Special theory of Relativity and Quantum Mechanics

SPH4U

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January 12, 2026

In unit 5, you are expecting to self-study special relativity and quantum thoery.

Contents

1 The Special Theory of Relativity

At the turn of the twentieth century, most of the physics community enjoyed a sense of accomplishment. Newtonian mechanics, provided the principles to the atomic level and established the idea of energy conservation. Maxwell's equation successfully unified the subjects of electricity, magnetism and optics. Light is a combination of oscillating electric and magnetic fields.

However, few famous physics experiments came to a similar statement. The speed of light is $3.0 \times 10^8 \frac{m}{s}$ at whatever frame of reference. Einstein developed the *Special Theory of Relativity* to explain why the speed of light is constant at different frames of reference.

1.1 Maxwell's assumption

He proposed that *electromagnetic wave* needed a medium through which to travel. The medium was called **ether**.

Definition 1.1 (ether)**.** *the proposed medium through which electromagnetic wave were once believed to propagate.*
Few properties of ether:

= *It had no mass*

= *It had no drag effect on the motions of the planets.*

= *It filled the vacuum of space.*

Maxwell stated that speed of light with respect to the ether will always be $c = 3.0 \times 10^8 \frac{m}{s}$ That meant that if you were at rest and observed a light source move relative to ether, you would measure the speed of light to be different from c. It means that speed of light is different from different perspective.

1.1.1 *Failure*

Experimental evidence did not support the idea that the speed of light varied with the speed of the inertial frame. No change in the speed of light with the motion of Earth. Experiments proved that electromagnetic waves do not require a medium in which to propagate, and the existence of ether could not be proven experimentally.

In the magnet-and-coil thought experiment, Maxwell's theory predicts that when a magnet moves toward a coil of wire, an electric field forms near the moving magnet. This electric field moves charges with the coil, thus inducing an electric current. However, Maxwell's theory also predicts that if the coil moves and the magnet remains at rest, a current exists in the coil, not because an electric field forms, but because the magnetic field exerts a force on the charges in the moving coil.

To einstein, it seemed illogical that selecting a frame of reference in which either the magnet or the coil is at rest would change the way we understand what is happening.

1.2 The Special theory of relativity

The *Special Theory of Relativity* has two postulate:

Postulate 1 (The Principle of Relativity)**.** *The laws of physics are the same in all inertial frames of reference. No physics experiment can ever determine whether you are at rest or moving at a constant velocity.*

Postulate 2 (The Speed of Light Principle)**.** *There is at least one inertial frame of reference in which, for an observer at rest in this frame of reference, the speed of light, c, in a vacuum is independent of the motion of the source of the light.*

Postulate 1 implies that if Postulate 2 is true in one inertial frame of reference, it must be true in all frames of reference.

The consequence of this is that the speed of light must be constant and the same in all inertial frame of reference because the laws of physics do not prefer one frame of reference over another.

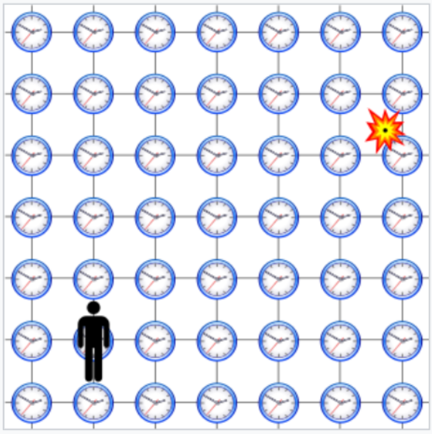
Theorem 1.2 (Special Theory of Relativity)**.** *All physical laws are the same in all inertial frames of reference, and the speed of light is independent of the motion of the light source or its observer in all inertial frames of reference.*

2 Time Dilation

Remark. Please go to Mr. Yang's video here! Here is just a brief personal review note about *time dilation* in that video.
Please check the textbook first, this note is only a summary!

Time Dilation, the model explains the slowing down of time in one reference frame moving relative to an observer in another reference frame.

2.0.1 Proper time vs non-proper time measurement



In special relativity, the observer measures events against an infinite latticework of synchronized clocks.

Definition 2.1 (Synchroinzed Clock Measurement/Proper time)**.** *A people measures a time at the same location/coordinate from his frame of reference!*

Definition 2.2 (Moving Clock Measurement/Non-proper time measurement)**.** *A people measures a time at the different location/coordinate from his frame of reference!*

2.0.2 Δt vs Δt_s

Δt_s is a Synchronized clock measurement of the time from observer 1's frame of reference. Δt is the moving clock measurement from a stationary perspective(Observer 2).

