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FDR Correction

Issues with FWER

- Methods that control the FWER ([Bonferroni](#), [RFT](#), [Permutation Tests](#)) provide a strong control over the number of false positives.
- While this is appealing the resulting thresholds often lead to tests that suffer from [low power](#).
- Power is critical in fMRI applications because the most interesting effects are usually at the edge of detection.

False Discovery Rate

- The **false discovery rate** (FDR) is a recent development in multiple comparison problems due to Benjamini and Hochberg (1995).
- While the FWER controls the probability of any false positives, the FDR controls the proportion of false positives among all rejected tests.

Notation

Suppose we perform tests on m voxels.

	Declared Inactive	Declared Active	
Truly inactive	U	V	m_0
Truly active	T	S	$m - m_0$
	$m - R$	R	m

U, V, T and S are unobservable random variables.

R is an observable random variable.



Definitions

- In this notation:

$$FWER = P(V \geq 1)$$

- False discovery rate:

$$FDR = E\left(\frac{V}{R}\right)$$

- The FDR is defined to be 0 if R=0.



Properties

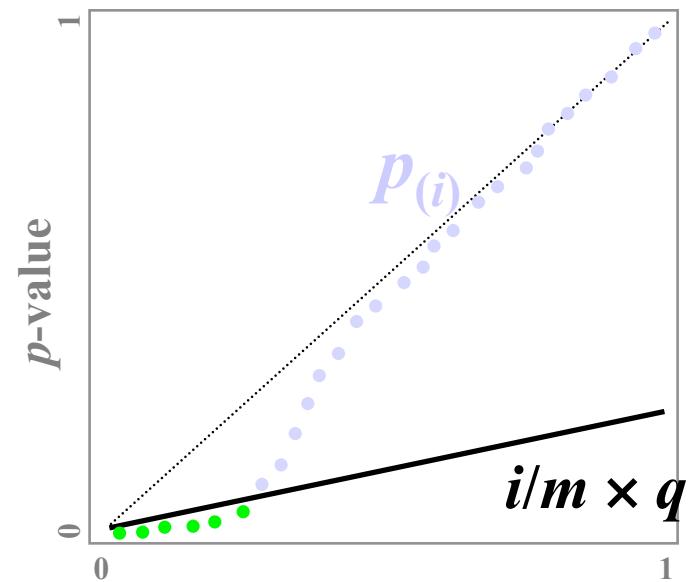
- A procedure controlling the FDR ensures that **on average** the FDR is no bigger than a pre-specified rate q which lies between 0 and 1.
- However, for **any given data set** the FDR need not be below the bound.
- An FDR-controlling technique **guarantees** control of the FDR in the sense that $\text{FDR} \leq q$.

BH Procedure

1. Select desired limit q on FDR (e.g., 0.05)
2. Rank p-values, $p_{(1)} \leq p_{(2)} \leq \dots \leq p_{(m)}$
3. Let r be largest i such that

$$p_{(i)} \leq i/m \times q$$

4. Reject all hypotheses corresponding to $p_{(1)}, \dots, p_{(r)}$.



Comments

- If all null hypothesis are true, the FDR is equivalent to the FWER.
- Any procedure that controls the FWER also controls the FDR. A procedure that controls the FDR only can be less stringent and lead to a gain in power.
- Since FDR controlling procedures work only on the p-values and not on the actual test statistics, it can be applied to any valid statistical test.

End of Module



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