Mapping

Idea: Map

from nD to mD

The object after mapping is a nD object living in mD space

From 1D to 1D

$x = x(t)$

if $x(t) = t$, then $\mathrm{d}x = \mathrm{d}t$

Magic factor$= 1$

From 1D to 2D

Curve C is given by $\vec{r}(t) = (x(t), y(t))$

Tagent vector $\vec{r}'(t) = (x_t, y_t)$

$\mathrm{d}C = |\vec{r}'(t)|\mathrm{d}t = \sqrt{x_t^2 + y_t^2}\mathrm{d}t$

Magic factor$= |\vec{r}'(t)|$

From 1D to 3D

Curve C is given by $\vec{r}(t) = (x(t), y(t), z(t))$

Tagent vector $\vec{r}'(t) = (x_t, y_t, z_t)$

$\mathrm{d}C = |\vec{r}'(t)|\mathrm{d}t = \sqrt{x_t^2 + y_t^2 + z_t^2}\mathrm{d}t$

Magic factor$= |\vec{r}'(t)|$

From 2D to 2D

Jacobian Matrix

Polar Coordinates

$(r, \theta) \to (x, y) = (r\cos\theta, r\sin\theta)$

$\mathrm{d}x\mathrm{d}y = r\mathrm{d}r\mathrm{d}\theta$

Magic factor$= J(r, \theta) = r$

From 2D to 3D

The object after mapping is a 2D surface living in 3D

Surface S is given by $\vec{r}(u,v) =< x(u,v), y(u,v), z(u,v) >$

$\vec{r}_u = \frac{\partial x}{\partial u}\vec{i} + \frac{\partial y}{\partial u}\vec{j} + \frac{\partial z}{\partial u}\vec{k}$ is the tangent vector in output space as u changes with v fixed

$\vec{r}_v = \frac{\partial x}{\partial v}\vec{i} + \frac{\partial y}{\partial v}\vec{j} + \frac{\partial z}{\partial v}\vec{k}$ is the tangent vector in output space as v changes with u fixed

$\mathrm{d}S = |\vec{r}_u \times \vec{r}_v| \mathrm{d}u\mathrm{d}v$

Magic factor$= |\vec{r}_u \times \vec{r}_v|$

Spherical Coordinates with Radius R

$(\varphi, \theta) \to (x, y, z) = (R \sin\varphi \cos\theta, R \sin\varphi \sin\theta, R \cos\varphi)$

$\mathrm{d}S = R^2 \sin(\varphi) \mathrm{d}\varphi \mathrm{d}\theta$

Magic factor$= |\vec{r}_\varphi \times \vec{r}_\theta| = R^2 \sin(\varphi)$

From 3D to 3D

Jacobian Matrix

Cylindrical Coordinates

$(r, \theta, z) \to (x, y, z) = (r \cos\theta, r \sin\theta, z)$

$\mathrm{d}x\mathrm{d}y\mathrm{d}z = r \mathrm{d}r\mathrm{d}\theta\mathrm{d}z$

Magic factor$= r$

Spherical Coordinates

$(\rho, \varphi, \theta) \to (x, y, z) = (\rho \sin(\varphi) \cos(\theta), \rho \sin(\varphi) \sin(\theta), \rho \cos(\varphi))$

$\mathrm{d}x\mathrm{d}y\mathrm{d}z = \rho^2 \sin(\varphi) d\rho d\varphi d\theta$

Magic factor $= J(\rho, \varphi, \theta) = \rho^2 \sin(\varphi)$