

Martin Lindquist

Department of Biostatistics  
Johns Hopkins  
Bloomberg School of Public Health

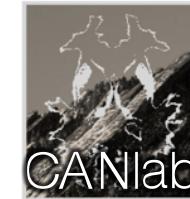
Tor Wager

Department of Psychology and  
Neuroscience and the  
Institute for Cognitive Science  
University of Colorado, Boulder

# The General Linear Model Applied to fMRI



University of Colorado **Boulder**



# REVIEW: KEY CONCEPTS

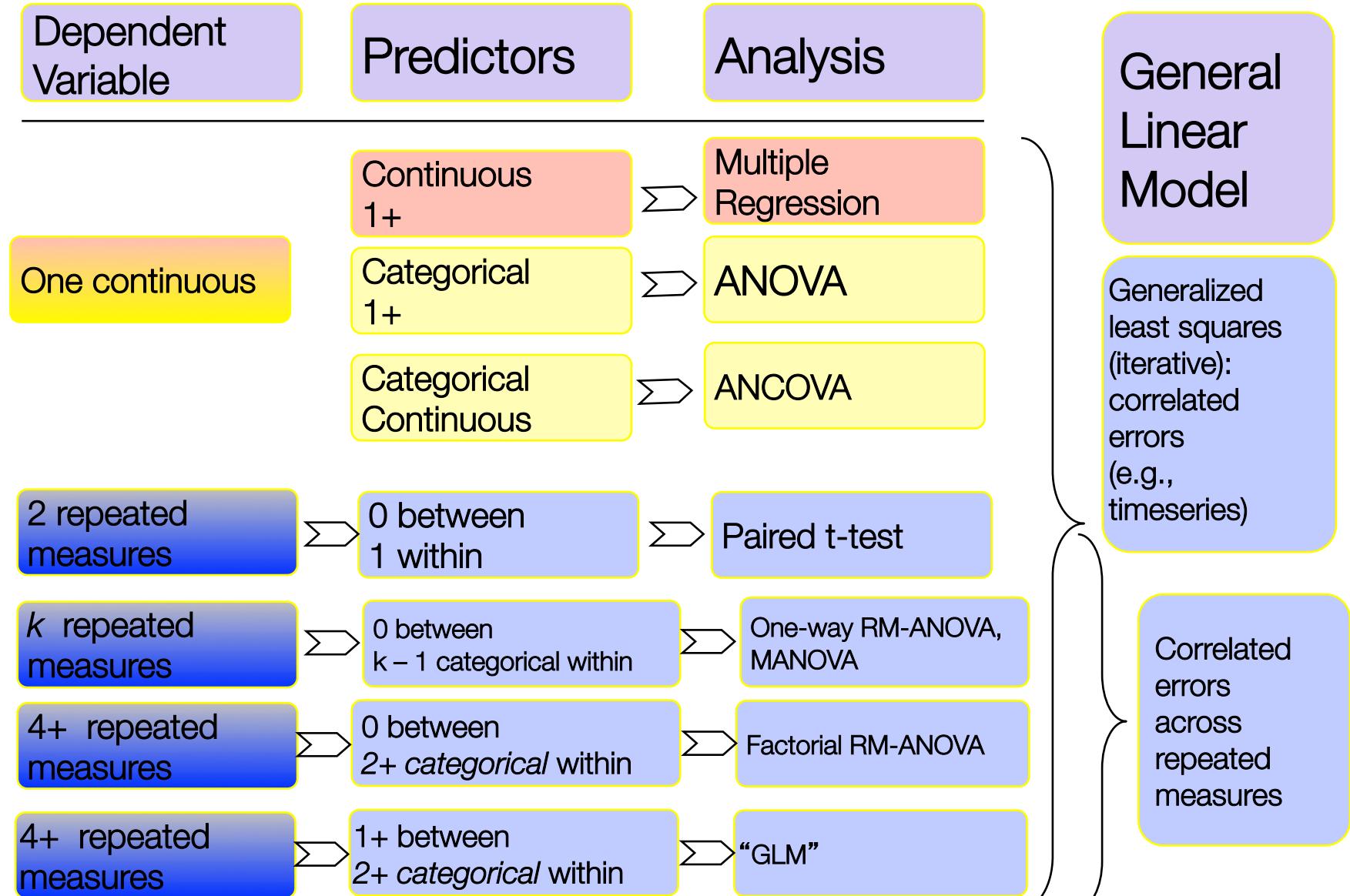
# General Linear Model

---

- The **general linear model** (GLM) approach treats the data as a linear combination of model functions (predictors) plus noise (error).
- The model functions are assumed to have **known** shapes (e.g., straight line, or known curve), but their amplitudes are **unknown** and need to be estimated.
- The GLM framework encompasses many of the commonly used techniques in fMRI data analysis (and data analysis more generally).



# The GLM Family



# The structural model for the GLM

Simple or multiple regression, t-tests, ANOVA, ANCOVA

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 & X_{11} & \cdots & X_{1p} \\ 1 & X_{21} & \cdots & X_{2p} \\ \vdots & \vdots & & \vdots \\ 1 & X_{np} & \cdots & X_{np} \end{bmatrix} \times \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

