Physics

SPH4U

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Here is the note for my SPH4U class. Thanks for Mr. McCumber for the materials and teaching.

Contents

1 Review of Describing and Graphing Motion

1.1 Position: $\vec{d}$

Position is the **straight-line distance** from a fixed reference point to a location, with a direction to the location from the reference point.

1.2 displacement: $\Delta\vec{d}$

Displacement is the **change of position**
Formula:

$$\Delta\vec{d} = \vec{d_2} - \vec{d_1}$$

or

$$n = \text{the amount of displacement you want to add}$$

$$\Delta\vec{d_{tot}} = \sum_{i=1}^{n} \Delta\vec{d_i}$$

1.3 Velocity: $\vec{v}$

Velocity is the **rate of change of position**

$$\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$$

1.4 Acceleration: $\vec{a}$

Acceleration is the *rate of change* of velocity, always in the form of $\frac{m}{s^2}$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

1.5 Graphing motion

For a **Position vs Time** graph:

- **For a position/displacement vs time graph, the velocity = the *slope***
- A slope of zero = The object is not moving
- Instantaneous velocity ($\vec{v}_{inst}$) = the slope of **tangent** line to the graph at that point in time
- Average velocity ($\vec{v}_{avg}$) = the slope of **secant** line for that time interval

For a **Velocity vs Time** graph:

- Can get an object's instantaneous velocity directly from the graph
- slope = **acceleration**
- Displacement = The **area** between the graph and the time-axis for that time interval

2 Equations of Motion

To start off, there are five equations that are used in the calculation of motion

$$\vec{v_f} = \vec{v_i} + \vec{a} * \Delta t$$

$$\Delta\vec{d} = \frac{1}{2}(\vec{v_i} + \vec{v_f}) * \Delta t$$

$$\Delta\vec{d} = \vec{v_i}\Delta t + \frac{1}{2}\vec{a} * \Delta t^2$$

$$\Delta\vec{d} = \vec{v_f}\Delta t - \frac{1}{2}\vec{a} * \Delta t^2$$

$$\vec{v_f}^2 = \vec{v_i}^2 + 2\vec{a}\Delta\vec{d}$$

2.1 Format requirements for answering Motion questionss

(i) You should always include a diagram that contains every known information from the question

(ii) $\vec{v}$ should be presented for at least two decimal places

(iii) Always list steps in your answer

(iv) When using quadratic solving function on calculator, always write **Using Quadratic Eq* in your answer

(v) When you form an equation system, you should label 1.2.3. on each equation in the system

(vi) Follow the Sig Digit rules

- ! For **add** and **subtract**, keep the least **decimal places**
- ! For **multiply** and **divide**, keep the least amount of **signficiant digit**

3 Adding and Subtracting 2-Dimensional Vectors

3.1 Vector addition and subtraction key words

+ Addition: Find "the **resultant**", "the total", or "the net".

- Subtraction: Find "the **difference**" or "the change in".

3.2 Steps for solving a vector problem

(i) Read the question carefully

(ii) Show unit conventions

(iii) Write "givens" (It helps to roughly sketch each vector and their compoents)

(iv) Set direction conventions

(v) Solve for each compoents (ex. $\Delta\vec{d_{1y}}, \Delta\vec{d_{2x}}$)

(vi) Choose one component direction (ex. Just the 'x' direction) and solve the equations for that direction

(vii) Repeat with the other direction

(viii) Sketch your resulting x and y vectors, joining them head-to-tail.

(ix) Calculate the magnitude and direction of the resultant. (Trigonometry)

(x) State the final answer, including the real-world direction

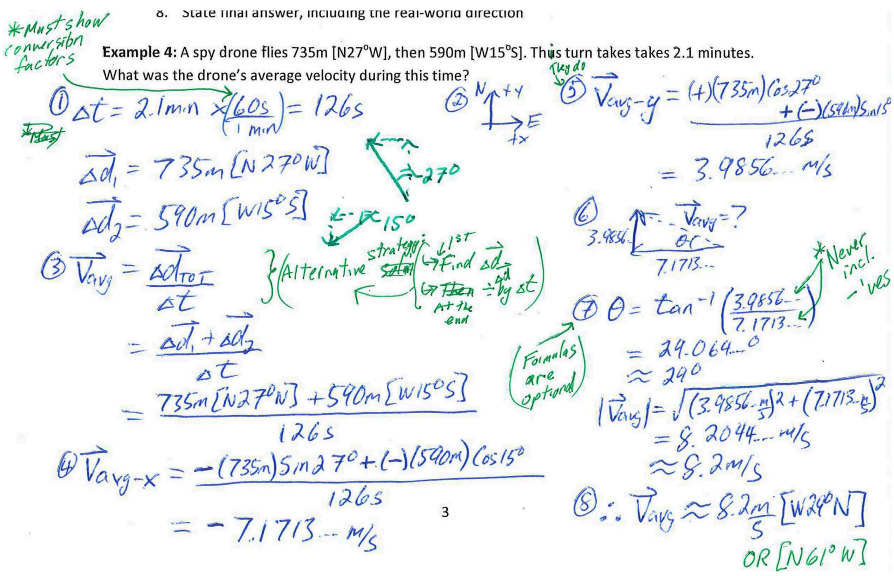
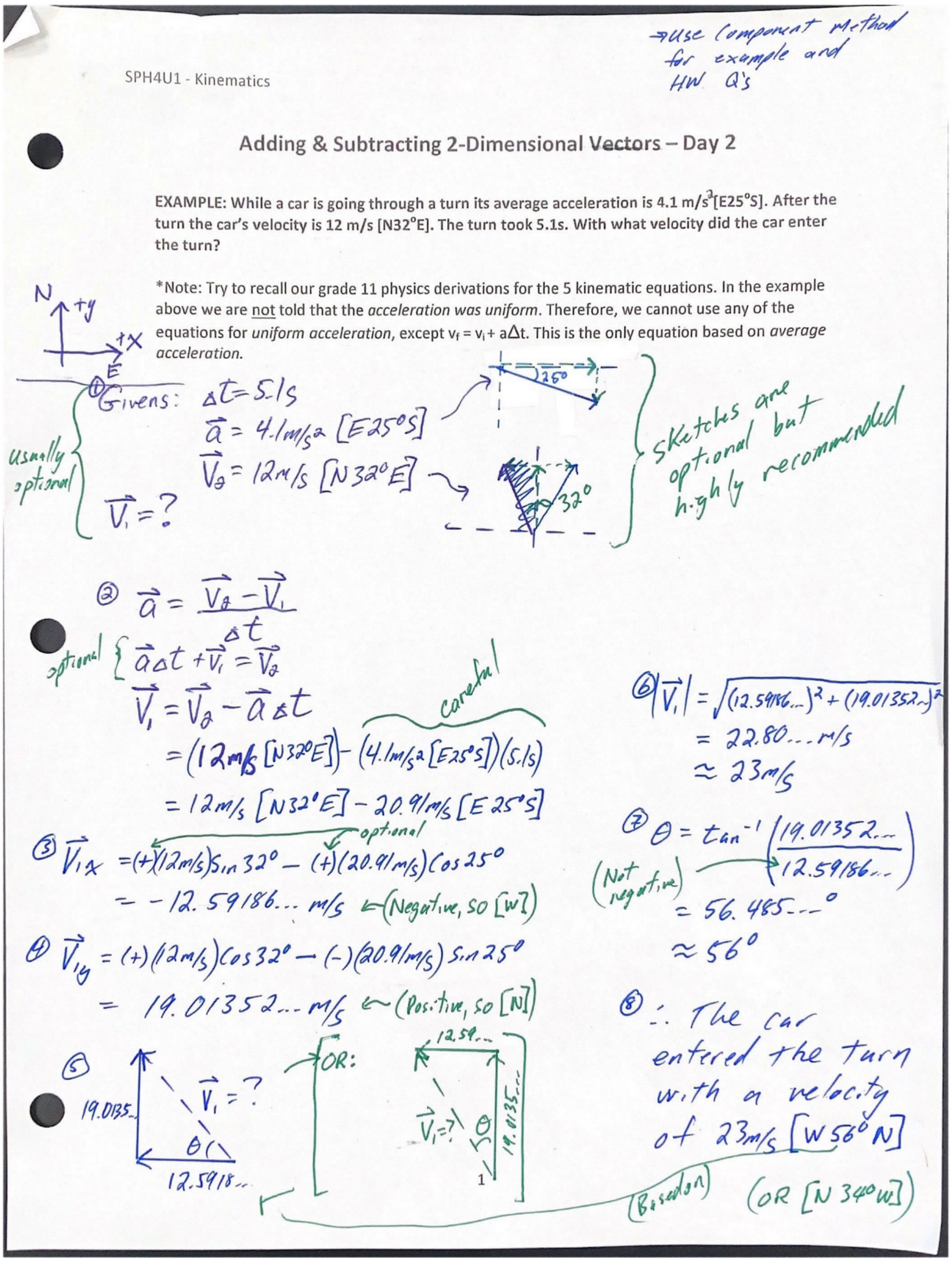


Figure 1: A sample answer for vector question

3.3 Another question type

Here is another question type:



→use component method for example and HW Q's

SPH4U1 - Kinematics

Adding & Subtracting 2-Dimensional Vectors – Day 2

EXAMPLE: While a car is going through a turn its average acceleration is 4.1 m/s^2[E25°S]. After the turn the car's velocity is 12 m/s [N32°E]. The turn took 5.1s. With what velocity did the car enter the turn?

*Note: Try to recall our grade 11 physics derivations for the 5 kinematic equations. In the example above we are not told that the *acceleration was uniform*. Therefore, we cannot use any of the equations for *uniform acceleration*, except $v_f = v_i + a\Delta t$. This is the only equation based on *average acceleration*.

N +y, +x E

usually optional

① Givens: $\Delta t = 5.1s$

$\vec{a} = 4.1 m/s^2$ [E25°S]

$\vec{V}_2 = 12 m/s$ [N32°E]

$\vec{V}_1 = ?$

25°, 32°

sketches are optional but highly recommended

② $\vec{a} = \dfrac{\vec{V}_2 - \vec{V}_1}{\Delta t}$

optional $\{ \vec{a}\Delta t + \vec{V}_1 = \vec{V}_2$

$\vec{V}_1 = \vec{V}_2 - \vec{a}\Delta t$

careful

$= (12 m/s [N32°E]) - (4.1 m/s^2 [E25°S])(5.1s)$

$= 12 m/s [N32°E] - 20.91 m/s [E25°S]$

optional

③ $\vec{V}_{1x} = (+)(12 m/s)\sin 32° - (+)(20.91 m/s)\cos 25°$

$= -12.59186... m/s$ ← (Negative, so [W])

④ $\vec{V}_{1y} = (+)(12 m/s)\cos 32° - (-)(20.91 m/s)\sin 25°$

$= 19.01352... m/s$ ← (Positive, so [N])

⑤ 19.0135..., $\vec{V}_1 = ?$, θ, 12.5918...

OR: 12.59..., $\vec{V}_1 = ?$, θ, 19.0135...

⑥ $|\vec{V}_1| = \sqrt{(12.5918...)^2 + (19.01352...)^2}$

$= 22.80... m/s$

$\approx 23 m/s$

⑦ $\theta = \tan^{-1}\left(\dfrac{19.01352...}{12.59186...}\right)$

(Not negative)

$= 56.485...°$

$\approx 56°$

⑧ ∴ The car entered the turn with a velocity of 23 m/s [W56°N]

(Based on) (OR [N34°W])

Figure 2: Remainder, multiply first

4 Frame of Reference

4.1 1 Dimension Frame of Reference

Definition 4.1 (Frame of Reference)**.** The motion of objects is measured based on a perspective/location that we considered to be motionless/stopped. This perspective, from which object's motion are measured is called Frame of reference.

