

# OSSD - Calculus and Vector

MCV4U

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**Unit 1: Rate of change.** In the first unit, we will discuss the concept of Rate of Change, Continuity and Limits. Also, the introduction to the Derivatives will be discussed.

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# 1 Slope

## 1.1 Measuring Rates of Change on a Table or a Graph

**Definition 1.1** (slope/Rate of Change). the ratio of the vertical distance to the horizontal distance between two points.

In Grade 9, we all studied the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$m$  represents the slope because linear function is written as:

$$f(x) = mx + b$$

### 1.1.1 Interval of the function

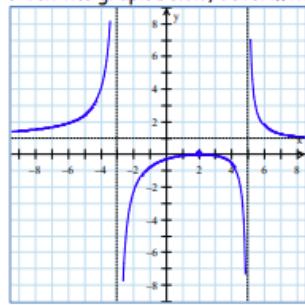
When you are asked to give the interval of a function, one must response in one of these forms:

$$x \in (-\infty, a) \cap (b, \infty)$$

*Remark.* We look the function from  $-\infty$  to  $\infty$

**Example 1.2.** Here is a question by Mr.Hoken

8. Given the graph below, determine the regions on the function where:



- a) the  $y$  values are increasing  
 $x \in (-\infty, -3)$  or  $-3 < x < 2, x \in \mathbb{R}$
- b) the rate of change (r.o.c.) is increasing  
 $x < -3 \cup x > 5, x \in \mathbb{R}$
- c) the  $y$  values are negative and the slopes are positive  
 $-3 < x < 2, x \in \mathbb{R}$
- d) the slopes are negative and the r.o.c. is decreasing  
 $x \in (2, 5), x \in \mathbb{R}$

The answer for a) should be  $x \in (-\infty, -3)$

The answer for b) should be  $x \in (-\infty, -3) \cup (5, \infty)$

### 1.1.2 Determine approximation for the slope of the tangent

To start off, we need to two points which have equal difference of  $x$  to the point that tangent line cross over.

Let's assume we have points 1, 2, and 3. Point 2 touches the tangent line. Solve for the slope between point 1 and point 2:

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

Solve for the slope between point 2 and point 3:

$$m_2 = \frac{y_3 - y_2}{x_3 - x_2}$$

The slope at point 2 that touches tangent line should be:

$$m = \frac{m_2 + m_1}{2}$$

### 1.1.3 Trends and increasing/decreasing

Here are the summary Mr. Hoken made for this class

- An increasing trend in y values corresponds to positive slopes of tangents in that interval
- A decreasing trend in y values corresponds to negative slopes of tangents in that interval
- "Rate of Change" in y values refers to your slope values. We use standard expressions to describe the trend in Slope values
  - \* "The rate of change is increasing" means that the slope values are climbing.  
ex.  $(0.5, 3, 4, 5, 6, 20, \dots)$  or  $(-10, -9, \dots, 1, 2)$
  - \* "The rate of change is decreasing" means that the slope values are dropping.  
ex.  $(25, 11, 9, 5, \dots, -3, -7, -20)$

## 1.2 Measuring Rates of Change Using a Defined Relation

### 1.2.1 Difference Quotient

**Definition 1.3** (Difference Quotient). a measure of the average rate of change of the function over an interval. In other words, it is a simplified expression for the slope of a secant. It can be expressed as

$$m = \frac{f(x+h) - f(x)}{h}$$

where  $h \neq 0$

Let's proof the difference Quotient

*Proof.* We have the formula of the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Assume we have two points:  $(x, f(x))$  and  $(x+h, f(x+h))$ , sub these two points into the formula of slope.

$$\begin{aligned} m &= \frac{f(x+h) - f(x)}{(x+h) - x} \\ m &= \frac{f(x+h) - f(x)}{h} \end{aligned}$$

□

Let's look at an example.

**Example 1.4.** Determine a general expression for the slope given the function  $f(x) = -x^2 + 4x + 5$

$$\begin{aligned} m &= \frac{f(x+h) - f(x)}{h} \\ m &= \frac{-(x+h)^2 + 4(x+h) + 5 - (-x^2 + 4x + 5)}{h} \\ m &= \frac{-x^2 - 2xh - h^2 + 4x + 4h + 5 + x^2 - 4x - 5}{h} \\ m &= \frac{-2xh - h^2 + 4h}{h} \\ m &= -2x + 4 - h, h \neq 0 \end{aligned}$$

### 1.2.2 Approximating the slope of the tangent

Recall:

- The slope of a secant line will give the average rate of change of the 'y' values over a givern interval of 'x'.
- The slope of a tangent line will give the instantaneous rate of change of the 'y' value at a givern value of 'x'.

For difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

if  $h$  is extremely small, we can approximate the slope of the tangent line.

In later sections, we will discuss about principle of limits. At that time, you will have a better understanding about today's topic.

## 2 Continuity and Limits

### 2.1 The Concept of Limit

**Definition 2.1** (Limits). Assume we have function  $f(x)$ ,  $x$ , and

$$\lim_{x \rightarrow a} f(x) = L$$

Using the  $\delta - \epsilon$  language:

$$\forall \epsilon > 0, \exists \delta > 0 \text{ such that } 0 < |a - x| < \delta \implies |f(x) - L| < \epsilon$$

Remember,  $f(a)$  does not have to be defined.

For limits, we are solving for the expected value instead of defined value. When the following conditions are met, the limits exists

—

$$\lim_{x \rightarrow a^-} f(x) \text{ must exist}$$

—

$$\lim_{x \rightarrow a^+} f(x) \text{ must exist}$$

—

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$$

**Definition 2.2** (Continuity). a function which there are no "breaks" or "gaps" in the function.

These conditions must be met

(i)

$$\lim_{x \rightarrow a} f(x) \text{ exist}$$

(ii)

$$f(a) \text{ must be defined}$$

(iii)

$$\lim_{x \rightarrow a} f(x) = f(a)$$

### 3 Reference Sheet

#### 3.1 Assume Knowledge

##### 3.1.1 Log Laws

$$\log_b A + \log_b B = \log_b(A \times B)$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b M^k = k \log_b M$$

$$\log_b b^k = k$$

$$b^{\log_b k} = k$$

##### 3.1.2 Trig Laws

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

sin Law:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine Law:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Unit Circle:

