

Grade 12 Math of Data Management

MDM4U

Qinghao Hu

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Chapter 1

Probability Distributions

1.1 Probability Distributions

1.1.1 Definitions for Probability Distributions

Definition 1.1.1 (Random Variable)

A variable whose values are numerical outcomes of a random phenomenon, such as a probability experiment.

Definition 1.1.2 (Discrete random variable)

A random variable that has a *FINITE* number of possible values in a given interval. Examples include number of books, shoe sizes, and report card marks

Definition 1.1.3 (Continuous variable)

A random variable that can have an *INFINITE* number of possible values in a given interval. Examples include height, time, distance and money

Definition 1.1.4

Probability Distribution The use of it illustrate the *PROBABILITY* of all possible outcomes of an experiment. The illustration may be in the form of a table of values, a graph, or an equation

Definition 1.1.5

The probability that a discrete random variable X have a particular value x is expressed as $P(X = x)$ or $P(x)$

Definition 1.1.6

The expected value, $E(x)$ of a random variable X is the predicted *MEAN* of all possible outcomes of a probability experiment. If X is discrete, then

$$E(x) = \sum x_i P(x_i)$$

and

$$\sigma = \sqrt{\sum (x - E(x))^2 P(x)}$$

1.2 Uniform Distributions

1.2.1 Different Distributions

Distributions of data can be classified by considering the general shape of its graph. This picture is a table of distributions which is copied from the teacher's note

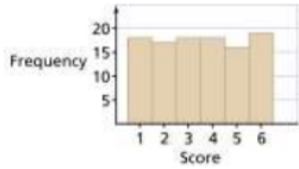
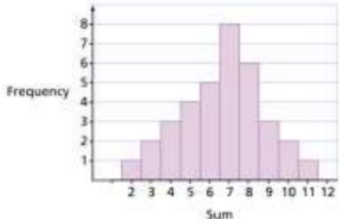
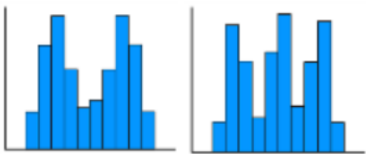
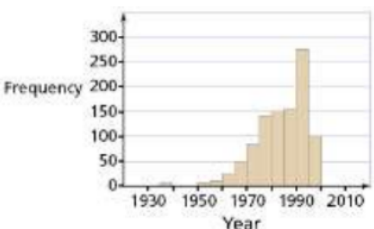
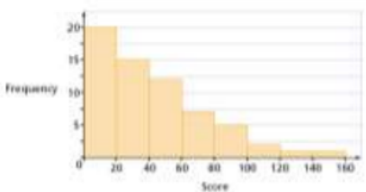
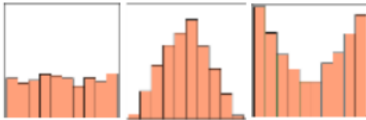
Distribution Type	Description	Example(s)
Uniform	height of <u>EACH</u> bar in the distribution histogram is roughly equal	
Mound-shaped	an interval with the <u>HIGHEST</u> frequency, frequencies of other intervals decrease as you move along either side	
Multimodal	Bimodal distributions has two <u>PEAKS</u> and trimodal distributions has three peaks	
Left-skewed (Negatively skewed)	mean is skewed to the <u>LEFT</u> hence the distribution is asymmetrical with a left-direction (i.e. a "left-tail" exists)	
Right-skewed (Positively skewed)	mean is skewed to the <u>RIGHT</u> hence the distribution is asymmetrical with a right-direction (i.e. a "right-tail" exists)	
Symmetric	shows mirror <u>SYMMETRY</u> about the centre of the distribution	

Figure 1.1: Thanks to Mr Tang

1.2.2 Characteristics of Uniform Distribution

If an distribution is considered Uniform, it will have following characteristics:

1. Each outcome is EQUALLY likely in any single trial of experiment

2. If X is discrete and n is the number of possible outcomes in the probability experiment, then

$$P(X = x) = P(x) = \frac{1}{n}$$

$$E(x) = \frac{\sum x_i}{n}$$

$$\sigma = \sqrt{\frac{1}{n} \sum (x - E(x))^2}$$

3. If X is continuous with values in range from a to b , then the expected value $E(x)$ will be $\frac{a+b}{2}$

1.3 Binnomial Distributions

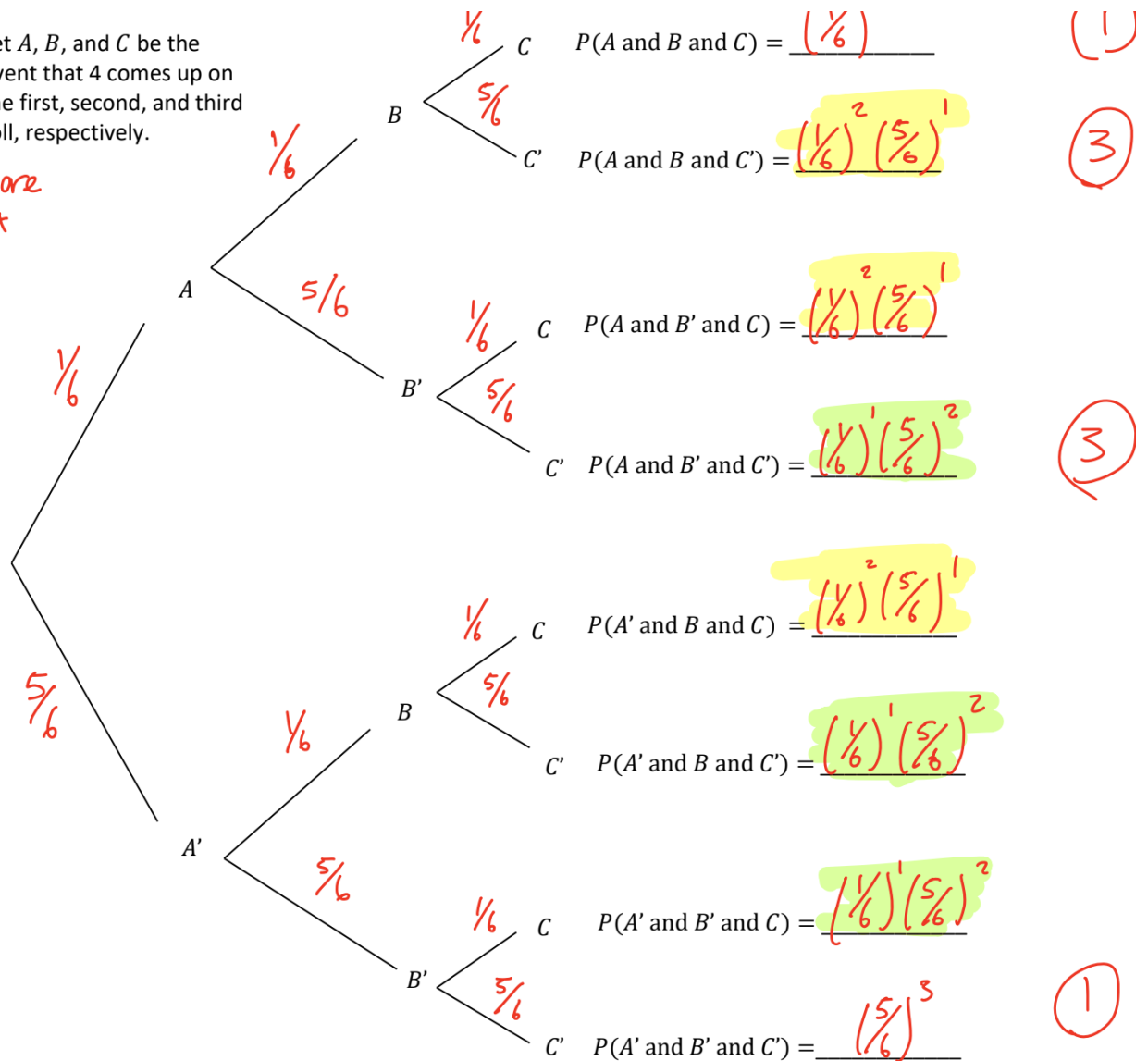
1.3.1 Example

Example 1

A probability experiment involves rolling a regular 6-sided die three times. Let the random variable X represent all possible number of 4's rolled. Complete the tree diagram, fill in the blanks, then show the distribution of X in a table.

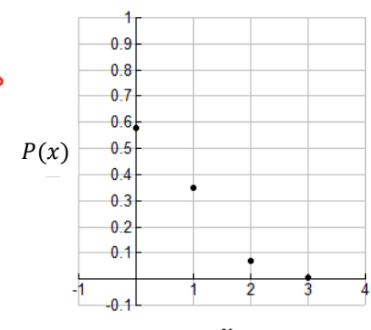
Let A , B , and C be the event that 4 comes up on the first, second, and third roll, respectively.