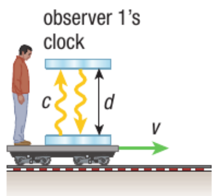


Figure 1: From the inertial FOR which has recorded Δt_s

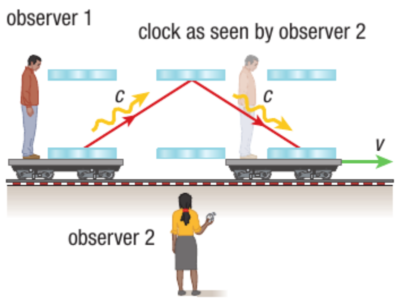


Figure 2: From the inertial FOR which has recorded Δt

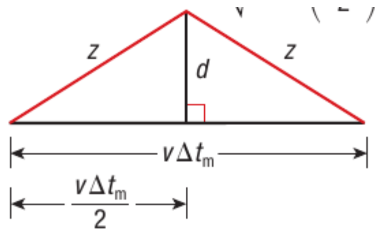


Figure 3: Path of light from observer 2's FOR

2.0.3 Derivation of Special Relativity equation

Lemma 2.3. *The time can be determined by this formula:*

$$\Delta t = \frac{distance}{speed}$$

Proof. To start off, we need to solve for z:

$$z = \sqrt{d^2 + (\frac{v\Delta t}{2})^2} \tag{1}$$

Then, let's solve for Δt

$$\Delta t = \frac{2z}{c}$$
$$\Delta t = \frac{2}{c} \times \sqrt{d^2 + (\frac{v\Delta t}{2})^2}$$
$$(\Delta t)^2 = \frac{4}{c^2} \times (d^2 + (\frac{v\Delta t}{2})^2)$$
$$\Delta t^2 = (\frac{2d}{c})^2 + (\frac{v\Delta t}{c})^2$$
$$\Delta t^2 = (\Delta t_s)^2 + (\frac{v\Delta t}{c})^2$$
$$\Delta t^2 - \frac{v^2\Delta t^2}{c^2} = (\Delta t_s)^2$$
$$\Delta t^2(1 - \frac{v^2}{c^2}) = (\Delta t_s)^2$$
$$\Delta t^2 = \frac{(\Delta t_s)^2}{1 - (\frac{v^2}{c^2})}$$
$$\Delta t = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$\square$

According to the theory of special relativity, $\Delta t > \Delta t_s$. As a result,

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \geq 1$$

2.0.4 Lorentz factor γ

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If we look at the factor of the γ, it only matters when the speed of the rocket is fast enoguh, or at least $0.3c$!

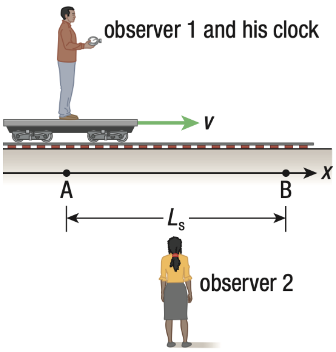
3 Length Contraction, Simultaneity, and Relativistic Momentum

3.1 Length Contraction

In the previous section, we discussed about time dilation. In the same paper, Herr Einstein stated that the length of a moving object is different from this object's moving frame and stationary rest frame.

Definition 3.1 (proper length(L_s)). *the length of an object or distance between two points as measured by an observer who is stationary relative to the object or distance*

Example 1. *Assume we want to measure the length of the car from observer 1 's perspective and observer 2's perspective.*



3.1.1 How can we solve this little problem

Lemma 3.2. *Length = speed × time*

$$L = v\Delta t$$

From 3.2, we understand that the length of the object can be easily find by calculating the time used by the object to pass point B.

From observer 1's perspective, the time that this object pass point B is Δt (measure at front and back accroding to his FOR, so this is a non-proper measurement)

From observer 2's perspective, the time that this object pass point B is Δt_s (time measure at the same point relative to the observer 2)

As a result,

$$L_s = v\Delta t$$

and

$$L = v\Delta t_s$$

Let's start from the time dilation formula:

$$\Delta t = \frac{\Delta t_s}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$v\Delta t = \frac{v\Delta t_s}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$L_s = L \times \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$L = L_s\sqrt{1-\frac{v^2}{c^2}} \tag{2}$$

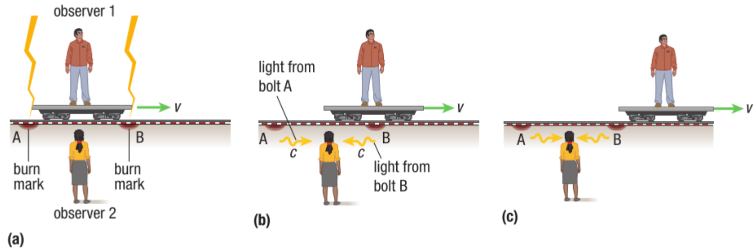
3.1.2 Muons and Evidence for Length Contraction and Time Dilation

Muons:

- Travel at speeds of about $0.99c$
- About 207 times as massive as electrons
- Decay in $2.2ms$
- Source: Cosmic radiation that collides with atoms in Earth's upper atmosphere
- Should decay after 660m, but in reality, 4800m
- Use special reality to explain this
- Due to time dilation, the clock of the Muons run relative slow.

3.2 Relativity of Simultaneity

It's really hard to explain it here. I suggest you to read the textbook first!



