Grade 12 Physics

SPH4U

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Contents

Chapter 1

Unit 1A

1.1 Review of Describing and Graphing Motion

1.1.1 Position: $\vec{d}$

Position is the **straight-line distance** from a fixed reference point to a location, with a direction to the location from the reference point.

1.1.2 displacement: $\Delta\vec{d}$

Displacement is the **change of position**
Formula:

$$\Delta\vec{d} = \vec{d_2} - \vec{d_1}$$

or

$$n = \text{the amount of displacement you want to add}$$

$$\Delta\vec{d_{tot}} = \sum_{i=1}^{n} \Delta\vec{d_i}$$

1.1.3 Velocity: $\vec{v}$

Velocity is the **rate of change of position**

$$\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$$

1.1.4 Acceleration: $\vec{a}$

Acceleration is the *rate of change* of velocity, always in the form of $\frac{m}{s^2}$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

1.1.5 Graphing motion

For a **Position vs Time** graph:

- **For a position/displacement vs time graph, the velocity = the *slope***
- A slope of zero = The object is not moving
- Instantaneous velocity ($\vec{v}_{inst}$) = the slope of **tangent** line to the graph at that point in time
- Average velocity ($\vec{v}_{avg}$) = the slope of **secant** line for that time interval

For a **Velocity vs Time** graph:

- Can get an object's instantaneous velocity directly from the graph
- slope = **acceleration**
- Displacement = The **area** between the graph and the time-axis for that time interval

1.2 Equations of Motion

To start off, there are five equations that are used in the calculation of motion

$$\vec{v_f} = \vec{v_i} + \vec{a} * \Delta d$$

$$\Delta\vec{d} = \frac{1}{2}(\vec{v_i} + \vec{v_f}) * \Delta t$$

$$\Delta\vec{d} = \vec{v_i}\Delta t + \frac{1}{2}\vec{a} * \Delta t^2$$

$$\Delta\vec{d} = \vec{v_f}\Delta t - \frac{1}{2}\vec{a} * \Delta t^2$$

$$\vec{v_f}^2 = \vec{v_i}^2 + 2\vec{a}\Delta\vec{d}$$

1.2.1 Format requirements for answering Motion questionss

1. You should always include a diagram that contains every known information from the question
2. $\vec{v}$ should be presented for at least two decimal places
3. Always list steps in your answer
4. When using quadratic solving function on calculator, always write **Using Quadratic Eq* in your answer
5. When you form an equation system, you should label 1.2.3. on each equation in the system
6. Follow the Sig Digit rules

   ! For **add** and **subtract**, keep the least **decimal places**
   ! For **multiply** and **divide**, keep the least amount of **signficiant digit**

1.3 Adding and Subtracting 2-Dimensional Vectors

1.3.1 Vector addition and subtraction key words

+ Addition: Find "the **resultant**", "the total", or "the net".

- Subtraction: Find "the **difference**" or "the change in".

1.3.2 Steps for solving a vector problem

1. Read the question carefully
2. Show unit conventions
3. Write "givens" (It helps to roughly sketch each vector and their compoents)
4. Set direction conventions
5. Solve for each compoents (ex. $\Delta\vec{d_{1y}}, \Delta\vec{d_{2x}}$)
6. Choose one component direction (ex. Just the 'x' direction) and solve the equations for that direction
7. Repeat with the other direction
8. Sketch your resulting x and y vectors, joining them head-to-tail.
9. Calculate the magnitude and direction of the resultant. (Trigonometry)
10. State the final answer, including the real-world direction

