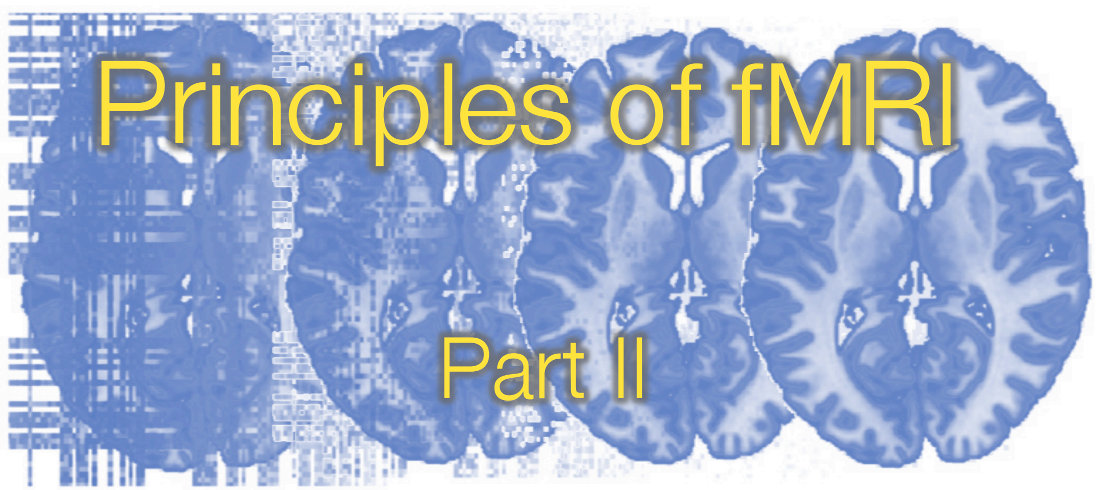

Principles of fMRI

Part II

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Bird's Eye View

- Goal
  - Induce human subject to do or experience the psychological states you're studying
  - Effectively detect brain signals related to those psychological states

- Design method
  - You control what to present and when
  - Two kinds of considerations: Psychological and statistical

Bird's Eye View

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Eight principles of fMRI design

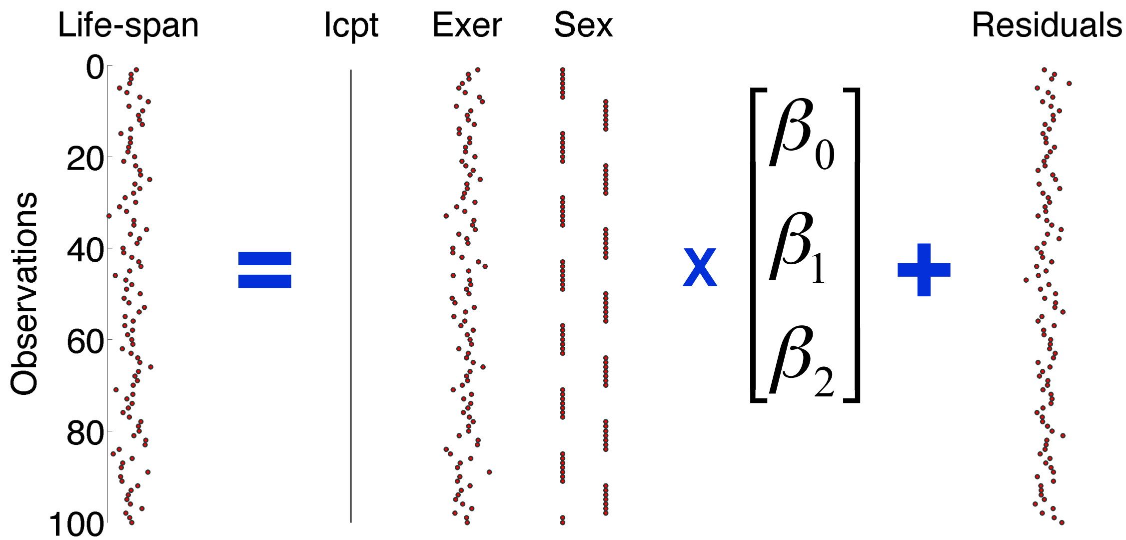
1) Sample size.
2) Scan time.
3) Number of conditions.
4) Grouping of events.
5) Temporal frequencies.
6) Randomization.
7) Nonlinearity.
8) Optimization.

Structural Model for the GLM

$$\mathbf{Y} = \mathbf{X}\beta + \varepsilon$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & \cdots & X_{1p} \\ 1 & X_{21} & \cdots & X_{2p} \\ \vdots & \vdots & & \vdots \\ 1 & X_{np} & \cdots & X_{np} \end{bmatrix} \times \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Outcome Data **Design matrix** **Model parameters** **Residuals**



$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

Algebraic Foundations: Design Efficiency

- Statistical power depends on the magnitude of an effect (e.g., contrast magnitude (c'bhat)) divided by it's standard error, a measure of variability due to noise

- For a t-test:

$$t = \frac{\hat{\beta}}{\hat{s}(\beta)}$$

A closer look at the standard error:

$$\hat{s}(\beta_i) = \underbrace{\hat{\sigma}}_{\text{Residual noise}} \underbrace{\sqrt{\left(X^T X\right)^{-1}_{ii}}}_{\text{Design inefficiency}}$$

Data is not required to estimate this!

