

# **Grade 12 Math of Data Management**

MDM4U

Qinghao Hu

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# Contents

|          |   |          |
|----------|---|----------|
| <b>1</b> | <b>Probability Distributions</b>                    | <b>2</b> |
| 1.1      | Probability Distributions . . . . .                 | 3        |
| 1.1.1    | Definitions for Probability Distributions . . . . . | 3        |
| 1.2      | Uniform Distributions . . . . .                     | 4        |
| 1.2.1    | Different Distributions . . . . .                   | 4        |
| 1.2.2    | Characteristics of Uniform Distribution . . . . .   | 4        |
| 1.3      | Binomial Distributions . . . . .                    | 6        |
| 1.3.1    | Example . . . . .                                   | 6        |
| 1.3.2    | Definitions . . . . .                               | 7        |
| 1.4      | Geometric Distributions . . . . .                   | 8        |
| 1.4.1    | Example . . . . .                                   | 8        |
| 1.4.2    | Definitions . . . . .                               | 8        |
| 1.5      | Hypergeometric Distributions . . . . .              | 9        |
| 1.5.1    | Example . . . . .                                   | 9        |
| 1.5.2    | Definitions . . . . .                               | 9        |

## Chapter 1

# Probability Distributions

## 1.1 Probability Distributions

### 1.1.1 Definitions for Probability Distributions

#### Definition 1.1.1 (Random Variable)

A variable whose values are numerical outcomes of a random phenomenon, such as a probability experiment.

#### Definition 1.1.2 (Discrete random variable)

A random variable that has a FINITE number of possible values in a given interval. Examples include number of books, shoe sizes, and report card marks

#### Definition 1.1.3 (Continuous variable)

A random variable that can have an INFINITE number of possible values in a given interval. Examples include height, time, distance and money

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#### Definition 1.1.4

Probability Distribution The use of it illustrate the PROBABILITY of all possible outcomes of an experiment. The illustration may be in the form of a table of values, a graph, or an equation

#### Definition 1.1.5

The probability that a discrete random variable  $X$  have a particular value  $x$  is expressed as  $P(X = x)$  or  $P(x)$

#### Definition 1.1.6

The expected value,  $E(x)$  of a random variable  $X$  is the predicted MEAN of all possible outcomes of a probability experiment. If  $X$  is discrete, then

$$E(x) = \sum x_i P(x_i)$$

and

$$\sigma = \sqrt{\sum (x - E(x))^2 P(x)}$$

## 1.2 Uniform Distributions

### 1.2.1 Different Distributions

Distributions of data can be classified by considering the general shape of its graph. This picture is a table of distributions which is copied from the teacher's note

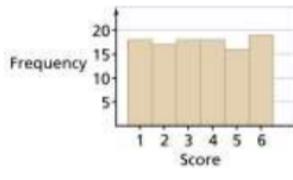
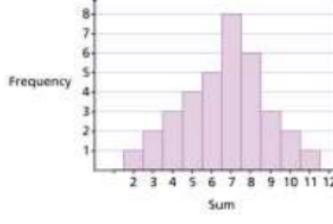
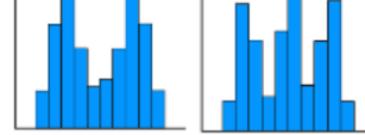
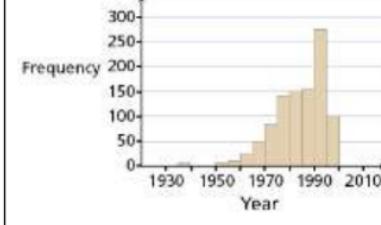
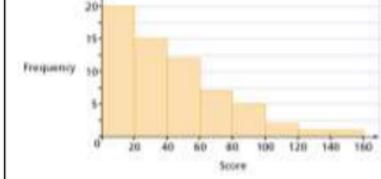
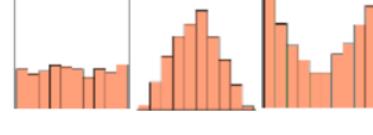
| Distribution Type                   | Description   | Example(s)  |
|-------------------------------------|---|---|
| Uniform                             | height of <u>EACH</u> bar in the distribution histogram is roughly equal  |    |
| Mound-shaped                        | an interval with the <u>HIGHEST</u> frequency, frequencies of other intervals decrease as you move along either side          |    |
| Multimodal                          | Bimodal distributions has two <u>PEAKS</u> and trimodal distributions has three peaks   |   |
| Left-skewed<br>(Negatively skewed)  | mean is skewed to the <u>LEFT</u> hence the distribution is asymmetrical with a left-direction (i.e. a "left-tail" exists)    |   |
| Right-skewed<br>(Positively skewed) | mean is skewed to the <u>RIGHT</u> hence the distribution is asymmetrical with a right-direction (i.e. a "right-tail" exists) |   |
| Symmetric                           | shows mirror <u>SYMMETRY</u> about the centre of the distribution   |  |

