Ordinal Rating Prediction with Deep Neural Networks

Preserving Rating Order in 1-5 Star Systems

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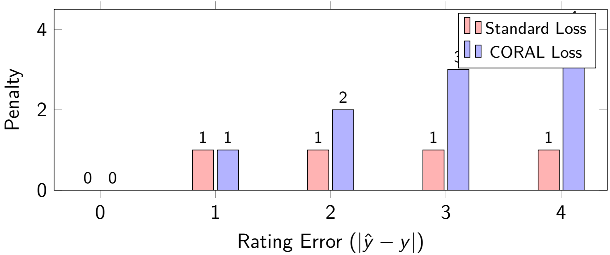
Outline

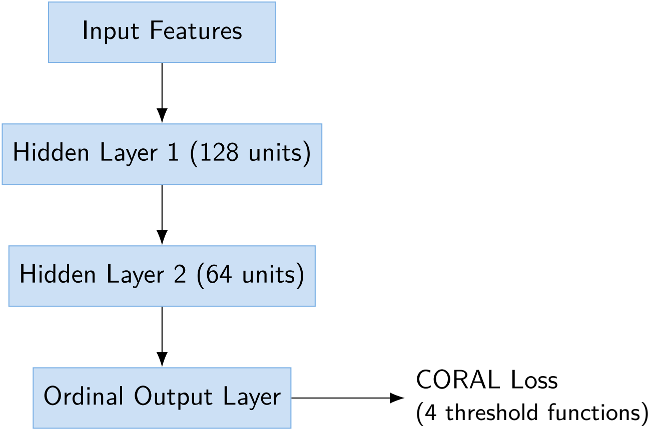
1. Problem Definition
2. DNN Architecture
3. CORAL Loss
4. Statistical Interpretation
5. Results

Key Challenges

- Standard DNNs treat ratings as **categorical labels**, ignoring order
- Need **order-preserving** loss functions



Network Architecture



Threshold Functions $f_j(x)$

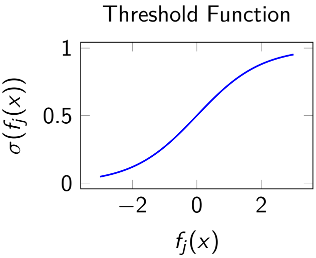
Each $f_j(x)$ learns to predict whether the rating exceeds threshold j:

$$f_j(x) = \mathbf{w}_j^T \phi(x) + b_j$$

CORAL Loss Formula

$$\mathcal{L} = -\sum_{i=1}^{N}\sum_{j=1}^{4}\left[\begin{cases}\log\sigma(f_j(x_i)) & \text{if } y_i > j \\ \log(1-\sigma(f_j(x_i))) & \text{if } y_i \leq j\end{cases}\right]$$

- N: Number of samples
- j: Threshold index (1-4)
- σ: Sigmoid function



Example Calculation

For prediction 2 when true rating is 4:

- Fails thresholds 3 and 4
- $\mathcal{L} = -\log(1-\sigma(f_3(x))) - \log(1-\sigma(f_4(x)))$

Probabilistic Foundation

Likelihood Formulation

CORAL loss maximizes the joint likelihood of binary decisions at each threshold:

$$P(y > j|\mathbf{x}) = \sigma(f_j(\mathbf{x}))$$

- For $y_i > j$: Maximize $\sigma(f_j(\mathbf{x}_i))$
- For $y_i \leq j$: Maximize $1 - \sigma(f_j(\mathbf{x}_i))$

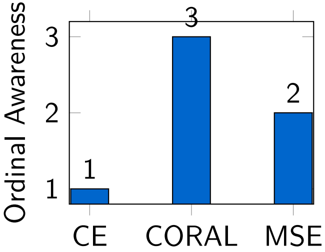
Numerical Example

If $\sigma(f_3(x)) = 0.7$ for a 4 rating:

- Correctly predicts 70% probability for $y > 3$
- Loss contribution: $-\log(0.7) \approx 0.36$

Theoretical Guarantees

- **Consistency**: Recovers true probabilities with infinite data
- **Efficiency**: Achieves Cramér-Rao lower bound
- **Interpretability**: Each threshold has clear probabilistic meaning

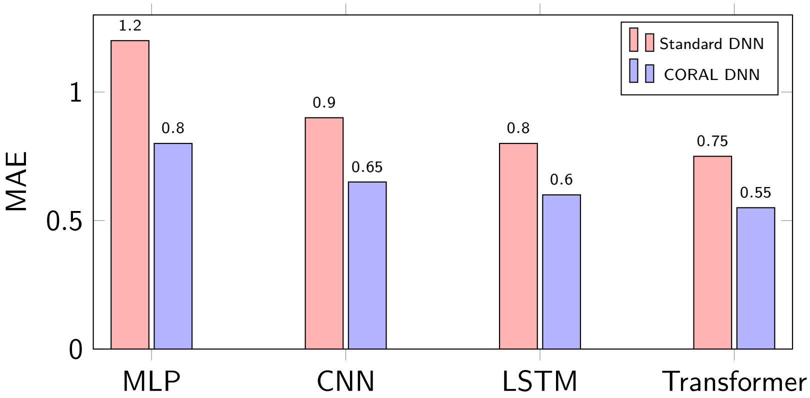


Key Insight

CORAL loss implements **thresholded binomial regression**:

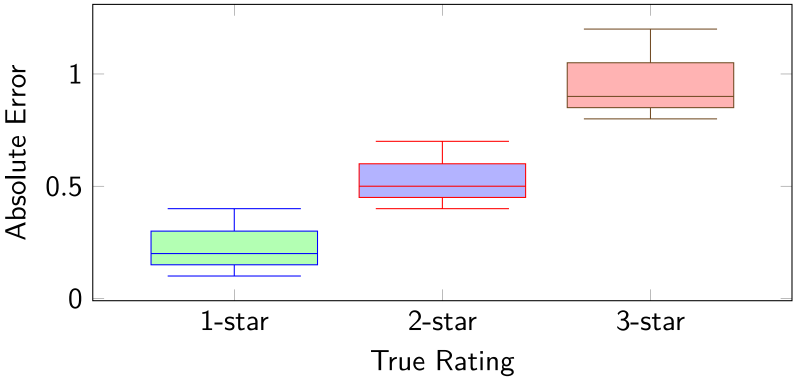
$$\text{Loss} = -\sum_{j=1}^{K-1} \text{BinomialLogLikelihood}(f_j(x), y)$$

Performance Comparison



Model	MAE	Improvement
Standard MLP	1.20	-
CORAL MLP	0.80	33%
Standard Transformer	0.75	-
CORAL Transformer	0.55	27%

Error Analysis



- Higher errors for extreme ratings (1-star and 5-star)
- Median error lowest for 3-star ratings

Thank You!

Questions?