Curl

Idea:

Convert Vector field $\vec{F}(x,y,z) = (P,Q,R) : R^3 \to R^3$ to a 3D vector.

$\nabla \times \vec{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \quad \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \quad \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)$

(see: Nabla)

Geometric Intuition

Consider $\vec{F} = (P(x,y), Q(x,y), 0)$

Then $\nabla \times \vec{F} = (0, 0, (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}))$

$\frac{\partial Q}{\partial x} > 0$ and $\frac{\partial P}{\partial y} < 0$ should contribute to a counterclockwise rotation.

$\frac{\partial Q}{\partial x} < 0$ and $\frac{\partial P}{\partial y} > 0$ should contribute to a clockwise rotation.

$\nabla \times \vec{F} = 0 \leftrightarrow \vec{F}$ is Conservative

$\nabla \times \vec{F} = 0 \leftarrow \vec{F}$ is Conservative

Clairaut's Theorem

$\nabla \times \vec{F} = 0 \rightarrow \vec{F}$ is Conservative