LibADMM: A Library of Alternating Direction Method of Multipliers for Compressed Sensing

Version 1.0

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https://github.com/canyilu/LibADMM

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TABLE 1: Applicability of the LibADMM package

$ \text{Sparse models } \\ \text{min}_{x} \ r(\mathbf{x}) \\ \text{s.i. } \mathbf{A}\mathbf{x} = \mathbf{b} \\ \text{min}_{x} \ r(\mathbf{x}) \\ \text{s.i. } \mathbf{A}\mathbf{x} = \mathbf{b} \\ \text{min}_{x} \ r(\mathbf{x}) \\ \text{s.i. } \mathbf{A}\mathbf{x} = \mathbf{b} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{\ \mathbf{x}\ _{1} + \lambda_{2}\ _{2}^{2}}{\ \mathbf{x}\ _{1} + \lambda_{2}\ _{2}^{2}} \\ \text{s.i. } \mathbf{A}\mathbf{x} = \mathbf{b} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{\ \mathbf{x}\ _{1} + \lambda_{2}\ _{2}^{2}}{\ \mathbf{x}\ _{1} + \lambda_{2}\ _{2}^{2}} \\ \text{s.i. } \mathbf{A}\mathbf{x} = \mathbf{b} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{\ \mathbf{x}\ _{1} + \lambda_{2}\ _{2}^{2}}{\ \mathbf{x}\ _{1} + \lambda_{2}\ _{2}^{2}} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{x}\ _{1}^{2} \\ \text{s.i. } \mathbf{A}\mathbf{x} + \mathbf{e} = \mathbf{b} \\ \text{l.} \ (\mathbf{e}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{l.} \ (\mathbf{e}) = \ \mathbf{e}\ _{1} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(\mathbf{x}) = \frac{1}{2} \ \mathbf{e}\ _{2}^{2} \\ \text{min}_{x} \ r(x$	Model	Problem		Function	Description and Reference
$Sparse \begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	Sparse		$r(\mathbf{x}) = \ \mathbf{x}\ _1$	11	ℓ_1 [14]
$Sparse \begin{tabular}{ c c c c c c c c c c c c c c c c c c c$		$\min_{\mathbf{x}} r(\mathbf{x})$	$r(\mathbf{x}) = \sum_{g \in \mathcal{G}} \ \mathbf{x}_g\ _2$	groupl1	Group Lasso [17]
$\begin{aligned} &\text{Sparse} \\ &\text{models} \\ &\text{models} \\ &\text{models} \\ &\frac{r(\mathbf{x}) = \left\ \mathbf{A} \mathrm{Diag}(\mathbf{x})\right\ _{*}}{r(\mathbf{x}) = \frac{1}{2}\ \mathbf{x}\ _{\mathrm{Sp}}^{2}} & \text{ksupport} & k \mathrm{support} \\ &\text{ksupport} & k \mathrm{support} \mathrm{morm} [6] \\ &\text{min}_{\mathbf{x},\mathbf{e}} l(\mathbf{e}) + \lambda r(\mathbf{x}) \\ &\text{s.t.} \mathbf{A} \mathbf{x} + \mathbf{e} = \mathbf{b} \end{aligned} & l(\mathbf{e}) = \ \mathbf{e}\ _{1} \\ &\frac{11R}{\mathrm{group11R}} & \mathrm{Reg. Group Lasso} \\ &\frac{\mathrm{elasticnetR}}{\mathrm{elasticnetR}} & \mathrm{Reg. Fused Lasso} \\ &\text{ksupportR} & \mathrm{Reg. Fused Lasso} \\ &\text{ksupport Rome} \\ &\frac{\mathrm{Reg. Fused Lasso}}{\mathrm{ksupport norm}} \end{aligned} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{L}\ _{+} + \lambda l(\mathbf{S}), \mathrm{s.t.} \mathbf{X} = \mathbf{L} + \mathbf{S}}{\mathrm{min}_{\mathbf{X}} \ \mathbf{X}\ _{+} \mathrm{s.t.} \mathbf{F}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{L}\ _{+} + \lambda l(\mathbf{E}), \mathrm{s.t.} \mathbf{A} = \mathbf{BX} + \mathbf{E}}{\mathrm{min}_{\mathbf{X}} \ \mathbf{L}\ _{+} \mathrm{s.t.} \mathbf{K}} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{Z}\ _{+} + \lambda l(\mathbf{E}), \mathrm{s.t.} \mathbf{A} = \mathbf{BX} + \mathbf{E}}{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{Z}\ _{+} + \lambda l(\mathbf{E}), \mathrm{s.t.} \mathbf{A} = \mathbf{BX} + \mathbf{E}} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} + \lambda l(\mathbf{E}), \mathrm{s.t.} \mathbf{A} = \mathbf{BX} + \mathbf{E}}{\mathrm{min}_{\mathbf{L},\mathbf{S},\mathbf{L}} \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} \\ &$			$r(\mathbf{x}) = \ \mathbf{x}\ _1 + \lambda_2 \ \mathbf{x}\ _2^2$	elasticnet	Elastic net [19]