Will observer 1 see both lightning at the same time? NO

Observer 2 observes that the railway car moves to the right, and because observer 1 is moving, the flash at B will reach him before the flash from A. Even with the distortions of time and space that arise from relativity, events do not occur out of sequence. So observer 1 will see the lightning strike at B before the lightning strike at A. The speeds of the light pulses from A and B are the same (a consequence of Einstein's pos- tulates), and the distances that the pulses travel are the same. Therefore, observer 1 must conclude that the light pulses were not emitted at the same time.

Let's assume clock there are three clocks, A, B and C.

Clock A, B are stationary at point A and B. Observer 1 hold the clock C

From stationary frame of reference, clock c elapse 10 seconds, while 20s elapse on A and B.

From observer 1's frame of reference, clock c elapse 10 seconds, while 5s elapse on A and B.

Where did the "extra" 15 seconds go? The answer is that while clocks A and B are synchroinzed in their frame of reference, they are not synchroinzed in the frame of reference of C.

When C get to A, the reading on A should be 0 seconds, B should be 15 seconds!

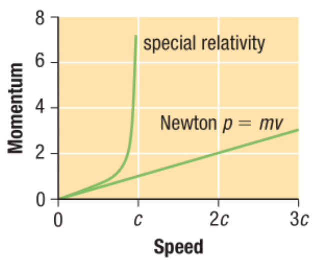
3.2.1 The Twin Paradox

The rock one is not inertial frame of reference. It has acceleration!

3.3 Relativistic Momentum

In Newtonian physics, momentum can be calculated as this, $\vec{p} = m\vec{v}$

However, as v approaches the speed of light, we have to take special relativity into account.



The effects of time dilation and length contraction are not included in the Newtonian momentum used in classical mechanics. To account for the relativistic effects on the momentum of objects moving near the speed of light, Einstein showed that proper time should be used to calculate momentum. This amounts to using a clock that travels along with the object. At the same time, an observer who watches the object moving with speed v should take the measurement of length. The proper time is given by this expression: $\Delta t_s = \Delta t_m \sqrt{1 - \frac{v^2}{c^2}}$.

Let us derive for the formula:

$$\vec{p} = m_s \vec{v}$$
$$\vec{p} = m_s \frac{\Delta x}{\Delta t_s}$$
$$p = \frac{m_s \Delta x}{\Delta t \sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \frac{m_s v}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

Definition 3.3 (Rest mass). *the mass of the object are measured at rest with respect to the observer, also called a proper mass. In the equation, m_s is a proper mass.*

Definition 3.4 (Relativistic mass). *the mass of an object measured by an observer moving with speed v with respect to the object.*

Relative mass proper mass equation

$$m = m_s \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

4 Mass-Energy Equivalence

In the previous section, we discussed the relativistic Momentum. The relativistic mass can be describe as this:

$$m_{\text{relativistic}} = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m is the mass of the object at rest.

Using the equations of special relativity, Herr Einstein concluded that the total energy, E_{total} for an object with rest mass m moving with speed v is equal to:

$$E_{\text{total}} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

When the object is at rest,

$$E_{\text{rest}} = mc^2$$

Definition 4.1 (rest energy (E_{rest}))**.** *the amount of energy an object at rest has with respect to an observer*

Definition 4.2 (relativistic kinetic energy(E_k))**.** *the energy of an object in excess of its rest energy*

The E_k can be solved by this:

$$E_{\text{total}} - E_{\text{rest}} = E_k$$
$$E_k = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2$$

Unlike various potential energy, where a force is acting on an object without moving it, rest energy is the property of matter itself. Like space and time form space-time theory, The conservation of energy principle is now the principle of *conservation of mass-energy*, which states that the rest enery is equal to rest mass times the speed of light squared!

5 Introduction to Quantum Theory

5.1 Why we need quantum theory

Physicists such as J.J. Thomson discovered that Newton's laws failed to explain the behaviour of electrons and atoms. Similarly, although Maxwell correctly described electromagnetic phenomena in the every day world, his equations failed to describe the microscopic world. This microscopic world is called the quantum world, where *quantum* refers to a very small increment of energy. The study of the behaviour of these very small bundles of energy, called *quantum theory*.

5.2 Particles and Waves

According to Newton's laws and Maxwell's equations, energy can be carried from one point to another in two ways: by particles and by waves. However, this is not accurate in the quantum world.

Definition 5.1 (Interference Effect)**.** *The net effect of the combination of two or more wave trains moving on intersecting or coincident paths.*

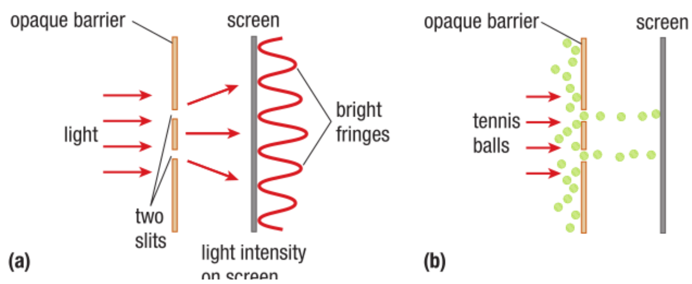
5.2.1 Differences between particle and wave

Wave

- Do show interference effects
- Waves do not deliver energy in discrete quantities. Wave deliver their energy continuously over time and spread out over the screen. (a) is a wave.