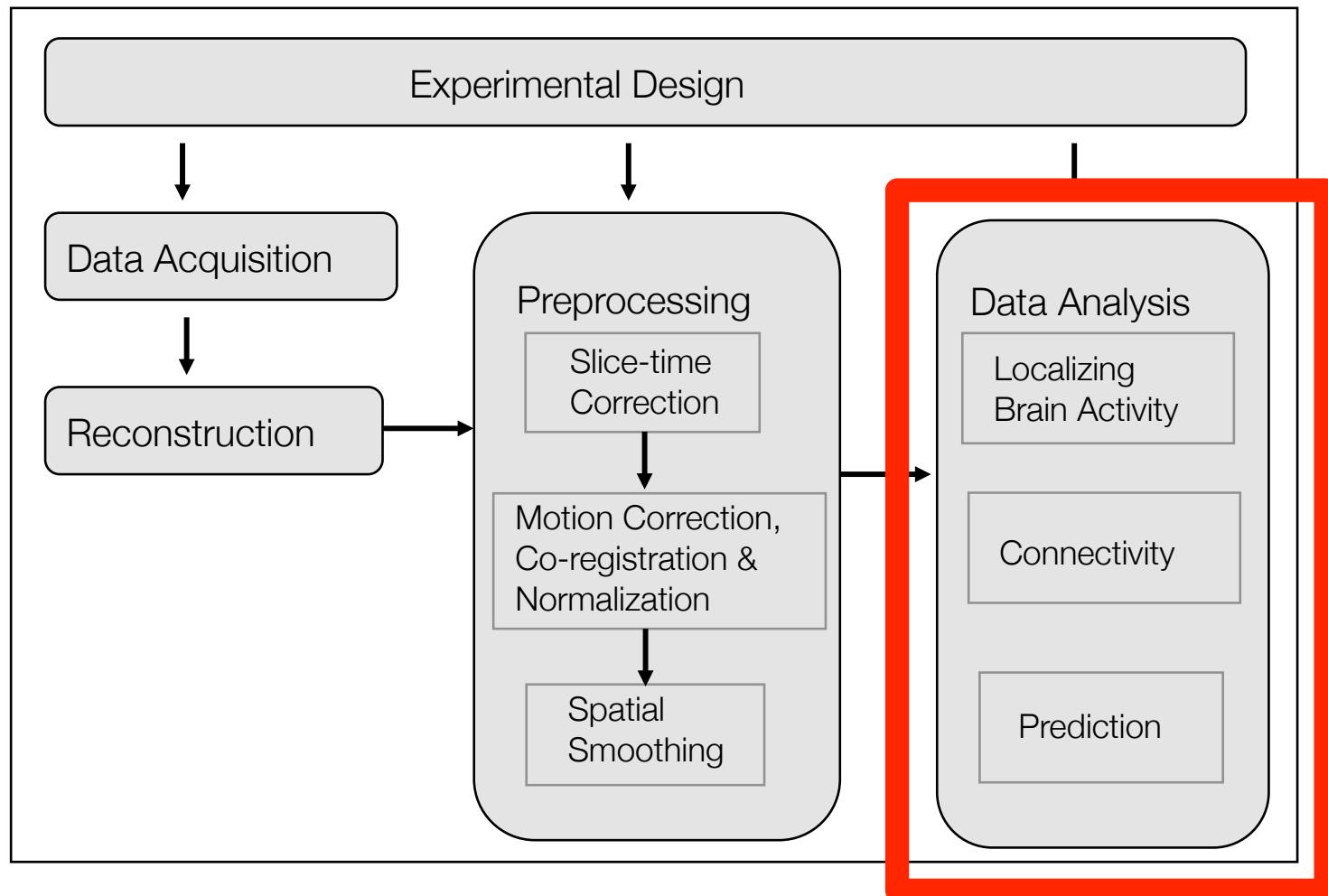
Observed Data

Design matrix

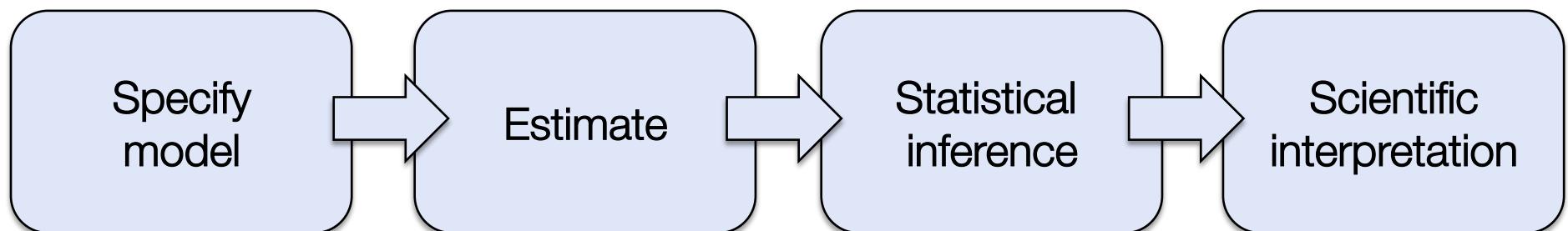
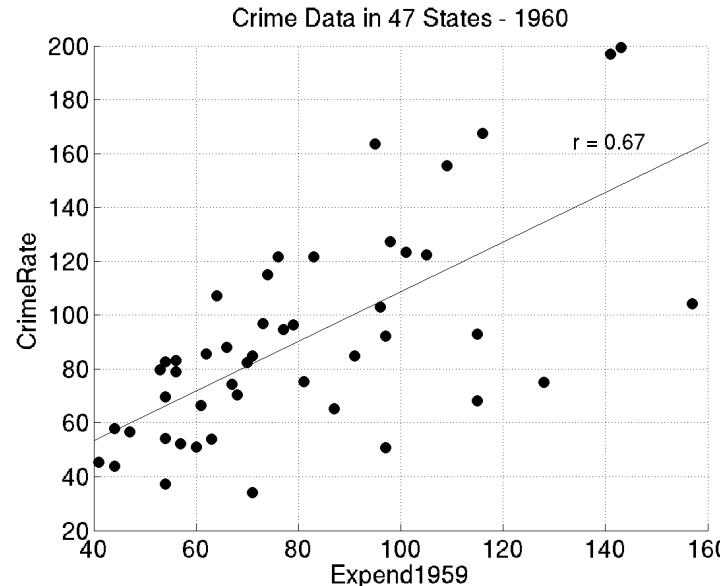
Model parameters

