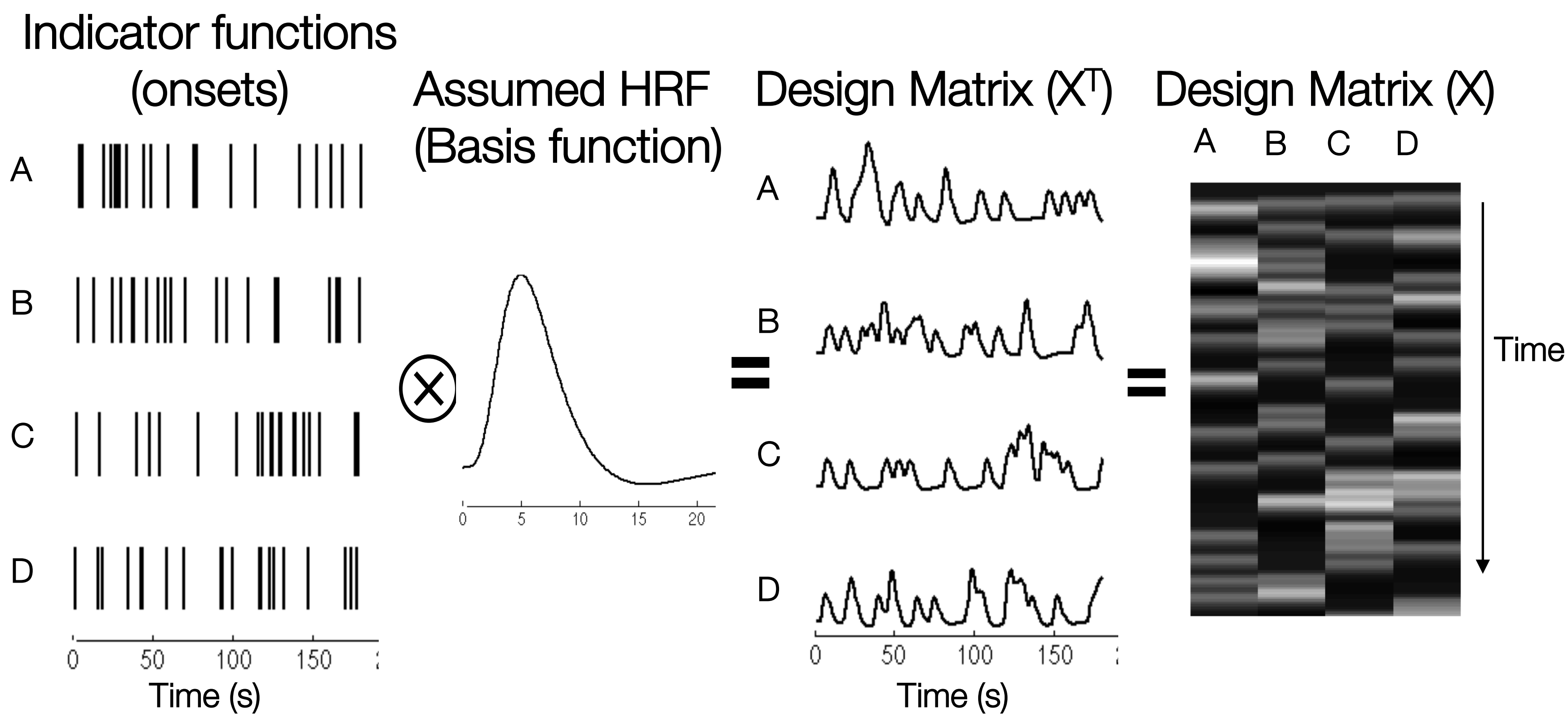
$$C = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

Then C' X is [A - B]
e.g., a simple difference between two conditions

$$C = \begin{bmatrix} .5 & .5 & .5 \\ .5 & -.5 & -.5 \\ -.5 & .5 & -.5 \\ -.5 & -.5 & .5 \end{bmatrix}$$

Then C' X has three columns, corresponding to main effects and interactions

Model building for multiple predictors



Assumptions!

Assume neural activity function is correct

Assume HRF is correct

Assume LTI system

We will look at how to relax these later