Particle

- Do not show interference effects
- Deliver energy in discrete quantities
- Double slit experiment, (b) is a particle.



The energy carried by a wave is described by its intensity, which equals the amount of energy the wave transports per unit time across a surface of unit area. For the light wave in figure 1(a), the amount of energy absorbed by the screen depends on the intensity of the wave and the absorption time. The amount of absorbed energy can take on any non-negative value.

5.2.2 An interference Experiment with Electrons

The separation of particles and waves is not found in the quantum world. There are only two types of behaviour are possible: waves exhibit interference; particles do not.

However, in reality, electron can exhibit interference, a property that classical theory says is possible only for waves.

Definition 5.2 (wave-particle duality)**.** *The property of matter that defines is dual nature of displaying both wave-like and particle-like characteristics.*

The following properties is followed in the quantum world:

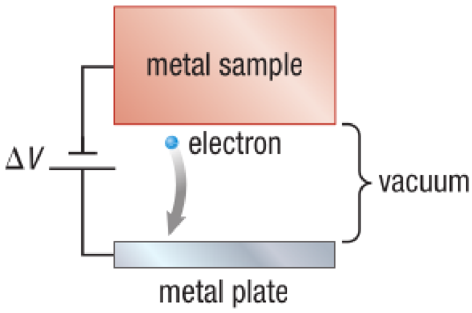
- All quantum objects, including electromagnetic radiation and electrons, can exhibit interference.
- All quantum objects, including electromagnetic radiation and electrons, transfer energy in distinct, or discrete, amounts. There discrete "parcels" of energy are quanta.

6 Photons and the Quantum Theory of Light

6.1 The Work Function

Definition 6.1 (Work Function). *The minimum energy required to remove a single electron from a piece of metal.*

The work function can be calculated through this experiment:



We can manipulate the Δv in a circuit.

When the electron is pulled out of the metal piece, we can record the Δv, and calculate the work function through this formula:

$$W = e\Delta v$$

6.2 The Photoelectric Effect

Another way to extract electrons from a metal is by shining light onto it. Light striking a metal surface is absorbed by the electrons.

Definition 6.2 (The Photoelectric Effect). *If an electron absorbs an amount of light energy above the metal's work function, it ejects from the metal in a phenomenon.*

Experimental studies of the photoelectric effect carried out around 1900 revealed that no electrons are emitted unless the light's frequency is greater than the threshold frequency

Definition 6.3 (Threshold frequency (f_0)). *the minimum frequency at which electrons are ejected from a metal.*

Through the experiment, scientists discovered that thershold frequency is independent of the intensity of the light, which is conflict to the classical physics (energy carried by a light wave is proportional to the intensity of light). Experiments found that when the frequency is below the threshold frequency, however, no electrons are ejected, no matter how great the light intensity.

6.2.1 Einstein's Quantum Theory of Light

Definition 6.4 (Photon). *A thought particle which two important properties*

- *Do not have any mass*

- *Exhibit interference effects*

According to Einstein, each photon carries a parcel of maximum kinetic energy according to the following equation:

$$E_{photon} = hf \tag{5}$$

Where f is the frequency of the wave and h is the planck constant.

$h = 6.63 \times 10^{-34} J \times s$
High intensity of light only means high amount of protons collide with metal

The absorption of light by an electron is just like a collision between two particles, a photon and an electron. Each photon is only responsible for removing one electron from the piece of matel.

If

$$\text{Energy of the Proton} < W$$

even with high intensity, no electron will be eject

In any other cases, electron will be ejected. As a result, we get the formula:

$$W = hf_0$$

and the E_k of the electron can be calculated as this:

$$E_k = E_{photon} - W$$
$$E_k = hf - hf_0$$
$$E_k = h(f - f_0)$$

This is a straight line. Hence, the kinetic energy of an ejected electrons should be linearly proportional to f.

6.3 Photons Possess Energy and Momentum

Einstein's quantum theory states that light energy can only be absorbed or emitted in discrete parcels, that is, as single photons. The classical theory of electromagnetic waves predicts that a light wave with Energy E also carries a certain amount of momentum:

$$P = \frac{E}{c}$$

Einstein's quantum theory predicts that the momentum of a single photon is:

$$P_{Photon} = \frac{hf}{c} \tag{6}$$

The wavelength of a light wave is related to is frequency as:

$$f\lambda = c$$

We can sub this equation for c in the photon momentum equation:

$$P_{Photon} = \frac{hf}{c}$$
$$P_{Photon} = \frac{hf}{f\lambda}$$
$$P_{Photon} = \frac{h}{\lambda} \quad (7)$$

6.3.1 Evidence of Photon Momentum

Instead of using visible light, Compton directed a beam of high-energy X ray photons at a thin metal foil. The foil ejected both electrons and lower-energy X-ray photons. This effect, in which incident X-ray photons lose energy and scatter of a metal foil along with free electrons, is called the **Compton effect**

X-ray photon acts like a particle in an elastic collision with an electron in the metal.