Does not depend on data, as long as the form of the model is correct.

- What is it?

Algebraic Foundations: Design Efficiency

- To minimize standard error:

$$\hat{s}(\beta_i) = \hat{\sigma}\sqrt{\left(X^T X\right)^{-1}_{ii}}$$

Reduce noise

Make design more efficient

Efficiency

$$e = \frac{1}{\sqrt{\left(X^T X\right)^{-1}_{ii}}}$$

Understanding and maximizing efficiency is one of the foundations of experimental design

Maximizing average efficiency for effects you care about is called A-optimality. It is directly related to statistical power.

There are other criteria, too (e.g., D-optimality).

- What is it?

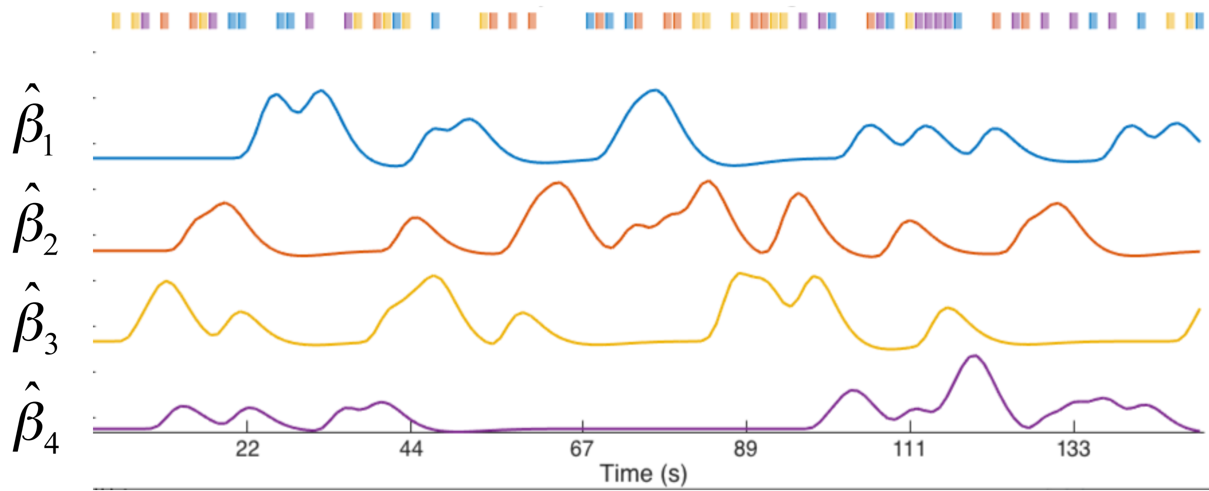
Efficient designs:

Algebraic Foundations: Design Efficiency

Efficiency

$$e = \frac{1}{\sqrt{\left(X^T X\right)^{-1}_{ii}}}$$

Example event-related design with 4 event types:



Dale 1999; Josephs & Henson 2000; Friston et al. 2000; Wager & Nichols 2003; Liu 2004; Hayasaka et al. 2007; Mumford & Nichols 2008

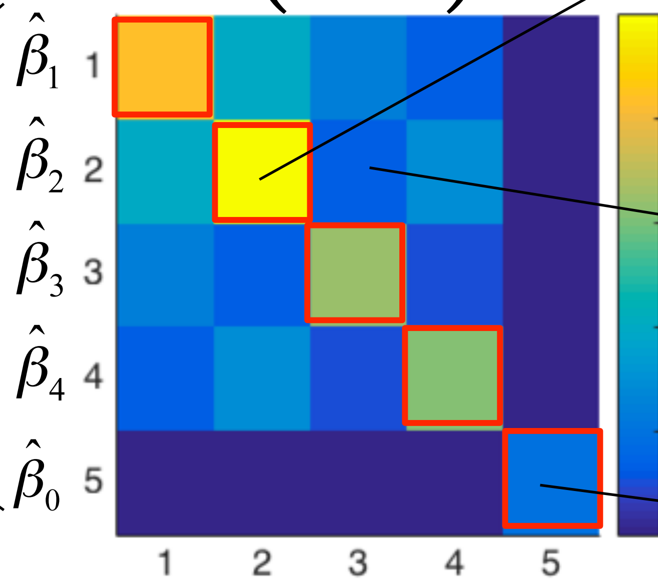
- What is it?