For certain Chase and Collision questions, you can use frame of reference to solve them

Example 4.2. How to write givens

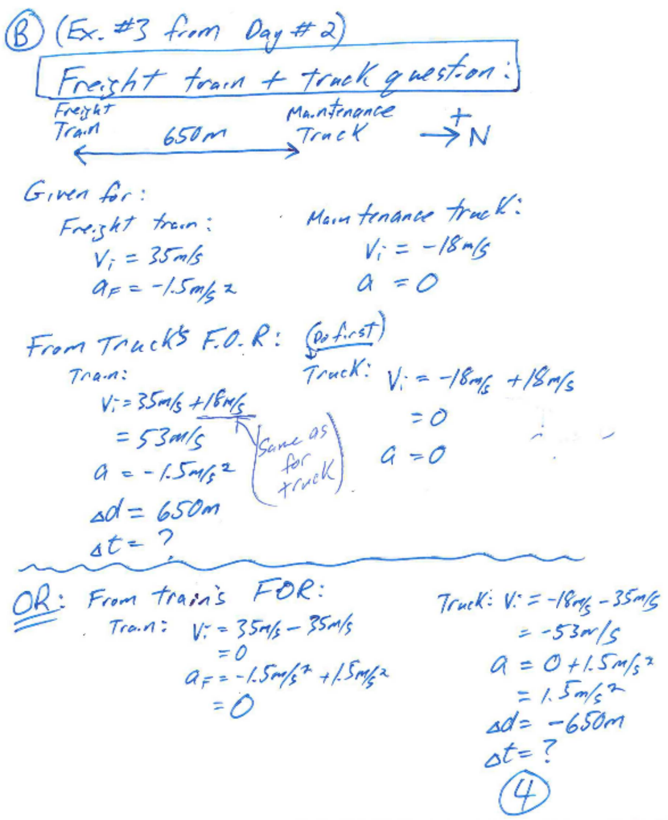


Figure 3: You should always write it

5 Relative Velocities in Two Dimensions

5.1 Recall

(i) All velocities are related to an F.O.R that we consider to be motionless or stopped

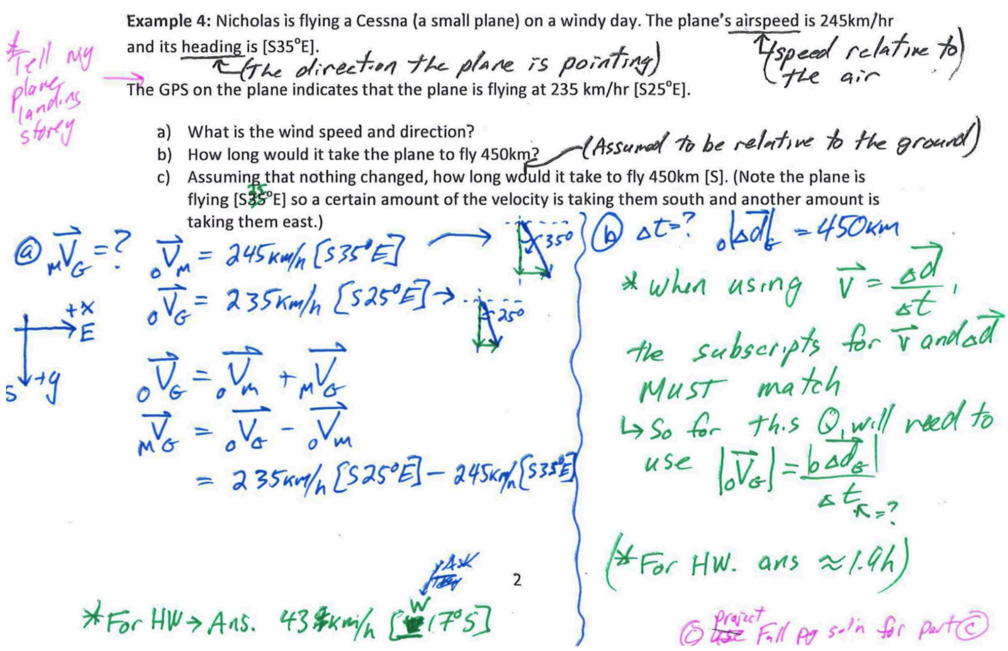
(ii) A frame of reference is just a perspective from which we observe/or measure the motion of objects

5.2 Definition

We sometimes refer to something that another thing can travel through as *medium*. We also sometimes refer to the thing that is moving through the medium as *object(O)*, and the ground use the symbol *(*G)

Therefore the equation can be written as:

$$v\vec{OG} = v\vec{OM} + v\vec{MG}$$



Example 4: Nicholas is flying a Cessna (a small plane) on a windy day. The plane's airspeed is 245km/hr and its heading is [S35°E].

The GPS on the plane indicates that the plane is flying at 235 km/hr [S25°E].

a) What is the wind speed and direction?

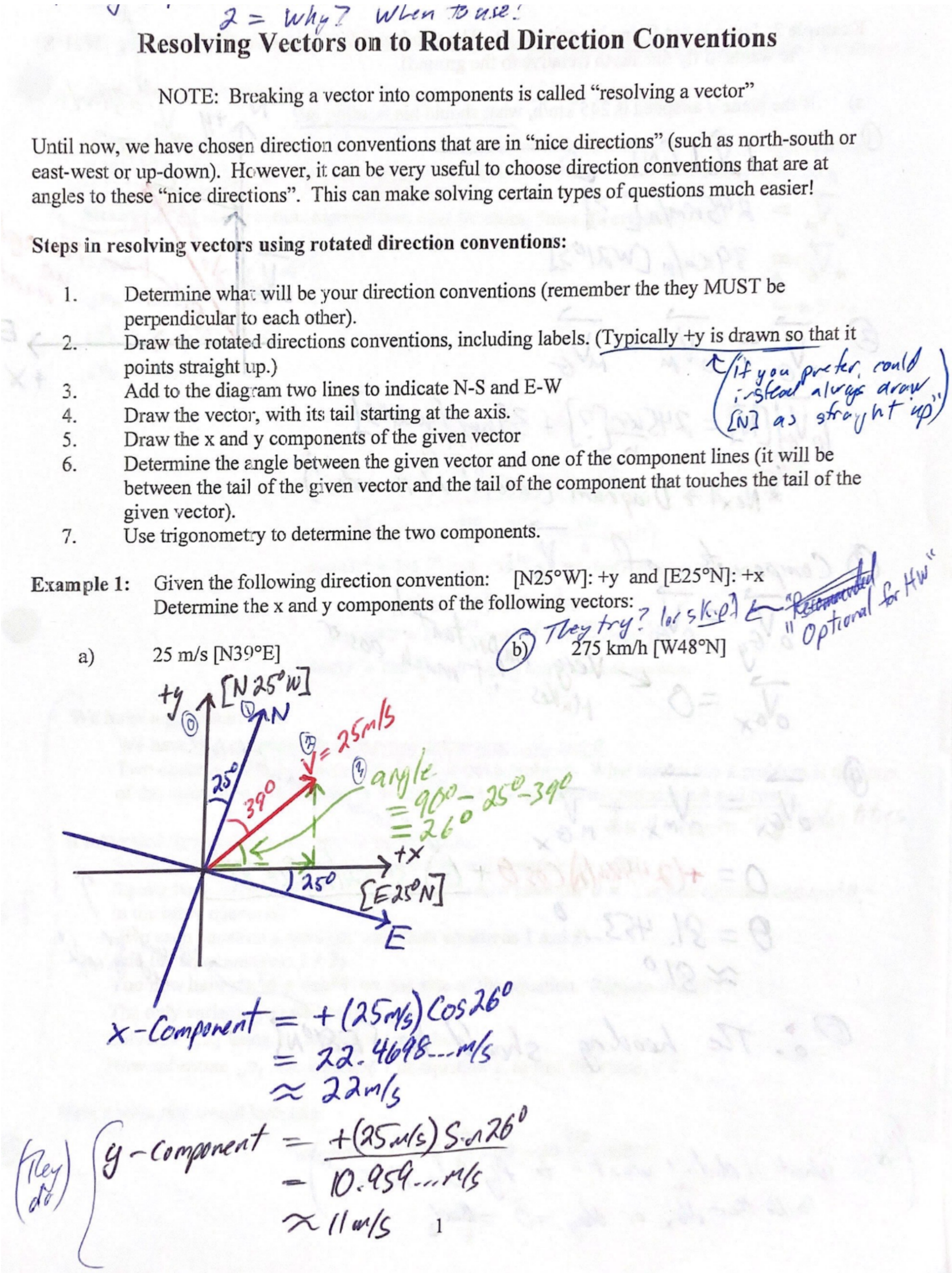
b) How long would it take the plane to fly 450km?

c) Assuming that nothing changed, how long would it take to fly 450km [S]. (Note the plane is flying [S35°E] so a certain amount of the velocity is taking them south and another amount is taking them east.)

2

Figure 4: An example question

6 F.O.R in 2-D



2 = why? When to use!

Resolving Vectors on to Rotated Direction Conventions

NOTE: Breaking a vector into components is called "resolving a vector"

Until now, we have chosen direction conventions that are in "nice directions" (such as north-south or east-west or up-down). However, it can be very useful to choose direction conventions that are at angles to these "nice directions". This can make solving certain types of questions much easier!

Steps in resolving vectors using rotated direction conventions:

1. Determine what will be your direction conventions (remember the they MUST be perpendicular to each other).
2. Draw the rotated directions conventions, including labels. (Typically +y is drawn so that it points straight up.) ("If you prefer, could instead always draw [N] as straight up")
3. Add to the diagram two lines to indicate N-S and E-W
4. Draw the vector, with its tail starting at the axis.
5. Draw the x and y components of the given vector
6. Determine the angle between the given vector and one of the component lines (it will be between the tail of the given vector and the tail of the component that touches the tail of the given vector).
7. Use trigonometry to determine the two components.

Example 1: Given the following direction convention: [N25°W]: +y and [E25°N]: +x
Determine the x and y components of the following vectors:

a) 25 m/s [N39°E]

b) 275 km/h [W48°N] They try? (or skip) ← ~~"Recommended~~ "Optional for HW"

x-Component $= +(25\,m/s)\cos 26^0 = 22.4698...\,m/s \approx 22\,m/s$

(They do) y-Component $= +(25\,m/s)\sin 26^0 = 10.959...\,m/s \approx 11\,m/s$

1

Figure 5: Teacher's note

7 Review of Netwon's Laws of Montion

7.1 Netwon's First Laws

Inertia is an object's resistance to a change in its state of uniform Motion.

Example 7.1. An object at rest will remain at rest And an object in motion will continue to move in a straight line constant speed UNLESS a non-zero net force acts on the object

7.2 Newton's second Law

Newton's 2^{nd} Law is the formula that explains the behaviour of object when the forces on the object are not zero.
We can orgnize to these formulas:

$$\vec{a} = \frac{\sum \vec{F}}{m} \tag{1}$$

and

$$\sum \vec{F} = m * \vec{a} \tag{2}$$

$\vec{a}$ = acceleration of the object
m = mass of the object in kg
$\sum \vec{F}$ = The sum of the net force

7.3 Newton's Third Law

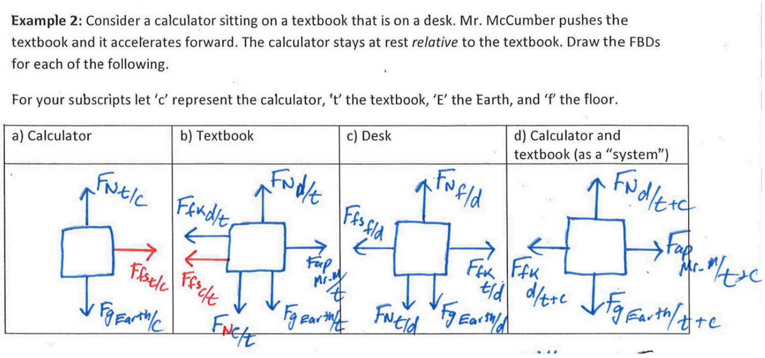
For every force, there is another force, which is *equal in magnitude* to the first force, but *opposite in direction*. These two forces will *act on separate objects*, unless they are "internal force"
This mean's that all forces Always come in pairs, but two forces may not be acting on the same object.

To fit the Newton's 3^{rd} Law pair forces, the two force must:

(i) Be the same type of force

(ii) $F_{\vec{A/B}} = -F_{\vec{B/A}}$

7.4 Free Body Diagrams (FBD)



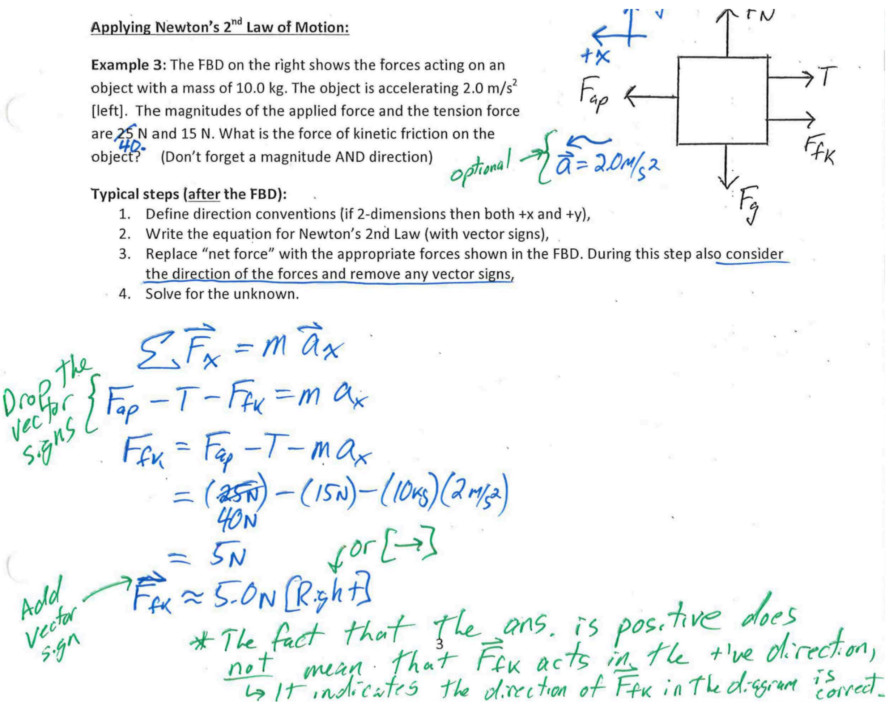
Example 2: Consider a calculator sitting on a textbook that is on a desk. Mr. McCumber pushes the textbook and it accelerates forward. The calculator stays at rest *relative* to the textbook. Draw the FBDs for each of the following.

For your subscripts let 'c' represent the calculator, 't' the textbook, 'E' the Earth, and 'f' the floor.

Figure 6: An example Free Body Diagram

7.5 Application of Newton's second Law

Here is an example:



Applying Newton's 2nd Law of Motion:

Example 3: The FBD on the right shows the forces acting on an object with a mass of 10.0 kg. The object is accelerating 2.0 m/s^2 [left]. The magnitudes of the applied force and the tension force are ~~25~~ 40 N and 15 N. What is the force of kinetic friction on the object? (Don't forget a magnitude AND direction)

Typical steps (after the FBD):

1. Define direction conventions (if 2-dimensions then both +x and +y),
2. Write the equation for Newton's 2nd Law (with vector signs),
3. Replace "net force" with the appropriate forces shown in the FBD. During this step also consider the direction of the forces and remove any vector signs,
4. Solve for the unknown.

optional $\vec{a} = 2.0\,m/s^2$

$$\Sigma \vec{F}_x = m\,\vec{a}_x$$

Drop the vector signs

$$F_{ap} - T - F_{fk} = m\,a_x$$

$$F_{fk} = F_{ap} - T - m\,a_x$$

$$= (\cancel{25N}\ 40N) - (15N) - (10kg)(2\,m/s^2)$$

$$= 5N$$

Add vector sign

$$\vec{F}_{fk} \approx 5.0N\ [Right] \text{ or } [\rightarrow]$$

* The fact that the ans. is positive does not mean that $\vec{F}_{fk}$ acts in the +'ve direction, ↳ It indicates the direction of F_{fk} in the diagram is correct.