The electron absorb low-frequency x-ray and collide with high-frequency x-ray.

After the collision, the photon emerges from the collision with lower energy and a different momentum. The electron deflects with the kinetic energy and momentum lost by the photon.

Definition 6.5 (Compton effect)**.** *The scattering of a photon by a free or weakly bound electron, in which total energy and momentum are conserved, leading to a change in the photon's wavelength.*

Compton's data indicated that the effect conserves both energy and momentum. Compton had to use equations of special relativity to analyze the collsion, including the equation for relativistic momentum. He would not have obtained the correct results without using special relativity. In this way, Einstein's ideas on relativity and the speed of light influenced work that confirmed Einstein's ideas about the behaviour and characteristics of photons.

6.3.2 Photon Interactoin

When a photon comes into ocntact with matter, an interaction takes place. Five main interactions can occur:

1. A photon may simply reflect, as when photons of visible light undergo perfectly elastic collisions with a mirror.
2. A photon may free an electron and be absorbed in the process, as in the photoelectric effect
3. A photon may emerge with less energy and momentum after freeing an electron. After this interaction with matter, the photon still travels at the speed of light but with less energy and a low frequency. This is the Compton effect.
4. A photon may be absorbed by an individual atom and elevate an electron to a higher energy level within atom. The electron remains within the atom but is in what is called an excited state.
5. A photon can undergo pair creaton, where it becomes converted into two particles with mass. This process conserves energy and momentum because all the energy of the photon becomes converted into the kinetic energy of the new particles and their rest mass energy.

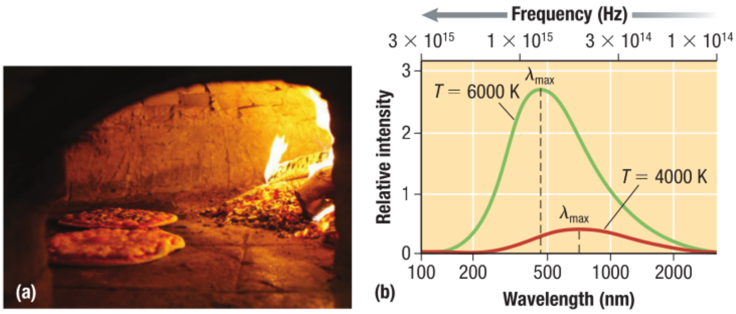
Definition 6.6 (Pair Creation). *the transformation of a photon into two particles with mass.*

6.4 Blackbody Radiation

Definition 6.7 (Blackbody). *An object that absorbs all radiation reaching it.*

Definition 6.8 (Blackbody Radiation). *Radiation emitted by a blackbody.*

The specific problem that puzzled Planck is represented by the glowing oven. This oven emits radiation over a range of wave-lengths and frequencies. To the eye, the colour of the oven is determined by the wavelength of the largest radiation intensity.



Experiments prior to Planck's work showed that the intensity curve has the same shape for a wide variety of objects. The blackbody intensity falls to zero at both long and short wavelengths, corresponding to low and high frequencies, respectively, with a peak in the middle. Planck tried to explain this behaviour.

At this time, physicists knew that electromagnetic waves from standing waves as they reflect back and forth inside an oven. These standing waves are just like the standing waves on a string. Standing waves on a string have frequencies that follow the pattern:

$$f_n = nf_0$$

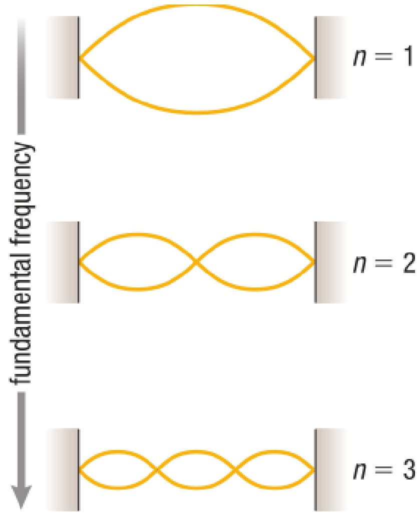
, where f_0 is the fundamental frequency and $n \in \mathbb{Z}$

According to the classical physics, each of these standing waves carries energy, and as their frequency increases, so does the total energy. As a result, the classical theory predicts that the blackbody intensity should become infinite as the frequency approaches infinite values.

6.4.1 Planck's Hypothesis

The classical theory would be in conflict with the experimental intensity curves in the graph. The true intensity falls to zero at high frequencies.