Algebraic Foundations: Design Efficiency

$$\left(X^T X\right)^{-1}$$

Efficiency

$$e = \frac{1}{\sqrt{\left(X^T X\right)^{-1}_{ii}}}$$



Higher values: Less efficient, lower power

Higher values: Parameter estimates are correlated

Intercept

What makes diagonal values smaller (and designs efficient)?

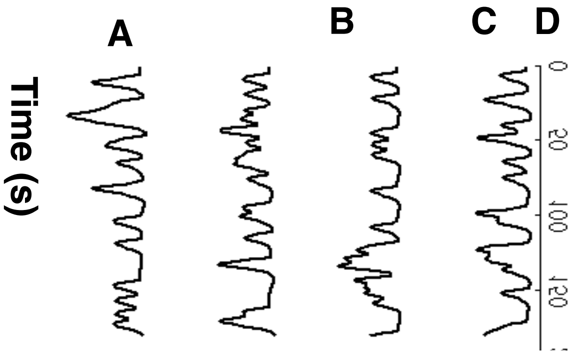
- Large rise and fall in predictors (predictor variance)
- Low covariance among predictors (orthogonal predictors)
- Large sample sizes (variance proportional to sqrt(n))

Design Efficiency in fMRI

Formula is more complex, principle is the same

Factor in contrasts, high-pass filtering and autocorrelation

Design Matrix (X)

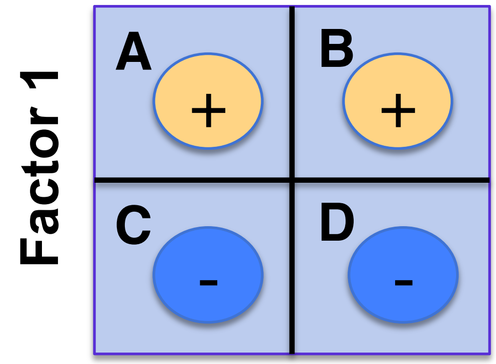


Contrast weights

$$\begin{pmatrix} 1 & 1 & -1 & -1 \end{pmatrix} * \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{bmatrix}$$

Factor 2

Factor 1



Contrast: main effect of Factor 1

Variance/ covariance of contrast values

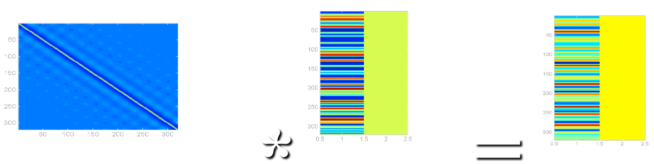
$$\operatorname{var}(C^T\hat{\beta}) = \hat{\sigma}^2 C^T \left(X^T X\right)^{-1} C \qquad t = \frac{c^T \hat{\beta}}{\hat{s}(c^T \beta)}$$

- C can be a matrix (one column per contrast); contrasts will be evaluated independently of one another

Design efficiency in fMRI

- Formula is more complex, principle is the same.

- Factor in filtering
  - Define: high-pass filtering matrix = K
  - Filtered design matrix = Z



K * **X** = **Z**

$$\mathrm{var}(C^T\hat{\beta}) = \hat{\sigma}^2 C^T \left(Z^T Z\right)^{-1} C$$

Design efficiency in fMRI

- Formula is more complex, principle is the same
- Factor in autocorrelation
  - Define: autocorrelation matrix = V

$$\mathrm{var}(C^T\hat{\beta}) = \hat{\sigma}^2 C^T Z^+ K V K^T Z^{+T} C^T$$

$$Z^+ = pinv(Z) = \left(Z^T Z\right)^{-1} \text{ when not rank deficient}$$

$$e = \frac{1}{\sqrt{\mathrm{var}(C^T\hat{\beta})}}$$

- Proportional to statistical power, and can be converted to power given effect sizes

See Friston et al., 2000; Zarahn, 2001

Efficiency in a Multi-level Setting

Principles of fMRI

In a group analysis, efficiency (and power) depend on within-person and between-person variance

$$\hat{S}(c^T\beta) = \sqrt{\frac{\hat{\sigma}^2_w c^T \left(X^T X\right)^{-1}_{ii} c + c^T \hat{\sigma}^2_B c}{N}} = \sqrt{\frac{\hat{s}^2_W + \hat{s}^2_B}{N}}$$

Sample size

Variance/ covariance of contrast values

Within-person standard errors (1/efficiency)

Variance/covariance of individual differences between people

- Increasing within-person efficiency (e.g., by collecting more data) helps up to a point
- Increasing sample size always helps
- The greater the between-person variance, the more sample size is important.

Wager & Nichols, 2003

Sample Size and Scan Time

- Efficiency at the 1st level (single-subject) increases as the square root of the number of images

- However: Power in group analysis is limited by number of subjects collected, and this is often a bigger constraint on power

- Even if efficiency at 1st level is infinite, power and efficiency at group level limited by $\sqrt{\frac{\hat{s}_B^2}{N}}$

End of Module

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