Residuals

# Data Processing Pipeline



# Simple regression: A simple linear model with one predictor



Specify  
model

Estimate

Statistical  
inference

Scientific  
interpretation

**Simplification:**  
“linear relationship”

**Find slope,  
intercept**

**Test slope:  
P-value**

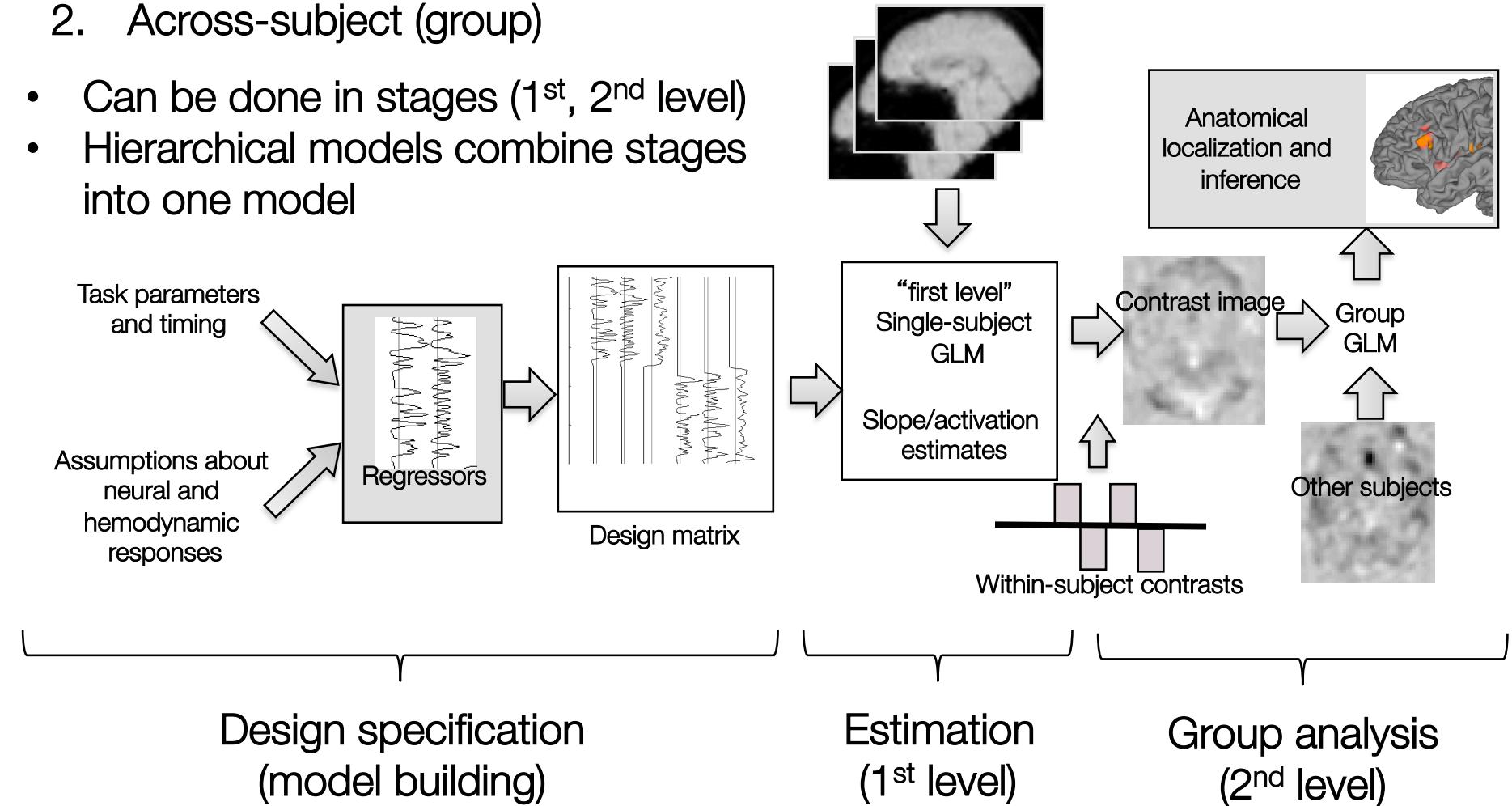
**Meaning of  
relationship?**



# Overview of the GLM analysis process

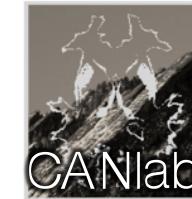
Typically a two-level hierarchical analysis

1. Within-subject (individual)
  2. Across-subject (group)
- Can be done in stages (1<sup>st</sup>, 2<sup>nd</sup> level)
  - Hierarchical models combine stages into one model





