Divergence

Idea:

Convert Vector field $\vec{F}(x,y,z) = (P,Q,R) : R^n \to R^n$ to a **number**.

$\nabla \cdot \vec{F} = \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \right)$

(see: Nabla)

Geometric Intuition

$\frac{\partial P}{\partial x} > 0$ and $\frac{\partial Q}{\partial y} > 0$ should contribute to an outward expansion.

$\frac{\partial P}{\partial x} < 0$ and $\frac{\partial Q}{\partial y} < 0$ should contribute to an inward contraction.

$\nabla \cdot \left(\nabla \times \vec{F} \right) = 0$

(see: Curl)

Clairaut's Theorem