Figure 1.1: Thanks to Mr Tang

### 1.2.2 Characteristics of Uniform Distribution

If an distribution is considered Uniform, it will have following characteristics:

1. Each outcome is EQUALLY likely in any single trial of experiment

2. If  $X$  is discrete and  $n$  is the number of possible outcomes in the probability experiment, then

$$\begin{aligned} P(X = x) &= P(x) = \frac{1}{n} \\ E(x) &= \frac{\sum x_i}{n} \\ \sigma &= \sqrt{\frac{1}{n} \sum (x - E(x))^2} \end{aligned}$$

3. If  $X$  is continuous with values in range from  $a$  to  $b$ , then the expected value  $E(x)$  will be  $\frac{a+b}{2}$

## 1.3 Binomial Distributions

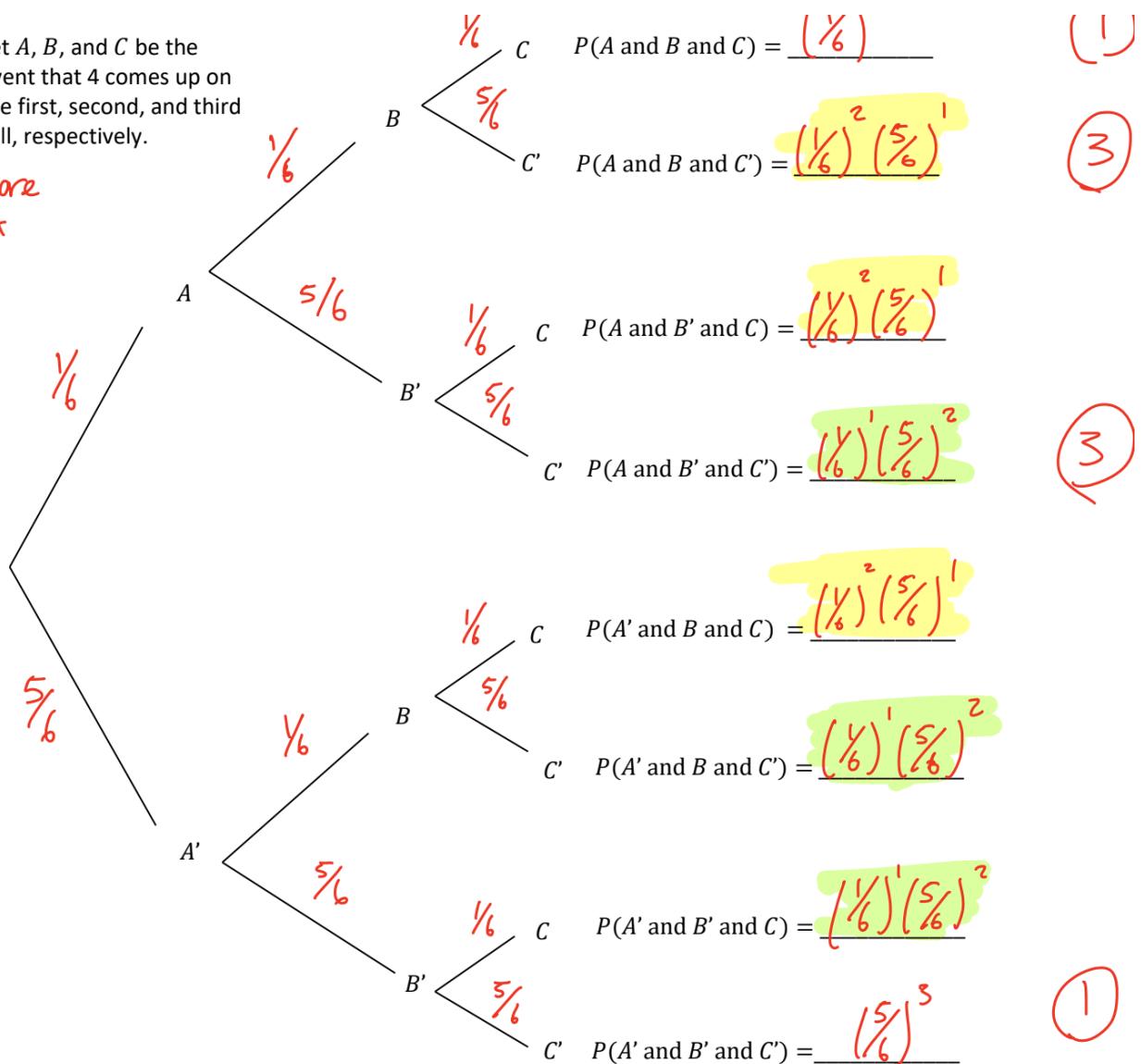
### 1.3.1 Example

#### Example 1

A probability experiment involves rolling a regular 6-sided die three times. Let the random variable  $X$  represent all possible number of 4's rolled. Complete the tree diagram, fill in the blanks, then show the distribution of  $X$  in a table.

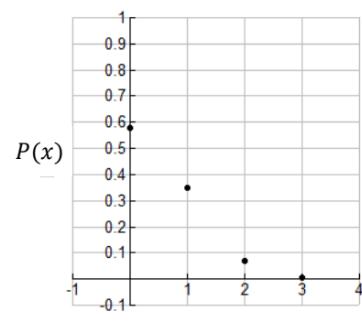
Let  $A$ ,  $B$ , and  $C$  be the event that 4 comes up on the first, second, and third roll, respectively.

$A, B, C$  are independent events



| $x$    | 0  | 1   | 2   | 3  |
|--------|--|---|---|--|
| $P(x)$ | $\binom{3}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^3 = \frac{125}{216}$ | $\binom{3}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2 = \frac{75}{216}$ | $\binom{3}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^1 = \frac{15}{216}$ | $\binom{3}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^0 = \frac{1}{216}$ |

$$\binom{n}{x} p^x (1-p)^{n-x}$$



This question is a typical example of **Binomial Distribution**

### 1.3.2 Definitions

#### Definition 1.3.1

The distribution of a discrete random variable  $X$  is a binomial probability distribution if:

1. The probability experiment is repeated a **FIXED** number of times (In the example,  $n = 3$ )
  2. The outcome of each trial can be **CATEGORIZED** into successful or failed outcomes
  3. The random variable  $X$  represents the number of **successes**
  4. Each trial of the experiment is **Independent**
- 

#### Theorem 1.3.2

If the distribution of  $X$  is a binomial probability,  $N$  is the number of independent trials of the experiment and  $p$  is the probability of success of each independent trial, then:

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad (1.1)$$

$$E(x) = np \quad (1.2)$$

$$\sigma = \sqrt{np(1-p)} \quad (1.3)$$

*Remark.* The binomial distribution is only work for Discrete random variable!

## 1.4 Geometric Distributions

### 1.4.1 Example

#### Example 2

Here is an example from our teacher's note:

- A standard 6-sided die is rolled. Let  $X$  represent the number of unsuccessful rolls (i.e. the waiting time) before a 4 appears.