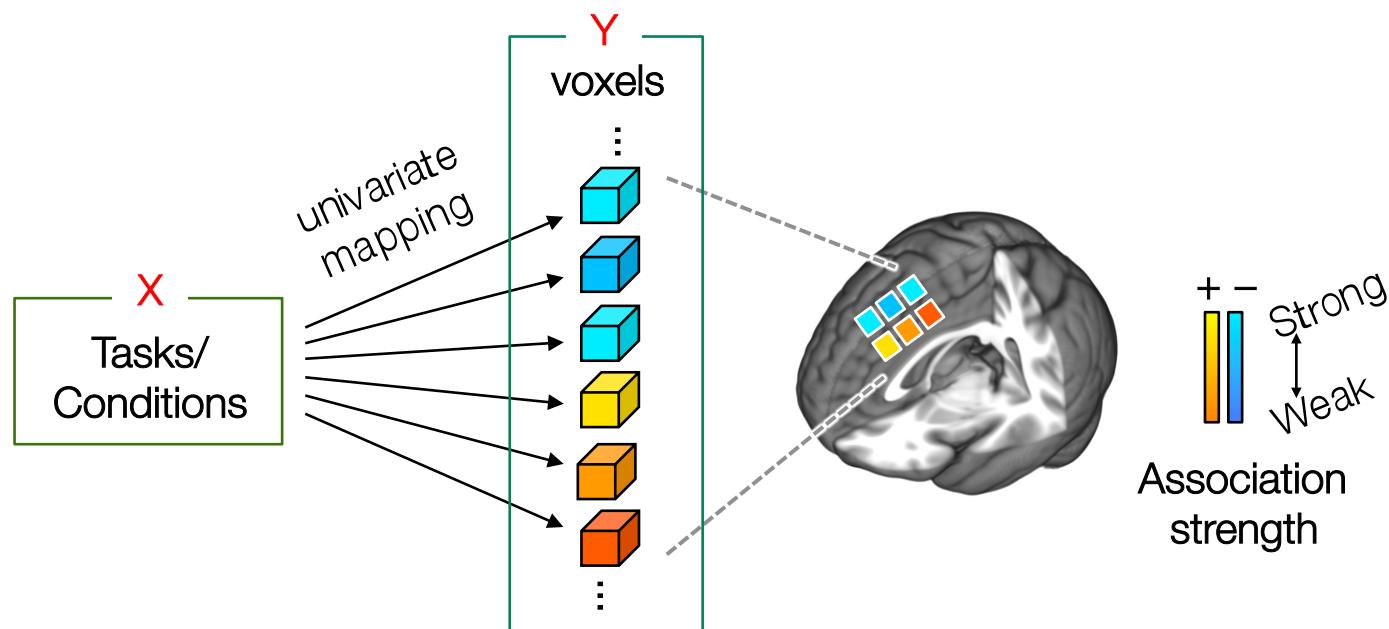
University of Colorado **Boulder**



# REGRESSION IN FMRI

# Mass Univariate Approach

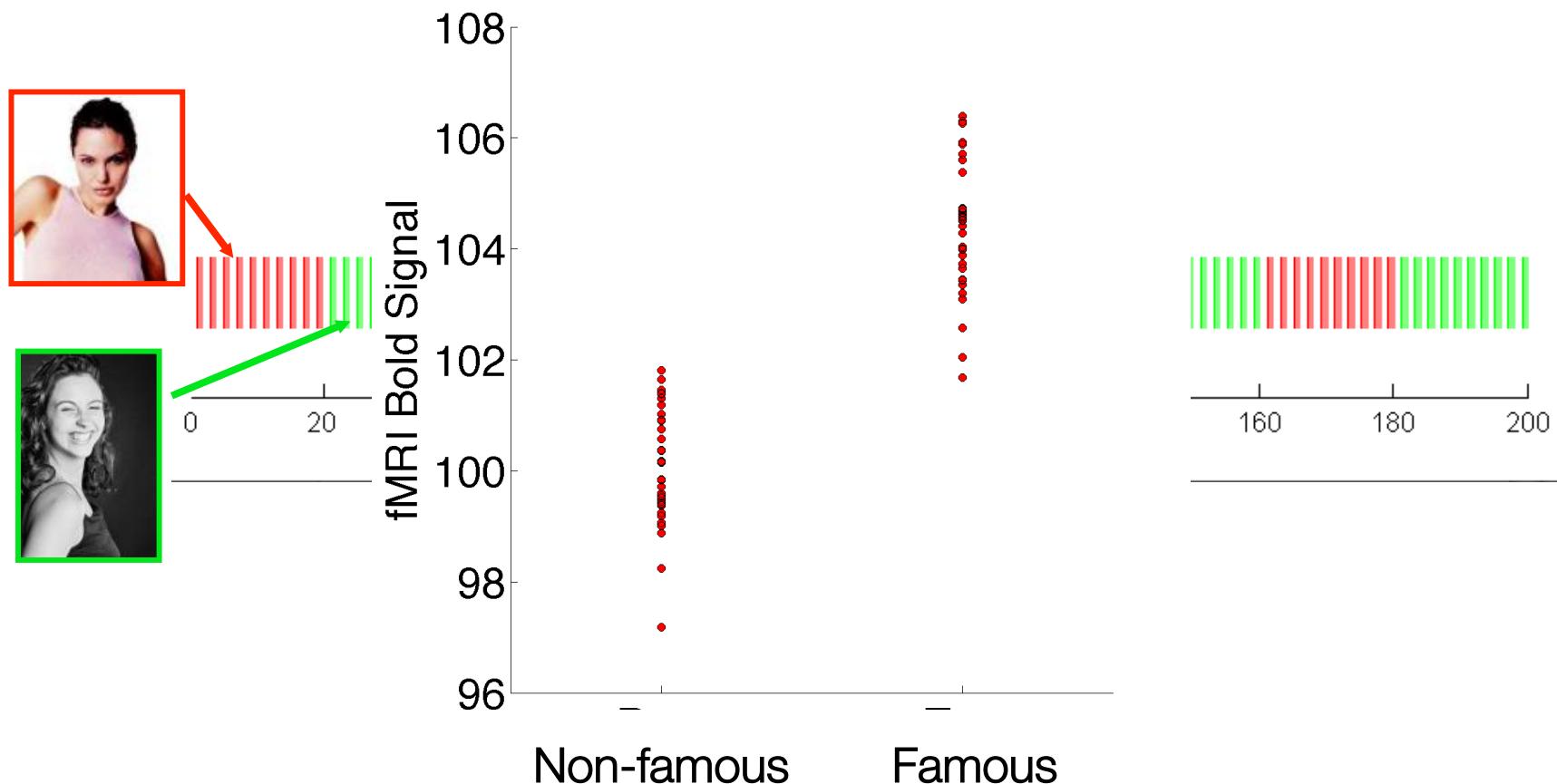
- Typically analysis is performed by constructing a separate model for each voxel
  - Brain activity in one voxel is the outcome ( $Y$ )
  - Stimulus, task, and/or behavioral variables are the predictors ( $X$ )
  - ‘Mass univariate approach’: Assumes voxels are independent, each its own separate test



Woo & Wager 2015; Image credit: Wani Woo

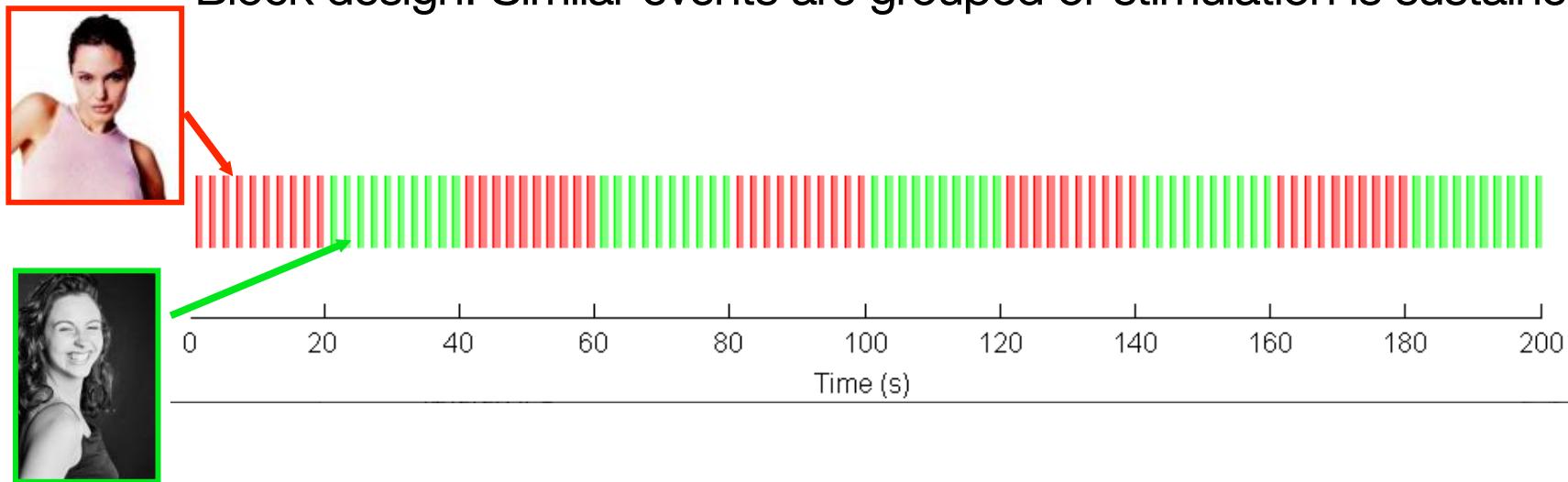
# First-level GLM: Single-voxel, single-subject

- Consider an experiment of alternating blocks of viewing famous and non-famous faces

