Stoke's Theorem

Idea: Fundamental Theorem of Calculus

$\vec{F}(x, y, z) =< P, Q, R >$

$$\int_C \vec{F} \cdot \mathrm{d}\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \vec{n} \mathrm{d}S = \iint_S (\nabla \times \vec{F}) \cdot \mathrm{d}\vec{S}$$

where C is the boundary curve of Open surface S.

$\vec{n}$ is the unit normal vector of $\mathrm{d}S$. (see: Orientation > Surface)

(see: Vector line integral, Flux)

The region couldn't be a 3D object, because they have infinite normal vectors.

Special case: Green's Theorem > Stoke's Theorem Representation

Geometric Intuition

Sum up rotations inside = rotation of the outside (boundary) (which is a Vector line integral)

(see: Curl for rotation inside, Vector line integral > Rotation for rotation outside, Fundamental Theorem of Calculus > Collecting Tangential component along Curve for special case of surface in 2D)

Special Case: S Being a Closed Surface

$$\int_C \vec{F} \cdot \mathrm{d}\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \vec{n} \mathrm{d}S = \iint_S (\nabla \times \vec{F}) \cdot \mathrm{d}\vec{S} = 0$$

(see: Orientation > Curve Enclosing Open Surface?)
Divide the Closed Surface (e.g. a ball) into top half and bottom half

For top half, the boundary curve have to be in counterclockwise direction, so that if Flux of Curl is positive, meaning the vector field on the boundary would be counterclockwise, then $\vec{F} \cdot \mathrm{d}\vec{r}$ is positive

For bottom half, the boundary curve have to be in clockwise direction, so that if Flux of Curl is positive, meaning the vector field on the boundary would be clockwise, then $\vec{F} \cdot \mathrm{d}\vec{r}$ is positive

Since two boundaries are in opposite direction, $\int_C \vec{F} \cdot \mathrm{d}\vec{r}$ would cancel.