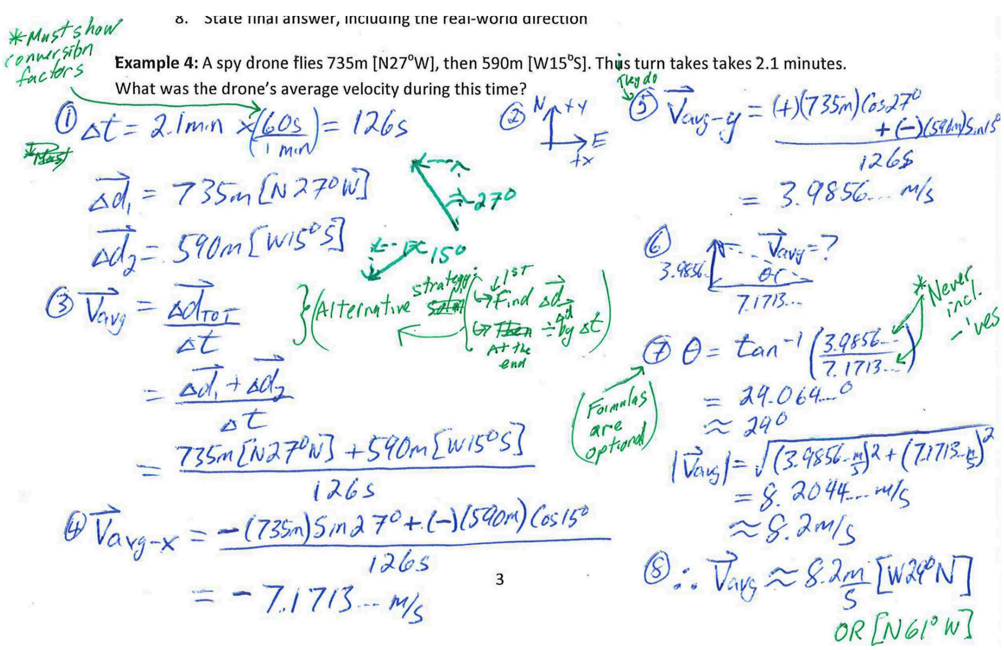
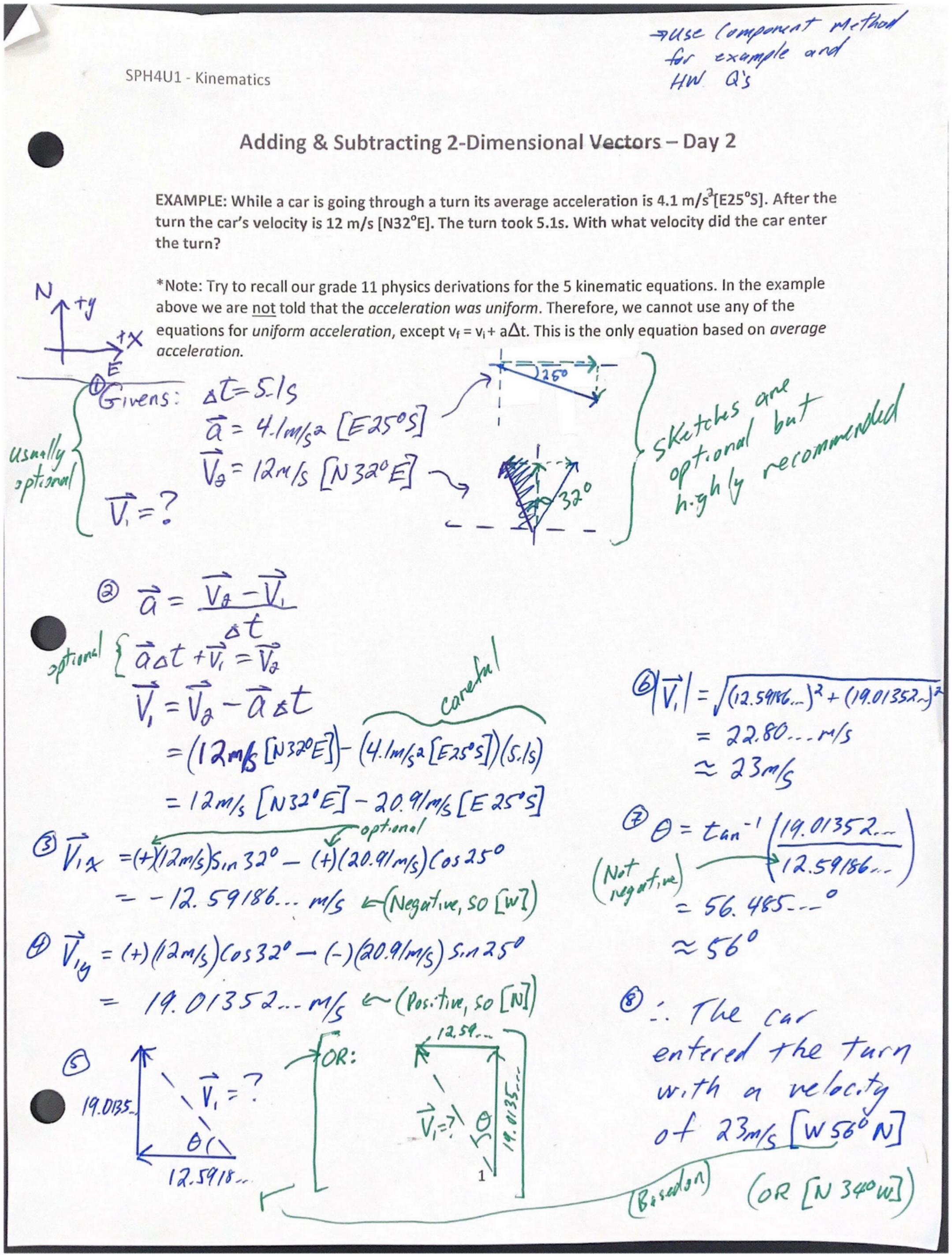


Figure 1.1: A sample answer for vector question

1.3.3 Another question type



→ use component method for example and HW Q's

SPH4U1 - Kinematics

Adding & Subtracting 2-Dimensional Vectors – Day 2

EXAMPLE: While a car is going through a turn its average acceleration is 4.1 m/s² [E25°S]. After the turn the car's velocity is 12 m/s [N32°E]. The turn took 5.1s. With what velocity did the car enter the turn?

*Note: Try to recall our grade 11 physics derivations for the 5 kinematic equations. In the example above we are not told that the *acceleration was uniform*. Therefore, we cannot use any of the equations for *uniform acceleration*, except $v_f = v_i + a\Delta t$. This is the only equation based on *average acceleration*.