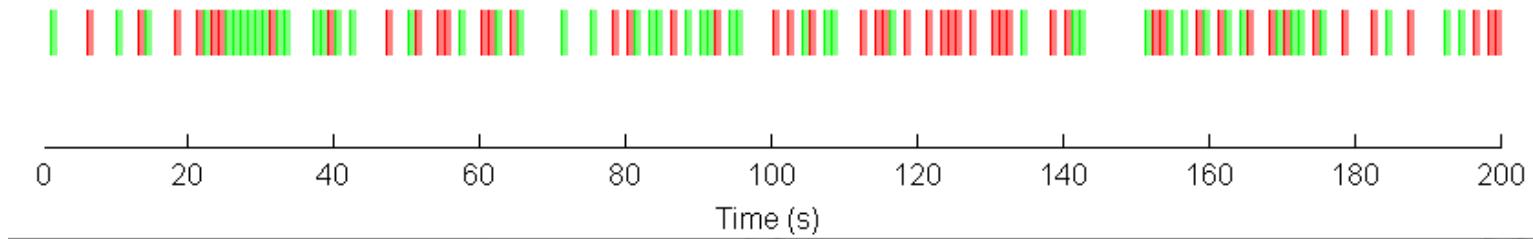


# First-level GLM: Blocked and event-related designs

- Block design: Similar events are grouped or stimulation is sustained

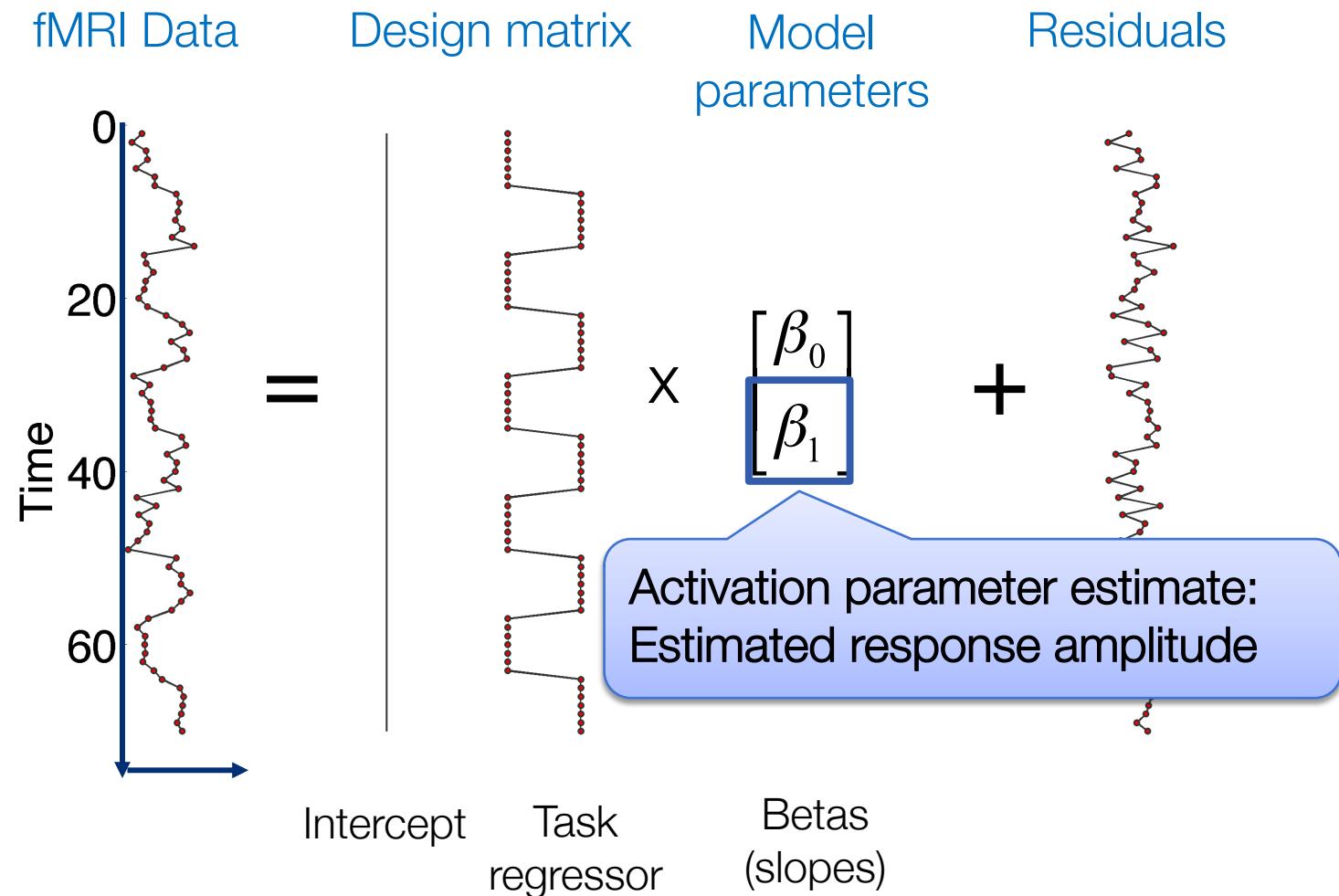


Event-related design: Brief events of different types are intermixed



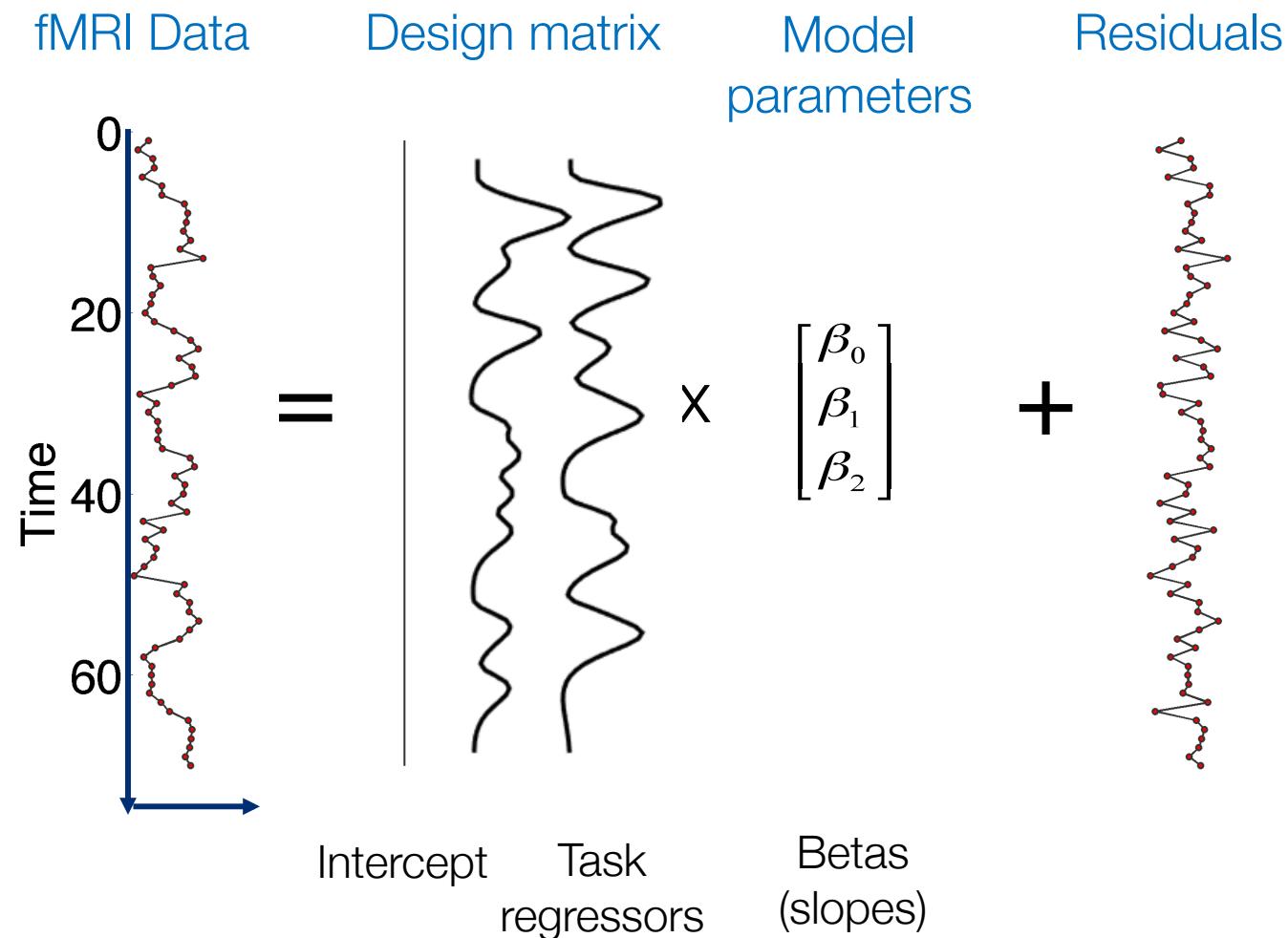
# First-level GLM: Single-voxel, single-subject

- A basic design matrix with one predictor (regressor) of