Planck resolved this disagreement by hypothesizing that the energy in a blackbody comes in discrete parcels (quanta). He believed that each parcel has energy equal to hf_n, where f_n is one of the standing wave frequencies and h is a universal constant. This explanation can correctly predit the behaviour of wave in the experiment. However, he could give no reason or justification for his assumption about standing-wave quanta.



6.5 Wien's Law

The wavelength at which the radiation intensity of a blackbody is largest is donoted by λ_{max} and is determined by the temperature, T, of the blackbody through ane expression called Wien's law.

$$\lambda_{max} = \frac{2.90 \times 10^{-3} m * K}{T} \tag{8}$$

7 Wave Properties of Classical Particles

7.1 Wave-like Properties of Classical Particles

In 1924, Louis de Broglie first suggested that all classical particles have wave-like Properties. In the previous section, you read that a photon has a momentum given by

$$P_{photon} = \frac{h}{\lambda}$$

De Broglie turned this result around and hypothesized that a particle with momentum p has a wavelength of

$$\lambda = \frac{h}{p} \tag{9}$$

Definition 7.1 (de Broglie wavelength)**.** *The wavelength associated with the motion of a particle possessing momentum of magnitude p*

If a particle has a wavelength, the particle should exhibit interference just as wave do.

According to his equation, a longer wavelength means that the value for momentum has to be small.

Definition 7.2 (Matter wave)**.** *The wave-like behaviour of particles with mass*

In 1927, physicists Clinton Davisson and Lester Germer performed an experiment in which they aimed a beam of electrons at a crystal target. the atoms in the target were space at regular intervals, acting as a series of slits for the electrons. Just as with the diffraction of light, the Davisson-Germer experiment exhibts interference when the wavelength of electrons is similar to the spacing between the atoms in the crystal. The diffraction technique used in the Davisson=Germer experiment is still used today as a way to measure molecule spacing within a crystal.

It is possible to determine the de Broglie wavelength of larger objects, such as baseballs. However, the momentum of large objects tends to be so large that it implies an incredibly small wavelength. That is why we are unable to see the interference of these objects.

7.2 Interpreting the Double-Slit Experiment

7.2.1 Collapse Interpretation

The electron leaves its source behaving as a particle, but then it spreads out and travels as a wave until it is measured at the screen.

This law claims that an electron physically changes from a particle to a wave and back again. These two behaviours and the physical laws that go with them alternative in a way that is not predicted by quantum mechanics.

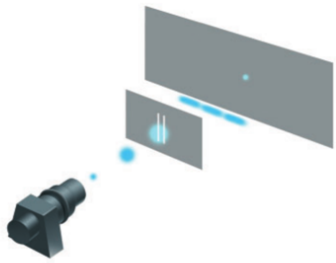


Figure 2 In the collapse interpretation, each electron in a double-slit experiment travels as a spread-out wave.

7.2.2 Pilot Wave Interpretation

The electron is just a simple particle whose motion is described by a single law. The motion of the electron depends on a mysterious pilot wave. To obtain the interference pattern in the double-slit experiment, the behaviour of the pilot wave must depend on everything everywhere in the universe, including future events. For example, the pilot wave "knows" whether one or two slits are open, and whether or not a detector is turned on at the screen.

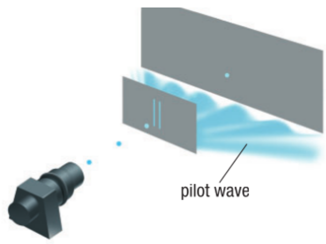


Figure 3 In the pilot wave interpretation, the electrons in a double-slit experiment are particles whose motion depends on a pilot wave.

7.2.3 Many Worlds Interpretation

A parallel universe exists for each of the electron's possible states. The Universe constantly splits into many versions of itself.

7.2.4 Copenhagen Interpretation

This interpretation view interprets the physical laws in terms of information about actual measurement made on a quantum-mechanical system. Certain questions do not have answers, such as what electrons are "doing" as they travel to the detection screen. You can only ask what the results will be if you do a certain experiment.

7.3 The Wave Function: A mathematical Description of Wave-Particle Duality

A wave function gives the probability for a particle to take any possible path, or for the particle to show up at any possible location on the detection screen in the double-slit experiment.

Wave fuction can be calculated by an equation from Erwin Schrodinger. Researchers use the Schrodinger equation to determine the wave function and how it varies with time.

Consider an electron confined to a particular region of space. A classical particle moving around inside the box would simply travel back and forth, bouncing from one wall to another. The wave function for a particle-wave inside this box is described by standing waves, similar to those you would see on a string.