$\begin{aligned} &\text{Sparse} \\ &\text{models} \\ &\text{models} \\ &\text{models} \\ &\frac{r(\mathbf{x}) = \left\ \mathbf{A} \mathrm{Diag}(\mathbf{x})\right\ _{*}}{r(\mathbf{x}) = \frac{1}{2}\ \mathbf{x}\ _{\mathrm{Sp}}^{2}} & \text{ksupport} & k \mathrm{support} \\ &\text{ksupport} & k \mathrm{support} \mathrm{morm} [6] \\ &\text{min}_{\mathbf{x},\mathbf{e}} l(\mathbf{e}) + \lambda r(\mathbf{x}) \\ &\text{s.t.} \mathbf{A} \mathbf{x} + \mathbf{e} = \mathbf{b} \end{aligned} & l(\mathbf{e}) = \ \mathbf{e}\ _{1} \\ &\frac{11R}{\mathrm{group11R}} & \mathrm{Reg. Group Lasso} \\ &\frac{\mathrm{elasticnetR}}{\mathrm{elasticnetR}} & \mathrm{Reg. Fused Lasso} \\ &\text{ksupportR} & \mathrm{Reg. Fused Lasso} \\ &\text{ksupport Rome} \\ &\frac{\mathrm{Reg. Fused Lasso}}{\mathrm{ksupport norm}} \end{aligned} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{L}\ _{+} + \lambda l(\mathbf{S}), \mathrm{s.t.} \mathbf{X} = \mathbf{L} + \mathbf{S}}{\mathrm{min}_{\mathbf{X}} \ \mathbf{X}\ _{+} \mathrm{s.t.} \mathbf{F}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{L}\ _{+} + \lambda l(\mathbf{E}), \mathrm{s.t.} \mathbf{A} = \mathbf{BX} + \mathbf{E}}{\mathrm{min}_{\mathbf{X}} \ \mathbf{L}\ _{+} \mathrm{s.t.} \mathbf{K}} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{Z}\ _{+} + \lambda l(\mathbf{E}), \mathrm{s.t.} \mathbf{A} = \mathbf{BX} + \mathbf{E}}{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{Z}\ _{+} + \lambda l(\mathbf{E}), \mathrm{s.t.} \mathbf{A} = \mathbf{BX} + \mathbf{E}} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} + \lambda l(\mathbf{E}), \mathrm{s.t.} \mathbf{A} = \mathbf{BX} + \mathbf{E}}{\mathrm{min}_{\mathbf{L},\mathbf{S},\mathbf{L}} \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}} \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} + \lambda \ \mathbf{Z}\ _{+} \\ &\frac{\mathrm{min}_{\mathbf{L},\mathbf{S}, \ \mathbf{Z}\ _{+} \\ &$		s.t. $\mathbf{A}\mathbf{x} = \mathbf{b}$	$r(\mathbf{x}) = \ \mathbf{x}\ _1 + \lambda_2 \sum_{i=2}^{p} x_i - x_{i-1} $	fusedl1	Fused Lasso [15]
models $ \begin{aligned} & \underset{\text{min}_{\mathbf{x},\mathbf{e}}}{\text{min}_{\mathbf{x},\mathbf{e}}} l(\mathbf{e}) + \lambda r(\mathbf{x}) \\ & \underset{\text{s.t.}}{l} l(\mathbf{e}) = \ \mathbf{e}\ _1 \\ & \underset{\text{s.t.}}{\text{s.t.}} \mathbf{A} \mathbf{x} + \mathbf{e} = \mathbf{b} \end{aligned} \begin{aligned} & \underset{\text{l.e.}}{l} l(\mathbf{e}) = \ \mathbf{e}\ _1 \\ & \underset{\text{s.t.}}{\text{group11R}} \end{aligned} $				tracelasso	Trace Lasso [12]
min_{\mathbf{x},\mathbf{e}} \ l(\mathbf{e}) + \lambda r(\mathbf{x}) \\ s.t. \ \mathbf{A}\mathbf{x} + \mathbf{e} = \mathbf{b} \\ l(\mathbf{e}) = \frac{1}{2} \ \mathbf{e}\ _2^2 \\ \frac{group11R}{stable plant} \ \frac{group11R}{Reg.} \ \frac{group1asso}{Reg.} \ \frac{Reg. Group Lasso}{Reg. Elastic net} \\ \frac{fused11R}{tracelassoR} \ \frac{Reg. fused Lasso}{Reg. frace Lasso} \\ \frac{min_{\mathbf{L},\mathbf{S}}}{Reg.} \ \frac{\ \mathbf{L}\ _* + \lambda l(\mathbf{S}), \ s.t. \ \mathbf{X} = \mathbf{L} + \mathbf{S}}{Reg. \ MN} \\ \frac{min_{\mathbf{X}} \ \mathbf{X}\ _*, \ s.t. \mathcal{P}_{\Omega}(\mathbf{X}) = \mathcal{P}_{\Omega}(\mathbf{M})}{Reg. \ MN} \\ \frac{min_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}}{nin_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}} \\ \frac{min_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}}{nin_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}} \\ \frac{min_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}}{nin_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}} \\ \frac{min_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}}{nin_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}} \\ \frac{min_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}}{nin_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}} \\ \frac{min_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}}{nin_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}} \\ \frac{min_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}}{nin_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \ s.t. \ \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}} \\ \frac{group11R}{tracelassoR} \ \frac{Reg. low-rank}{Reg. low-rank tensor completion lossed on} \\ \frac{low-rank}{tin_{\mathbf{X},\mathbf{E}} \ \mathbf{E}			$r(\mathbf{x}) = \frac{1}{2} \ \mathbf{x}\ _{ksp}^2$	ksupport	k support norm [6]
$ \begin{aligned} & \min_{\mathbf{x},\mathbf{e}} \ l(\mathbf{e}) + \lambda r(\mathbf{x}) & l(\mathbf{e}) = \ \mathbf{e}\ _1 \\ & \text{s.t. } \mathbf{A} \mathbf{x} + \mathbf{e} = \mathbf{b} \end{aligned} \end{aligned}$	models		•	11R	Reg. ℓ_1
s.t. $\mathbf{A}\mathbf{x} + \mathbf{e} = \mathbf{b}$	models			groupl1R	Reg. Group Lasso