- a) Complete the given table.

| $x$    | 0             | 1   | 2   | 3   | 4   |
|--------|---------------|---|---|---|---|
| $P(x)$ | $\frac{1}{6}$ | $(\frac{5}{6})(\frac{1}{6}) = \frac{5}{36}$ | $(\frac{5}{6})^2(\frac{1}{6}) = \frac{25}{216}$ | $(\frac{5}{6})^3(\frac{1}{6}) = \frac{125}{1296}$ | $(\frac{5}{6})^4(\frac{1}{6}) = \frac{625}{7776}$ |

- b) What is the probability that 4 appears within the first 3 rolls?

$$\begin{aligned} & P(0) + P(1) + P(2) \\ &= \frac{1}{6} + \frac{5}{36} + \frac{25}{216} = \frac{96}{216} \end{aligned}$$

### 1.4.2 Definitions

#### Definition 1.4.1

The distribution of a discrete random variable  $X$  is a geometric probability distribution if:

1. each trial of this experiment is **INDEPENDENT**
2. the outcome of each trial can be categorized into **Successful** or failed outcomes
3. the random variable  $X$  represents the number of failed outcomes **before** a success occurs

#### Theorem 1.4.2

If the distribution of  $X$  is a geometric probability distribution then:

$$P(x) = (1 - p)^x p \quad (1.4)$$

$$E(x) = \frac{1 - p}{p} \quad (1.5)$$

$$\sigma = \frac{\sqrt{1 - p}}{p} \quad (1.6)$$

## 1.5 Hypergeometric Distributions

### 1.5.1 Example

#### Example 3

An example from Teacher's note!

- There are ten red and eight green dice in a container. Six dice are drawn at random (without replacement).

Let the random variable  $X$  represent the number of red dice drawn.

Display the probability distribution of  $X$  in a table then determine  $E(X)$ .

| $x$    | 0   | 1  | 2  | 3  | 4  | 5  | 6   |
|--------|---|--|--|--|--|--|---|
| $P(x)$ | $\frac{\binom{10}{0} \binom{8}{6}}{\binom{18}{6}}$<br>$= \frac{1}{663}$ | $\frac{\binom{10}{1} \binom{8}{5}}{\binom{18}{6}}$<br>$= \frac{20}{663}$ | $\frac{\binom{10}{2} \binom{8}{4}}{\binom{18}{6}}$<br>$= \frac{75}{442}$ | $\frac{\binom{10}{3} \binom{8}{3}}{\binom{18}{6}}$<br>$= \frac{80}{221}$ | $\frac{\binom{10}{4} \binom{8}{2}}{\binom{18}{6}}$<br>$= \frac{70}{221}$ | $\frac{\binom{10}{5} \binom{8}{1}}{\binom{18}{6}}$<br>$= \frac{24}{221}$ | $\frac{\binom{10}{6} \binom{8}{0}}{\binom{18}{6}}$<br>$= \frac{5}{442}$ |

$$E(X) = \sum x_i P(x_i)$$

$$= 0\left(\frac{1}{663}\right) + 1\left(\frac{20}{663}\right) + \dots + 6\left(\frac{5}{442}\right)$$

$$= \frac{10}{3} \text{ red dice}$$

- Fill in the blanks.

OR  
≡

$$E(X) =$$

$$= \left(\frac{10}{18}\right)(6)$$

$$= \frac{10}{3} \text{ red dice}$$

### 1.5.2 Definitions

#### Definition 1.5.1

The distribution of a discrete random variable  $X$  is a hypergeometric probability distribution if:

- the probability experiment is repeated a **fixed** number of times
- the outcome of each trial can be categorized into **success** or failed outcomes
- the random variable  $X$  represents the number of **successes**
- each trial of experiment is **dependant**

#### Theorem 1.5.2

If the distribution of  $X$  is hypergeometric with  $n$  dependent trials, and  $a$  represents the number of successful outcomes initially among a total of  $N$  possible outcomes in the beginning, then

$$P(x) = \frac{\binom{a}{x} \binom{N-a}{n-x}}{\binom{N}{n}} \quad (1.7)$$

$$E(x) = \frac{an}{N} \quad (1.8)$$

$$\sigma = \sqrt{n\left(\frac{a}{N}\right)\left(\frac{N-a}{N}\right)\left(\frac{N-n}{N-1}\right)} \quad (1.9)$$