N +y, +x E

usually optional

① Givens: $\Delta t = 5.1s$

$\vec{a} = 4.1 m/s^2$ [E25°S]

$\vec{V}_2 = 12 m/s$ [N32°E]

$\vec{V}_1 = ?$

25°, 32°

sketches are optional but highly recommended

② $\vec{a} = \dfrac{\vec{V}_2 - \vec{V}_1}{\Delta t}$

optional $\{ \vec{a}\Delta t + \vec{V}_1 = \vec{V}_2$

$\vec{V}_1 = \vec{V}_2 - \vec{a}\Delta t$

$= (12 m/s [N32°E]) - \underbrace{(4.1 m/s^2 [E25°S])(5.1s)}_{\text{careful}}$

$= 12 m/s$ [N32°E] $- 20.91 m/s$ [E25°S]

optional

③ $\vec{V}_{1x} = (+)(12 m/s)\sin 32° - (+)(20.91 m/s)\cos 25°$

$= -12.59186... m/s$ ← (Negative, so [W])

④ $\vec{V}_{1y} = (+)(12 m/s)\cos 32° - (-)(20.91 m/s)\sin 25°$

$= 19.01352... m/s$ ← (Positive, so [N])

⑤ 19.0135..., $\vec{V}_1 = ?$, θ, 12.5918...

OR: 12.59..., $\vec{V}_1 = ?$, θ, 19.0135...

⑥ $|\vec{V}_1| = \sqrt{(12.59186...)^2 + (19.01352...)^2}$

$= 22.80... m/s$

$\approx 23 m/s$

⑦ $\theta = \tan^{-1}\left(\dfrac{19.01352...}{12.59186...}\right)$ (Not negative)

$= 56.485...°$

$\approx 56°$

⑧ ∴ The car entered the turn with a velocity of 23 m/s [W56°N] (Based on)

(OR [N34°W])

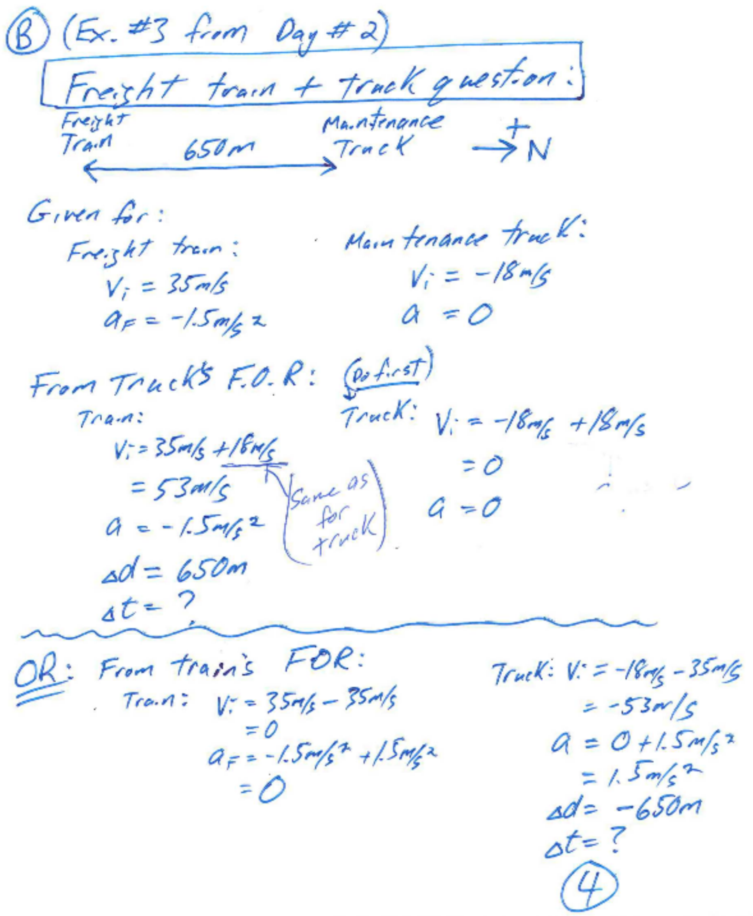
Figure 1.2: Remainder, multiply first

1.4 Frame of Reference

1.4.1 1 Dimension Frame of Reference

For certain Chase and Collision questions, you can use frame of reference to solve them

Example. How to write givens



(B) (Ex. #3 from Day #2)

Freight train + truck question:

Freight Train 650m Maintenance Truck $\overset{+}{\rightarrow}$ N

Given for:

Freight train:
$v_i = 35 m/s$
$a_F = -1.5 m/s^2$

Maintenance truck:
$v_i = -18 m/s$
$a = 0$

From Truck's F.O.R: (Do first)

Truck: $v_i = -18 m/s + 18 m/s$
$= 0$
$a = 0$

Train:
$v_i = 35 m/s + 18 m/s$
$= 53 m/s$
$a = -1.5 m/s^2$
$\Delta d = 650 m$
$\Delta t = ?$

Same as for truck

OR: From train's FOR:

Train: $v_i = 35 m/s - 35 m/s$
$= 0$
$a_F = -1.5 m/s^2 + 1.5 m/s^2$
$= 0$

Truck: $v_i = -18 m/s - 35 m/s$
$= -53 m/s$
$a = 0 + 1.5 m/s^2$
$= 1.5 m/s^2$
$\Delta d = -650 m$
$\Delta t = ?$