$ \begin{array}{c} \text{S.f. } \mathbf{A} \mathbf{x} + \mathbf{e} = \mathbf{b} & l(\mathbf{e}) = \frac{2}{2} \ \mathbf{e} \ _2^2 \\ & \text{tracelassoR} & \text{Reg. Trace Lasso} \\ & \text{ksupportR} & \text{Reg. } k \text{ support norm} \\ & \text{Reg. } k \text{ support norm} \\ & \text{Reg. } k \text{ support norm} \\ & \text{min_{\mathbf{L},\mathbf{S}}} \ \mathbf{L} \ _* + \lambda l(\mathbf{S}), \text{ s.t. } \mathbf{X} = \mathbf{L} + \mathbf{S} \\ & \text{min_{\mathbf{X}}} \ \mathbf{X} \ _* + \lambda k(\mathbf{E}), \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M} \\ & \text{min_{\mathbf{X},\mathbf{L}}} \ \mathbf{X} \ _* + \lambda l(\mathbf{E}), \text{ s.t. } \mathbf{A} = \mathbf{B} \mathbf{X} + \mathbf{E} \\ & \text{min_{\mathbf{X},\mathbf{L}}} \ \mathbf{X} \ _* + \lambda l(\mathbf{E}), \text{ s.t. } \mathbf{A} = \mathbf{B} \mathbf{X} + \mathbf{E} \\ & \text{min_{\mathbf{X},\mathbf{L}}} \ \mathbf{Z} \ _* + \lambda \ \mathbf{X} \ _* + \lambda l(\mathbf{E}) \\ & \text{s.t. } \mathbf{X} + \mathbf{L} \mathbf{X} - \mathbf{X} = \mathbf{E} \\ & \text{min_{\mathbf{X},\mathbf{L}}} \ \mathbf{X} \ _* + \lambda \mathbf{X} \ \mathbf{X} \ _* + \lambda l(\mathbf{E}) \\ & \text{s.t. } \mathbf{X} + \mathbf{L} \mathbf{X} - \mathbf{X} = \mathbf{E} \\ & \text{min_{\mathbf{L},\mathbf{S},\mathbf{L}}} \ \mathbf{X} \ _* + \lambda \mathbf{X} \ \mathbf{X} \ _* + \lambda l(\mathbf{E}) \\ & \text{s.t. } \mathbf{A} = \mathbf{B} \mathbf{X} + \mathbf{E} \\ & \text{min_{\mathbf{L},\mathbf{S},\mathbf{L}}} \ \mathbf{X} \ _* + \lambda \mathbf{X} \ \mathbf{X} \ _* + \lambda l(\mathbf{E}) \\ & \text{s.t. } \mathbf{A} = \mathbf{B} \mathbf{X} + \mathbf{E} \\ & \text{min_{\mathbf{L},\mathbf{S},\mathbf{L}}} \ \mathbf{X} \ _* + \lambda \mathbf{X} \ \mathbf{X} \ _* + \lambda l(\mathbf{E}) \\ & \text{s.t. } \mathbf{X}_{1} = \mathbf{L} \mathbf{X}_{1} + \mathbf{L}_{1} + \lambda \mathbf{X} \ \mathbf{X} \ _* + \lambda l(\mathbf{E}) \\ & \text{s.t. } \mathbf{X}_{1} = \mathbf{L} \mathbf{X}_{1} + \mathbf{L}_{1} + \lambda \mathbf{X} \ \mathbf{X} \ _* + \lambda l(\mathbf{E}) \\ & \text{s.t. } \mathbf{X}_{1} = \mathbf{L} \mathbf{X}_{1} + \mathbf{L}_{1} + \lambda \mathbf{X} \ \mathbf{X} \ _* + \lambda l(\mathbf{E}) \\ & \text{s.t. } \mathbf{X}_{1} = \mathbf{L}_{1} + \lambda \mathbf{X} \ \mathbf{X} \ _* + \lambda l(\mathbf{E}) \\ & \text{s.t. } \mathbf{X}_{1} = \mathbf{L}_{1} \mathbf{X}_{1} \ _* + \lambda \mathbf{X} \ \mathbf{X} \ _* + \lambda l(\mathbf{E}) \\ & \text{min_{\mathbf{L},\mathbf{S},\mathbf{L}}} \ \mathbf{L} \ _* + \lambda \mathbf{L} \ \mathbf{L}_{1} \ _* + \lambda l(\mathbf{E}) \\ & \text{min_{\mathbf{L},\mathbf{S},\mathbf{L}}} \ \mathbf{L} \ _* + \lambda \mathbf{L} \ \mathbf{L}_{1} \ _* + \lambda \mathbf{L} \ \mathbf{L}_{2} \ _* \\ & \text{min_{\mathbf{L},\mathbf{S},\mathbf{L}}} \ \mathbf{L} \ _* \ \mathbf{L}_{1} \ _* + \lambda l(\mathbf{E}) \\ & \text{min_{\mathbf{L},\mathbf{S},\mathbf{L}}} \ \mathbf{L} \ _* \ \mathbf{L}_{1} \ _* \ \mathbf{L}_{1} \ _* \\ & \text{min_{\mathbf{L},\mathbf{S},\mathbf{L}}} \ \mathbf{L} \ _* \ \mathbf{L}_{1} \ _* \ _* \ \mathbf{L}_{1} \ _* \\ & \text{min_{\mathbf{L},\mathbf{S},\mathbf{L}}} \ \mathbf{L} \ _* \ \mathbf{L}_{1} \ _* \ _* \ \mathbf{L}_{1} \ _* \\ & \text{min_{\mathbf{L},\mathbf{S},\mathbf{L}}} \ \mathbf{L} \ _* \ \mathbf{L}_{1} \ _* \ \mathbf{L}_{1} \ _* \\ & \text{min_{\mathbf{L},\mathbf{L},\mathbf{L},\mathbf{L},\mathbf{L},\mathbf{L},\mathbf{L},\mathbf{L}$				elasticnetR	Reg. Elastic net
				fusedl1R	Reg. Fused Lasso
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				tracelassoR	Reg. Trace Lasso
$ \begin{array}{c} \min_{\mathbf{X}} \ \mathbf{X}\ _{*}, \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{X}) = \mathcal{P}_{\Omega}(\mathbf{M}) & \text{1rmc} & \text{Low-rank matrix completion } [1] \\ \min_{\mathbf{X}, \mathbf{E}} \ \mathbf{X}\ _{*} + \lambda l(\mathbf{E}), \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M} & \text{1rmcR} & \text{Reg. Low-rank matrix completion} \\ \min_{\mathbf{M}, \mathbf{X}, \mathbf{E}} \ \mathbf{X}\ _{*} + \lambda l(\mathbf{E}), \text{ s.t. } \mathbf{A} = \mathbf{B}\mathbf{X} + \mathbf{E} & \text{1rr} & \text{Low-rank representation } [8] \\ \min_{\mathbf{X}, \mathbf{L}} \ \mathbf{Z}\ _{*} + \ \mathbf{L}\ _{*} + \lambda \lambda l(\mathbf{E}) & \text{1stlrr} & \text{Latent low-rank representation } [9] \\ \text{s.t. } \mathbf{X} = \mathbf{L}\mathbf{X} - \mathbf{X} = \mathbf{E} & \text{1stlrr} & \text{Low-rank