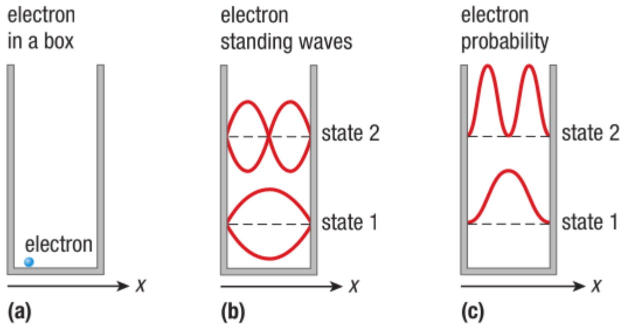


Figure 4 (a) A thought experiment of an electron that is trapped in a box. (b) The electron wave probability function forms a standing particle–wave similar to the standing waves on a string fastened to the walls of the box. The electron wavelength must therefore "fit" into the box as it would for a standing wave. (c) The quantum-mechanical probabilities of finding the electron at different locations in the box correspond to each of the two wave functions in (b).

The figure 4(b) shows two possible wave function solutions corresponding to electrons with different kinetic energies. The wavelengths of these standing waves are different, since the wavelength of an electron depends its kinetic energy. In the case of mechanical waves in classical mechanics, these two solutions correspond to two standing waves with different wavelength.

After get the wave function for a particular situation, such as for the electron in (b), you can try to calculate the position and speed of the electron. However, the results do not give a simple single value for x. The probability of finding the electron at certain value of x is large in some regions and small in others. The probability distribution is different for each wave function.

7.4 The Heisenberg Uncertainty Principle

Theorem 7.3 (Heisenberg Uncertainty Principle)**.** *A mathematical statement that says that if Δx is the uncertainty in a particle's position, and Δp is the uncertainty in its momentum, then*

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

where h is Planck's constant

This principle says that there is a limit to how accurately simultaneous measurements of the position and momentum of a quantum object can be.

If you measure the position of a quantum object with great accuracy, then you can only measure its momentum with little accuracy.

The act of measuring the system itself disturbs the system.

8 The Standard Model of Elementary Particles

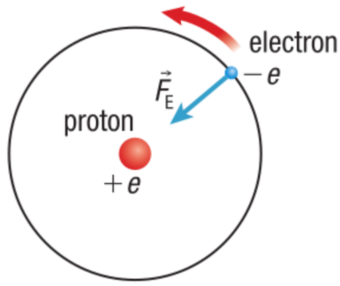
8.1 Ernest Rutherford

In 1909, Ernest Rutherford and his students did an experiment. They aimed high-speed, positively charged particles at a thin sheet of gold foil. Under the traditional theory, Rutherford and his team expected most of the particles to pass through the foil.

Instead, they discovered that a small number of particles were deflected. Rutherford realized that this result meant that all the positive charge in an atom must be concerntrated in a very small volume.

Rutherford suggested that the atom is like a miniature solar system, with electrons orbiting the nucleus just as planets orbit the sun. The model asl proposed that they could move in any orbit.

The strong force hold the protons together.

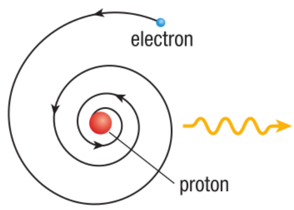


The electrons must move in orbits to avoid "*falling*" into the nucleus as a result of the electric force.

8.2 Problems with the Planetary Model

Maxwell's classical theory of electromagnetic predicts that an electron emits electromagnetic radiation when it orbits a proton. The radiation carries away energy. If the electron in a hydrogen atom loses energy in this way, Newtonian mechanics predicts that it will spiral inward to the nucleus.

If this model were correct, all atoms would collapse, which is not the case. Physicists were unable to modify the planetary model to make the atoms stable.



8.3 The Bohr Model of the Atom

Bohr proposed a quantum-mechanical approach to the motion of electrons within the atom. He was inspired by the Planck-Einstein introduction of quanta into the theory of electromagnetic radiation. Bohr's theory went against the well-established classical laws of mechanics and electromagnetism.

Bohr proposed that an electron in an atom can have only certain orbits with particular values for the radius of each orbit. The special values of the orbital radius meant that the electron could only have special values of potential energy and kinetic energy. The total energy could take on only certain discrete, quantized values. Each value of energy corresponds to what is now called an energy level.

Bohr's model was partially successful. It provided a physical model of the hydrogen atom. The model matched the internal energy levels to the levels observed in a hydrogen spectrum. At the same time, the model accounted for the stability of the hydrogen atom.

However, Bohr's model, was incomplete. When applied to atoms with many electrons, the model broke down.

An explanation for the BOhr model came from de Broglie's matter waves, which were developed 10 years after Bohr's work. As mentioned in previous chapter, the electrons in a given energy level have special values of kinetic energy. Therefore, those electrons have certain values of momentum.

Using de Broglie's model, the allowed electron orbits in hydrogen correspond exactly to those orbits in which electron waves from circular standing waves around the nucleus.