(4)

Figure 1.3: You should always write it

1.5 Relative Velocities in Two Dimensions

1.5.1 Recall

1. All velocities are related to an F.O.R that we consider to be motionless or stopped

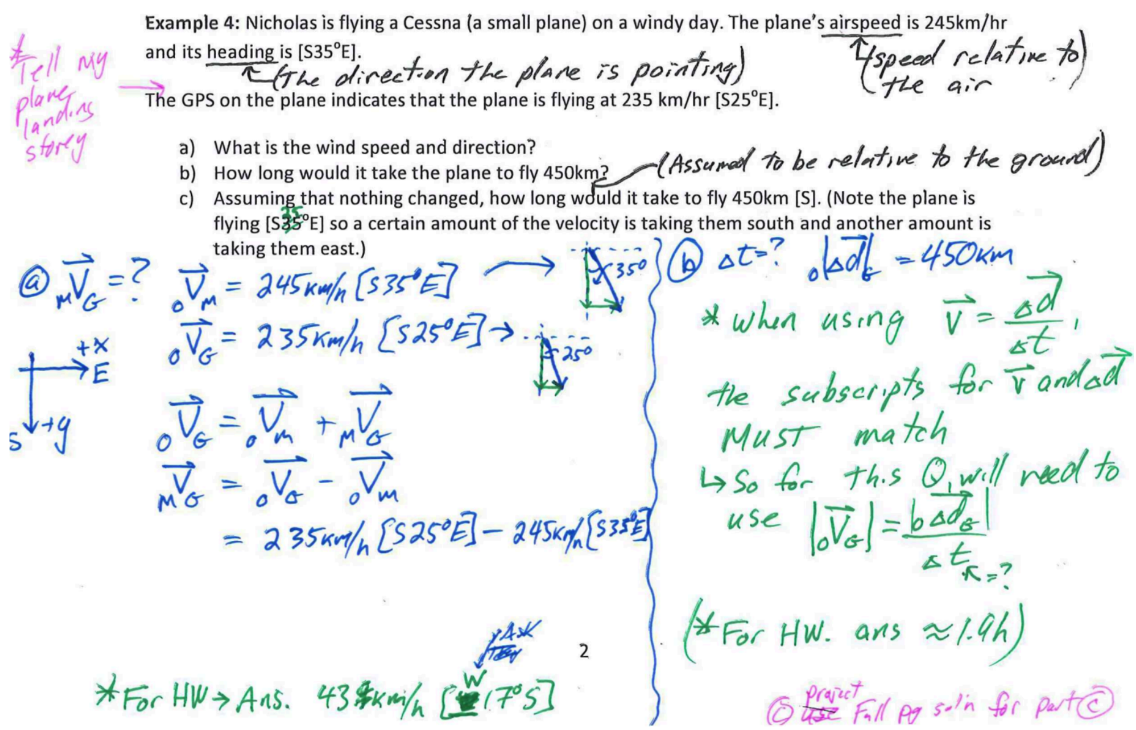
2. A frame of reference is just a perspective from which we observe/or measure the motion of objects

1.5.2 Definition

We sometimes refer to something that another thing can travel through as *medium*. We also sometimes refer to the thing that is moving through the medium as *object(O)*, and the ground use the symbol *(*G)

Therefore the equation can be written as:

$$v_{\vec{OG}} = v_{\vec{OM}} + v_{\vec{MG}}$$



Example 4: Nicholas is flying a Cessna (a small plane) on a windy day. The plane's airspeed is 245km/hr and its heading is [S35°E].

(The direction the plane is pointing)

(speed relative to the air)

*Tell my plane landing story

The GPS on the plane indicates that the plane is flying at 235 km/hr [S25°E].

a) What is the wind speed and direction?
b) How long would it take the plane to fly 450km? (Assumed to be relative to the ground)
c) Assuming that nothing changed, how long would it take to fly 450km [S]. (Note the plane is flying [S35°E] so a certain amount of the velocity is taking them south and another amount is taking them east.)

ⓐ $_M\vec{V}_G = ?$ $\quad _O\vec{V}_M = 245\,km/h\,[S35°E]$ $\quad _O\vec{V}_G = 235\,km/h\,[S25°E]$

+x E, S +y

$_O\vec{V}_G = {_O\vec{V}_M} + {_M\vec{V}_G}$

$_M\vec{V}_G = {_O\vec{V}_G} - {_O\vec{V}_M}$

$= 235\,km/h\,[S25°E] - 245\,km/h\,[S35°E]$

*For HW → Ans. 43 km/h [W 17° S]

ⓑ $\Delta t = ?$ $\quad |_O\Delta\vec{d}_G| = 450\,km$

* when using $\vec{V} = \frac{\Delta \vec{d}}{\Delta t}$, the subscripts for $\vec{V}$ and $\Delta\vec{d}$ must match

↳ So for this Q, will need to use $|_O\vec{V}_G| = \frac{|_O\Delta\vec{d}_G|}{\Delta t_{\leftarrow ?}}$

(*For HW. ans ≈ 1.9h)

