Curl

Idea:

Convert Vector field $\vec{F}(x,y,z) = (P,Q,R): R^3 o R^3$ to a 3D vector.

$$abla imes ec{F} = \left(rac{\partial R}{\partial y} - rac{\partial Q}{\partial z} \,, \quad rac{\partial P}{\partial z} - rac{\partial R}{\partial x} \,, \quad rac{\partial Q}{\partial x} - rac{\partial P}{\partial y}
ight)$$

(see: Nabla)

Geometric Intuition

Consider $\vec{F} = (P(x,y),Q(x,y),0)$

Then
$$abla imes ec{F} = (0,0,(rac{\partial Q}{\partial x} - rac{\partial P}{\partial y}))$$

 $rac{\partial Q}{\partial x}>0$ and $rac{\partial P}{\partial y}<0$ should contribute to a counterclockwise rotation.

 $\frac{\partial Q}{\partial x}<0$ and $\frac{\partial P}{\partial y}>0$ should contribute to a clockwise rotation.

$$abla imes ec{F} = 0 \leftrightarrow ec{F}$$
 is Conservative

$$abla imes ec{F} = 0 \leftarrow ec{F} ext{ is } ext{Conservative}$$

Clairaut's Theorem

$$abla imes ec{F} = 0
ightarrow ec{F}$$
 is Conservative