OSSD - Calculus and Vector

MCV4U

Qinghao Hu

February 15, 2026

Unit 1: Rate of change. In the first unit, we will discuss the concept of Rate of Change, Continuity and Limits. Also, the the introduction to the Derivatives will be discussed.

Contents

1 Continuity and Limits

1.1 The Concept of Limit

Definition 1.1 (Limits)**.** Assume we have function $f(x)$, x, and

$$\lim_{x\to a} f(x) = L$$

Using the $\delta - \epsilon$ language:

$$\forall\epsilon > 0, \exists\delta > 0 \text{ such that } 0 < |a - x| < \delta \implies |f(x) - L| < \epsilon$$

Remember, $f(a)$ does not have to be defined.

For limts, we are solving for the expected value instead of defined value. When the following conditions are met, the limits exists

–
$$\lim_{x\to a^-} f(x) \text{ must exist}$$

–
$$\lim_{x\to a^+} f(x) \text{ must exist}$$

–
$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$$

Definition 1.2 (Continuity)**.** a function which there are no ”breaks” or ”gaps” in the function.

These conditions must be met

(i)
$$\lim_{x\to a} f(x) \text{ exist}$$

(ii)
$$f(a) \text{ must be defined}$$

(iii)
$$\lim_{x\to a} f(x) = f(a)$$

1.2 Continuity and Limits

Definition 1.3 (Limits Law)**.** Assume we have function $f(x)$ and $g(x)$, which

$$\lim_{x \to a} f(x) \text{ and } \lim_{x \to a} g(x) \text{ both exist}$$

We also have constant c We get these properties:

(i)
$$\lim_{x \to a} c = c$$

(ii)
$$\lim_{x \to a} x = a$$

(iii)
$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

(iv)
$$\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

(v)
$$\lim_{x \to a} [f(x) * g(x)] = \lim_{x \to a} f(x) * \lim_{x \to a} g(x)$$

(vi)
$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

(vii)
$$\lim_{x \to a} [f(x)]^n = (\lim_{x \to a} f(x))^n$$

(viii)
$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{(\lim_{x \to a} f(x))}$$

Somes cases you should be careful

Example 1.4. Evaluate and simplify

$$\lim_{x \to a} \sqrt{x}$$

Recall: if a limit exist

(i)
$$\lim_{x \to a^-} f(x) \text{ exists}$$

(ii)
$$\lim_{x \to a^+} f(x) \text{ exists}$$

(iii)
$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

In this case,

$$\lim_{x\to 0^-} \sqrt{x} \text{ does not exist}$$

because

$$D : x \in \mathcal{R} | x \geq 0$$

Example 1.5. Evaluate and simplify:

$$\lim_{x\to 1} x^2 - 5x$$

Remark. Please be carefull with the bracket, don't mix with

$$\lim_{x\to 1}(x^2 - 5x)$$

The solution should be:

$$\begin{aligned} &= (\lim_{x\to 1} x)^2 - 5x \\ &= (1)^2 - 5x \\ &= 1 - 5x \end{aligned}$$

2 Reference Sheet

2.1 Assume Knowledge

2.1.1 Log Laws

$$\log_b A + \log_b B = \log_b(A \times B)$$
$$\log_b(\tfrac{M}{N}) = \log_b M - \log_b N$$
$$log_b M^k = k \log_b M$$
$$log_b b^k = k$$
$$b^{\log_b k} = k$$

2.1.2 Trig Laws

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$
$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \cot^2\theta = \csc^2\theta$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$$

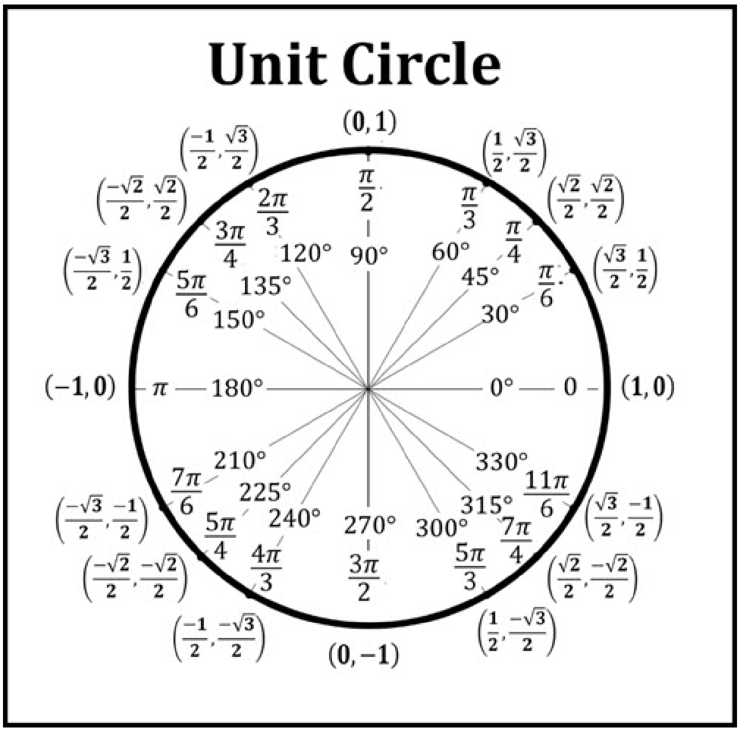
sin Law:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine Law:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Unit Circle:


Unit Circle
(0,1)
(−1/2, √3/2)
(1/2, √3/2)
(−√2/2, √2/2)
(√2/2, √2/2)
(−√3/2, 1/2)
(√3/2, 1/2)
π/2
2π/3
π/3
3π/4
π/4
5π/6
π/6
90°
120°
60°
135°
45°
150°
30°
(−1,0)
π
180°
0°
0
(1,0)
210°
330°
7π/6
11π/6
225°
315°
5π/4
7π/4
240°
300°
4π/3
5π/3
270°
3π/2
(−√3/2, −1/2)
(√3/2, −1/2)
(−√2/2, −√2/2)
(√2/2, −√2/2)
(−1/2, −√3/2)
(1/2, −√3/2)
(0,−1)