ⓒ Project Full pg soln for part ⓒ

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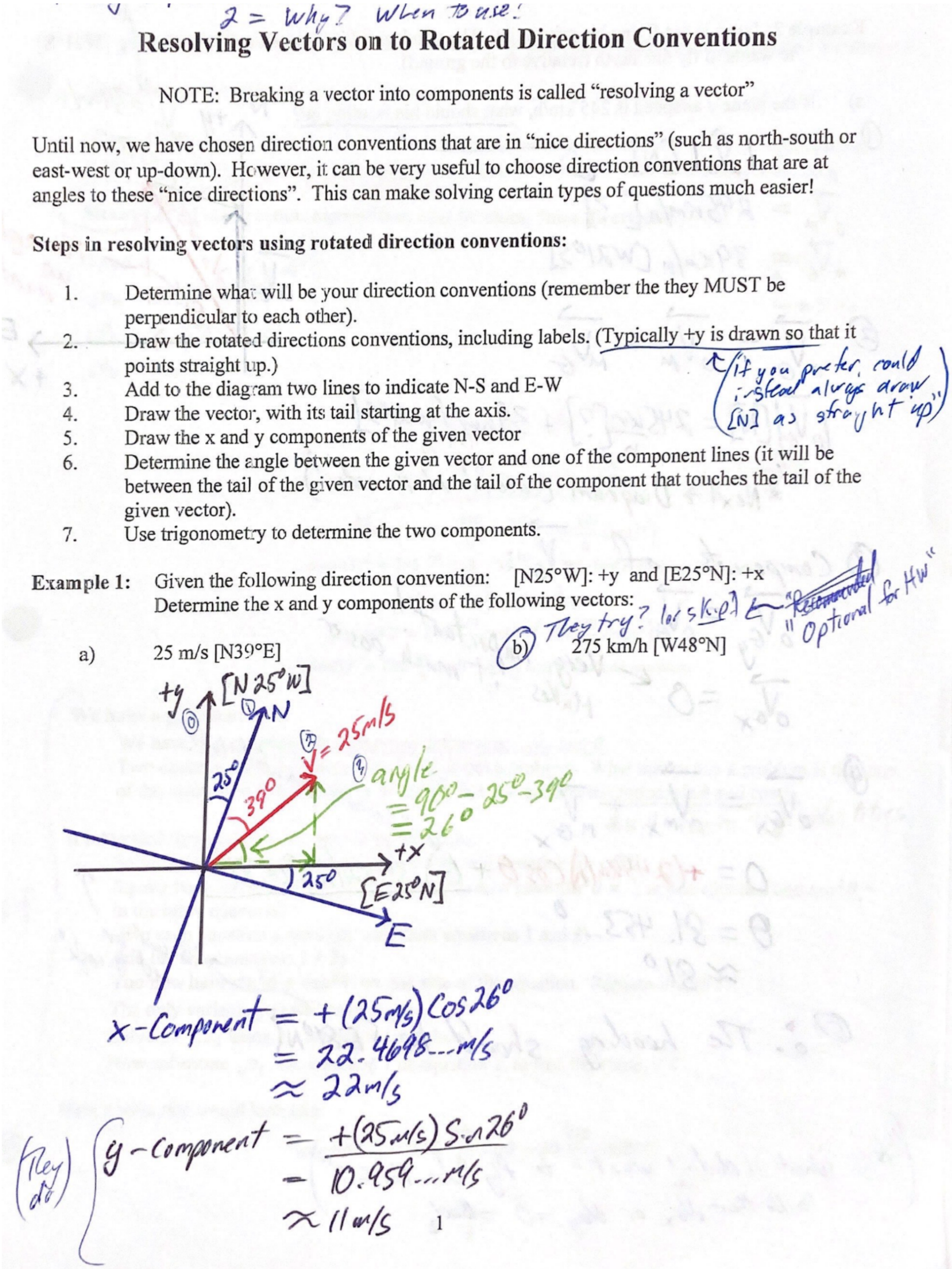
Figure 1.4: An example question

1.6 F.O.R in 2-D



2 = why? When to use!

Resolving Vectors on to Rotated Direction Conventions

NOTE: Breaking a vector into components is called "resolving a vector"

Until now, we have chosen direction conventions that are in "nice directions" (such as north-south or east-west or up-down). However, it can be very useful to choose direction conventions that are at angles to these "nice directions". This can make solving certain types of questions much easier!

Steps in resolving vectors using rotated direction conventions:

1. Determine what will be your direction conventions (remember the they MUST be perpendicular to each other).
2. Draw the rotated directions conventions, including labels. (Typically +y is drawn so that it points straight up.) (If you prefer, could instead always draw [N] as "straight up")
3. Add to the diagram two lines to indicate N-S and E-W
4. Draw the vector, with its tail starting at the axis.
5. Draw the x and y components of the given vector
6. Determine the angle between the given vector and one of the component lines (it will be between the tail of the given vector and the tail of the component that touches the tail of the given vector).
7. Use trigonometry to determine the two components.