A, B, C are independent events



x	0	1	2	3
$P(x)$	$\binom{3}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^3$ $= \frac{125}{216}$	$\binom{3}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2$ $= \frac{75}{216}$	$\binom{3}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^1$ $= \frac{15}{216}$	$\binom{3}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^0$ $= \frac{1}{216}$

$$\binom{n}{x} p^x (1-p)^{n-x}$$



This question is a typical example of **Binominal Distribution**

1.3.2 Definitions

Definition 1.3.1

The distribution of a discrete random variable X is a binomial probability distribution if:

1. The probability experiment is repeated a *FIXED* number of times (In the example, $n = 3$)
 2. The outcome of each trial can be *CATEGORIZED* into successful or failed outcomes
 3. The random variable X represents the number of **successes**
 4. Each trial of the experiment is **Independent**
-

Theorem 1.3.2

If the distribution of X is a binomial probability, N is the number of independent trials of the experiment and p is the probability of success of each independent trial, then:

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad (1.1)$$

$$E(x) = np \quad (1.2)$$

$$\sigma = \sqrt{np(1-p)} \quad (1.3)$$

Remark. The binomial distribution is only work for Discrete random variable!

1.4 Geometric Distributions

1.4.1 Example

Example 2

Here is an example from our teacher's note:

A standard 6-sided die is rolled. Let X represent the number of unsuccessful rolls (i.e. the waiting time) before a 4 appears.

a) Complete the given table.

x	0	1	2	3	4
$P(x)$	$\frac{1}{6}$	$(\frac{5}{6})(\frac{1}{6}) = \frac{5}{36}$	$(\frac{5}{6})^2(\frac{1}{6}) = \frac{25}{216}$	$(\frac{5}{6})^3(\frac{1}{6}) = \frac{125}{1296}$	$(\frac{5}{6})^4(\frac{1}{6}) = \frac{625}{7776}$

b) What is the probability that 4 appears within the first 3 rolls?

$$P(0) + P(1) + P(2) = \frac{1}{6} + \frac{5}{36} + \frac{25}{216} = \frac{96}{216}$$

1.4.2 Definitions

Definition 1.4.1

The distribution of a discrete random variable X is a geometric probability distribution if:

1. each trial of this experiment is **INDEPENDENT**
2. the outcome of each trial can be categorized into **Successful** or failed outcomes
3. the random variable X represents the number of failed outcomes **before** a success occurs

Theorem 1.4.2

If the distribution of X is a geometric probability distribution then:

$$P(x) = (1 - p)^x p \quad (1.4)$$

$$E(x) = \frac{1 - p}{p} \quad (1.5)$$

$$\sigma = \frac{\sqrt{1 - p}}{p} \quad (1.6)$$

1.5 Hypergeometric Distributions

1.5.1 Example

Example 3

An example from Teacher's note!

- There are ten red and eight green dice in a container. Six dice are drawn at random (without replacement). Let the random variable X represent the number of red dice drawn. Display the probability distribution of X in a table then determine $E(X)$.

x	0	1	2	3	4	5	6
$P(x)$	$\frac{\binom{10}{0}\binom{8}{6}}{\binom{18}{6}}$ $= \frac{1}{663}$	$\frac{\binom{10}{1}\binom{8}{5}}{\binom{18}{6}}$ $= \frac{20}{663}$	$\frac{\binom{10}{2}\binom{8}{4}}{\binom{18}{6}}$ $= \frac{75}{442}$	$\frac{\binom{10}{3}\binom{8}{3}}{\binom{18}{6}}$ $= \frac{80}{221}$	$\frac{\binom{10}{4}\binom{8}{2}}{\binom{18}{6}}$ $= \frac{70}{221}$	$\frac{\binom{10}{5}\binom{8}{1}}{\binom{18}{6}}$ $= \frac{24}{221}$	$\frac{\binom{10}{6}\binom{8}{0}}{\binom{18}{6}}$ $= \frac{5}{442}$

$$E(x) = \sum x_i P(x_i)$$

$$= 0\left(\frac{1}{663}\right) + 1\left(\frac{20}{663}\right) + \dots + 6\left(\frac{5}{442}\right)$$

$$= \frac{10}{3} \text{ red dice}$$

OR

$$E(x) =$$

$$= \left(\frac{10}{18}\right)(6)$$

$$= \frac{10}{3} \text{ red dice}$$

- Fill in the blanks.

1.5.2 Definitions

Definition 1.5.1

The distribution of a discrete random variable X is a hypergeometric probability distribution if:

- the probability experiment is repeated a **fixed** number of times
- the outcome of each trial can be categorized into **success** or failed outcomes
- the random variable X represents the number of **successes**
- each trial of experiment is **dependant**

Theorem 1.5.2

If the distribution of X is hypergeometric with n dependent trials, and a represents the number of successful outcomes initially among a total of N possible outcomes in the beginning, then

$$P(x) = \frac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}} \quad (1.7)$$

$$E(x) = \frac{an}{N} \quad (1.8)$$

$$\sigma = \sqrt{n\left(\frac{a}{N}\right)\left(\frac{N-a}{N}\right)\left(\frac{N-n}{N-1}\right)} \quad (1.9)$$