Figure 7:

8 Review of Projectile Motion

8.1 basic

A simple projectile is an object that has a single, *non-uniform* acting on it. This single force must be a *non-contact* force between objects without the two objects being contact. Ex. Gravity forces, magnetic forces, electric forces

There are some kind of simple questions:

(i) An object dropped

(ii) A soccer ball is kicked

(iii) A bullet is fired from a gun

(iv) An electron is moving through a uniform electrical field (Gravity is negligible)

Remainder: The object only accelerate in the direction of the net force. In any direction perpendicular to net force, the object main a constant velocity.

Some important points to remember:

- At maximum height all projectiles have a **vertical** velocity equal to **zero**
- When an object starts and ends at the same vertical location, the $\Delta\vec{d}_y = 0$
- When an object is dropped or launched horizontally, then $\vec{v_{y1}} = 0$

8.2 Special formula

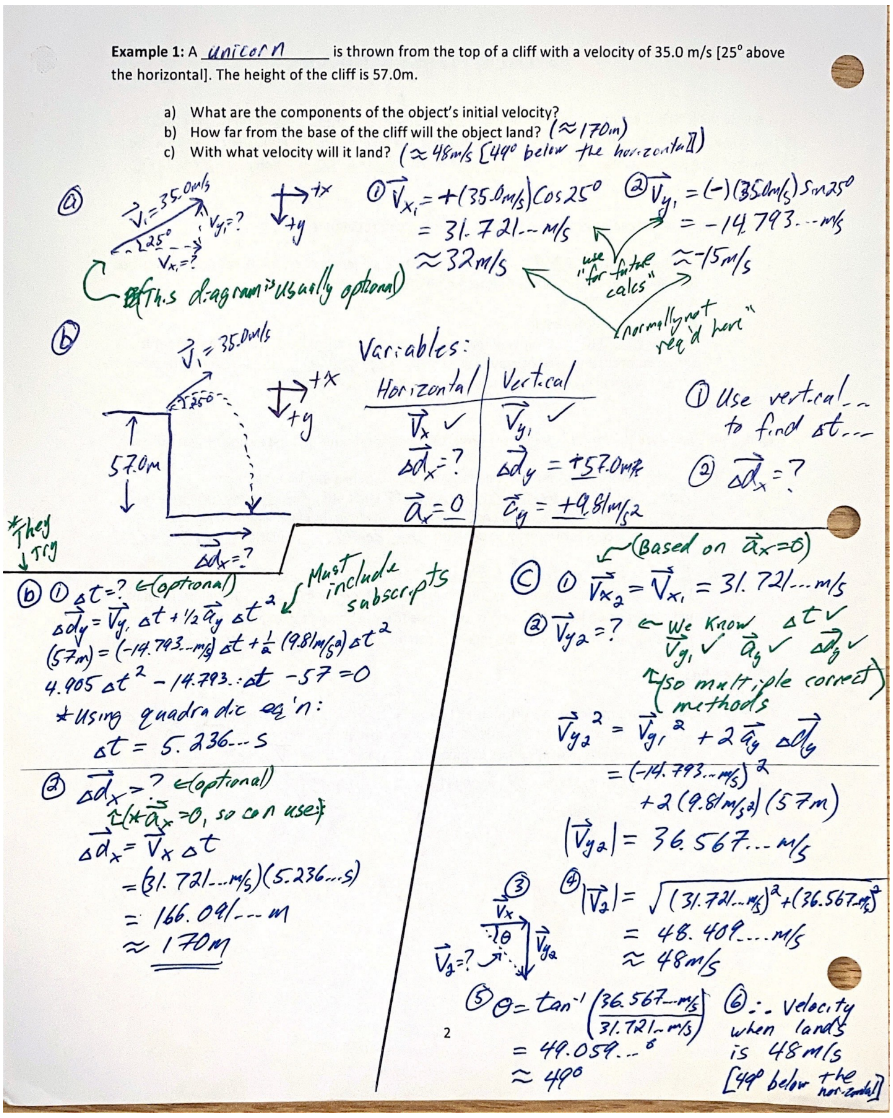
$$R = \frac{v_i^2 * sin2\theta}{g} \tag{3}$$

R is the range of the projectile (Horizontal distance in m)
v_i is the launch speed of the projectile in (m/s)
θ is the launch angle of the projectile
g is acceleration due to gravity

The formula 3 can only be used if the project starts and ends at the same vertical location

8.3 An example question

Remainder, you should always label your direction conventions


Example 1: A unicorn is thrown from the top of a cliff with a velocity of 35.0 m/s [25° above the horizontal]. The height of the cliff is 57.0m.

a) What are the components of the object's initial velocity?

b) How far from the base of the cliff will the object land? (≈170m)

c) With what velocity will it land? (≈48m/s [49° below the horizontal])

Figure 8:

9 Friction

In high school we deal with two types of friction: static and Kinetic

9.1 Kinetic Friction

Used when the two surfaces that are in contact are *slides relative to each other*.

$$F_{fk} = \mu_{fk} * F_n \tag{4}$$

Kinetic friction on an object can make an object slow down or speed up

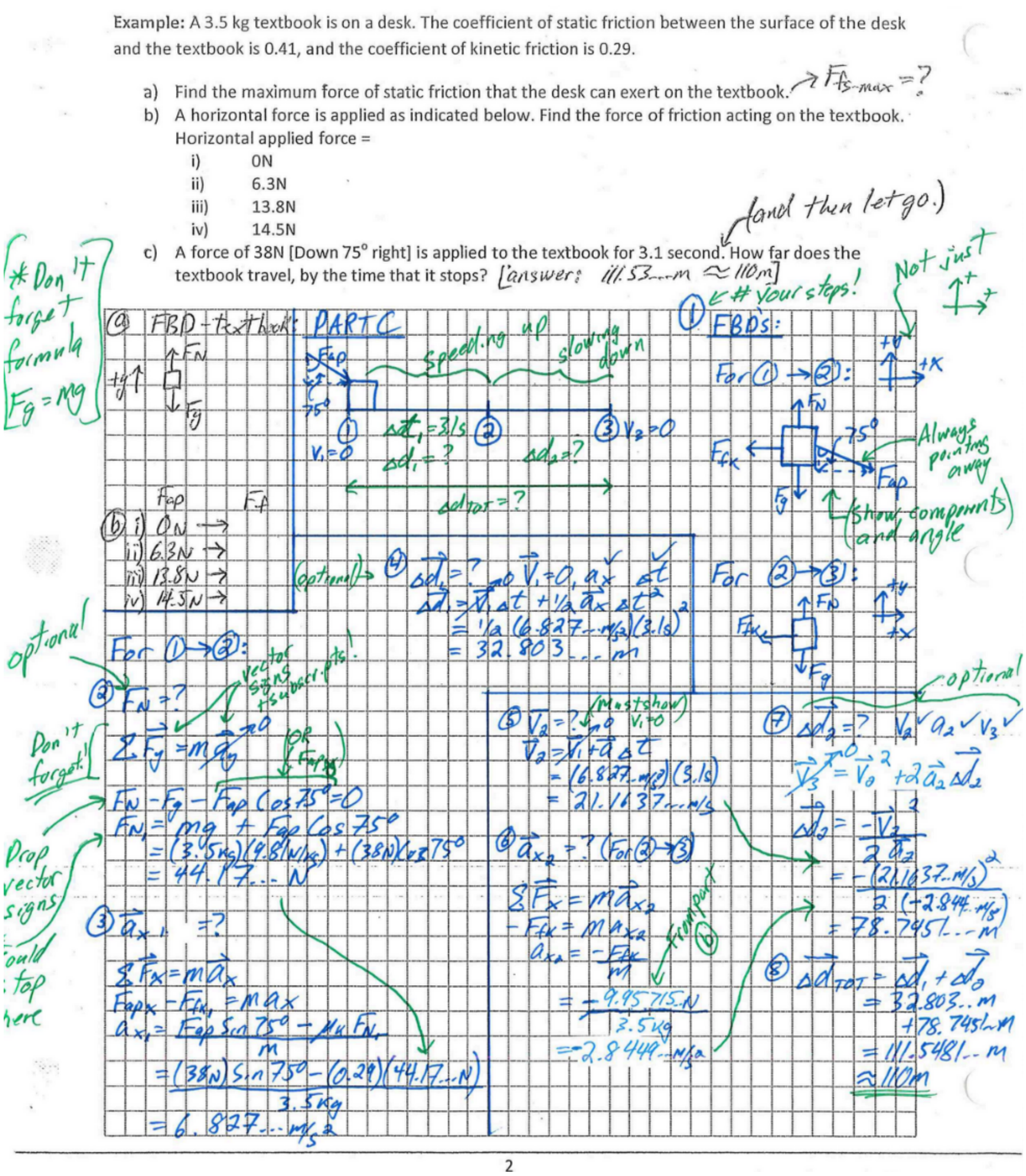
9.2 Static Friction

Used when the two surfaces are Not Sliding relative to each other

$$F_{fs.max} = \mu_{fs} * F_n \tag{5}$$

9.3 Remainder

Don't forget to mention to prove $\underline{F_N = F_g}$



Example: A 3.5 kg textbook is on a desk. The coefficient of static friction between the surface of the desk and the textbook is 0.41, and the coefficient of kinetic friction is 0.29.

a) Find the maximum force of static friction that the desk can exert on the textbook. → $F_{fs\text{-}max} = ?$

b) A horizontal force is applied as indicated below. Find the force of friction acting on the textbook. Horizontal applied force =

- i) 0N
- ii) 6.3N
- iii) 13.8N
- iv) 14.5N

c) A force of 38N [Down 75° right] is applied to the textbook for 3.1 second (and then let go.) How far does the textbook travel, by the time that it stops? [answer: 111.53...m ≈ 110m]

* Don't forget formula $F_g = mg$

(a) FBD - textbook

(b) i) 0N ii) 6.3N iii) 13.8N iv) 14.5N

PART C: speeding up, slowing down

① $\Delta t_1 = 3.1s$ ② ③ $v_2 = 0$; $v_1 = 0$; $\Delta d_1 = ?$; $\Delta d_2 = ?$; $\Delta d_{TOT} = ?$

① ← # your steps! FBD's: Not just ↑+ →+

For ① → ②: Always pointing away; Show components and angle

For ② → ③: optional

For ① → ②:

② $F_N = ?$

$\Sigma \vec{F}_y = m\vec{a}_y$ (vector signs + subscripts!) Don't forget!

$F_N - F_g - F_{ap}\cos 75° = 0$

$F_N = mg + F_{ap}\cos 75°$

$= (3.5kg)(9.81 N/kg) + (38N)\cos 75°$

$= 44.17...N$

Drop vector signs — could stop here

③ $\vec{a}_{x_1} = ?$

$\Sigma \vec{F}_x = m\vec{a}_x$

$F_{apx} - F_{fk_1} = ma_x$

$a_{x_1} = \dfrac{F_{ap}\sin 75° - \mu_k F_{N_1}}{m}$

$= \dfrac{(38N)\sin 75° - (0.29)(44.17...N)}{3.5kg}$

$= 6.827... m/s^2$

(optional) ④ $\Delta \vec{d}_1 = ?$; $\vec{v}_1 = 0$, a_x, Δt

$\Delta \vec{d}_1 = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}_x \Delta t^2$

$= \frac{1}{2}(6.827... m/s^2)(3.1s)^2$

$= 32.803... m$

⑤ $\vec{v}_2 = ?$ (Must show $v_1 = 0$)

$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$

$= (6.827... m/s^2)(3.1s)$

$= 21.1637... m/s$

⑥ $\vec{a}_{x_2} = ?$ (For ② → ③)

$\Sigma \vec{F}_x = m\vec{a}_{x_2}$

$-F_{fk} = ma_{x_2}$

$a_{x_2} = \dfrac{-F_{fk}}{m}$ (from part b)

$= \dfrac{-9.95715 N}{3.5 kg}$

$= -2.8449... m/s^2$

⑦ $\Delta \vec{d}_2 = ?$ v_2 ✓ a_2 ✓ v_3 ✓

$\vec{v}_3^{\,2} = \vec{v}_0^{\,2} + 2\vec{a}_2 \Delta \vec{d}_2$

$\Delta \vec{d}_2 = \dfrac{-\vec{v}_2^{\,2}}{2\vec{a}_2}$

$= -\dfrac{(21.1637... m/s)^2}{2(-2.844... m/s^2)}$

$= 78.7451... m$

⑧ $\Delta \vec{d}_{TOT} = \Delta \vec{d}_1 + \Delta \vec{d}_2$

$= 32.803... m + 78.7451... m$

$= 111.5481... m$

$\approx 110 m$

2

Figure 9:

10 Tension, compression and Pulleys

Internal Forces are Netwon's third Law pair of forces that act inside of an object or a system of objects that are in contact with each other.