Example 1: Given the following direction convention: [N25°W]: +y and [E25°N]: +x
Determine the x and y components of the following vectors:

a) 25 m/s [N39°E]

b) 275 km/h [W48°N] They try? (or skip) ← ~~"Recommended~~ "Optional for HW"

x-Component $= +(25\,m/s)\cos 26°$
$= 22.4698...\,m/s$
$\approx 22\,m/s$

(They do) y-Component $= +(25\,m/s)\sin 26°$
$= 10.959...\,m/s$
$\approx 11\,m/s$

1

Figure 1.5: Teacher's note

1.7 Review of Netwon's Laws of Montion

1.7.1 Netwon's First Laws

Inertia is an object's resistance to a change in its state of uniform Motion.

Example. An object at rest will remain at rest And an object in motion will continue to move in a straight line constant speed UNLESS a non-zero net force acts on the object

1.7.2 Newton's second Law

Newton's 2^{nd} Law is the formula that explains the behaviour of object when the forces on the object are not zero.
We can orgnize to these formulas:

$$\vec{a} = \frac{\sum \vec{F}}{m} \tag{1.1}$$

and

$$\sum \vec{F} = m * \vec{a} \tag{1.2}$$

$$\vec{a} = \text{acceleration of the object}$$
$$m = \text{mass of the object in kg}$$
$$\sum \vec{F} = \text{The sum of the net force}$$

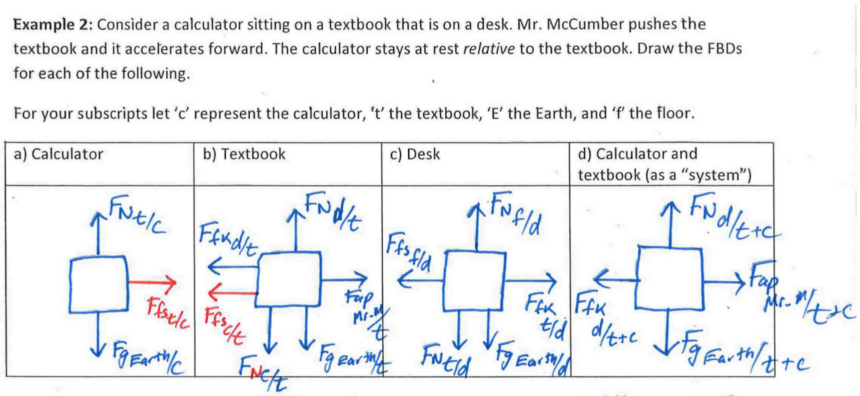
1.7.3 Newton's Third Law

For every force, there is another force, which is *equal in magnitude* to the first force, but *opposite in direction*. These two forces will *act on separate objects*, unless they are "internal force"
This mean's that all forces Always come in pairs, but two forces may not be acting on the same object.

To fit the Newton's 3^{rd} Law pair forces, the two force must:

1. Be the same type of force
2. $\vec{F_{A/B}} = -\vec{F_{B/A}}$

1.7.4 Free Body Diagrams (FBD)



Example 2: Consider a calculator sitting on a textbook that is on a desk. Mr. McCumber pushes the textbook and it accelerates forward. The calculator stays at rest *relative* to the textbook. Draw the FBDs for each of the following.

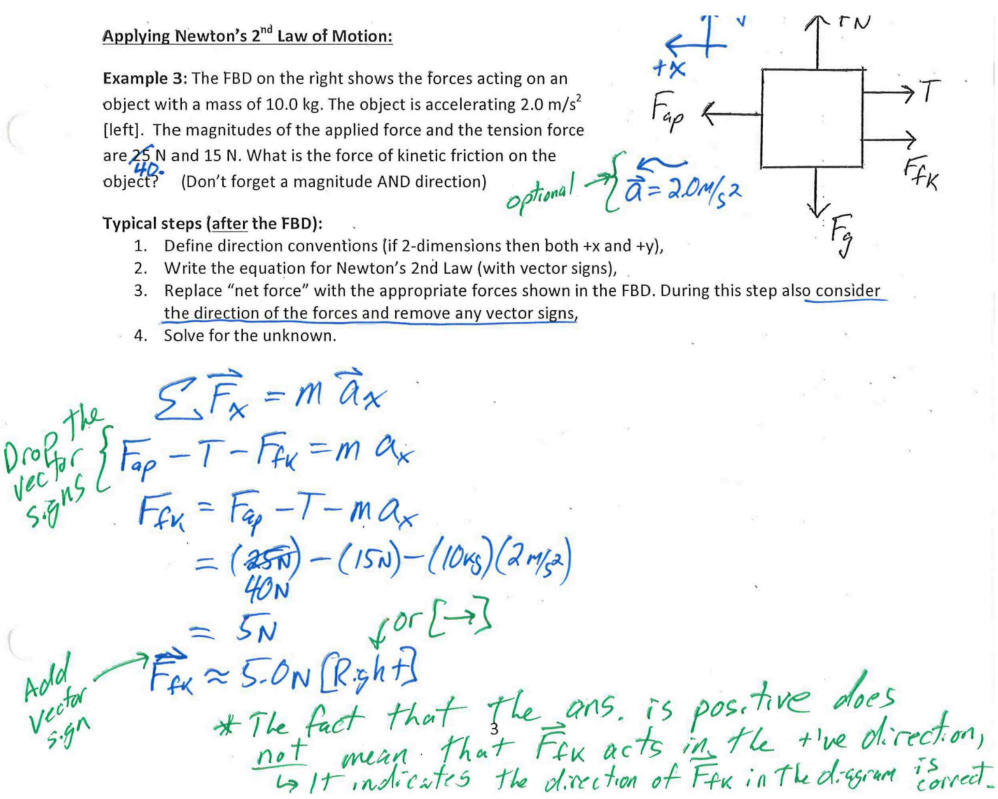
For your subscripts let 'c' represent the calculator, 't' the textbook, 'E' the Earth, and 'f' the floor.