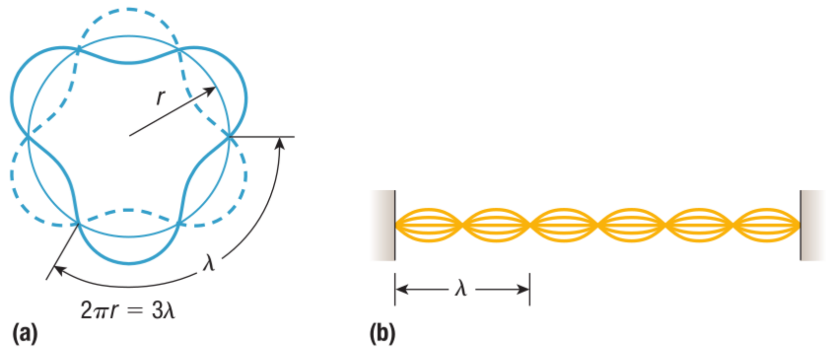


Figure 3 (a) The standing-wave pattern for an electron wave in a stable orbit of hydrogen requires that the orbital circumference equal a whole number of wavelengths. (b) The standing-wave pattern for a string fixed at both ends requires that the distance between supports equal a whole number of wavelengths.

8.4 Twenty-First-Century Physics and Antimatter

Definition 8.1 (Antimatter). *a form of matter in which each particle has the same mass and an opposite charge as its counterpart in ordinary matter*

Two examples of antimatter are

Anti-protons
Anti-neutrons

Although the neutron and anti-neutron are both neutral, they are different particles. Neutron is made up of a certain combination of quarks, and the anti-neutron is made up of the corresponding combination of anti-quarks

Table 1 Some Properties of Electrons, Protons, Neutrons, and Their Anti-particles

Particle	Symbol	Mass (kg)	Mass (MeV/c^2)	Charge
electron	e^-	9.109×10^{-31}	0.511	-1
positron	e^+	9.109×10^{-31}	0.511	$+1$
proton	p	1.673×10^{-27}	938	$+1$
anti-proton	$\bar{p}$	1.673×10^{-27}	938	-1
neutron	n	1.675×10^{-27}	940	0
anti-neutron	$\bar{n}$	1.675×10^{-27}	940	0

Anti-particles offer researchers a chance to see special relativity at work at the microscopic level. For example, when an electron encounters its anti-particle, the positron, the two undergo a reaction that destroys both particles.

8.5 The Standard Model

Definition 8.2 (Quark). *an elementary particle that makes up protons, neutrons, and other hadrons.*
Was first discovered in collision experiments involving protons

Definition 8.3 (Hadrons). *A class of particles that contains the neutron, the proton, and the pion; composed of combinations of quarks and anti-quarks*

- *Hadrons composed of three quarks are called baryons.*
- *A quark and an anti-quark can also combine to form a particle. Hadrons composed of just two quarks are called mesons.*

Definition 8.4 (Leptons). *A class of particles that includes the electron, the muon, the tauon, and the three types of neutrinos; not composed of smaller particles*

Definition 8.5 (Standard model). *the modern theory of fundamental particles and their interactions*

When a high-energy electron collides with a proton, the way that the electron scatters (that is, its outgoing direction and energy) gives information about how mass and charge are distributed inside the proton.

All hadrons are composed of quarks, so the interactions between quarks determine the properties of hadrons and how they relate to one another. The two most important hadrons are the proton and the neutron, so the behaviour of quarks also determines the properties of nuclei.

Quarks are charged, so they act on each other through the electric force. They also interact through the strong force mentioned earlier. Quarks bind together to form protons and neutrons (nucleons), and the strong force is responsible for holding protons and neutrons together to make nuclei.

8.5.1 Leptons

Leptons are naturally grouped into three pairs:

- Electron and Electron neutrino
- Muon and Muon neutrino
- Tau and Tau Neutrino

The muon and the tau are not stable. Electrons are stable, but the behaviour of neutrinos is more complicated. Neutrinos travel through space, they change from one to another of the three types of neutrinos, an effect called neutrino oscillation.

8.5.2 Bosons: Force Mediating Particles

The fundamental forces of nature are the ways in which individual particles interact with each other. Every interaction in the universe can be described using only three forces.

- Strong nuclear force
- Weak nuclear force
- electromagnetism

The strong nuclear force holds the subatomic particles of the nucleus together.

The weak nuclear force causes radioactive decay and starts the process of hydrogen fusion and other nuclear process in star

The electromagnetic force is responsible for the attraction and repulsion among electrical charges.

Definition 8.6 (Fermion)**.** *A fundamental particle that forms matter*
Including quark and leptons

Definition 8.7 (Bosons)**.** *The particle responsible for transmitting electromagnetic, strong, and weak force.*

From this view,

- The electromagnetism force is "mediated" by the photon, which means that photons transmit the electromagnetic force acting on charged particles.
- The strong nuclear force is mediated between quarks by particles called Gluons. Eight different types of gluons exist, and the force exerted between two quarks depends on type of each quark.
- The weak nuclear force is mediated by a family of three particles called the W^+, W^- and Z bosons. Unlike the photon and gluons, which have zero mass, the W^+, W^- and Z bosons have mass.

According to the standard model, another boson, the Higgs boson, exists.

Definition 8.8 (Higgs Bosons)**.** *the theoretical particle thought to play a role in giving mass to other particles.*