interest (e.g., Task – Control, Famous vs. Non-famous) :



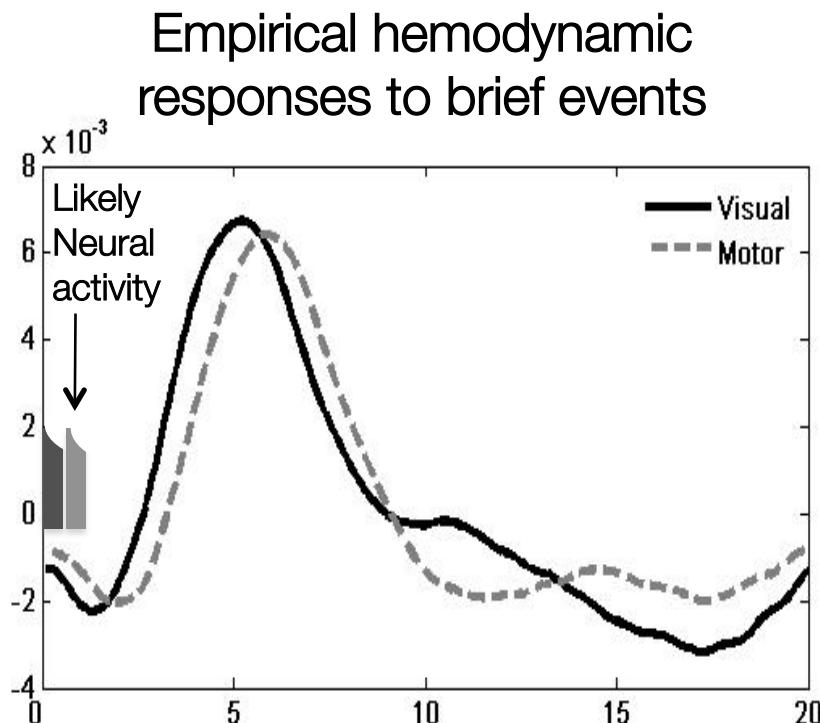
# First-level GLM: Single-voxel, single-subject

- A basic design matrix with two predictors (e.g., Famous and Non-famous) :



# Accounting for hemodynamic delay

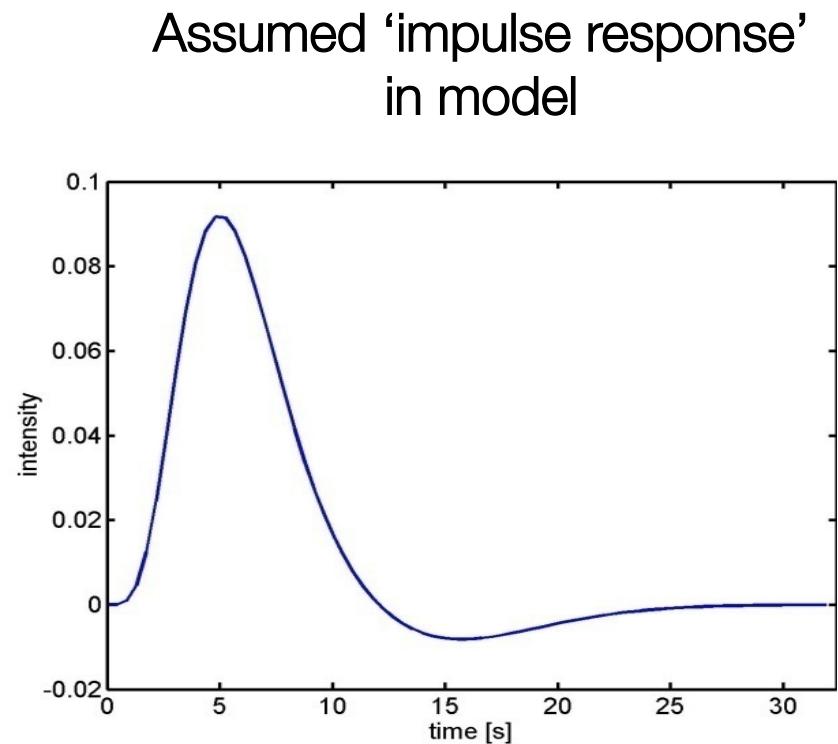
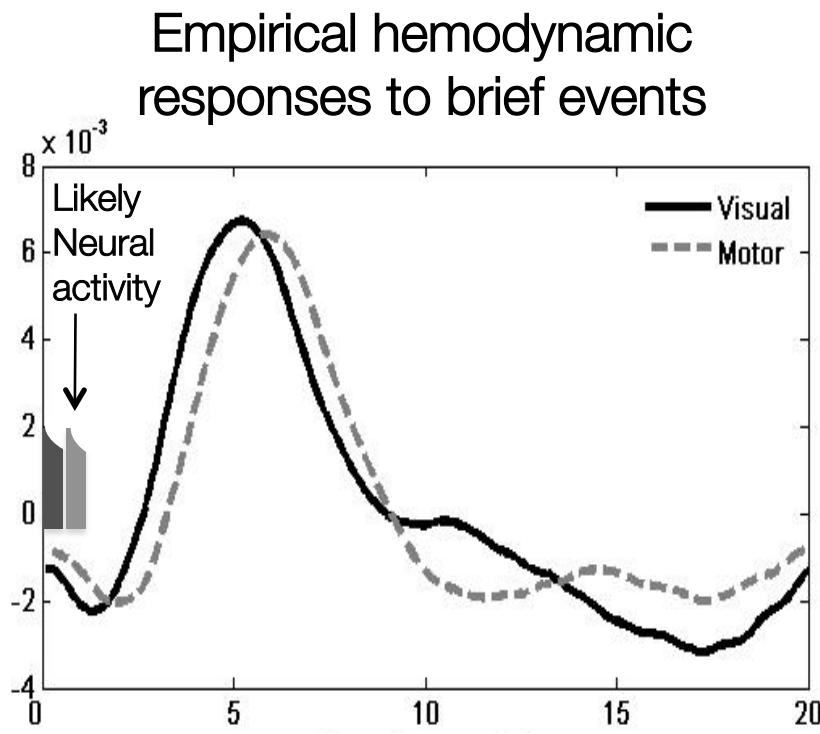
- BOLD responses are delayed and dispersed relative to neural activity



