# Flux

Idea:

The flux of a Vector field $\vec{F}$ through a surface $S$ is defined as

$$\iint_S \vec{F} \cdot \vec{n} \mathrm{d}S$$

where $\vec{n}$ is the unit normal vector of that surface

There's no 3D version of it, because 3D objects have infinite normal vectors.

# Geometric Intuition

The amount of the vector field flowing through the surface.

# Surface Living in 3D

Mapping > From 2D to 3D: Surface $S$ is given by $\vec{r}(u,v) =< x(u,v), y(u,v), z(u,v) >$

Unit normal is $\frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|}$

$$\iint_S \vec{F} \cdot \vec{n} \mathrm{d}S = \iint_S \vec{F} \cdot \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|} \mathrm{d}S$$

Also, $\mathrm{d}S = |\vec{r}_u \times \vec{r}_v| \mathrm{d}u\mathrm{d}v$, so

$$\iint_S \vec{F} \cdot \vec{n} \mathrm{d}S = \iint_D \vec{F} \cdot \frac{\vec{r}_u \times \vec{r}_v}{\left|\vec{r}_u \times \vec{r}_v\right|} (\left|\vec{r}_u \times \vec{r}_v\right| \mathrm{d}u\mathrm{d}v) = \iint_D \vec{F} \cdot (\vec{r}_u \times \vec{r}_v) \mathrm{d}u\mathrm{d}v$$