Principles of fMRI

Part II

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Granger Causality

Granger Causality

- Granger causality is a technique that was originally developed in economics that has recently been applied to connectivity studies.

- It does not rely on the *a priori* specification of a structural model, but rather is an approach for quantifying the usefulness of past values from various brain regions in predicting current values in other regions.

Set Up

- Let x and y be two time courses of length *T* extracted from two brain regions.

- Each time course is modeled using a linear autoregressive model of the *M*th order

$$x[n] = \sum_{m=1}^{M} a[m]x[n-m] + \varepsilon_x[n]$$

$$y[n] = \sum_{m=1}^{M} b[m]y[n-m] + \varepsilon_y[n]$$

- Here ε_x and ε_y are both white noise.

- Next, expand each model using the autoregressive terms from the other signal.

$$x[n] = \sum_{m=1}^{M} a_x[m]x[n-m] + \sum_{m=1}^{M} b_x[m]y[n-m] + \varepsilon_x[n]$$

$$y[n] = \sum_{m=1}^{M} b_y[m]y[n-m] + \sum_{m=1}^{M} a_y[m]x[n-m] + \varepsilon_y[n]$$

- The current value depends both on the past M values of its own time course, but also on the past M values of the other time course.

Interpretation

- Using these models one can test whether the history of x has predictive value on the current value of y (and vice versa).

- If the model fit is significantly improved by the inclusion of the cross-autoregressive terms, it provides evidence that the history of one of the time courses can be used to predict the current value of the other and a "Granger-causal" relationship is inferred.

Measuring Influence

- Geweke (1982) proposed a measure of linear dependence $F_{x,y}$ between x[n] and y[n] which implements Granger causality in terms of vector autoregressive models.

- $F_{x,y}$ is a measure of the total linear dependence between x and y.
  - If nothing about the current value of x (or y) can be explained by a model containing all values of y (or x) then $F_{x,y}$ will be 0.

- The term $F_{x,y}$ can be decomposed into the sum of three components:

$$F_{x,y} = F_{x\to y} + F_{y\to x} + F_{x\cdot y}$$

- $F_{x\to y}$ and $F_{y\to x}$ are measures of linear directed influence from x to y and y to x, respectively.
  - If past values of x improve the prediction of the current value of y, then $F_{x\to y} > 0$. A similar interpretation holds for $F_{y\to x}$.

- $F_{x\cdot y}$ is a measure of the undirected instantaneous influence between the series.
  - The improvement in the prediction of the current value of x (or y) by including the current value of y (or x) in a linear model already containing the past values of x and y.

Computation

- Let,

$$x[n] = \sum_{m=1}^{M} a[m]x[n-m] + \varepsilon_x[n] \qquad \mathrm{var}(\varepsilon_x[n]) = \mathbf{\Sigma}_1$$

$$y[n] = \sum_{m=1}^{M} b[m]y[n-m] + \varepsilon_y[n] \qquad Var(\varepsilon_y[n]) = \mathbf{T}_1$$

- Further, let $\mathbf{q}[n] = \begin{bmatrix} x[n] \\ y[n] \end{bmatrix}$ where

$$\mathbf{q}[n] = \sum_{m=1}^{M} \mathbf{A}_q[m]\mathbf{q}[n-m] + \varepsilon_q[n] \qquad Var(\varepsilon_q[n]) = \mathbf{Y} = \begin{bmatrix} \mathbf{\Sigma}_2 & \mathbf{C} \\ \mathbf{C}^T & \mathbf{T}_2 \end{bmatrix}$$

Computation

- Total linear dependence between x and y:

$$F_{x,y} = F_{x \rightarrow y} + F_{y \rightarrow x} + F_{x \cdot y}$$

where

$$F_{x,y} = \ln\left(|\mathbf{\Sigma}_1| \cdot |\mathbf{T}_1| / |\mathbf{Y}| \right) \qquad F_{x \rightarrow y} = \ln\left(|\mathbf{T}_1| / |\mathbf{T}_2| \right)$$

$$F_{x \cdot y} = \ln\left(|\mathbf{\Sigma}_2| \cdot |\mathbf{T}_2| / |\mathbf{Y}| \right) \qquad F_{y \rightarrow x} = \ln\left(|\mathbf{\Sigma}_1| / |\mathbf{\Sigma}_2| \right)$$

Interpretation

Principles of fMRI

- If past values of x improve on the prediction of the current value of y, then $F_{x \to y}$ is large.
- A similar interpretation, but in the opposite direction, holds for $F_{y \to x}$.
- The difference between the two terms can be used to infer which regions history is more influential on the other. This difference is referred to as Granger Causality.