- Function of blood oxygenation, flow, volume (Buxton et al, 1998)
- Peaks at 4-6s poststimulus; baseline after 20-30s
- Initial undershoot (Malonek & Grinvald, 1996)
- Similar across brain regions, but not always!

# Accounting for hemodynamic delay

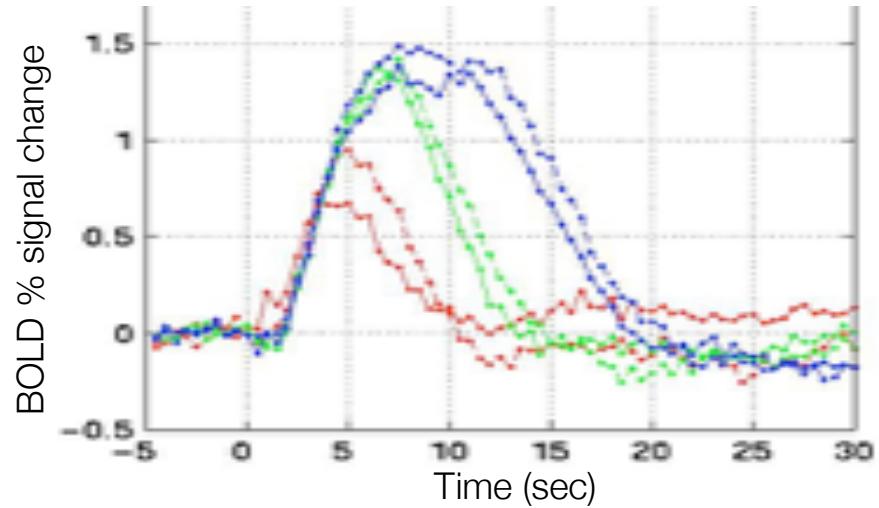
- BOLD responses are delayed and dispersed relative to neural activity



Common model: Fixed linear combination of 2 gamma functions

# Building GLM Predictors: LTI system

How to turn assumed neural responses into a predictor in a GLM model?



- Solution: Assume a linear time invariant (LTI) system.
  - Here the neuronal activity acts as the input or impulse and the HRF acts as the impulse response function.
  - Single solution for brief neural events or sustained epochs.
- The fMRI signal at time  $t$ ,  $x(t)$ , is modeled as the convolution of a stimulus function  $v(t)$  and the hemodynamic response  $h(t)$ , that is,

$$x(t) = (v * h)(t)$$

# Building GLM Predictors: LTI system

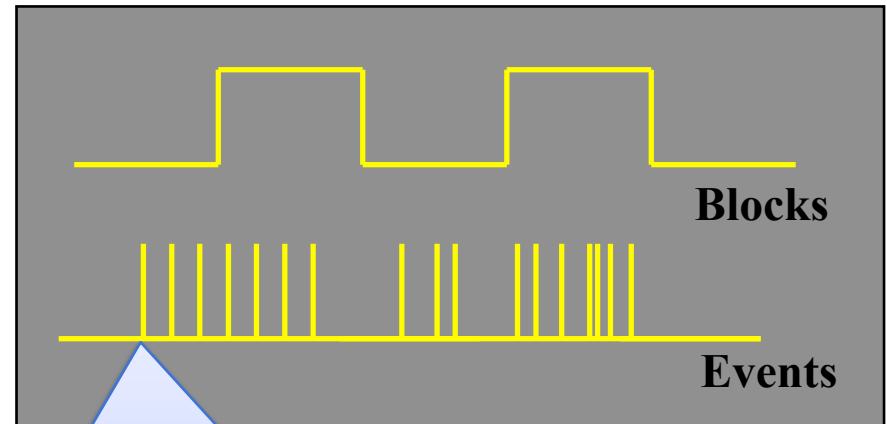
## Linear Time-Invariant system

LTI specified solely by

- ⌚ Stimulus function of experiment

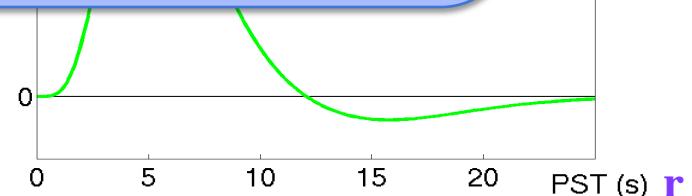
...Convolved with

- ⌚ Hemodynamic Response Function (HRF), assuming instantaneous impulse



Linear: Same HRF for each event no matter what came before

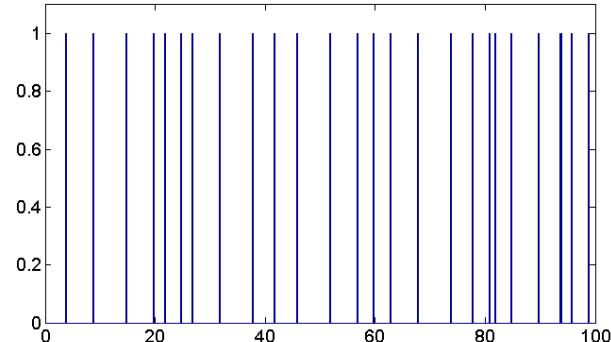
Time-invariant: Same response across time



