Euclid

EU

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1 Algebra

Theorem 1.1 (Roots of Quadratic Equation)**.** *The roots of Quadratic Equation can be determined by the Quadratic Formula, or more formally, Sridharacharya Formula*

Assume r_1 *and* r_2 *are both roots of the Quadratic Equation* $ax^2 + bx + c = 0$

$$r_1 = \frac{-(b) + \sqrt{b^2 - 4ac}}{2a}, r_2 = \frac{-(b) - \sqrt{b^2 - 4ac}}{2a}$$

r_1 *and* r_2 *may be imagery numbers if* $\sqrt{b^2 - 4ac} < 0$

Theorem 1.2 (Vieta's Formula)**.** *Assume we have a polynomial formula with degree of n*

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

and $r_1, r_2, \cdots, r_n$ *are the roots of the polynomial.*
We can get:

$$r_1 + r_2 + \cdots + r_{n-1} + r_n = \frac{a_{n-1}}{a_n}$$

$$(r_1r_2+r_1r_3+\cdots+r_1r_{n-1}+r_1r_n)+(r_2r_3+r_2r_4+\cdots+r_2r_{n-1}+r_2r_n)+\cdots+r_{n-1}r_n = \frac{a_{n-2}}{a_n}$$

$$r_1r_2\cdots r_{n-1}r_n = (-1)^n \frac{a_0}{a_n}$$

Proposition 1.3 (Simon's Favorite factoring trick)**.** *for example:*

$$mn - 2m - 4n + 8 = (m - 4) \times (n - 2)$$

2 Sequence and Series

2.1 Arithemtic Sequences and Series

Theorem 2.1 (Sum of Arithemtic Sequences and Series)**.** *Assume we have a sequence with n terms:*

$$a + (a + d) + (a + 2d) + (a + (n - 1)d)$$

The sum of the sequence is

$$\frac{2a + (n - 1)d}{2} * n$$

2.2 Geometric Sequences and Series

Theorem 2.2 (Sum of Geometric Sequences and Series)**.** *Assume we have a geometric sequence with n terms:*

$$a + ar + \cdots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$$

The sum of the geometric sequence is

$$\frac{a(r^n - 1)}{r - 1} \ or \ \frac{a(r^n - 1)}{1 - r}$$

If a geometric sequence has infinite amount of terms and $|r| \leq 1$, the sum of this sequence can be expressed as

$$\frac{a}{1 - r}$$