and sparse representation } [9] \\ \text{s.t. } \mathbf{X} = \mathbf{B}\mathbf{X} + \mathbf{E} & \text{1rsr} & \text{Low-rank and sparse representation } [9] \\ \text{s.t. } \mathbf{X} = \mathbf{B}\mathbf{X} + \mathbf{E} & \text{1rsr} & \text{Low-rank and sparse representation } [18] \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} + \lambda 1 \ \mathbf{X}\ _{1} + \lambda 2 l(\mathbf{E}) & \text{1rsr} & \text{Low-rank and sparse representation } [18] \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} + \lambda 1 \ \mathbf{X}\ _{1} + \lambda 2 l(\mathbf{E}) & \text{1rsr} & \text{Low-rank and sparse representation } [18] \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} + \lambda 1 \ \mathbf{L}\ _{1} + \lambda 2 l(\mathbf{E}) & \text{1rsr} & \text{Low-rank and sparse representation } [18] \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} + \lambda 1 \ \mathbf{L}\ _{1} + \lambda 2 l(\mathbf{E}) & \text{1rsr} & \text{Low-rank and sparse representation } [18] \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} + \lambda 1 \ \mathbf{L}\ _{*} + \lambda 2 l(\mathbf{E}) & \text{1rsr} & \text{Low-rank and sparse representation } [18] \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} + \lambda 1 \ \mathbf{L}\ _{*} + \lambda 2 l(\mathbf{E}) & \text{1rsr} & \text{Low-rank and sparse representation } [18] \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} + \lambda 1 \ \mathbf{L}\ _{*} + \lambda 2 l(\mathbf{E}) & \text{1rsr} & \text{Low-rank and sparse representation } [18] \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} + \lambda \ \mathbf{L}\ _{*} \ \mathbf{L}\ _{*} \ \mathbf{L}\ _{*} \\ \text{s.t. } \mathbf{L}\ _{*} \ \mathbf{L}\ _{*} \ \mathbf{L}\ _{*} \ \mathbf{L}\ _{*} \\ \text{s.t. } \mathbf{L}\ _{*} \ \mathbf{L}\ _{*} \ \mathbf{L}\ _{*} \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} \ \mathbf{L}\ _{*} \ \mathbf{L}\ _{*} \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} \ \mathbf{L}\ _{*} \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} \ \mathbf{L}\ _{*} \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ \mathbf{L}\ _{*} \\ \text{mint}_{\mathbf{L}, \mathbf{S}_{i}} \ $				ksupportR	Reg. k support norm
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$\min_{\mathbf{L},\mathbf{S}} \ \mathbf{L}\ _* + \lambda l(\mathbf{S}), \text{ s.t. } \mathbf{X} = \mathbf{L} + \mathbf{S}$		rpca	Robust PCA [2]
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$\min_{\mathbf{X}} \ \mathbf{X}\ _*, \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{X}) = \mathcal{P}_{\Omega}(\mathbf{M})$		lrmc	Low-rank matrix completion [1]
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$\min_{\mathbf{X}, \mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M}$		lrmcR	Reg. Low-rank matrix completion
matrix $S.t. XZ + LX - X = E$ $Iattiff$ Latent low-rank representation [9] $Iattiff$		$\min_{\mathbf{X}, \mathbf{E}} \ \mathbf{X}\ _* + \lambda l(\mathbf{E}), \text{ s.t. } \mathbf{A} = \mathbf{B}\mathbf{X} + \mathbf{E}$		lrr	Low-rank representation [8]
matrix $ \begin{aligned} & \underset{\text{min}_{\mathbf{X},\mathbf{E}}}{\min_{\mathbf{X},\mathbf{E}}} \ \mathbf{X}\ _{*} + \lambda_{1} \ \mathbf{X}\ _{1} + \lambda_{2} l(\mathbf{E}) \\ & \text{s.t. } \mathbf{A} = \mathbf{B}\mathbf{X} + \mathbf{E} \end{aligned} & \text{lrsr} & \text{Low-rank and sparse representation [18]} \\ & \underset{\text{min}_{\mathbf{L}_{i},\mathbf{S}_{i}}}{\min_{\mathbf{L}_{i},\mathbf{S}_{i}}} \ \mathbf{L}\ _{*} + \lambda_{2} \sum_{i=1}^{m} \ \mathbf{S}_{i}\ _{1} \\ & \text{s.t. } \mathbf{X}_{i} = \mathbf{L} + \mathbf{S}_{i}, i = 1, \cdots, m, \mathbf{L} \geq 0, \mathbf{L} 1 = 1 \\ & \underset{\text{min}_{\mathbf{L}_{i},\mathbf{E}_{i}}}{\min_{\mathbf{L}_{i},\mathbf{E}_{i}}} \ \mathbf{L}\ _{*} + \lambda \ \mathbf{E}(\mathbf{E}_{i}) + \alpha \ \mathbf{Z}\ _{2,1} \\ & \text{s.t. } \mathbf{X}_{i} = \mathbf{X}_{i} \mathbf{Z}_{i} + \mathbf{E}_{i}, i = 1, \cdots, K \end{aligned} & \text{mlap} & \text{Multi-task low-rank affinity pursuit [4]} \\ & \underset{\text{min}_{\mathbf{L},\mathbf{S}}}{\min_{\mathbf{L}_{i},\mathbf{S}}} \ \mathbf{L}\ _{*} + \lambda \ \mathbf{C} \circ \mathbf{S}\ _{1}, \text{ s.t. } \mathbf{A} = \mathbf{L} + \mathbf{S}, 0 \leq \mathbf{L} \leq 1 \\ & \underset{\text{min}_{\mathbf{L},\mathbf{S}}}{\min_{\mathbf{P}}} \mathbf{P}, \mathbf{L} \rangle + \lambda \ \mathbf{P}\ _{1}, \text{ s.t. } 0 \leq \mathbf{P} \leq \mathbf{I}, \text{Tr}(\mathbf{P}) = k \end{aligned} & \text{sparsesc} & \text{Sparse spectral clustering [3]} \\ & \underset{\text{min}_{\mathbf{L},\mathbf{S}}}{\min_{\mathbf{P}}} \sum_{i=1}^{k} \alpha_{i} \ \mathcal{L}_{i(i)}\ _{*} + \ \mathbf{S}\ _{1}, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S} \end{aligned} & \text{trpca_snn} & \text{Tensor robust PCA based on sum of nuclear norm [5]} \\ & \underset{\text{tensor}}{\min_{\mathbf{X},\mathbf{E}}} \sum_{i=1}^{k} \alpha_{i} \ \mathcal{X}_{i(i)}\ _{*}, \text{ s.t. } \mathcal{P}_{\Omega}(\mathcal{X}) = \mathcal{P}_{\Omega}(\mathcal{M}) \end{aligned} & \text{lrtc_snn} & \text{lcow-rank tensor completion based on sum of nuclear norm [10]} \\ & \underset{\text{tensor}}{\min_{\mathbf{L},\mathbf{S}}} \ \mathcal{L}\ _{*} + \lambda \ \mathcal{S}\ _{1}, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S} \end{aligned} & \text{trpca_tnn} & \text{Tensor Robust PCA based on sum of nuclear norm [10]} \\ & \underset{\text{s.t. } \mathcal{P}_{\Omega}(\mathcal{X}) + \mathcal{E} = \mathcal{M}}{\min_{\mathbf{L},\mathbf{S}}} \ \mathcal{L}\ _{*} + \lambda \ \mathcal{S}\ _{1}, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S} \end{aligned} & \text{trpca_tnn} & \text{Tensor Robust PCA based on tensor nuclear norm [10]} \\ & \underset{\text{s.t. } \mathcal{P}_{\Omega}(\mathcal{X}) + \mathcal{E} = \mathcal{M}}{\min_{\mathbf{L},\mathbf{S}}} \ \mathcal{L}\ _{*} + \lambda \ \mathcal{L}\ _$	Low rank			latlrr	Latent low-rank representation [9]
models $ \begin{array}{lllllllllllllllllllllllllllllllllll$	LOW-FallK	s.t. $XZ + LX - X = E$			
models $ \begin{array}{l} \text{min}_{\mathbf{L}_i,\mathbf{S}_i} \ \mathbf{L}\ _* + \lambda \sum_{i=1}^m \ \mathbf{S}_i \ _1 \\ \text{s.t. } \mathbf{X}_i = \mathbf{L} + \mathbf{S}_i, \ i = 1, \cdots, m, \ \mathbf{L} \geq 0, \ \mathbf{L} 1 = 1 \\ \hline \min_{\mathbf{Z}_i,\mathbf{E}_i} \sum_{i=1}^K (\ \mathbf{Z}_i \ _* + \lambda \mathbf{L}(\mathbf{E}_i)) + \alpha \ \mathbf{Z} \ _{2,1} \\ \text{s.t. } \mathbf{X}_i = \mathbf{X}_i = \mathbf{Z}_i + \mathbf{L}_i, \ i = 1, \cdots, K \\ \hline \min_{\mathbf{L}_i,\mathbf{S}} \ \mathbf{L} \ _* + \lambda \ \mathbf{L} \otimes \mathbf{S} \ _1, \ \text{s.t. } \mathbf{A} = \mathbf{L} + \mathbf{S}, 0 \leq \mathbf{L} \leq 1 \\ \hline \min_{\mathbf{D}_i,\mathbf{S}} \ \mathbf{L} \ _* + \lambda \ \mathbf{L} \otimes \mathbf{S} \ _1, \ \text{s.t. } \mathbf{A} = \mathbf{L} + \mathbf{S}, 0 \leq \mathbf{L} \leq 1 \\ \hline \min_{\mathbf{D}_i,\mathbf{S}} \ \mathbf{L} \ _* + \lambda \ \mathbf{L} \otimes \mathbf{S} \ _1, \ \text{s.t. } \mathbf{A} = \mathbf{L} + \mathbf{S}, 0 \leq \mathbf{L} \leq 1 \\ \hline \min_{\mathbf{D}_i,\mathbf{S}} \ \mathbf{L} \ _* + \lambda \ \mathbf{L} \otimes \mathbf{S} \ _1, \ \text{s.t. } \mathbf{L} \otimes \mathbf{L} \leq 1 \\ \hline \min_{\mathbf{D}_i,\mathbf{S}} \ \mathbf{L} \otimes $	matrix	$\min_{\mathbf{X},\mathbf{E}} \ \mathbf{X}\ _* + \lambda_1 \ \mathbf{X}\ _1 + \lambda_2 l(\mathbf{E})$		lrsr	Low-rank and sparse representation [18]
models s.t. $\mathbf{X}_i = \mathbf{L} + \mathbf{S}_i, i = 1, \cdots, m, \mathbf{L} \geq 0, \mathbf{L} 1 = 1$ min $_{\mathbf{Z}_i, \mathbf{E}_i} \sum_{i=1}^K (\ \mathbf{Z}_i\ _* + \lambda l(\mathbf{E}_i)) + \alpha \ \mathbf{Z}\ _{2,1}$ mlap Multi-task low-rank affinity pursuit [4] s.t. $\mathbf{X}_i = \mathbf{X}_i \mathbf{Z}_i + \mathbf{E}_i, i = 1, \cdots, K$ min $_{\mathbf{L}, \mathbf{S}} \ \mathbf{L}\ _* + \lambda \ \mathbf{C} \circ \mathbf{S}\ _1, \text{ s.t. } \mathbf{A} = \mathbf{L} + \mathbf{S}, 0 \leq \mathbf{L} \leq 1$ igc Improved graph clustering [3] min $_{\mathbf{P}} \langle \mathbf{P}, \mathbf{L} \rangle + \lambda \ \mathbf{P}\ _1, \text{ s.t. } 0 \leq \mathbf{P} \leq \mathbf{I}, \text{Tr}(\mathbf{P}) = k$ sparsesc Sparse spectral clustering [13] Tensor robust PCA based on sum of nuclear norm [5]					