According to the Netwon's third law, when we analyze the system, the sum of the forces equals 0. Thus, these forces will have no impact on the net force acting on the system and will have ***no effect on the system's acceleration***

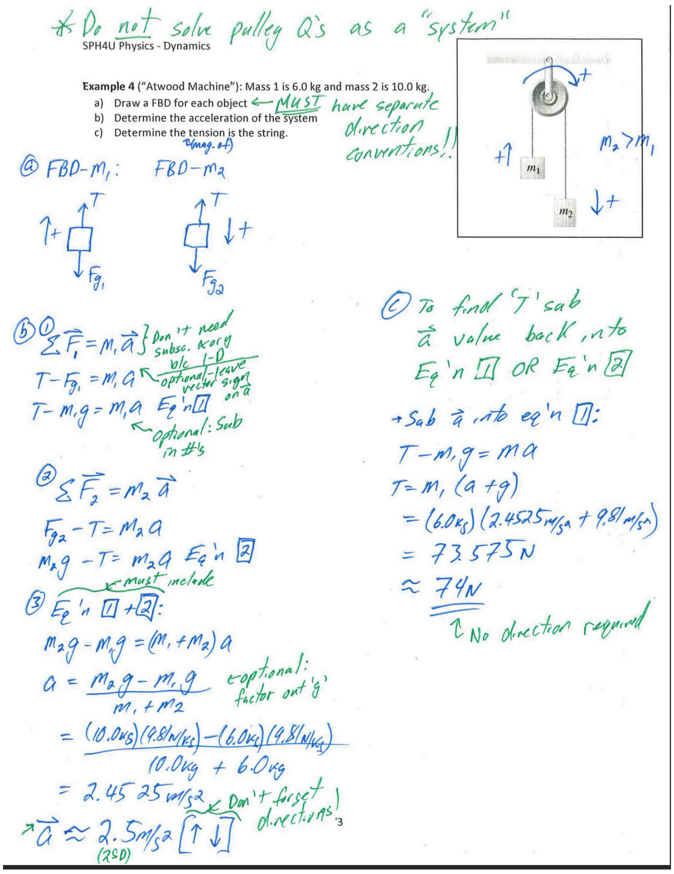
10.1 Tention: T

If one object is pulling the other object then you are dealing with **Tension**.

10.2 Compression: C

If one object is pushing the other object then you are dealing with **Compression**

Here is an example question



* Do not solve pulley Q's as a "system"

SPH4U Physics - Dynamics

Example 4 ("Atwood Machine"): Mass 1 is 6.0 kg and mass 2 is 10.0 kg.

a) Draw a FBD for each object ← MUST have separate direction conventions!!
b) Determine the acceleration of the system
c) Determine the tension is the string. (mag. of)

$m_2 > m_1$

ⓐ FBD-m_1: FBD-m_2

ⓑ ① $\Sigma \vec{F}_1 = m_1\vec{a}$ } Don't need subsc. for g b/c 1-D

$T - F_{g_1} = m_1 a$ ← optional: leave vector sign on a

$T - m_1 g = m_1 a$ Eq'n $\boxed{1}$ ← optional: Sub in #'s

② $\Sigma \vec{F}_2 = m_2\vec{a}$

$F_{g_2} - T = m_2 a$

$m_2 g - T = m_2 a$ Eq'n $\boxed{2}$ ← must include

③ Eq'n $\boxed{1} + \boxed{2}$:

$m_2 g - m_1 g = (m_1 + m_2) a$

$a = \dfrac{m_2 g - m_1 g}{m_1 + m_2}$ ← optional: factor out 'g'

$= \dfrac{(10.0\text{kg})(9.81\text{N/kg}) - (6.0\text{kg})(9.81\text{N/kg})}{10.0\text{kg} + 6.0\text{kg}}$

$= 2.4525\ \text{m/s}^2$ ← Don't forget directions!

$\vec{a} \approx 2.5\ \text{m/s}^2\ [\uparrow\downarrow]$ (2SD)

ⓒ To find 'T' sub $\vec{a}$ value back into Eq'n $\boxed{1}$ OR Eq'n $\boxed{2}$

→ Sub $\vec{a}$ into eq'n $\boxed{1}$:

$T - m_1 g = m a$

$T = m_1(a + g)$

$= (6.0\text{kg})(2.4525\ \text{m/s}^2 + 9.81\ \text{m/s}^2)$

$= 73.575\ \text{N}$

$\approx 74\ \text{N}$

↑ No direction required

3

Figure 10: Pulley question example

11 Inclined plane with Friction

To deal with inclined plans with friction, you need to determine the direction that the system will likely to accelrate.

Think of this as **tug of war** between these 2 forces,. Whichever force has a greater magnitude will be the direction that the system will likely to **accelerate**

11.1 How to determine the direction that the system will likely to accelerate

Look at the forces acting on the system. (Active forces are forces that are *trying to make the system move*) Do not include internal forces

Along the potential line of the system's acceleration:

– Find the sum of the active forces, which are in one Dimension

– Find the sum of the active forces which are acting in the opposite direction

Compare the magnitude of these two force sums, and whichever direction has the larger magnitude of active forces is the directions that system is likely to accelerate

11.2 Example template

Active forces[label directions]
Calculate for it
Process$\cdots$
Active forces[Label directions]
Calculate for it
$\therefore$ the system will more likely to move in which direction. The friction will move $\cdots$

12 Proportionality

Definition 12.1. Proportionality is concerned witht he relationship between two different variables in an equation. It is the method of showing the **effective change** in the values of the **independent variables** will have on the **dependent variable**

Proportionality is entirely based on **multiplication**. If the value of the independent variable is multiplied by a **specific factor**, then the value of the dependent variable will be multiplied by some mathematical operation of that factor.

Direct Proportionality

In this graph, the proportionality will make a straight line

Mathematical expression:

$$x \propto y$$

Square Proportionality

Read "x is proportional to the square of y"

The graph looks like half of a quadratic function

$$x \propto y^2$$

Square root proportionality

Half of a square root function

"x is proportional to the square root of y"

$$x \propto \sqrt{y}$$

Inverse square proportionality

Looks like a rational function

$$x \propto \frac{1}{y}$$
$$x \propto \frac{1}{y^2}$$
$$x \propto \frac{1}{y^3 1}$$

13 Fictitious Forces and Apparent Weight

13.1 Compare Inertial and non-inertial F.O.R

13.1.1 Inertial F.O.R

One that is not accelerating. It moves at a constant speed

13.1.2 Non-Inertial F.O.R

This Frame of Reference is accelerating

13.2 Fictitious Forces

Fictitious forces are also called apparent forces or perceived forces
Explanation: When the object is viewed from a non-intertial F.O.R, we created fictitious force to explain the motion and behavior
The fictitious forces will always act in the direction opposite to the direction of acceleration of the frame of reference.

The magnitude of each fictitious force can be calculated by:

$$F_{fict} = m|a_{F.\vec{O}.R}|$$

Perceived acceleration could be represented by $a_{\vec{per}}$
Note: The object's actual acceleration would be measured relative to an inertial FOR

13.3 Apparent Weight

Technically, this would be the sum of the normal force and the force of friction that a surface exerts on an object. In high school, we treat *apparent weight* as just the *normal force a surface exerts* exerts on the object.

13.4 Some of the formulas

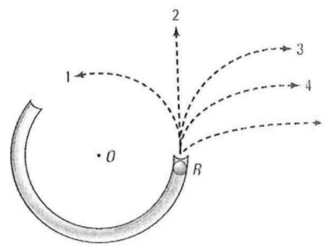
$$\sum \vec{F} = ma_{\vec{per}}$$

14 Lecture 2.5

14.1 Uniform Circular Motion

Direction: The velocity of an object at any point along a circle has a direction that is tangential to the circle

Question: If an object is attached to a string, swung in a circular motion and then the string is released, which of the five paths shown here will the object take?
ANS: Path 2



14.2 Centripetal acceleration

From the Newton second law, we understand that An object will accelerate in the same direction as the net force.

If the centripetal force is directed toward the centre of the circle, then what direction is the acceleration in? ANS: Toward the circle

In other words, the acceleration will always perpendicular to the velocity of the object.

14.3 Formulas

Formula 1:

$$\vec{a_c} = \frac{4\pi^2 R}{T^2}$$

$$\vec{a_c} = 4\pi^2 R f^2$$

$$\vec{a_c} = \frac{V^2}{R}$$

$\vec{a_c}$ is the acceleration of the object in $\frac{m}{s^2}$
R is the radius of the circular path that the object is moving around (in m)
T is the period of the object's motion
v is the speed of the object in (m/s)

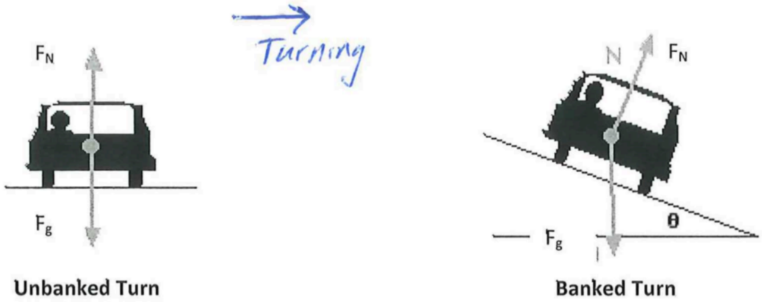
For clockwise:

$$\text{direction of acceleration} = \text{direction of velocity} + 90 \text{ degree}$$

else:

$$\text{direction of acceleration} = \text{direction of velocity - } 90 \text{ degree}$$

15 Motion of a car on Banked Turn



15.1 Forces

For unbanked Turn, Static friction contribute to the centripetal force
For Banked Turn, both static friction and normal force contribute to the centripetal force

15.2 Critical Speed

Definition 15.1. Critical speed the minimum speed needed at which a vehicle can travel around a curve, baked road without relying on static friction

The formula for critical speed is defined as:

$$v = \sqrt{R * tan\theta * g}$$

$$v = \text{Critical Speed}$$
$$R = \text{The radius of the banked turn}$$
$$g = \text{The acceleration by Gravity}$$

Above the critical speed, the car wants to go up. At this case, friction must act down the bank to prevent sliding outward
Below the critical speed, the car wants to go down. At this case, friction must act up the bank to prevent sliding inward

16 Universal Gravitation, Gravitational field

16.1 Force of Gravity

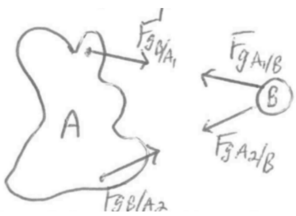
The formula for the **Force of Gravity** acting between two objects is:

$$Fg = \frac{G * m_1 * m_2}{R^2}$$

Fg = the magnitude of the force of gravity that m_1 exerts on m_2 and m_2 exerts on m_1
G = Universal Gravitational Constant
m_1 = the mass of one of the objects (in kg)
m_2 = the mass of the other object (in kg)
R is the distance separating the objects' center of mass in (m)

Remainder: **Altitude** refers to the distance between the Earth's surface and the object!

In reality, every particle in A exerts a force of gravity on every particle in B. If the objects (A and B) are relatively close together and large (relative to their separation distance) then these forces are not parallel.



*As the distance separating the two objects increases, the forces become closer to parallel

The **formula works** best for two objects who seperation distance is very large relative to their sizes, or when both object are perfect sphere

The **formula works well** for a very, very large sphere (whose mass is uniformly distributed through out) and a relatively small object on its surface

You can not use this formula when one object is inside of another object!

16.2 Gravational Fields

Definition 16.1. **A force field** is a region surrounding an object in which the object is capable of exerting a force on another object

A Gravational field is a region surrounding an object in which the object is capable of exerting a force of gravity on another object.

16.3 Differences between strength of gravity and acceleration

Acceleration due to gravity:

Units: $\frac{m}{s^2}$
What does it imply?: When an object is in free fall, it will accelerate at that rate
When is it true: Only when Fg is the only force on the object

Gravitational field strength:

Units: $\frac{N}{kg}$
What does it imply?: When an object is in free fall, it will accelerate at that rate
When is it true: Gravity is exertings a force of $|\vec{g}|$ Newtons for each Kg of mass

Specific types of field strengths are additive. The net gravitational field strength at a location is the sum of all the individual strengths of gravitational fields at that location OR $\sum \vec{g} = \vec{g_1} + \vec{g_2}$

When you need to calculate magnitude of Gravational field from that object M exerts on m:

$$Fg_{M/m} = \frac{GMm}{R^2} \tag{6}$$

$$Fg_{M/m} = mg \tag{7}$$

Add 6 and 7

$$g = \frac{GM}{R^2} \tag{8}$$

g is the magnitude of the grav field strength of M, at a specific location (in N/kg)
R is the distance that the location is from M's centre of mass(in m)
G is the universal gravitational constant ($G = 6.67 * 10^{-11} \frac{Nm^2}{Kg^2}$)

17 Satellites

A satellite is an object that orbits around another object

There are **natural** satellites and **artifical** object

- The moon is a **natural** satellite of the Earth
- The international space station (ISS) is an **artifical** object

17.1 Netwon's Cannon

His idea was: *if a cannon is placed on the top of a very tall mountain, and if you could ignore air resistance. The cannon shoots a cannonball horizontal*

At the idea speed: the distance the cannonball has fall **equals** the distance that the Earth has **turned away**

If $v < v_{idea}$, the distance between the ball and Earth's surface will **decrease**

If $v > v_{idea}$, the distance between the ball and Earth's surface will **increase**

The cannon must has a constant speed and travel in the perfect circular path

17.2 Geosynchronous

They have the same orbital period as the **rotational speed** of the object they are on the ground

