Surface integral

Idea: Fundamental Theorem of Calculus

(Or Area integral)
The relationship between surface integrals and surface area is much the same as the relationship between line integrals and arc length.

Object in 2D

Mapping > From 2D to 2D: $(u, v) \rightarrow (x, y)$

Integrate a function over a 2D object in input space, object living in 2D output space

$$\iint_S f(x, y) \mathrm{d}x \mathrm{d}y = \iint_D f(x(u, v), y(u, v)) \left| \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial y}{\partial u} \frac{\partial x}{\partial v} \right| \mathrm{d}u \mathrm{d}v$$

but the object could be a line living in 2D output space?

Object in 3D

Mapping > From 2D to 3D: $\vec{r}(u, v) = x(u, v)\vec{i} + y(u, v)\vec{j} + z(u, v)\vec{k}$

Integrate a function over a 2D object in input space, object living in 3D output space

$$\iint_S f(x, y, z) \mathrm{d}S = \iint_D f\left(x(u, v), y(u, v), z(u, v)\right. \left|\vec{r}_u \times \vec{r}_v\right| \mathrm{d}u \mathrm{d}v$$