s.t. $\mathbf{X}_i = \mathbf{L} + \mathbf{S}_{i,-1} \times \mathbf{I} + \mathbf{N}_{i} \times \mathbf{I} = \mathbf{I}, \cdots, m, \mathbf{L} \geq 0, \mathbf{L} \mathbf{I} = \mathbf{I}$ $\min_{\mathbf{Z}_i, \mathbf{E}_i} \sum_{i=1}^K (\ \mathbf{Z}_i\ _* + \lambda \mathbf{I}(\mathbf{E}_i)) + \alpha \ \mathbf{Z}\ _{2,1}$ s.t. $\mathbf{X}_i = \mathbf{X}_i \mathbf{Z}_i + \mathbf{E}_{i,i} i = 1, \cdots, K$ $\min_{\mathbf{L}, \mathbf{S}} \ \mathbf{L}\ _* + \lambda \ \mathbf{C} \circ \mathbf{S}\ _1, \text{ s.t. } \mathbf{A} = \mathbf{L} + \mathbf{S}, 0 \leq \mathbf{L} \leq 1 \qquad \text{igc} \qquad \text{Improved graph clustering [3]}$ $\min_{\mathbf{P}} \langle \mathbf{P}, \mathbf{L} \rangle + \lambda \ \mathbf{P}\ _1, \text{ s.t. } 0 \leq \mathbf{P} \leq \mathbf{I}, \text{Tr}(\mathbf{P}) = k \qquad \text{sparsesc} \qquad \text{Sparse spectral clustering [13]}$ $\min_{\mathbf{L}, \mathbf{S}} \sum_{i=1}^k \alpha_i \ \mathbf{L}_{i(i)}\ _* + \ \mathbf{S}\ _1, \text{ s.t. } \mathbf{X} = \mathbf{L} + \mathbf{S} \qquad \text{trpca_snn} \qquad \text{Tensor robust PCA based on sum of nuclear norm [5]}$ $\min_{\mathbf{X}} \sum_{i=1}^k \alpha_i \ \mathbf{X}_{i(i)}\ _*, \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{X}) = \mathcal{P}_{\Omega}(\mathbf{M}) \qquad \text{lrtc_snn} \qquad \text{Low-rank tensor completion based on sum of nuclear norm [10]}$ $\min_{\mathbf{X}, \mathbf{E}} \sum_{i=1}^k \alpha_i \ \mathbf{X}_{i(i)}\ _* + \lambda \ (\mathbf{E}) \qquad \text{s.t. } \mathcal{P}_{\Omega}(\mathbf{X}) + \mathbf{E} = \mathbf{M} \qquad \text{lrtc_snn} \qquad \text{Tensor Robust PCA based on tensor nuclear norm [11]}$ $\min_{\mathbf{L}, \mathbf{E}} \ \mathbf{L}\ _* + \lambda \ \mathbf{S}\ _1, \text{ s.t. } \mathbf{X} = \mathbf{L} + \mathbf{S} \qquad \text{trpca_tnn} \qquad \text{Tensor Robust PCA based on tensor nuclear norm [11]}$ $\min_{\mathbf{X}, \mathbf{E}} \ \mathbf{L}\ _* + \lambda \ \mathbf{S}\ _1, \text{ s.t. } \mathbf{X} = \mathbf{L} + \mathbf{S} \qquad \text{trpca_tnn} \qquad \text{Tensor Robust PCA based on tensor nuclear norm [11]}$ $\min_{\mathbf{X}} \ \mathbf{E}\ _* \ \mathbf{X}\ _*, \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{X}) = \mathcal{P}_{\Omega}(\mathbf{M}) \qquad \text{lrtc_tnn} \qquad \text{Tensor Robust PCA based on tensor nuclear norm [11]}$ $\min_{\mathbf{X}} \ \mathbf{E}\ _* \ \mathbf{X}\ _*, \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{X}) = \mathcal{P}_{\Omega}(\mathbf{M}) \qquad \text{lrtc_tnn} \qquad \text{Tensor Robust PCA based on tensor nuclear norm [11]}$ $\min_{\mathbf{X}} \ \mathbf{E}\ _* \ \mathbf{X}\ _*, \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{X}) = \mathcal{P}_{\Omega}(\mathbf{M}) \qquad \text{lrtc_tnn} \qquad \text{Reg. low-rank tensor completion based on tensor nuclear norm [11]}$	models	s.t. $\mathbf{X}_i = \mathbf{L} + \mathbf{S}_i, i = 1, \cdots, m, \mathbf{L} \geq 0, \mathbf{L}1 = 1$		rmsc	Robust multi-view spectral clustering [16]
s.t. $\mathbf{X}_i = \mathbf{X}_i \mathbf{Z}_i + \mathbf{E}_i, i = 1, \cdots, K$	models				
s.t. $\mathbf{X}_i = \mathbf{X}_i \mathbf{Z}_i + \mathbf{E}_i, i = 1, \cdots, K$ $\min_{\mathbf{L},\mathbf{S}} \ \mathbf{L}\ _* + \lambda \ \mathbf{C} \circ \mathbf{S}\ _1, \text{ s.t. } \mathbf{A} = \mathbf{L} + \mathbf{S}, 0 \le \mathbf{L} \le 1$ $\min_{\mathbf{P}} \langle \mathbf{P}, \mathbf{L} \rangle + \lambda \ \mathbf{P}\ _1, \text{ s.t. } 0 \le \mathbf{P} \le \mathbf{I}, \text{Tr}(\mathbf{P}) = k$ $\min_{\mathbf{L},\mathbf{S}} \sum_{i=1}^k \alpha_i \ \mathcal{L}_{i(i)}\ _* + \ \mathbf{S}\ _1, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \sum_{i=1}^k \alpha_i \ \mathcal{L}_{i(i)}\ _* + \ \mathbf{S}\ _1, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \sum_{i=1}^k \alpha_i \ \mathcal{X}_{i(i)}\ _* + \ \mathbf{S}\ _1, \text{ s.t. } \mathcal{P}_{\Omega}(\mathcal{M})$ $\min_{\mathbf{L},\mathbf{S}} \sum_{i=1}^k \alpha_i \ \mathcal{X}_{i(i)}\ _* + \lambda \ (\mathcal{E})$ $\text{s.t. } \mathcal{P}_{\Omega}(\mathcal{X}) + \mathcal{E} = \mathcal{M}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathbf{S}\ _1, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathbf{S}\ _1, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathbf{S}\ _1, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1, \text{ s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1, \text{ s.t. } \mathcal{L} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1, \text{ s.t. } \mathcal{L} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1, \text{ s.t. } \mathcal{L} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1, \text{ s.t. } \mathcal{L} = \mathcal{L} + \mathcal{S}$ $\min_{\mathbf{L},\mathbf{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{L}\ _*$				mlan	Multi-task low-rank affinity pursuit [4]