The period is around **24 hrs**

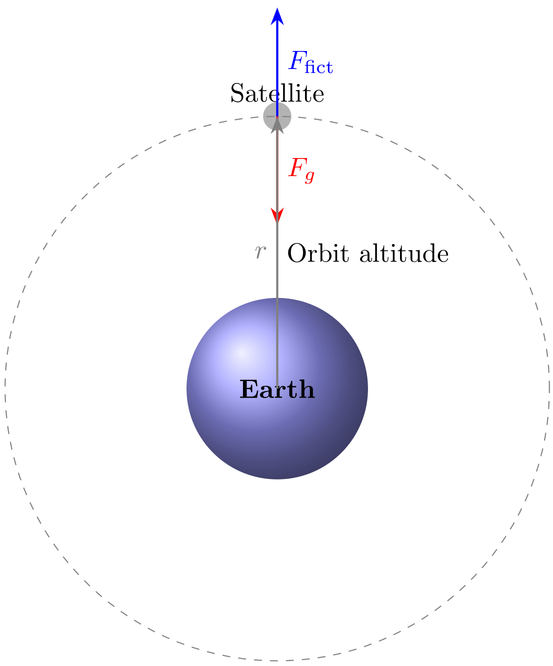
There is a special type of geosynchronous is called **Geostationary**

- "Hang above" a location on Earth's **Equator**
- They orbit in the same **direction** that the Earth rotates

17.3 Formulas related to satellite

We will derive each formulas in this handout:

To start off, let's draw the FBD for the satellite



Down is negative

Let's derive the formula for satellite:

$$\sum \vec{F} = m * \vec{a_{per}}$$
$$F_g - F_{fict} = 0$$
$$mg - m * |F_{FOR}| = 0$$
$$mg = ma_c$$
$$a_c = \frac{Gm}{R^2}$$

Sub in $a_c = \frac{V^2}{R}$:

$$\frac{v^2}{R} = \frac{GM}{R^2}$$
$$v = \sqrt{\frac{GM}{R}}$$
$$v = \text{orbital speed}$$
$$M = \text{mass of the object that is obited in (kg)}$$

Sub in $a_c = \frac{4\pi^2 R}{T^2}$

$$\frac{4\pi^2 R}{T^2} = \frac{GM}{R}$$
$$T^2 = \frac{4\pi^2 R^3}{GM}$$
$$T = \sqrt{\frac{4\pi^2 R^3}{GM}}$$

18 Rotating Frame of Reference

18.1 Little problem

When a person is standing (on Earth) there are two forces acting on them:

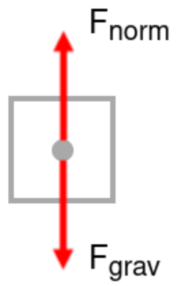


Figure 11: Normal force and Gravity

The normal force acting on the person is pushing force, thus a force of **compresion**. An object that has a compression force must be able to **withstand this force** or it will collapse. (See 18.1)

In the caes of a person, their muscles and their bones must be able to withstand this force, thus you **muskule-skeletal system develops** to withstand this force

When a person is in orbit around a planet, there is only the force of gravity acting on them! They are in **constant state of free fall**

FBD for a person orbiting a planet:

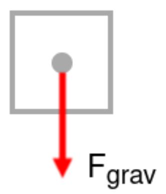


Figure 12: Force of Gravity

The person is missing the **normal force** (the compressive force) that they are used to feeling. Without the compressive force, the person's musculo=skeletal system starts to undergo **atrophication**

To solve this problem, we need to make sure that both two forces are acting on the person. The forces must be in **opposite direction** and one force must be a **compression force**

Solution 1

The spaceship is **accelerating** uniformly, in a straight line.

But, there are two problems:

(i) Run out of fuel

(ii) As an object speed up, its **mass increases**. If mass increases, to maintain the acceleration the netforce would also increase

Solution 2

Get a very very large hallow ring and make it spin

For an internal F.O.R, there will be both **Force of Normal** and **Force of Gravity** acting on it

From an internal F.O.R:

$$\sum \vec{F} = m\vec{a_{per}}$$
$$F_N - F_{fict} = 0$$
$$F_N = F_{fict}$$
$$F_N = m * |a_{F.O.R}|$$
$$F_N = ma_c$$

We can calculate the v of the people:

$$F_N = ma_c$$
$$mg = ma_c$$
$$g = a_c$$
$$g = \frac{v^2}{R}$$
$$v = \sqrt{gR}$$

So, in the spaceship rotating question, we can always assume:

$$F_N = F_g$$

We can also using

$$a_c = \frac{4\pi^2 R}{T^2}$$

To sub in

$$g = \frac{4\pi^2 R}{T^2}$$
$$T = \sqrt{\frac{4\pi^2 R}{g}}$$

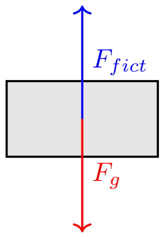
or

$$f = \sqrt{\frac{g}{4\pi^2 R}}$$

18.2 Perceived Acceleration in a Rotating Frame of Reference

When a person is stand on the equator, there are a_c to affect the ball

The **perceived** acceleration should be calculated like this:



Now, let's calculate the $\vec{a_{per}}$

$$\sum \vec{F} = m\vec{a_{per}} \tag{9}$$

$$F_N - F_{fict} = ma_{per} \tag{10}$$

$$mg - m\left|a_{F.O.R}\right| = ma_{per} \tag{11}$$

$$a_{per} = g - a_c \tag{12}$$

Note

You can always assume Earth's surface to be an inertial F.O.R unless:

– You need to be **extremely accurate**

– The question states **at the equator**

19 Work and Kinetic Energy

19.1 Kinetic Energy

Definition 19.1 (Kinetic Energy)**.** is the energy of motion. There are two types of kinetic energy

Type 1: Traslational Kinetic Energy

The kinetic energy that an object has because it is *moving from on location to another*. In can be describe by this formula:

$$E_k = \frac{1}{2}mv^2 \tag{13}$$

E_k is the translational kinetic energy of the object (in J or kgm^2/s^2)
m is the mass in kg
v is the speed in m/s

Because all motion is relative, an object's speed (and therefore E_k) *depends on the choose Frame of Reference*

Notes:

Do not solve the conservation of energy problem involving a change of Frame of Reference. Start from your perspective

E_k is a scalar, not a vector

Type 2: Rotational Kinetic Energy

Not testable, don't give a shit about this qusetion.

19.2 Mechancial Work

Definition 19.2 (Mechancial Work)**.** Transfer of energy into E_k or the transfer of kinetic energy into another type of energy

Definition 19.3 (Potential Energy)**.** Energy which is stored in a system of objects due to forces acting in between thos objects

19.2.1 What does the sign of the work mean?

- *Positive* work on a system means it receives energy from its surroundings
- *Negative* work on a system means it gave energy to its surroundings
- *Negative* work occurs when the force has a component in the direction opposite the displacement

The formula to descirbe the work is:

$$W = F_{A/B} * \Delta\vec{d}_B * cos\theta \tag{14}$$

$W_{A/B}$ is the work that $F_{A/B}$ does on the object B. This is also the amount of E_k that object A transfers into object B when A exerts a force on B(in J)
$F_{A/B}$ is the magnitude of force that A exerts on B(in J)
θ is the angle between $F_{A/B}$ and B's displacement

Remark. Remainder: Only forces on the direction of displacement is responsible for the work

You should always write $\cos\theta$ when use the formula

20 Gravational Potential Energy

20.1 Some boring definitions

Definition 20.1 (Gravitation Potential Energy)**.** The energy storedf in a system of objects due to the force of gravity acting between those objects. In other words, the energy is stored collectively **among all** the objects in the system

Whne the force of gravity acting on the two objects causes this stored GPE to be converted into kinetic energy, the kinetic energy is not **shared evenlly** between these two objects. In the class Example, the earth effectively gets **zero** and the care effectively gets **all of them**. Due to this reason, when we have two objects with a very large difference in mass, we can always assume that the GPE is **stored only in the smaller object**

20.2 Formulas for GPE

Formula 1

$$\Delta E_g = mg\Delta h \tag{15}$$

ΔE_g is the change in Potential gravational energy(in J)
m is the mass of the object (in kg)
Δh is the change in height (in m)
g is the acceleration due to gravity (in m/s^2)

Fromula 2

$$E_g = mg\Delta h \tag{16}$$

$E - g$ is related to the GPE of the object

Remark. For all questions related to the **Gravational Potential Energy**, you must set your reference height in the diagram. Or, Mr McCumber will forget to add 0.5 for your test!

In reality, as h changes, the distance between the centres of the two objects changes, and therefore g will change. However, 3 SD's of the constant of acceleration due to gravity can handle changes with up and down 1 km

21 The law of the Conservation of Energy

21.1 Boring Definitions

Definition 21.1 (Law of the Conservation of Energy)**.** Energy will neither be created or destroy, only change from one form to another

Remark. The law of the conservation of energy only works in a **closed isiolated** system. In reality, the only true **closed and isolated** system is the Universe

Mechancial Energy

Definition 21.2 (Mechancial Energy)**.** The sum of the **kinetic energy** and **gravational potential energy**.

Mathematically:

$$E_m = E_g + E_k \tag{17}$$

21.2 Question solving techniques

When you solve a question about conversation of energy, always write this:

$$E_{m1} = E_{m2} \tag{18}$$

$$E_{k2} + E_{g2} = E_{k1} + E_{g1} + W_{ap} + W_f \tag{19}$$

Then, cross out terms which equal to **zero**

Remark. Remember to write this, or teacher will forget to add your marks!

22 String & Elastic Potential Energy

22.1 The Force of String

Definition 22.1 (Spring Force)**.** Can be wrote as F_{spring}. It is the force exerted by the spring on a object.

According to the Hooke's Law, the **force exerted by a string** is proportional to the string's displacement. So we can express the relationships between them by some formulas:

Vector Version:

$$\vec{F_x} = -k\Delta\vec{x} \tag{20}$$

Scalar Version:

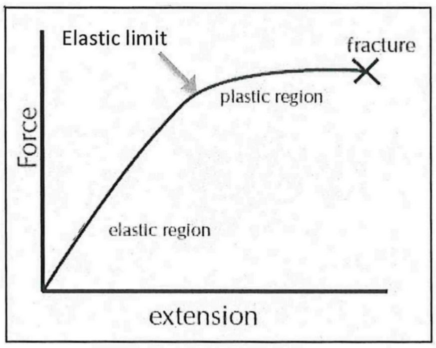
$$F_x = k\Delta x \tag{21}$$

F_x is the force exerted by the string on whatever stretches it.
k is the constant of string
x is the displacement of the spring from its unstratched

An essential feature of Hooke's law is that the direction of the spring force is **opposite** to the direction of displacement from equilibrium.

Remark. When you use the Scalar version, you must clearly understand the direction of the force in you heart.

Deeper explanation about Hooke's law



Definition 22.2 (Elastic Region)**.** Elastic objects obey Hooke's Law in this region. If the applied force removed, the object will naturally return back to its original shape

Definition 22.3 (Elastic Limit)**.** The maximum amount of deformation an object can withstand, and still return to its original shape.

Definition 22.4 (Plastic Region)**.** The object no longer obeys Hooke's Law. The object's shape is now permanetly changed.

Definition 22.5 (Fracture)**.** The maximum amount of shape change the object can take, prior to failing (breaking).

22.2 Elastic Potential Energy

Definition 22.6 (Elastic Potential Energy)**.** The potential energy due to the stretching or compressing of an elastic material

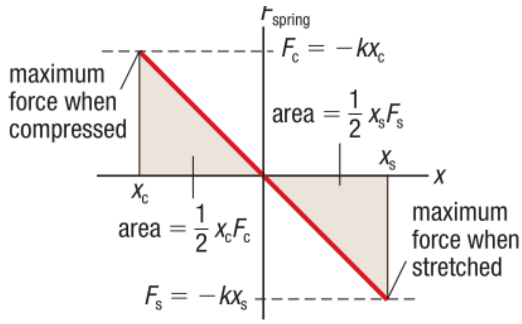


Figure 13: The work done by a variable force is equal to the area under the the force-displacement graph

Formula

$$W = \frac{1}{2} * \Delta x * F_{spring} \tag{22}$$

$$W = \frac{1}{2} * \Delta x * (k * \Delta x) \tag{23}$$

$$W = \frac{1}{2} * k * (\Delta x)^2 \tag{24}$$

The work done by the spring force is the negative of this amount, and is also the negative of the change in Potential Energy. That means that the work done stretching or compressing the spring is transformed into elastic potential energy.

$$E_e = \frac{1}{2} k (\Delta x)^2 \tag{25}$$

22.3 Ignore Gravational Potential Energy

We can ignore the Gravational Potential Energy in the vertial spring question if all of these conditions are met:

– The mass remains in contact with the spring

– We measure all changes in the length relative to the equilibrium position of the mass-spring system (ie. $x_{eq} = 0$)

23 Simple Harmonic Motion

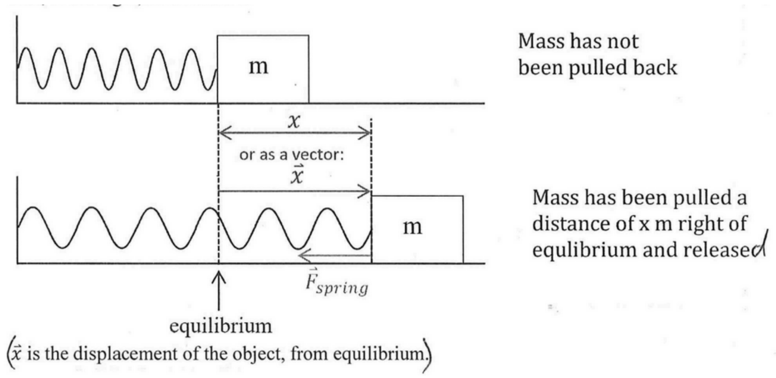
23.1 Definitions

23.1.1 Simple Harmonic Motion

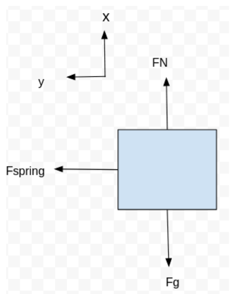
An object undergoes simple harmonic motion, if two conditions are met:

(i) The net force acting on the object is **directly proportional** to the object's displacement away from equilibrium (The location where $\sum F = 0$)

(ii) The direction the net force acts on the object must be **opposite** the object's displacement from equilibrium.