# Building GLM Predictors: Convolution

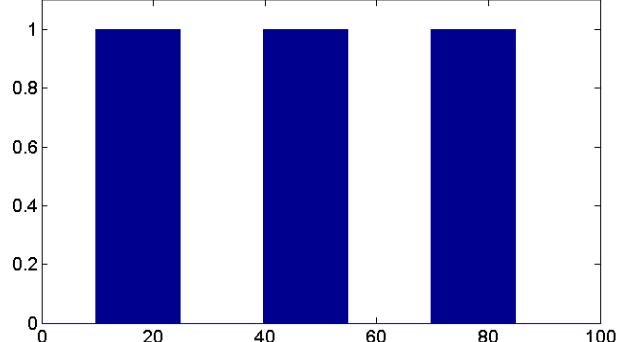
Neural response  
Function  
(assumed)

Event-Related Design



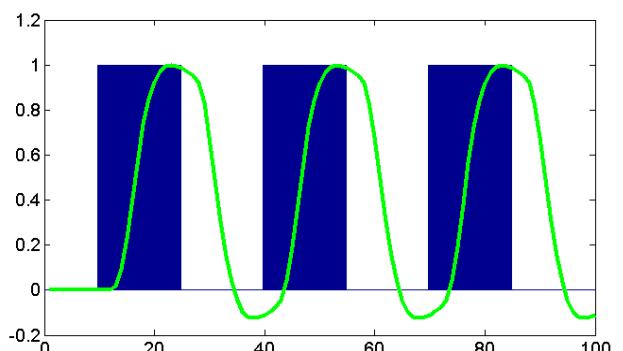
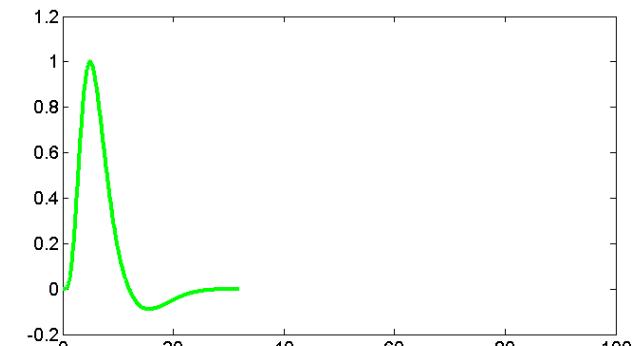
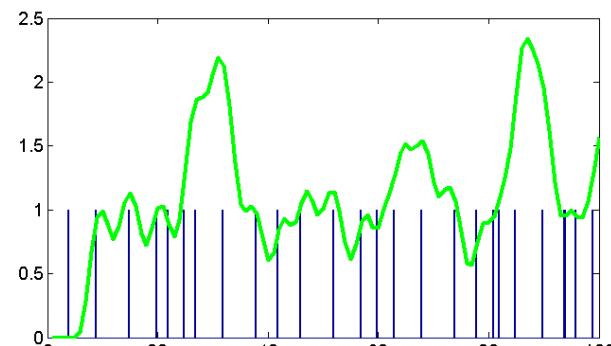
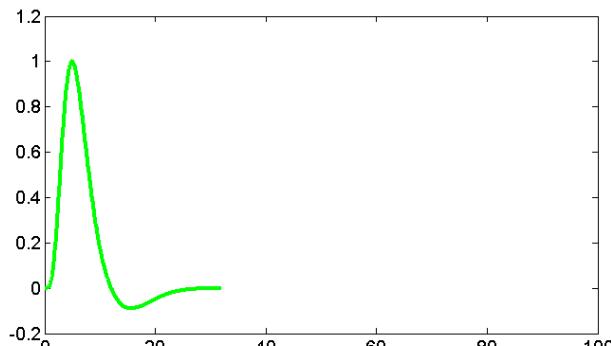
Hemodynamic  
Response  
Function

Block or epoch Design

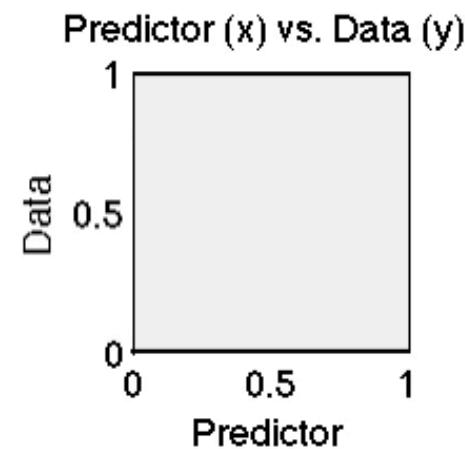


Predicted  
Response after  
convolution:

Predictor in GLM



# Regression in fMRI: One predictor



Copyright 2008 Tor Wager



[http://psych.colorado.edu/  
~tor](http://psych.colorado.edu/~tor)

# Model building for multiple predictors: Single-subject, single voxel

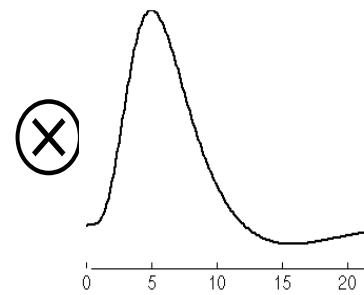


## Indicator functions

(onsets)



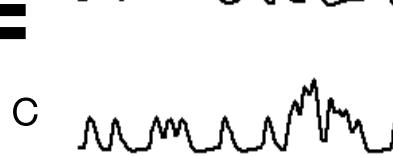
## Assumed HRF (Basis function)



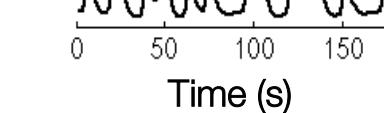
## Design Matrix ( $X$ )



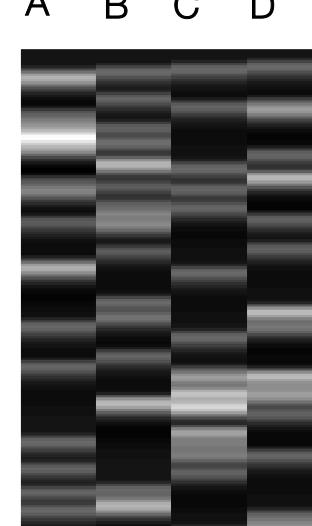
B



D



## Design Matrix ( $\mathbf{X}$ )



Time

# End of Module



@fMRIstats