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		s.t. $\mathbf{X}_i = \mathbf{X}_i \mathbf{Z}_i + \mathbf{E}_i, i = 1, \cdots, K$		шар	Mater task low rank annity pursuit [4]
				igc	1 01
		$\min_{\mathbf{P}} \langle \mathbf{P}, \mathbf{L} \rangle + \lambda \mathbf{P} _1$, s.t. $0 \leq \mathbf{P} \leq \mathbf{I}, \text{Tr}(\mathbf{P}) = k$	sparsesc	Sparse spectral clustering [13]
Low-rank tensor completion based on sum of nuclear norm [5]	tensor	$\min_{k \in \mathcal{K}} \nabla^k _{\mathcal{L}} \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $		trnga ann	Tensor robust PCA based on
Low-rank tensor min $\chi \sum_{i=1}^{k} \alpha_i \ \mathcal{X}_{i(i)} \ _*$, s.t. $\mathcal{P}_{\Omega}(\mathcal{X}) = \mathcal{P}_{\Omega}(\mathcal{M})$ 1rtc_snn sum of nuclear norm [10] Reg. low-tank tensor completion based on sum of nuclear norm $\min_{\mathcal{L}, \mathcal{S}} \ \mathcal{L} \ _* + \lambda \ \mathcal{S} \ _1$, s.t. $\mathcal{X} = \mathcal{L} + \mathcal{S}$ 1rtc_snn sum of nuclear norm [11] Tensor Robust PCA based on tensor nuclear norm [11] $\min_{\mathcal{L}, \mathcal{S}} \ \mathcal{L} \ _* + \lambda \ \mathcal{S} \ _1$, s.t. $\mathcal{P}_{\Omega}(\mathcal{X}) = \mathcal{P}_{\Omega}(\mathcal{M})$ 1rtc_tnn Low-rank tensor completion based on tensor nuclear norm [11] Reg. low-rank tensor completion based on tensor nuclear norm [11]		$ \text{min}_{\mathcal{L}, \mathcal{S}} \sum_{i=1}^{\alpha_i} \alpha_i \ \mathcal{L}_{i(i)} \ $	$ \mathcal{A}_{i}\rangle \ * + \ \mathcal{S} \ _1, \text{ s.t. } \mathcal{A} = \mathcal{L} + \mathcal{S}$	cipca_siii	sum of nuclear norm [5]
Low-rank tensor $\min_{\mathbf{X}, \mathbf{\mathcal{E}}} \sum_{i=1}^k \alpha_i \ \mathbf{\mathcal{X}}_{i(i)} \ _* + \lambda l(\mathbf{\mathcal{E}})$ sum of nuclear norm [10] Reg. low-tank tensor completion based on sum of nuclear norm $\min_{\mathbf{\mathcal{E}}, \mathbf{\mathcal{E}}} \ \mathbf{\mathcal{E}} \ _* + \lambda \ \mathbf{\mathcal{E}} \ _1$, s.t. $\mathbf{\mathcal{X}} = \mathbf{\mathcal{L}} + \mathbf{\mathcal{S}}$ trpca_tnn Tensor Robust PCA based on tensor nuclear norm [11] $\min_{\mathbf{\mathcal{X}}} \ \mathbf{\mathcal{X}} \ _*, \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{\mathcal{X}}) = \mathcal{P}_{\Omega}(\mathbf{\mathcal{M}})$ $\min_{\mathbf{\mathcal{X}}} \ \mathbf{\mathcal{X}} \ _* + \lambda l(\mathbf{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{\mathcal{X}}) + \mathbf{\mathcal{E}} = \mathbf{\mathcal{M}}$ $\lim_{\mathbf{\mathcal{X}}} \ \mathbf{\mathcal{X}} \ _* + \lambda l(\mathbf{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{\mathcal{X}}) + \mathbf{\mathcal{E}} = \mathbf{\mathcal{M}}$ $\lim_{\mathbf{\mathcal{X}}} \ \mathbf{\mathcal{X}} \ _* + \lambda l(\mathbf{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{\mathcal{X}}) + \mathbf{\mathcal{E}} = \mathbf{\mathcal{M}}$ $\lim_{\mathbf{\mathcal{X}}} \ \mathbf{\mathcal{X}} \ _* + \lambda l(\mathbf{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{\mathcal{X}}) + \mathbf{\mathcal{E}} = \mathbf{\mathcal{M}}$ $\lim_{\mathbf{\mathcal{X}}} \ \mathbf{\mathcal{X}} \ _* + \lambda l(\mathbf{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{\mathcal{X}}) + \mathbf{\mathcal{E}} = \mathbf{\mathcal{M}}$ $\lim_{\mathbf{\mathcal{X}}} \ \mathbf{\mathcal{X}} \ _* + \lambda l(\mathbf{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{\mathcal{X}}) + \mathbf{\mathcal{E}} = \mathbf{\mathcal{M}}$ $\lim_{\mathbf{\mathcal{X}}} \ \mathbf{\mathcal{X}} \ _* + \lambda l(\mathbf{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\mathbf{\mathcal{X}}) + \mathbf{\mathcal{E}} = \mathbf{\mathcal{M}}$		$\min_{\boldsymbol{\mathcal{X}}} \sum_{i=1}^k \alpha_i \ \boldsymbol{\mathcal{X}}_{i(i)}\ _*, \text{ s.t. } \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) = \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{M}})$		lrtc_snn	Low-rank tensor completion based on
tensor tensor $\min_{\mathbf{x}, \mathbf{\mathcal{E}}} \sum_{i=1}^{n} \alpha_i \ \mathbf{\mathcal{X}}_{i(i)} \ _* + \lambda l(\mathbf{\mathcal{E}})$ and $\sum_{s.t.} \mathcal{P}_{\Omega}(\mathbf{\mathcal{X}}) + \mathbf{\mathcal{E}} = \mathbf{\mathcal{M}}$ lensor $\sum_{s.t.} \mathcal{P}_{\Omega}(\mathbf{\mathcal{X}}) + \mathbf{\mathcal{E}} = \mathbf{\mathcal{M}}$ le					sum of nuclear norm [10]
tensor tensor $\min_{\mathcal{L},\mathcal{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1$, s.t. $\mathcal{X} = \mathcal{L} + \mathcal{S}$ trpca_tnn Tensor Robust PCA based on tensor nuclear norm [11] $\min_{\mathcal{X}} \ \mathcal{X}\ _*$, s.t. $\mathcal{P}_{\Omega}(\mathcal{X}) = \mathcal{P}_{\Omega}(\mathcal{M})$ lrtc_tnn Low-rank tensor completion based on tensor nuclear norm [11] $\min_{\mathcal{X}} \ \mathcal{X}\ _* + \lambda \ \mathcal{S}\ _$				lrtcR_snn	Reg. low-tank tensor completion based on
models					sum of nuclear norm
models $\min_{\boldsymbol{\mathcal{X}}} \ \boldsymbol{\mathcal{X}}\ _{*}, \text{ s.t. } \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) = \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{M}}) $ $\lim_{\boldsymbol{\mathcal{X}}} \ \boldsymbol{\mathcal{X}}\ _{*}, \text{ s.t. } \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) = \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{M}}) $ $\lim_{\boldsymbol{\mathcal{X}}} \ \boldsymbol{\mathcal{X}}\ _{*} + \lambda l(\boldsymbol{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) + \boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{M}} $ $\lim_{\boldsymbol{\mathcal{X}}} \ \boldsymbol{\mathcal{X}}\ _{*} + \lambda l(\boldsymbol{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) + \boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{M}} $ $\lim_{\boldsymbol{\mathcal{X}}} \ \boldsymbol{\mathcal{X}}\ _{*} + \lambda l(\boldsymbol{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) + \boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{M}} $ $\lim_{\boldsymbol{\mathcal{X}}} \ \boldsymbol{\mathcal{X}}\ _{*} + \lambda l(\boldsymbol{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) + \boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{M}} $ $\lim_{\boldsymbol{\mathcal{X}}} \ \boldsymbol{\mathcal{X}}\ _{*} + \lambda l(\boldsymbol{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) + \boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{M}} $ $\lim_{\boldsymbol{\mathcal{X}}} \ \boldsymbol{\mathcal{X}}\ _{*} + \lambda l(\boldsymbol{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) + \boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{M}} $		$\min_{\mathcal{L}, \mathcal{S}} \ \mathcal{L}\ _* + \lambda \ \mathcal{S}\ _1$, s.t. $\mathcal{X} = \mathcal{L} + \mathcal{S}$		trpca_tnn	Tensor Robust PCA based on
$\min_{\boldsymbol{\mathcal{X}}} \ \boldsymbol{\mathcal{X}}\ _*$, s.t. $\mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) = \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{M}})$ lrtc_tnn Low-rank tensor completion based on tensor nuclear norm [11] $\min_{\boldsymbol{\mathcal{X}}} \mathcal{E}_{\Omega} \ \boldsymbol{\mathcal{X}}\ _* + \lambda l(\boldsymbol{\mathcal{E}})$, s.t. $\mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) + \boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{M}}$ lrtcR tnn Reg. low-rank tensor completion based on					tensor nuclear norm [11]
tensor nuclear norm [11] $\min_{\boldsymbol{\mathcal{X}}} \boldsymbol{\mathcal{E}} \ \boldsymbol{\mathcal{X}}\ _* + \lambda l(\boldsymbol{\mathcal{E}}), \text{ s.t. } \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) + \boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{M}}$ lrtcR tnn Reg. low-rank tensor completion based on		$\min_{m{\mathcal{X}}} \ m{\mathcal{X}}\ _*, ext{ s.t. } \mathcal{P}_{\Omega}(m{\mathcal{X}}) = \mathcal{P}_{\Omega}(m{\mathcal{M}})$		lrtc_tnn	Low-rank tensor completion based on
$ \min \mathbf{v} \in \mathcal{X} _* + \lambda l(\mathbf{\mathcal{E}}), \text{ s.t. } P_{\mathbf{\mathcal{O}}}(\mathcal{X}) + \mathbf{\mathcal{E}} = \mathcal{M}$					tensor nuclear norm [11]
tensor nuclear norm [11]		$\min_{\mathcal{X}, \mathcal{E}} \ \mathcal{X}\ _* + \lambda l(\mathcal{E}), \text{ s.t. } \mathcal{P}_{\Omega}(\mathcal{X}) + \mathcal{E} = \mathcal{M}$		lrtcR_tnn	Reg. low-rank tensor completion based on
[]					tensor nuclear norm [11]

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