Let see the FBD for m:



$$\sum \vec{F} = \vec{F_{spring}} \tag{26}$$
$$\sum m\vec{a} = -k\vec{x} \tag{27}$$
$$\vec{a} = \frac{-k\vec{x}}{m} \tag{28}$$

Changing to proportionality:

$$\text{Net Force} \propto -\vec{x} \tag{26}$$

23.1.2 Expectations

Definition 23.1 (Simple Harmonic Motion)**.** Go through repeated oscillations at the same amplitude.

If there was **friction** acting on the object, it would no longer undergo simple harmonic motion. We would call the motion of the object: **damped Harmonic Motion**

Some Real life examples:

- Car shock absorbers
- A guitar string
- A pendalam or a string
- Bungee jumping

23.2 How can we solve form the acceleration of SHM

Returning to our FBD of the mass and the net force statement from it:

$$\sum \vec{F} = m\vec{a} \tag{27}$$
$$\vec{F_s} = m\vec{a} \tag{28}$$
$$-k\vec{x} = m\vec{a} \tag{29}$$
$$\vec{a} = \frac{-kx}{m} \tag{30}$$

$\vec{a}$ is the acceleration of the mass (in m/s^2)
k is the force constant of the spring (in N/m)
$\vec{x}$ is the displacement of the mass from its equilibrium position (in m)
m is the mass of the object that is attached to the spring (in kg)

Remark. Because the acceleration is a vector, we need to make a direction convention to use the equation.

23.3 The period of the Simple Harmonic Motion

The y-component of the uniform circular motion is similar to the acceleration of the simple Harmonic Motion

For the object going around circle:

$$a_c = \frac{4\pi^2 R}{T^2} \tag{31}$$

For the mass on the end of the spring:

$$\vec{a} = -\frac{k\vec{x}}{m} \tag{31}$$
$$|\vec{a}| = \frac{kx}{m} \tag{32}$$

When the object is on the top/bottom of the perfect circular motion, the acceleration is equal to the magnitude of the acceleration of the object at the equilibrium position:

$$a_c = |a| \tag{32}$$
$$\frac{4\pi^2 R}{T^2} = \frac{kx}{m} \tag{33}$$
$$T = +/- \sqrt{\frac{m * 4\pi^2 R}{kx}} (\text{At this time } R = x) \tag{34}$$
$$T = 2\pi * \sqrt{\frac{m}{k}} \tag{35}$$

24 Linear Momentum & Impulse

24.0.1 Linear Momentum

Linear Momentum is the product of an object's mass and its velocity:

$$\vec{p} = m\vec{v} \tag{36}$$

$\vec{p}$ is the Momentum in $(kg * \frac{m}{s})$

Newton called momentum "the **true Quantity of motion**". Why? Momentum is a combination of an object's **inertia**(its mass basically) and what it is doing (its **velocity**). He felt that it provided a more complete picture of what was required to cause a specific change in what an object was doing.

24.0.2 Impulse

Impulse is the **product of that force** acting on an object and the **duration** of time that the force acted on the object.

$$\vec{J} = \vec{F} * \Delta t \tag{37}$$

$\vec{J}$ = the impulse in (N*s)

The formula has a similar limitation to the formula for the work done on an object. Both formulas assume that the force acting on the object.

Thus, if the force acting on the object is not constant, we can find the impulse that the force provides by finding the area between the line/curve on a **Force vs Gravity graph**

Let's see some formula:

$$\sum \vec{J} = \sum \vec{F} * \Delta t$$
$$\sum \vec{J} = (m * \vec{a}) * \Delta t$$
$$\sum \vec{J} = (m * \frac{\vec{v_2} - \vec{v_1}}{\Delta t}) * \Delta t$$
$$\sum \vec{J} = m * \vec{v_2} - m * \vec{v_1}$$
$$\sum \vec{J} = \vec{p_1} - \vec{p_1}$$

$$\sum \vec{J} = \Delta\vec{p} \tag{38}$$

25 Conservation of Momentum

Definition 25.1. Two or more objects interact and **exert forces from each other**. The forces in the interaction are a Newton's Third Law pair of forces (ie $\vec{F_{A/B}} = -\vec{F_{B/A}}$)

25.1 Equation

Consider a person standing on any icy surface throws a heavy object horizontally:

FBD for the person (Left) and the object (Right)

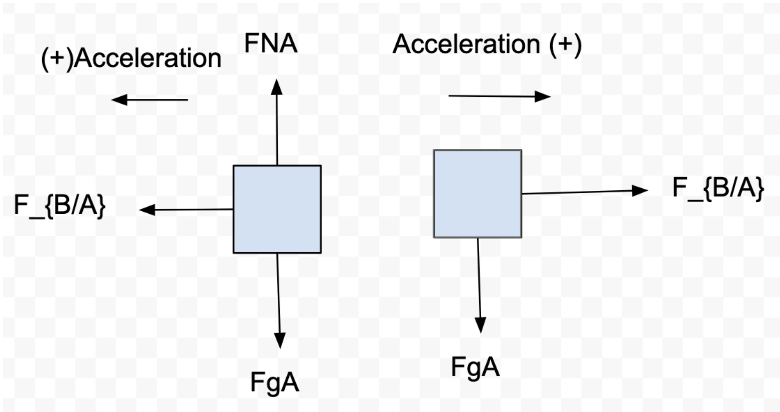


Figure 14: We will assume that the interaction forces are essentially the net force acting on the object

Equation for the object A:

$$\begin{aligned}\sum \vec{F_A} &= m_A * \vec{a_A} \\ \vec{F_{B/A}} &= m_A \vec{a_A} \\ \vec{F_{B/A}} &= m_A \frac{\vec{v_{fA}} - \vec{v_{iA}}}{\Delta t_A}\end{aligned} \tag{39}$$

Equation for the object B:

$$\begin{aligned}\sum \vec{F_B} &= m_B * \vec{a_B} \\ \vec{F_{A/B}} &= m_B \vec{a_B} \\ \vec{F_{A/B}} &= m_B \frac{\vec{v_{fB}} - \vec{v_{iB}}}{\Delta t_B}\end{aligned} \tag{40}$$

Lemma 25.2. *From Newton's third law, we know* $\vec{F_{A/B}} + \vec{F_{B/A}} = 0$

We add 39 and 40 together:

$$m_A \frac{\vec{v_{fA}} - \vec{v_{iA}}}{\Delta t_A} + m_B \frac{\vec{v_{fB}} - \vec{v_{iB}}}{\Delta t_B} = 0$$

We know the time for both object should be the same.

$$m_A * \vec{v_{fA}} + m_B * \vec{v_{fB}} = m_A * \vec{v_{iA}} + m_B * \vec{v_{iB}}$$
$$\vec{P_{A2}} + \vec{P_{B2}} = \vec{P_{A1}} + \vec{P_{B1}} \tag{41}$$

41 is the law of **Conservation of Momentum**
Always write this line at the beginning of your analysis

Definition 25.3 (The Law of Conservation of Momentum)**.** The total momentum of a system of objects after an interaction is **equal** to the total momentum of the system before the interaction.

According to the 25.3, we can understand the total momentum of the system is **constant through out** the interaction.

The law assume that any other force that could **accelerate** any objects in the system during the interaction is **negligible**

26 Types of Collisions

26.1 Definitions

Collisions are typically classified based on the amount of **kinetic energy** the system has after the collision, in comparsion to the amount of kinetic energy the system had before the collision. In other words, **how does E_k' with E_k**?

26.2 Elastic Collisions

In an elastic collision, the kinetic energy of the system after the collision is **equal** to the kinetic energy of the system before the collision. In mathematics, the equation can be represented by:

$$E_k` = E_k$$

Remark. This does not mean the kinetic energy of the system after the collision is **equal** to the kinetic energy of the system before the collision (Unlike momentum)

26.2.1 Steps of the collision

Remark. This is on a horzontal frictionless surface, so we can ignore Gravational Potential Energy

Before the collision:The mechancial of energy is entirely in the form of kinetic energy

First half ot collision:The interaction forces cause the object to start deform. As the object deform, they transfer E_k to E_s.

At the approximate midpoint of the collision:The deformation of the object is at a maximum. E_S is the maxium and E_k is the minimum.

During the second half of the collision: The restoring forces are now doing *positive work* on the system, transferring elastic potential energy **back into** E_k

After the collision: The system's mechanical energy is now entirely E_k, at this time, $E_s = 0$. All E_s is transfered to E_k.

26.2.2 Head-on Collision

Before the Collision: System's mechancial energy is entirely E_k

During the first half of the collision: The spring get compressed. E_k is transformed into E_s.

At the mid-point of the collision:

- Spring is at the most compressed point.
- Distance between cars are minimumized
- $\vec{v_A} = \vec{v_B}$
- E_k is minimumized

- E_s is maximumized

During the second half of the collision:

- $\vec{v_A} < \vec{v_B}$
- Distance between the carts is increasing
- E_s is being transferred back into E_k

After the collision: The system is entirely E_k now

Remark. Elastic collisions **cannot occur** between visible objects in real life. At least some of the enery will be lost as thermal or sound.

26.3 Inelasic Collision

So for this collision, E_k‘ is less than E_k

It is impossible for a system to have more kinetic energy after the collision, than it had before the collision, unless:

(i) One of the object had **stored energy** before the colllision, which was transferred into kinetic enery during the collision.

(ii) An **external** force (such as force of gravity) is doing positive work on the system, during the collision.

26.3.1 Completely inelastic collision

In this collision, the maximum amount of kinetic energy that could be "lost" is lost as a result of the collision.

After the collusion, the objects involved in the collision will be **stack/attached together**

Apple and Arrow is an example of this question.

The following condition must be met for a Perfectly Inelastic Collision:

- $\vec{v_A}‘ = \vec{v_B}‘ = \vec{v}‘$

27 HEAD-ON Collisions

27.1 Equations

Under this section, I will derive Equations that is useful to solve for final velocity for each object when you have initial velocity for a HEAD-ON Collision question

The equation for $\vec{v_A}'$

$$\vec{v_A}' = \frac{m_A - m_B}{m_A + m_B} * \vec{v_A} \tag{42}$$

The equation for $\vec{v_B}'$

$$\vec{v_B}' = \frac{2m_A}{m_A + m_B} * \vec{v_A} \tag{43}$$

For questions which $\vec{v_B}$ is not zero, we need change Frame of Reference to $\vec{v_B}$ in order to solve the question. And change the FOR back later.

28 Real Gravitatonal Potential Energy

28.1 Real gravitional potential Energy

In earlier time in this unit, we studied a formula for gravitatonal Potential Energy

$$E_g = mgh \tag{44}$$

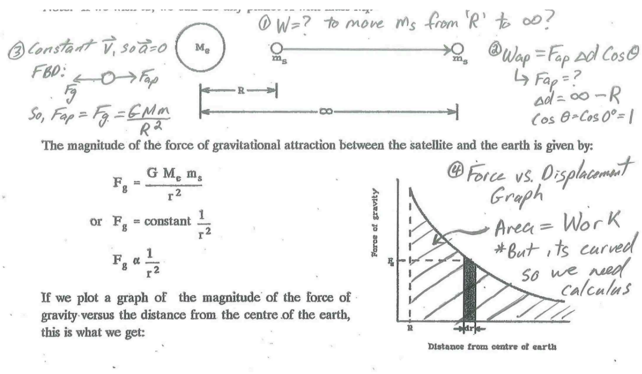
In fact, there are few problems with our old formula:

(i) It assumes that g is constat (it change as the distance above the Earth's surface changes)

(ii) the formula uses a reference location which we define. This is not an absolute location, where E_g is 0

So, let's derive for the new equation of gravitatonal Potential Energy

Consider a satellite of mass m_s at a large distance r from the centre of planet Eart (mass M_e)



We understand the work done by a force is basically the area under the curve one a Force vs displacement graph

So:

$$\begin{aligned}
\text{Work done} &= \int_{r=R}^{r=\infty} F_g dr \\
&= \int_{r=R}^{r=\infty} \frac{GM_E m_s}{R^2} dr \\
&= GM_E m_s * \int_{r=R}^{r=\infty} \frac{1}{R^2} dr \\
&\dots \\
W &= \frac{GM_E m_s}{R}
\end{aligned} \tag{45}$$

The work done is equal to the increase in gravitational potential energy prossessed by the satellite

$$W = \Delta E_g = E_{g\infty} - E_{gR} = \frac{GM_E m_s}{R}$$

$$E_{gR} = \frac{-GM_E m_s}{R} \tag{46}$$

28.2 Satellite

The Kineatic Energy of a satellite orbit around the earth could be calculated using thine formula:

$$E_k = \frac{GM_E m_s}{2R} \tag{47}$$

Then we can calculate the total mechancial energy of the system:

$$E_T = E_k + E_g \tag{48}$$

$$E_T = \frac{-GM_E m_s}{R} + \frac{GM_E m_s}{2R} \tag{49}$$

$$= -\frac{GM_E m_s}{2R} \tag{50}$$

The 50 is the *Total Mechancial Energy* of the satellite

29 Escape from a Gravational Field

Definition 29.1 (Binding Energy)**.** The **Minimum** E_k required for an object to escape a second object's gravational field. For the minimum E_k, required by the first object, the first object must reach an **infinite distance away** from the second object by the time that the second object's force of gravity **stop** the first object

Definition 29.2 (Escape Energy)**.** This is a specific type of binding energy. It is an object's binding energy when the object is originally **on the surface** of the second object

Example 29.3. The **escape energy** of an object from the Earth's surface is the minimum **additional** kinetic energy the object needs to have to escape the Earth's gravational field

Definition 29.4 (Escape Velocity)**.** The **Minimum speed** an object would need to have (when launched vertically) when it is on an object's surface to escape its gravitational field

29.0.1 Solve for Escape Velocity

Let's assume position 2 is the postion where the object escape the planet's gravitational field and position 1 is on the surface of that planet

$$E_{M2} = E_{M1} \tag{51}$$
$$0 = E_{T1} \tag{52}$$
$$0 = E_{k1} + E_{g1} \tag{53}$$
$$E_{k1} = -E_{g1} \tag{54}$$
$$\frac{1}{2}mv_1^2 = -\frac{GM_E m_s}{R_1} \tag{55}$$
$$v_1 = \sqrt{\frac{2GM_E}{R_1}} \tag{56}$$

The 56 is the formula for Escape Velocity

29.0.2 Potential Communication Question

Why do most rocket launches occur from locations close to the equator?

