AOPS AMC12 class note

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1 Class 1: Quadratics, Vieta, and Factorization

Today, we will look at quadratic functions, Vieta's formula and factorizations

1.1 Quadratic Equation

A quadratic equation is an equation of the form

$$ax^2 + bx + c = 0 \tag{1}$$

where a, b *and* c *are constants and* $a \neq 0$

1.2 Roots of Quadratic Equation

The roots of Quadratic Equation can be determined by the Quadratic Formula, or more formally, Sridharacharya Formula

Assume r_1 *and* r_2 *are both roots of the Quadratic Equation* $ax^2 + bx + c = 0$

$$r_1 = \frac{-(b) + \sqrt{b^2 - 4ac}}{2a}, r_2 = \frac{-(b) - \sqrt{b^2 - 4ac}}{2a} \tag{2}$$

r_1 *and* r_2 *may be imagery numbers if* $\sqrt{b^2 - 4ac} < 0$

1.3 Vieta's Formula

Assume we have a polynomial formula with degree of n

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \tag{3}$$

and $r_1, r_2, \cdots, r_n$ *are the roots of the polynomial*
we can get:

$$r_1 + r_2 + \cdots + r_{n-1} + r_n = \frac{a_{n-1}}{a_n} \tag{4}$$

$$(r_1 r_2 + r_1 r_3 + \cdots + r_1 r_{n-1} + r_1 r_n) + (r_2 r_3 + r_2 r_4 + \cdots + r_2 r_{n-1} + r_2 r_n) + \cdots + r_{n-1} r_n = \frac{a_{n-2}}{a_n} \tag{5}$$

$$r_1 r_2 \cdots r_{n-1} r_n = (-1)^n \frac{a_0}{a_n} \tag{6}$$

1.4 Factorizations

Some key factorizations that you should be familiar with are different of square, difference of cubes, and sum of cubes

one good example is the "Simon's favorite factoring trick"

for example:

$$\text{Factor the equation, } mn - 2m - 4n + 8 = 8$$

we can get:

(m - 4)(n - 2) = 8

2 Class 2: Solving equations and systems

Summary:
When dealing with several variables, look for ways of eliminating them to get what you want. And make sure you read the problem carefully so that you know what you want, and you do not end up doing more work than you have to.

3 Class 3: Sequences and Series

3.1 Arithemtic Sequences and Series

An arithmetic sequence is a sequence of the form $a, a + d, a + 2d$, and so on (for example, 7,11,15,19 is an arithmetic sequence). In other words, we begin with a first term a, and repeatedly add a common difference d to obtain the terms that follow.

Te sum of the arithmetic series with n terms is:

$$a + (a + d) + \cdots + [a + (n - 1)d] = \frac{2a + (n - 1)d}{2} * n \tag{7}$$

3.2 Geometric Sequences and Series

A geometric sequence is a sequence of the for a, ar, ar^2, and so on. In other words, we have a first term a, and repeatedly multiply by a common ratio r to obtain the terms that follow

The sum of the geometric series with n term is:

$$a + ar + \cdots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r} \tag{8}$$

The numerator can be viewed as the difference of two terms, a and ar^n. <u>Notice in particular that these are not the first term and last term.</u>

For $|r| < 1$, the sum of the infinite geometric series is:

$$a + ar + ar^2 + \cdots = \frac{a}{1 - r}$$

4 Class 4: Functions and Polynomials

Today we will look at the properties of certain functions, such as the "floor" function and logarith, as well as polynomials , which form a very special class of functions