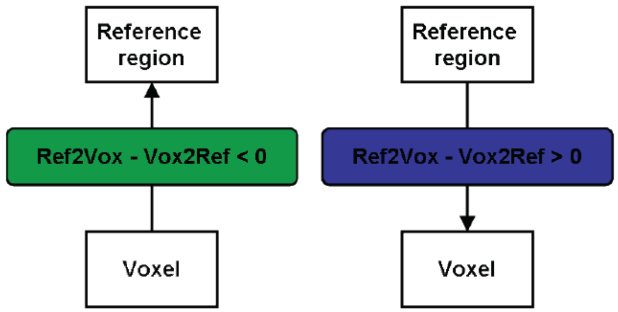
Granger Causality Map

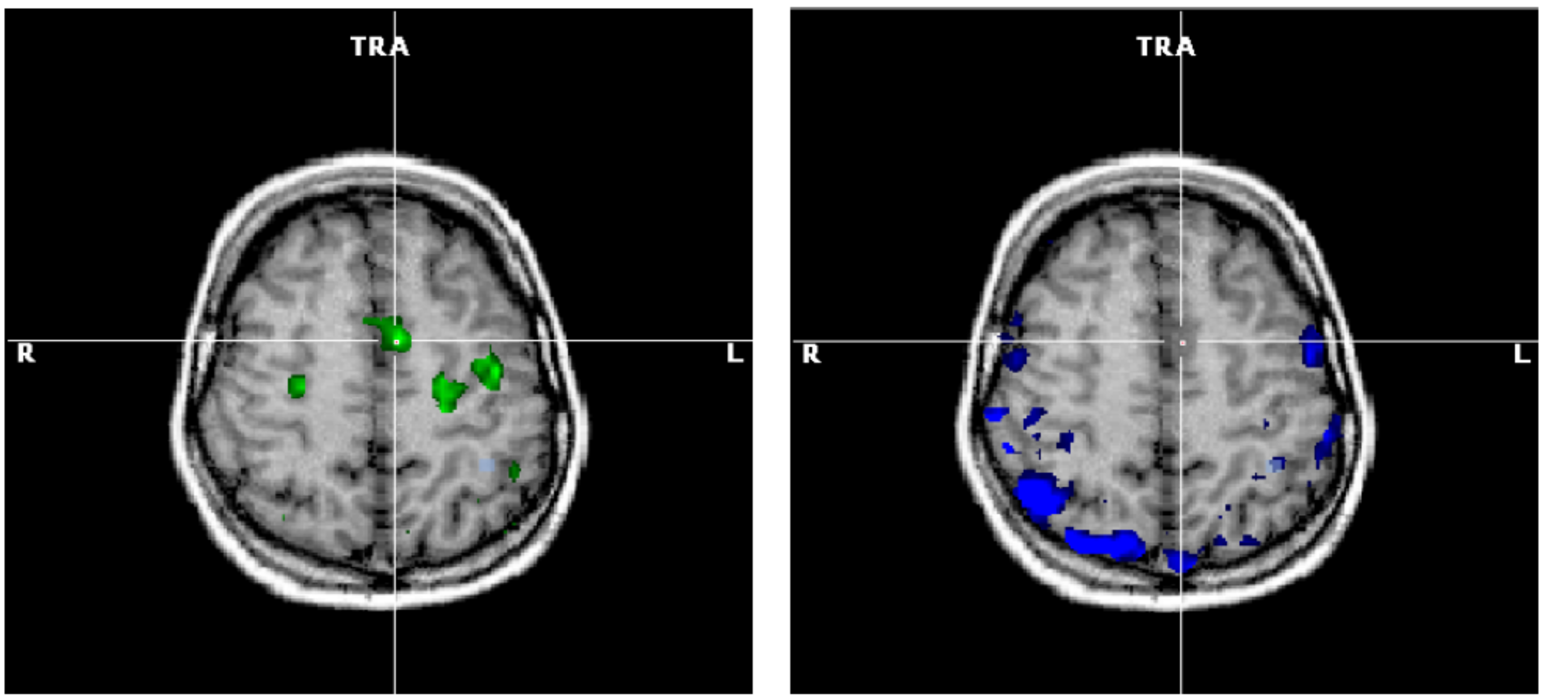
- A Granger Causality Map (GCM) is computed with respect to a single selected reference region (e.g., seed region).

- It maps both sources of influence to the reference region and targets of influence from the reference region over the brain.

Granger Causality Mapping

$$\mathrm{dGCM} = \mathrm{Ref2Vox} - \mathrm{Vox2Ref}$$





Roebroeck

Comments

- From the definition of Granger Causality it is clear that the idea of temporal precedence is used to identify the direction and strength of “causality” using information in the data.

- While it is reasonable that temporal precedence is a necessary condition for causation, it is certainly not a sufficient condition.

- Therefore to directly equate Granger causality and causality requires a leap of faith.

End of Module

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