Because on the Earth's surface, the rocket's speed is not actually 0. The rocket has whatever speed the Earth's surface has and the Earth is moving fastest at the equator.

Let's solve for the speed of the rocket at the equator:

$$v = \frac{2\pi R}{T} \tag{57}$$
$$v = \frac{2\pi 6.38 * 10^3}{24.0h} \tag{58}$$
$$v = \frac{2 * \pi * 6.38 * 10^3}{86400s} \tag{59}$$
$$v = 463.96 \cdots m/s \tag{60}$$

How does this help?
Assume position is surface, position 2 is on the orbit

$$E_{M2} = E_{M1} + E_{added}$$
$$E_{M2} = E_{k1} + E_{g1} + E_{added}$$
$$E_{added} = E_{M2} - E_{k1} - E_{g1}$$

Because we have initial E_{K1}, less E_{added} is needed. As a result, less fuel will be used. Cost less money

Remark. Assume that for an "Object on a surface", E_K is always 0 unless the questions explicitly says "At the equator"

30 Review of Electronstatics

In this section, we will briefly review basic Electronstatics which we learned from Grade 9 Science

30.1 Electric Charge

30.1.1 Electron

By the early 1900s, physicists had identified the subatomic particles called the electron and the proton as the basic units of charge. All protons carry the same amount of positive charge, e, and all electrons carry an equal but opposite charge, $-e$. Charges interact with each other in very specific ways governed by the **law of electric charges**

Theorem 30.1 (Law of Electric Charges). *Like charges repel each other; unlike charges attract.*

30.1.2 Charge of atom

Cation: a positive ion. # of protons > # of electrons
Anion: a negative ion. # of protons < # of electrons

The **Total Charge** is the sum of all the charges in that object and can be positive, negative or zero. The charge is equal to zero when the negative charge equals to negative charge.

Theorem 30.2 (Law of Conservation of Charge). *Charge can be transferred from one object to another, but the total charge of a closed system remains constant.*

30.1.3 Coulomb

The basic unit of charge is called the coulomb (C). The charge of electron, $-e$, is $-1.60 \times 10^{-19}C$, and the charge of a single proton, $+e$, is $1.60 \times 10^{-19}C$

Symbol e often donotes the magnitude of the charge of an electron or a proton.

The symbol q denotes the amount of charge, such as the total charge onf a small piece of paper. In other words, the total charge of a particle is q.

30.2 Conductors and Insulators

Definition 30.3 (Conductor). A conductor is a substance in which electrons can move easily among atoms.

Definition 30.4 (Insulator). any substance in which electrons are not free to move easily from one atom to another.

Insulator hold the electron when other electron come in. There are no free electrons in the insulator, and insulator does not allow the extra electrons to move about easily. Instead, these eadded electrons stay where they are initially placed.

30.3 Different methods of charging

30.3.1 Charging an Object by Friction

In reality, some object has stronger ability to hold on electrons than others. Assume we have two neutral object, when we rub these two objects, electrons will follow from the object with weaker hold on electrons to the other one with stronger hold on electrons.

30.3.2 Carging an object by Induced Charge Separation

Assume we have two objects, one with zero charge and other one with negative charge. When we put the negative object towards the positive object, electrons in the neutral object will repel to the electrons in the negative object. As a result, electrons in the neutral object will redistributed throughout the material. The positive side of the netural object is closer than the negative side of the object, in which makes the neutral object attrack to the negative object.

30.3.3 Charging by Contact

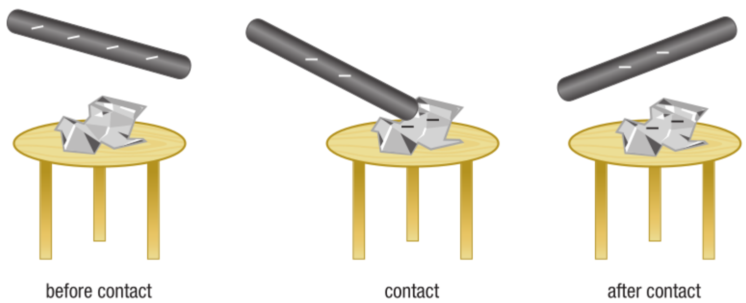


Figure 15: Picture from my textbook

30.3.4 Charging by Induction

Using a negative object to create a positive object

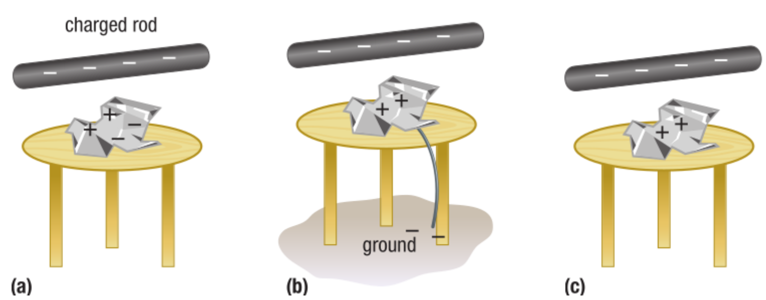


Figure 16: Picture from my textbook

31 Coulomb's Law

31.1 Background

Coulomb was a scientist who studied electricity in the early 1800's. He wanted to find out what factor affect the electrostatic force with two charged objects.

Coulomb based his experiment on Carendish's experiment.

To be able to perform the experiment Coulomb needed to electrically charge each of the pith balls and know the magnitude of the charge on each ball. His solution for this was to find the relative magnitude of the charge on each pitch ball.

31.2 Formula

By measuring the amount of force, the separation distance between the charged objects and the relative charge of the pith balls, Coulomb was abl to find the following relationships:

$$F_E \propto \frac{1}{R^2}$$
$$F_E \propto q_1 q_2$$

We can bring these proportionalities together:

$$|F_E| = \frac{k\,|q_1|\,|q_2|}{R^2}$$

F_E is the magnitude of the electrical force in between two point charges
q_a and q_b is the absolute value of the charge of each object (in C)
R is the separation distance betweeen the objects (in m)
k is Coulomb's law constant of proportionality ($k = 8.99 \times 10^9 \frac{Nm^2}{C^2}$)

Remark. When using equations for electrical forces, don't substitute in the sign of the charge. Find the direction of the force conceptually!

31.2.1 Point Charge

If the charged objects had a reasonably large size, then the electrical force on each object would cause the changes to *move inside* of each object. In a point charge, the charge is unable to move around.

31.2.2 Compared to $F_g = \frac{Gm_1m_2}{R^2}$

Similar

- $F \propto \frac{x}{R^2}$

Difference

- $k > G$
- F_E can be both attraction and repulson.

32 Electric Fields

Definition 32.1 (Field)**.** The region where an appropriate object would feel a force!

- If there's a gravitational field, a mass will feel a force.
- If there's an electric field, a charge will feel a force.
- If there is an magnetic field, a magnet (or a moving charge) will feel a force.

Visualizing Electric Fields - Field lines show how a small *positive* charge would move.

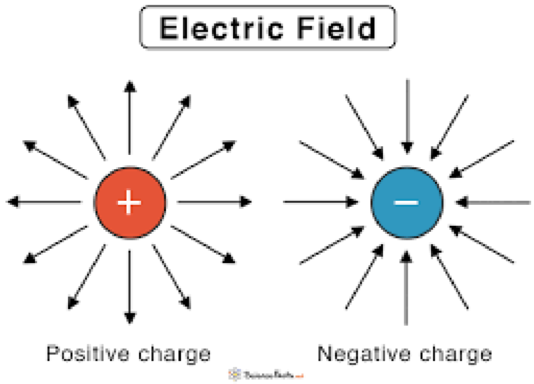


Figure 17: Electric field of Positive and negative charge

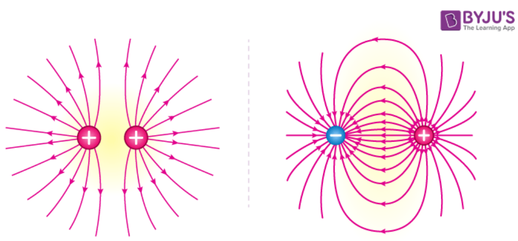


Figure 18: Electric field between two charges

32.0.1 Parallel Plates

Two charged metal plates that are parallel to each other → "parallel plates"

- The field strength outside of the plates is very weak and can be considered negligible
- The field lines in between the plates are equdistant stand.

32.0.2 Formulas

$$\mathcal{E} = \frac{k\,|q|}{R^2} \tag{61}$$

$\mathcal{E}$ is the magnitude of the electric field strength around a point charge (in $\frac{N}{C}$)
$k = 8.99 \times 10^9 \frac{Nm^2}{C^2}$
R is the distance away from the point charge (q) where you want to know the field strength (in m)

Remark. Electric fields are vector. The direction of the field will be based on the direction of force that would be exerted on a positively-charged object!

$$\vec{F_E} = q \times \vec{\mathcal{E}} \tag{62}$$

$\vec{F_E}$ is the magnitude of the electrical force exerted on q (in N)
q is the that is in the electric field(in C)
$\vec{\mathcal{E}}$ is the strength of the electrical field that the charge is in (in $\frac{N}{C}$)

33 Electric Potential Energy & Electric Potential

33.1 Electric Potential Energy

Definition 33.1 (Electric Potential Energy)**.** The energy stored in a system of two or more objects due to the electrical force acting in between the charges.

33.1.1 Formula

Formula for electrical potential energy stored in a system of two charges:

$$E_E = \frac{kq_Aq_B}{R}$$

Remark. Remember, always input the sign of q_A and q_B

You may notice, there is no negative sign for the formula of electrical potential energy compare to gravitional potential energy.

Gravity is always a force of attraction (this is what causes the negative in the formula)

However, electrical forces can either be forces of attraction or repulsion, which means that electrical energy can either be negative or positive.

$$\text{Repulsion:}+$$
$$\text{Attraction:}-$$

For electrical potential energy, electrical potential, or electric potential difference, always include *the sign of charges* into the formula

Now we have a new type of mechancial energy to add to our expression!

$$E_M = E_g + E_k + E_s + E_E$$

Remark. Gravitational Potential Energy is typically negligible in compression to electric Potential Energy

33.2 Electric Potential for Point Charges

Definition 33.2 (Electric Potential)**.** The electrical Potential per coulomb of charge at a location.

Let's discuss the difference between *electric field* and *electric potential*

Electric Field

- Can exist without there being an electrical force
- To have an electric force, a charge needs to be at a location where there is an electric field

Electric Potential

- Can exist without there being electrical potential energy
- To have elctric potential energy, a charge needs to be at a location where there is electrical potential

- $\mathcal{E}$
- $\frac{N}{C}$
- V
- $\frac{J}{C}$

33.2.1 Formula

Electric Field:

$$\vec{F_E} = \vec{\mathcal{E}} \times q \tag{63}$$

Remark. Do <u>not</u> substitue the sign of the charge

Electric Potential:

$$E_E = Vq \tag{64}$$

Remark. Substitue the sign of the charge

33.2.2 Calculate the electrical potential around a point charge

$$\begin{aligned} E_E &= Vq_1 \\ \frac{k \times q \times q_1}{R} &= Vq_1 \\ V &= \frac{k \times q}{R} \end{aligned} \tag{65}$$

V is the elec potential (of q) at a distance of R away from q

34 Electrical Potential Difference

Definition 34.1 (Electrical Potential Difference)**.** The change in the electrical potential between two points, and uses the symbol ΔV

34.0.1 Formula

Assume a charge q goes from a location where the electrical potential is V_1 to a location where the electrical potential is V_2

Electrical Potential Energy of q

Loaction 1:

$E_{E1} = q \times V_1$

Location 2:

$E_{E2} = q \times V_2$

$$\begin{aligned} \Delta E_E &= E_{E2} - E_{E1} \\ \Delta E_E &= q \times V_2 - q \times V_1 \\ \Delta E_E &= q \times \Delta V \end{aligned} \tag{66}$$

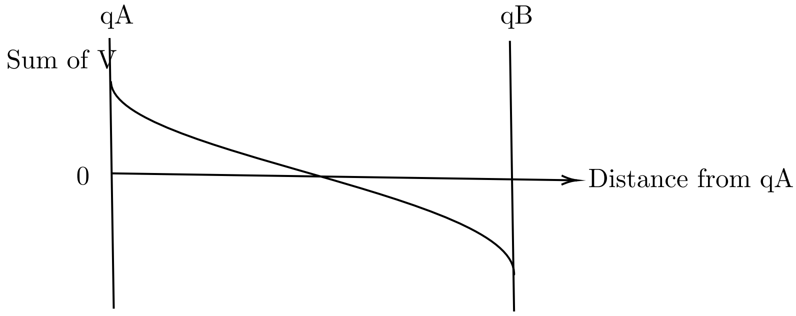
Remark. When use this formula, sub in all the signs!

Theorem 34.2. *Assume we have two charges, q_A (which is positive) and q_B(which is negative), the total electrical potential at a point in between q_A and q_B will be $V_A + V_B$*

If $|q_A| = |q_B|$ and $q_B = -q_A$, the electrical potential at the midpoint in between these two charges is 0.

$$\lim_{position \to q_A} V = \infty$$
$$\lim_{position \to q_B} V = -\infty$$
$$\lim_{position \to midpoint} V = 0$$

A graph of the electrical potential would look like:



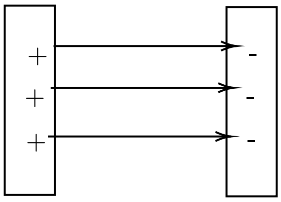
If we put a new charge at the midpoint q_A and q_B, we will find that $V = 0$ and $E_E = 0$. However, the charge would still move. To understand what will happen to this charge, we need to look at the energy gradient

Roll Down!

Remark. About *Energy Gradient*, please follow teacher's note!

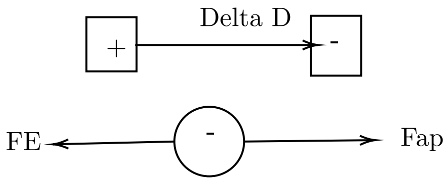
35 The electrical field in between two charged parallel plates

The electrical field in between two parallel plates will be uniform



35.0.1 Derive of $\mathcal{E} = \frac{\Delta V}{d}$

Consider a negative charge of q that starts at the positive plate and is pushed at a constant velocity toward the negative plate (by some force)



Lemma 35.1. $W_{F_{ap}} = \mathcal{E}qd$

To start of, let's solve for F_{ap}

$$\begin{aligned} \sum \vec{F} &= 0 \\ F_{ap} - F_E &= 0 \\ F_{ap} &= F_E \\ F_{ap} &= \mathcal{E}q \end{aligned} \tag{67}$$

Next, let's solve for $W_{F_{ap}}$

$$W_{F_{ap}} = F_{ap} \times \Delta d \times \cos\theta$$

Sub 67 in to this equation:

$$\begin{aligned} W_{F_{ap}} &= (\mathcal{E}q)\Delta d \times 1 \\ W_{F_{ap}} &= \mathcal{E}qd \end{aligned} \tag{68}$$

The applied force has transferred kinetic energy into q, but q's kinetic energy hasn't changed. The electrical force is doing negative work on q, transferring the kinetic energy that the applied force gave to q into electrical potential energy

Theorem 35.2. $\mathcal{E} = \frac{|\Delta V|}{d}$ *(This is only work for parallel plate question) You need to think about the direction conceptually!*

$$\Delta E_E = W_{F_{ap}}$$
$$\Delta E_E = \mathcal{E}qd$$
$$q\Delta V = \mathcal{E}qd$$
$$\Delta V = \mathcal{E}d$$
$$\mathcal{E} = \frac{\Delta V}{d}$$

When we deal with two charged parallel plate, we can say:

$$\Delta E_E = -\Delta E_k$$

When we release the electron, the energy will transfer from Electrical Potential Energy into Kinetic Energy. Kinetic energy increases and Electrical Potential Energy decreases.

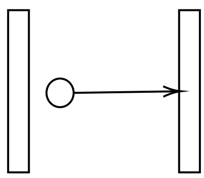
We can rearrange this equation:

$$\Delta E_E + \Delta E_k = 0$$

The formula for the electric potential difference for parallel plates is the same as for point charges:

$$\Delta E_E = q\Delta V \tag{69}$$

What if the charge between the plates doesn't travel as the way from one plate to another plate?



$$\Delta E_E = q\Delta V(\frac{x}{d}) \tag{70}$$

d equals to the total gap between these two parallel plates
x equals to the distance that the charged particle will move

36 Intro to Magnetism

The direction of the magnetic Field for a magnet: from *South Pole* to *North Pole*

36.1 Solenoid

Factors affecting the strength of a solenoid

- The current flow through the solenoid
- The number of turns in the solenoid
- The diameter of the solenoid
- The substance that fills the core of the solenoid

36.2 Magnetic Domain Theory

Any piece of ferromagnetic material contains tiny, rotatable magnets called *dipole*

Each *dipole* has a north and south pole, which are indivisible.

A magnetic domain is created when these dipoles rotate so that in a region of the material, a group of the dipoles point in the same direction

36.2.1 Factors affecting the magnetic field created by a permanent magnet

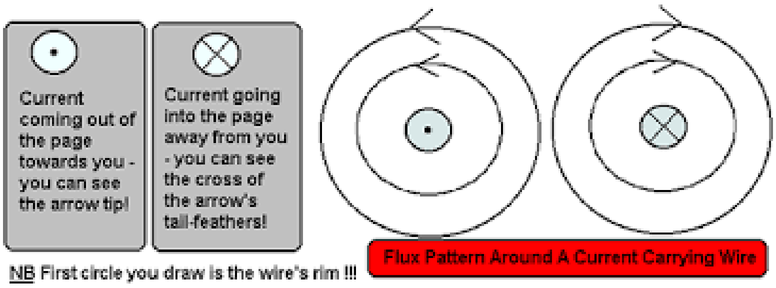
- can lose the magnetic field if it is dropped, jarred or heated (This may cause the dipoles to become randomly aligned).
- A permanent's magnet's field can be reversed. This may occur if the permanent magnet is placed in a stronger magnetic field.
- Any permanent magnet can be broken into smaller pieces.
- Any permanent magnet has a maximum possible magnetic field strength. once all of the dipoles in it are aligned

37 Magnetic Force on Moving Charges

37.1 Magnetic Force

37.1.1 Why electron creates a magnetic field

A stationary charged particle does not create a magnetic field. However, a charged particle that is moving creates a circular magnetic field around it. The magnetic field is perpendicular to the direction the particle is traveling.



Definition 37.1 (Magnetic Force)**.** The magnetic field that the moving particle creates can interact with another magnetic field that cause magnetic force to be exerted (on both the charged particle and the thing that is creating the other magnetic field)

Theorem 37.2. *The magnitude of the magnetic force exerted on the charged particle depends on four factors:*

(i) The charge of the particle ($F_M \propto q$)

(ii) The speed at which the particle is moving (relative to the magnetic field it is moving through)($F_M \propto v$)

(iii) The strength of the magnitude field that the charged particle is moving through

(iv) The angle in between the two magnetic fields ($0°$ leads to the maximum magnetic force, $90°$ leads to the minimum)

It is time consuming to determine the field direction around a charged particle that is moving. As a result, people tends to find the angle between the direction of current traveling and the magnetic field

Theorem 37.3. *The strength of force of magnetic is defined by this formula:*

$$F_M = |q|\, vB \sin\theta$$

F_M is the magnitude of the magnetic force on the charged particle (in Newton)
q is the charge of the particle(in C)
v is the speed of the particle (related to the magnetic field is it going through (in $\frac{m}{s}$))
θ is the angle in between the direction the particle is travelling and the magnetic field it is traveling through. B is the strength of the magnetic field that the particle is going through (in T or Tesla)

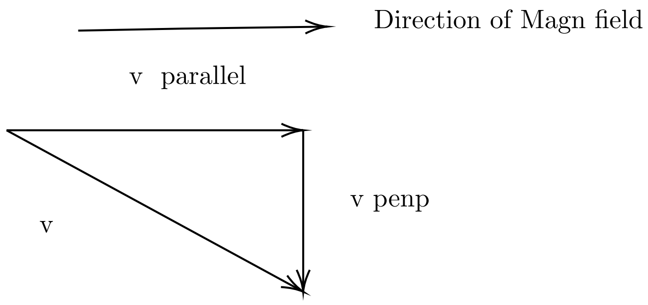
Remark. Do not include the signs for any variable in this formula, you need to determine the direction of F_M conceptually

37.2 Right Hand Rule

- For use with positvely-charged particles
- Point the thumb of your right hand in the direction the particle is moving
- Point the fingers of your right hand in the direction of the magnetic-field the particle is travelling
- The palm of your right hand will face the direction of the force on the positvely charged particle.

38 The motion of Charged Particles in Magnetic Fields

When a charged particle go through a magnetic field, we can seperate the motion in to two components.



38.0.1 What will the motion of the particle look like

– If the magnetic field is large and uniform, $\vec{v_{||}}$ will remain constant

– Because the magnetic force is always perpendicular to $\vec{v_{\perp}}$, it will not affect the magnitude of $\vec{v_{\perp}}$, it will only cause the direction to change.

– If the magnetic field doesn't change, the F_M will be constant

– Thus, we get a <u>corkscrew</u> motion.

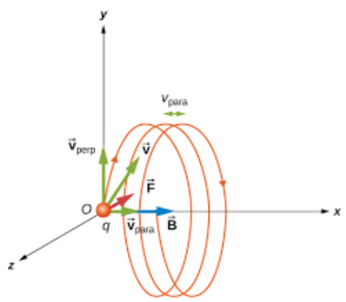


Figure 19: Motion of che charged particle

38.0.2 Aurora Borealis

The sun emits charged particles (ions), we call this the solar wind. When the solar wind reaches the Earth, the charged particles can become trapped in the Earth's mag field.

The ions flow the mag field towards one of the poles. The ions start to descend as they get closer to the poles (to regions where the air is more dense). When one of the ions collide with an air particle (either oxygen or nitrogen) light is given off. If enough collisions occur, we are able to see the light.

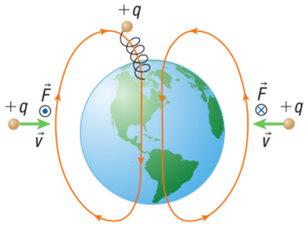


Figure 20: Aurora Borealis

38.0.3 Mass Spectrometers

Under the right conditions a particle that enters a magnetic field will undergo full uniform circular motion

Following two conditions must be met:

- The magnetic field must be relatively large and uniform.
- The particle must be travelling penpendicular.

Procedures

Mass spectrometer can be used to determine the mass of a particle with following steps:

- Something to inject particles into the spectrometer, at the correct plate of a particle.
- Something that causes the particles to become ionized
- A particle accelerator to "shoot" the particles into a magnetic field
- A large, uniform magnetic field
- A moveable ion detector (Used to determine where the ion emerges from the magnetic field)

Theorem 38.1. *If we can determine the q of the particle; Magnetic Field Strength (B); Potential Difference Δv and radius of the particle's circular motion, the mass of the particle can be calculated using this formula:*

$$m = \frac{|q| B^2 R^2}{2 |\Delta v|}$$

Please do not include the sign when you are using the formula.

Here is another formula that maybe useful:

$$\Delta \vec{F} = m\vec{a}$$
$$F_M = ma_c$$
$$|q| vB \sin\theta = m\frac{v^2}{R}$$

39 The Magnetic Force on a Straight Conductor

39.1 Definitions

Theorem 39.1. *The magnetic force on a straight conductor can be described by this formula:*

$$F_M = BIL\sin\theta$$

F_M *is the magnitude of the magnetic force on the straight conductor (in N)*
B *is the strength of the magnetic field that the conductor is in(in T)*
I *is the current flowing through the conductor in A*
L *is the length of the conductor that is the magnetic field (in m)*
θ *is the angle in between the current and the magnetic field*

39.1.1 Derive a formula for the magnetic force on the conductor

Lemma 39.2. *The magnitude of the magnetic force of one charged particle in the conductor can be evaluated by this formula:*

$$F_{M(single\ charge)} = |q|\, vB\sin\theta \tag{71}$$

Lemma 39.3. *If one of the charged particles travels from one end of the conductor to the other end, we can rewrite v into*

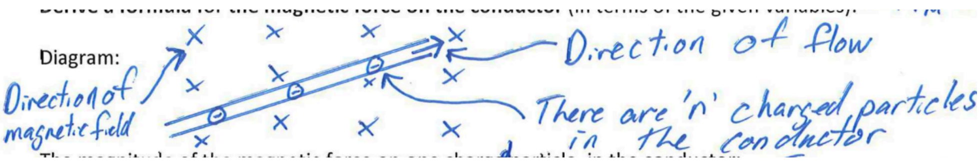
$$v = \frac{L}{\Delta t} \tag{72}$$

Lemma 39.4. *Current is defined as the amount of charge that passes through a location, per unit time. In the time of Δt, n charges pass through the end of the conductor*

$$I = \frac{nq}{\Delta t} \tag{73}$$

Proof. Assume we have a straight conductor with these parameters in magnetic field:

A length of L
A magnetic Field strength of B
A current of I
Angle between the current and the magnetic field: θ



The F_M on all of the charged particles in the conductor will be the sum of all the F_M on the individual charged particles. Each particle will experience an idential F_M because each particle has the same velocity and charge. Also the magnetic field that the conductor is in is uniform.

So we can multiple 71 by n:

$$F_{M(\text{on all charge})} = n \times |q|\, vB\sin\theta \tag{74}$$

Sub 72 into 74:

$$F_{M(\text{all charge})} = n \times \frac{|q|}{\Delta t} LB \sin\theta \tag{75}$$

Sub 73 into 75

$$F_{M(\text{all charge})} = ILB \sin\theta$$

□

39.1.2 The direction of the force

If you want to use the Right hand Rule # 3, you should point your thumb to the direction of the current flow

In the contrary, if you want to use Left hand Rule # 3, you should point your thumb to the direction of the electron flow

39.2 What is Tesla

1 Tesla is the strength of the magnetic field that will cause a force of 1 Newton to be exerted one a 1m long straight conductor that has a current of 1 A

Proof.

$$F_M = BIL \sin\theta (\theta = 90^\circ)$$
$$B = \frac{F_M}{IL}$$
$$1T = \frac{1N}{1A \times 1m}$$

□

If we have 2 Tesla of magnetic Field Strength:

$$2T = \frac{2N}{1A \times 1m}$$