a) Calculator	b) Textbook	c) Desk	d) Calculator and textbook (as a "system")

Figure 1.6: An example Free Body Diagram

1.7.5 Application of Newton's second Law

Here is an example:



Applying Newton's 2nd Law of Motion:

Example 3: The FBD on the right shows the forces acting on an object with a mass of 10.0 kg. The object is accelerating 2.0 m/s^2 [left]. The magnitudes of the applied force and the tension force are ~~25~~ 40 N and 15 N. What is the force of kinetic friction on the object? (Don't forget a magnitude AND direction)

optional $\overleftarrow{a} = 2.0\,m/s^2$

Typical steps (after the FBD):

1. Define direction conventions (if 2-dimensions then both +x and +y),
2. Write the equation for Newton's 2nd Law (with vector signs),
3. Replace "net force" with the appropriate forces shown in the FBD. During this step also consider the direction of the forces and remove any vector signs,
4. Solve for the unknown.

$$\sum \vec{F}_x = m\vec{a}_x$$

Drop the vector signs

$$F_{ap} - T - F_{fk} = m a_x$$

$$F_{fk} = F_{ap} - T - m a_x$$

$$= (\cancel{25N}\ 40N) - (15N) - (10kg)(2\,m/s^2)$$

$$= 5N$$

or $[\rightarrow]$

Add vector sign

$$\vec{F}_{fk} \approx 5.0N\ [\text{Right}]$$

* The fact that the ans. is positive does not mean that $\vec{F}_{fk}$ acts in the +'ve direction,
↳ It indicates the direction of F_{fk} in the diagram is correct.

Figure 1.7:

1.8 Review of Projectile Motion

1.8.1 basic

A simple projectile is an object that has a single, *non-uniform* acting on it. This single force must be a *non-contact* force between objects without the two objects being contact. Ex. Gravity forces, magnetic forces, electric forces

There are some kind of simple questions:

1. An object dropped
2. A soccer ball is kicked
3. A bullet is fired from a gun
4. An electron is moving through a uniform electrical field (Gravity is negligible)

Remainder: The object only accelerate in the direction of the net force. In any direction perpendicular to net force, the object main a constant velocity.

Some important points to remember:

- At maximum height all projectiles have a **vertical** velocity equal to **zero**
- When an object starts and ends at the same vertical location, the $\Delta\vec{d}_y = 0$
- When an object is dropped or launched horizontally, then $\vec{v_{y1}} = 0$

1.8.2 Special formula

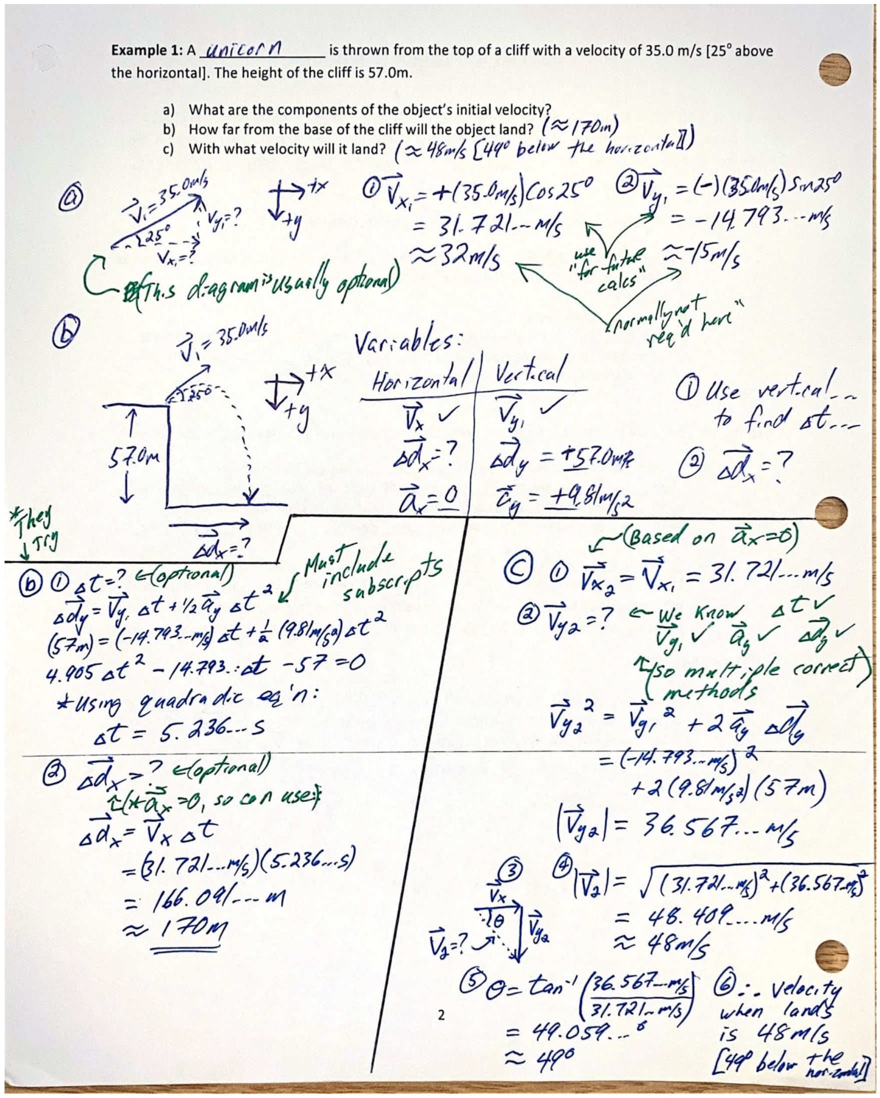
$$R = \frac{v_i^2 * sin2\theta}{g} \tag{1.3}$$

