Scalar line Integral

Idea: Fundamental Theorem of Calculus

1D to 1D

Integrate a Scalar function over a 1D object in input space, object living in 1D output space

The object is a Mapping > From 1D to 1D

$$\int_a^b f(x)\mathrm{d}x = \int_{t_{start}=a}^{t_{end}=b} f(x(t))\mathrm{d}t$$

1D to 2D

Integrate a Scalar function over a 1D object in input space, object living in 2D output space

The object is a Mapping > From 1D to 2D, while the function is $f(x, y) : R^2 \to R$

integrate $f(x, y)$ along the curve C: integral **with respect to arc length**

$$\int_C f(C)\mathrm{d}C = \int_{t_{start}}^{t_{end}} f(C)\mathrm{d}C = \int_{t_{start}}^{t_{end}} f(x(t), y(t))\mathrm{d}C = \int_{t_{start}}^{t_{end}} f(x(t), y(t))\sqrt{x_t^2 + y_t^2}\mathrm{d}t$$

Integrate in "pieces"

(It is a random name.)

Integrate a Scalar function $f(x, y) : R^2 \to R$ **with respect to** x as you move along the curve C

$$\int_C f(x, y)\mathrm{d}x = \int_C f(x(t), y(t))\frac{\mathrm{d}x}{\mathrm{d}y}\mathrm{d}t$$

1D to 3D

The object is a Mapping > From 1D to 3D.

The line integral of scalar function $f(x, y) : R^3 \to R$ over C is

$$\int_{t_{start}}^{t_{end}} f(x(t), y(t), z(t))\sqrt{x_t^2 + y_t^2 + z_t^2}\mathrm{d}t$$