

Orientation

Idea:

Closed Curve in 2D

Counterclockwise as positive direction, clockwise as negative direction.

If C is given by the vector function $\vec{r}(t)$, $a \leq t \leq b$ then the region D is always on the left as the point $\vec{r}(t)$ traverses C .

Normal of Curve in 2D

If $\vec{r}(t) = (x(t), y(t))$, then

Unit normal vector is $\vec{n}(t) = (\frac{y'(t)}{|\vec{r}'(t)|}, -\frac{x'(t)}{|\vec{r}'(t)|})$. (90° clockwise rotation)

The reason being, if C is described by \vec{r} and C is a closed curve, and \vec{r}_t is counterclockwise (see: [Orientation > Closed Curve in 2D](#)), then the normal vector will be pointing outward.

Surface

If S is a 2D region (surface) and \vec{n} is the unit normal vector of dS , then

$$\vec{n}dS = d\vec{S}$$

If the surface is a [Mapping > From 2D to 3D](#), then

$$\vec{n}dS = (\vec{r}_u \times \vec{r}_v)dudv$$

(see: [Flux > Surface Living in 3D](#) for example)

Curve Enclosing Open Surface?