R is the range of the projectile (Horizontal distance in m)
v_i is the launch speed of the projectile in (m/s)
θ is the launch angle of the projectile
g is acceleration due to gravity

The formula 1.3 can only be used if the project starts and ends at the same vertical location

1.8.3 An example question

Remainder, you should always label your direction conventions



Example 1: A unicorn is thrown from the top of a cliff with a velocity of 35.0 m/s [25° above the horizontal]. The height of the cliff is 57.0m.

a) What are the components of the object's initial velocity?
b) How far from the base of the cliff will the object land? ($\approx$170m)
c) With what velocity will it land? ($\approx$48m/s [49° below the horizontal])

ⓐ $\vec{v}_1 = 35.0$m/s, $V_{y_1} = ?$, $V_{x_1} = ?$, 25°, +x, +y

(This diagram is usually optional)

① $\vec{V}_{x_1} = +(35.0\text{m/s})\cos 25^\circ = 31.721\ldots\text{m/s} \approx 32\text{m/s}$

② $\vec{V}_{y_1} = (-)(35.0\text{m/s})\sin 25^\circ = -14.793\ldots\text{m/s} \approx -15\text{m/s}$

"use for future calcs"

"normally not req'd here"

ⓑ $\vec{v}_1 = 35.0$m/s, 25°, 57.0m, $\vec{\Delta d}_x = ?$, +x, +y

Variables:

Horizontal	Vertical
$\vec{V}_x$ ✓	$\vec{V}_{y_1}$ ✓
$\vec{\Delta d}_x = ?$	$\vec{\Delta d}_y = +57.0$m
$\vec{a}_x = 0$	$\vec{a}_y = +9.81\text{m/s}^2$

① Use vertical to find Δt

② $\vec{\Delta d}_x = ?$

*They try

ⓑ ① $\Delta t = ?$ ← (optional)

Must include subscripts

$\vec{\Delta d}_y = \vec{V}_{y_1}\Delta t + \frac{1}{2}\vec{a}_y \Delta t^2$

$(57\text{m}) = (-14.793\ldots\text{m/s})\Delta t + \frac{1}{2}(9.81\text{m/s}^2)\Delta t^2$

$4.905\Delta t^2 - 14.793\ldots\Delta t - 57 = 0$

*using quadratic eq'n:

$\Delta t = 5.236\ldots\text{s}$

② $\vec{\Delta d}_x = ?$ ← (optional)

(∵ $\vec{a}_x = 0$, so can use)

$\vec{\Delta d}_x = \vec{V}_x \Delta t$

$= (31.721\ldots\text{m/s})(5.236\ldots\text{s})$

$= 166.091\ldots\text{m}$

$\approx 170\text{m}$

ⓒ ① $\vec{V}_{x_2} = \vec{V}_{x_1} = 31.721\ldots\text{m/s}$ (Based on $\vec{a}_x = 0$)

② $\vec{V}_{y_2} = ?$ ← We know Δt ✓ $\vec{V}_{y_1}$ ✓ $\vec{a}_y$ ✓ $\vec{\Delta d}_y$ ✓

(so multiple correct methods)

$\vec{V}_{y_2}^{\,2} = \vec{V}_{y_1}^{\,2} + 2\vec{a}_y \vec{\Delta d}_y$

$= (-14.793\ldots\text{m/s})^2 + 2(9.81\text{m/s}^2)(57\text{m})$

$|\vec{V}_{y_2}| = 36.567\ldots\text{m/s}$

③ $\vec{V}_x$, θ, $\vec{V}_{y_2}$, $\vec{V}_2 = ?$

④ $|\vec{V}_2| = \sqrt{(31.721\ldots\text{m/s})^2 + (36.567\ldots\text{m/s})^2} = 48.409\ldots\text{m/s} \approx 48\text{m/s}$

⑤ $\theta = \tan^{-1}\left(\frac{36.567\ldots\text{m/s}}{31.721\ldots\text{m/s}}\right) = 49.059\ldots^\circ \approx 49^\circ$

⑥ ∴ Velocity when lands is 48m/s [49° below the horizontal]

2

Figure 1.8: