Grade 12 Physics

SPH4U

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Contents

Chapter 3

Unit 2: Energy and Momentum

3.1 Work and Kinetic Energy

3.1.1 Kinetic Energy

Definition 3.1.1. *Kinetic energy is the energy of motion. There are two types of kinetic energy*

Type 1: Traslational Kinetic Energy

The kinetic energy that an object has because it is *moving from on location to another*. In can be describe by this formula:

$$E_k = \frac{1}{2}mv^2 \tag{3.1}$$

E_k is the translational kinetic energy of the object (in J or kgm^2/s^2)
m is the mass in kg
v is the speed in m/s

Because all motion is relative, an object's speed (and therefore E_k) *depends on the choose Frame of Reference*

Notes:

Do not solve the conservation of energy problem involving a change of Frame of Reference. Start from your perspective

E_k is a scalar, not a vector

Type 2: Rotational Kinetic Energy

Not testable, don't give a shit about this qusetion.

3.1.2 Mechancial Work

Definition 3.1.2. *Mechanical work: Transfer of energy into E_k or the transfer of kinetic energy into another type of energy*

Definition 3.1.3. *Potential energy: Energy which is stored in a system of objects due to forces acting in between thos objects*

The formula to descirbe the work is:

$$W = F_{A/B} * \Delta\vec{d}_B * cos\theta \tag{3.2}$$

$W_{A/B}$ is the work that $F_{A/B}$ does on the object B. This is also the amount of E_k that object A transfers into object B when A exerts a force on B(in J)
$F_{A/B}$ is the magnitude of force that A exerts on B(in J)
θ is the angle between $F_{A/B}$ and B's displacement

Remainder: Only forces on the direction of displacement is responsible for the work

3.2 Gravational Potential Energy

3.2.1 Some boring definitions

Definition 3.2.1. *(Gravational Potential Energy): The energy storedf in a system of objects due to the force of gravity acting between those objects. In other words, the energy is stored collectively* ***among all*** *the objects in the system*

Whne the force of gravity acting on the two objects causes this stored GPE to be converted into kinetic energy, the kinetic energy is not **shared evenlly** between these two objects. In the class Example, the earth effectively gets **zero** and the care effectively gets **all of them**. Due to this reason, when we have two objects with a very large difference in mass, we can always assume that the GPE is **stored only in the smaller object**

3.2.2 Formulas for GPE

Formula 1

$$\Delta E_g = mg\Delta h \tag{3.3}$$

ΔE_g is the change in Potential gravational energy(in J)
m is the mass of the object (in kg)
Δh is the change in height (in m)
g is the acceleration due to gravity (in m/s^2)

Fromula 2

$$E_g = mg\Delta h \tag{3.4}$$

$E - g$ is related to the GPE of the object

Remark. For all questions related to the **Gravational Potential Energy**, you must set your reference height in the diagram. Or, Mr McCumber will forget to add 0.5 for your test!

3.3 The law of the Conservation of Energy

3.3.1 Boring Definitions

Definition 3.3.1. *(Law of the Conservation of Energy): Energy will neither created or destroy, only change from one form to another*

Remark. The law of the conservation of energy only works in a **closed isiolated** system. In reality, the only true **closed and isolated** system is the Universe

Mechancial Energy

Definition 3.3.2. *(Mechancial Energy): is the sum of the* ***kinetic energy*** *and g**gravational potential energy**.*

Mathematically:

$$E_m = E_g + E_k \tag{3.5}$$

3.3.2 Question solving techniques

When you solve a question about conversation of energy, always write this:

$$E_{m1} = E_{m2} \tag{3.6}$$

$$E_{k2} + E_{g2} = E_{k1} + E_{g1} + W_{ap} + W_f \tag{3.7}$$

Then, cross out terms which equal to **zero**

Remark. Remember to write this, or teacher will forget to add your marks!

3.4 String & Elastic Potential Energy

3.4.1 The Force of String

Definition 3.4.1. *(Spring Force): Can be wrote as F_{spring}. It is the force exerted by the spring on a object.*

According to the Hooke's Law, the **force exerted by a string** is proportional to the string's displacement. So we can express the relationships between them by some formulas:

Vector Version:

$$\vec{F_x} = -k\Delta\vec{x} \tag{3.8}$$

Scalar Version:

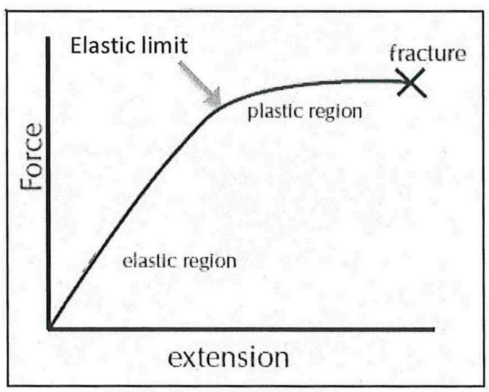
$$F_x = k\Delta x \tag{3.9}$$

F_x is the force exerted by the string on whatever stretches it.
k is the constant of string
x is the displacement of the spring from its unstratched

An essential feature of Hooke's law is that the direction of the spring force is **opposite** to the direction of displacement from equilibrium.

Remark. When you use the Scalar version, you must clearly understand the direction of the force in you heart.

Deeper explanation about Hooke's law



Definition 3.4.2. *(Elastic Region): Elastic objects obey Hooke's Law in this region. If the applied force removed, the object will naturally return back to its original shape*

Definition 3.4.3. *(Elastic Limit): The maximum amount of deformation an object can withstand, and still return to its original shape.*

Definition 3.4.4. *(Plastic region): The object no longer obeys Hooke's Law. The object's shape is now permanetly changed.*

Definition 3.4.5. *(Fracture): The maximum amount of shape change the object can take, prior to failing (breaking).*

3.4.2 Elastic Potential Energy

Definition 3.4.6. *(elastic potential energy): The potential energy due to the stretching or compressing of an elastic material*

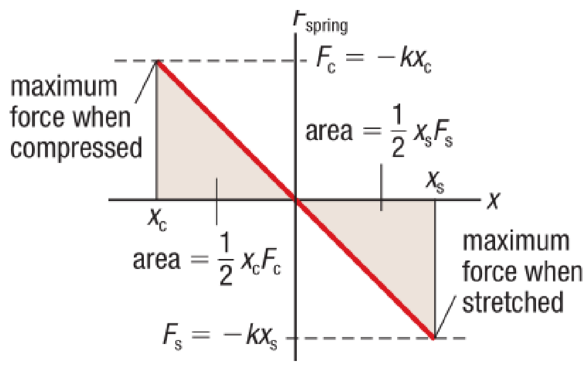


Figure 3.1: The work done by a variable force is equal to the area under the the force-displacement graph

Formula

$$W = \frac{1}{2} * \Delta x * F_{spring} \tag{3.10}$$

$$W = \frac{1}{2} * \Delta x * (k * \Delta x) \tag{3.11}$$

$$W = \frac{1}{2} * k * (\Delta x)^2 \tag{3.12}$$

The work done by the spring force is the negative of this amount, and is also the negative of the change in Potential Energy. That means that the work done stretching or compressing the spring is transformed into elastic potential energy.

$$E_e = \frac{1}{2}k(\Delta x)^2 \tag{3.13}$$

3.4.3 Ignore Gravational Potential Energy

We can ignore the Gravational Potential Energy in the vertial spring question if all of these conditions are met:

- The mass remains in contact with the spring
- We measure all changes in the length relative to the equilibrium position of the mass-spring system (ie. $x_{eq} = 0$)

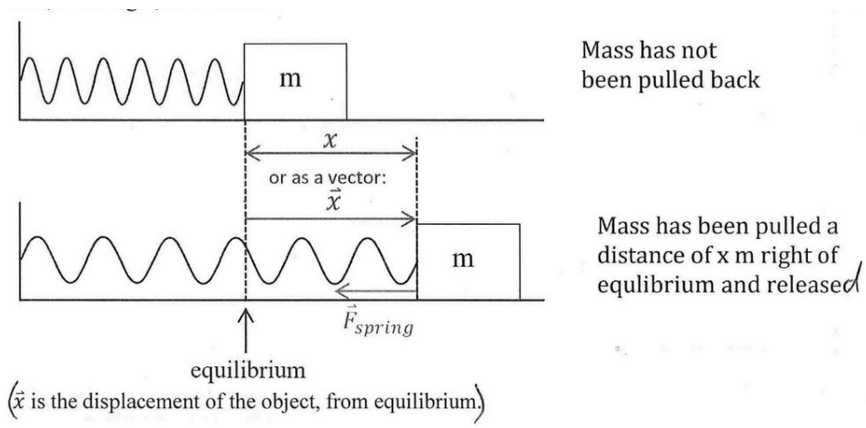
3.5 Simple Harmonic Motion

3.5.1 Definitions

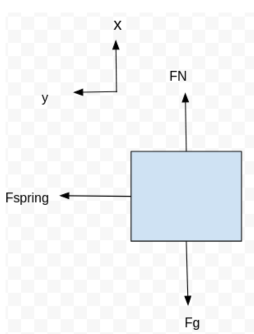
Simple Harmonic Motion

An object undergoes simple harmonic motion, if two conditions are met:

1. The net force acting on the object is **directly proportional** to the object's displacement away from equilibrium (The location where $\sum F = 0$)
2. The direction the net force acts on the object must be **opposite** the object's displacement from equilibrium.



Let see the FBD for m:



$$\sum \vec{F} = \vec{F_{spring}} \tag{3.14}$$
$$\sum \vec{F} = -k\vec{x} \tag{3.15}$$

Changing to proportionality:

$$\text{Net Force} \propto -\vec{x} \tag{3.14}$$

Expectations

If there was **friction** acting on the object, it would no longer undergo simple harmonic motion. We would call the motion of the object: **Damaged Harmonic Motion**

Some Real life examples:

- Car shock absorbers
- A guitar string
- A pendalam or a string

- Bungee jumping

3.5.2 How can we solve form the acceleration of SHM

Returning to our FBD of the mass and the net force statement from it:

$$\sum \vec{F} = m\vec{a} \tag{3.15}$$
$$\vec{F}_s = m\vec{a} \tag{3.16}$$
$$-kx = m\vec{a} \tag{3.17}$$
$$a = \frac{-kx}{m} \tag{3.18}$$

$\vec{a}$ is the acceleration of the mass (in m/s^2)
k is the force constant of the spring (in N/m)
$\vec{x}$ is the displacement of the mass from its equilibrium position (in m)
m is the mass of the object that is attached to the spring (in kg)

Remark. Because the acceleration is a vector, we need to make a direction convention to use the equation.

3.5.3 The period of the Simple Harmonic Motion

The y-component of the uniform circular motion is similar to the acceleration of the simple Harmonic Motion

For the object going around circle:

$$a_c = \frac{4\pi^2 R}{T^2} \tag{3.19}$$

For the mass on the end of the spring:

$$\vec{a} = -\frac{k\vec{x}}{m} \tag{3.19}$$
$$|\vec{a}| = \frac{kx}{m} \tag{3.20}$$

When the object is on the top/bottom of the perfect circular motion, the acceleration is equal to the magnitude of the acceleration of the object at the equilibrium position:

$$a_c = |a| \tag{3.20}$$
$$\frac{4\pi^2 R}{T^2} = \frac{kx}{m} \tag{3.21}$$
$$T = +/- \sqrt{\frac{m * 4\pi^2 R}{kx}} (\text{At this time } R = x) \tag{3.22}$$
$$T = 2\pi * \sqrt{\frac{m}{k}} \tag{3.23}$$

3.6 Linear Momentum & Impulse

Linear Momentum

Linear Momentum is the product of an object's mass and its velocity:

$$\vec{p} = m\vec{v} \tag{3.24}$$

$\vec{p}$ is the Momentum in $(kg * \frac{m}{s})$

Newton called momentum "the **true Quantity of motion**". Why? Momentum is a combination of an object's **inertia**(its mass basically) and what it is doing (its **velocity**). He felt that it provided a more complete picture of what was required to cause a specific change in what an object was doing.

Impulse

Impulse is the **product of that force** acting on an object and the **duration** of time that the force acted on the object.

$$\vec{J} = \vec{F} * \Delta t \tag{3.25}$$

$\vec{J}$ = the impulse in (N*s)

The formula has a similar limitation to the formula for the work done on an object. Both formulas assume that the force acting on the object.

Thus, if the force acting on the object is not constant, we can find the impulse that the force provides by finding the area between the line/curve on a **Force vs Gravity graph**

Let's see some formula:

$$\sum \vec{J} = \sum \vec{F} * \Delta t$$
$$\sum \vec{J} = (m * \vec{a}) * \Delta t$$
$$\sum \vec{J} = (m * \frac{\vec{v_2} - \vec{v_1}}{\Delta t}) * \Delta t$$
$$\sum \vec{J} = m * \vec{v_2} - m * \vec{v_1}$$
$$\sum \vec{J} = \vec{p_1} - \vec{p_1}$$

$$\sum \vec{J} = \Delta\vec{p} \tag{3.26}$$

3.7 Conservation of Momentum

Definition 3.7.1. *Two or more objects interact and* ***exert forces from each other****. The forces in the interaction are a Newton's Third Law pair of forces (ie* $\vec{F_{A/B}} = -\vec{F_{B/A}}$*)*

3.7.1 Equation

Consider a person standing on any icy surface throws a heavy object horizontally:

FBD for the person (Left) and the object (Right)

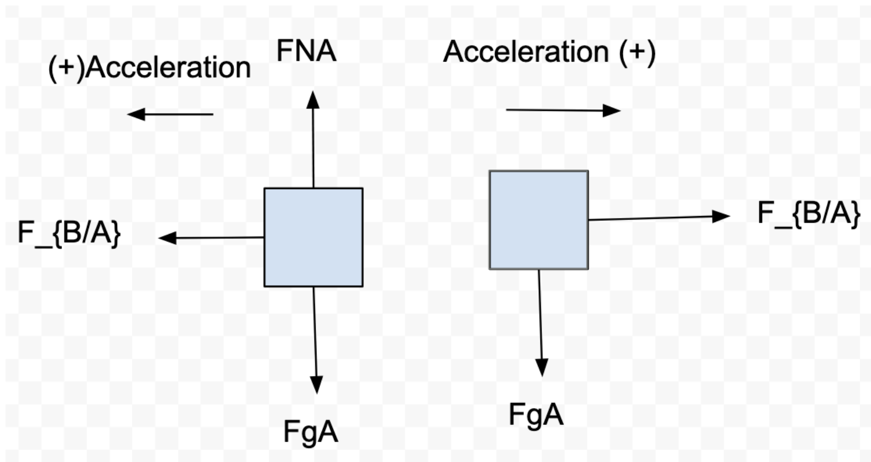


Figure 3.2: We will assume that the interaction forces are essentially the net force acting on the object

Equation for the object A:

$$\begin{aligned}\sum \vec{F_A} &= m_A * \vec{a_A} \\ \vec{F_{B/A}} &= m_A \vec{a_A} \\ \vec{F_{B/A}} &= m_A \frac{\vec{v_{fA}} - \vec{v_{iA}}}{\Delta t_A}\end{aligned} \tag{3.27}$$

Equation for the object B:

$$\begin{aligned}\sum \vec{F_B} &= m_B * \vec{a_B} \\ \vec{F_{A/B}} &= m_B \vec{a_B} \\ \vec{F_{A/B}} &= m_B \frac{\vec{v_{fB}} - \vec{v_{iB}}}{\Delta t_B}\end{aligned} \tag{3.28}$$

Lemma 3.7.2. *From Newton's third law, we know* $\vec{F_{A/B}} + \vec{F_{B/A}} = 0$

We add **??** and **??** together:

$$m_A \frac{\vec{v_{fA}} - \vec{v_{iA}}}{\Delta t_A} + m_B \frac{\vec{v_{fB}} - \vec{v_{iB}}}{\Delta t_B} = 0$$

We know the time for both object should be the same.

$$\begin{aligned}m_A * \vec{v_{fA}} + m_B * \vec{v_{fB}} &= m_A * \vec{v_{iA}} + m_B * \vec{v_{iB}} \\ \vec{P_{A2}} + \vec{P_{B2}} &= \vec{P_{A1}} + \vec{P_{B1}}\end{aligned} \tag{3.29}$$

?? is the law of **Conservation of Momentum**
Always write this line at the beginning of your analysis

Definition 3.7.3. *(The Law of Conservation of Momentum): The total momentum of a system of objects after an interaction is* ***equal*** *to the total momentum of the system before the interaction.*

According to the **??**, we can understand the total momentum of the system is **constant through out** the interaction.

The law assume that any other force that could **accelerate** any objects in the system during the interaction is **negligible**

3.8 Types of Collisions

3.8.1 Definitions

Collisions are typically classified based on the amount of **kinetic energy** the syste has after the collision, in comparsion to the amount of kinetic energy the system had before the collision. In other words, **how does E_k' with E_k?**

3.8.2 Elastic Collisions

In an elastic collision, the kinetic energy of the system after the collision is **equal** to the kinetic energy of the system before the collision. In mathematics, the equation can be represented by:

$$E_k` = E_k$$

Remark. This does not mean the kinetic energy of the system after the collision is **equal** to the kinetic energy of the system before the collision (Unlike momentum)

Steps of the collision

Remark. This is on a horzontal frictionless surface, so we can ignore Gravational Potential Energy

Before the collision:The mechancial of energy is entirely in the form of kinetic energy

First half ot collision:The interaction forces cause the object to start deform. As the object deform, they transfer E_k to E_s.

At the approximate midpoint of the collision:The deformation of the object is at a maximum. E_S is the maxium and E_k is the minimum.

During the second half of the collision: The restoring forces are now doing *positive work* on the system, transferring elastic potential energy **back into** E_k

After the collision: The system's mechanical energy is now entirely E_k, at this time, $E_s = 0$. All E_s is transfered to E_k.

Head-on Collision

Before the Collision: System's mechancial energy is entirely E_k

During the first half of the collision: The spring get compressed. E_k is transformed into E_s.

At the mid-point of the collision:

- Spring is at the most compressed point.
- Distance between cars are minimumized
- $\vec{v_A} = \vec{v_B}$
- E_k is minimumized
- E_s is maximumized

During the second half of the collision:

- $\vec{v_A} < \vec{v_B}$
- Distance between the carts is increasing

- E_s is being transferred back into E_k

After the collision: The system is entirely E_k now

Remark. Elastic collisions **cannot occur** between visible objects in rael life. At least some of the enery will be lost as thermal or sound.

3.8.3 Inelasic Collision

So for this collision, E_k‘ is less than E_k

It is impossible for a system to have more kinetic energy after the collision, than it had before the collision, unless:

1. One of the object had **stored energy** before the colllision, which was transferred into kinetic enery during the collision.
2. An **external** force (such as force of gravity) is doing positive work on the system, during the collision.

Completely inelastic collision

In this collision, the maximum amount of kinetic energy that could be ”lost” is lost as a result of the collision.

After the collusion, the objects involved in the collision will be **stack/attached together**

Apple and Arrow is an example of this question.

The following condition must be met for a Perfectly Inelastic Collision:

- $\vec{v_A}‘ = \vec{v_B}‘ = \vec{v}‘$

3.9 HEAD-ON Collisions

3.9.1 Equations

Under this section, I will derive Equations that is useful to solve for final velocity for each object when you have initial velocity for a HEAD-ON Collision question

The equation for $\vec{v_A}'$

$$\vec{v_A}' = \frac{m_A - m_B}{m_A + m_B} * \vec{v_A} \tag{3.30}$$

The equation for $\vec{v_B}'$

$$\vec{v_B}' = \frac{2m_A}{m_A + m_B} * \vec{v_A} \tag{3.31}$$

For questions which $\vec{v_B}$ is not zero, we need change Frame of Reference to $\vec{v_B}$ in order to solve the question. And change the FOR back later.

3.10 Real Gravitatonal Potential Energy

3.10.1 Real gravitional potential Energy

In earlier time in this unit, we studied a formula for gravitatonal Potential Energy

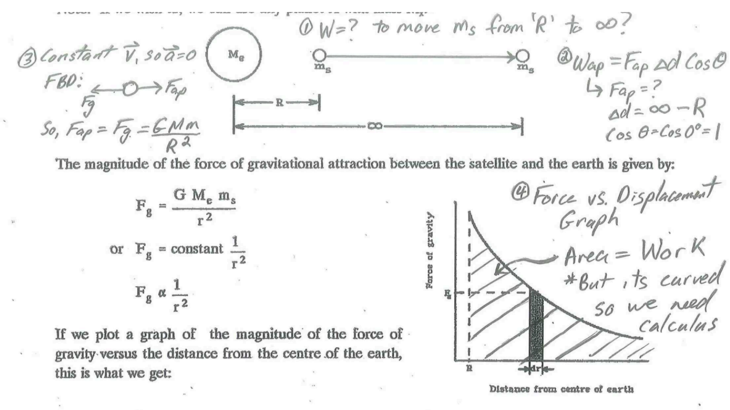
$$E_g = mgh \tag{3.32}$$

In fact, there are few problems with our old formula:

1. It assumes that g is constat (it change as the distance above the Earth's surface changes)
2. the formula uses a reference location which we define. This is not an absolute location, where E_g is 0

So, let's derive for the new equation of gravitatonal Potential Energy

Consider a satellite of mass m_s at a large distance r from the centre of planet Eart (mass M_e)



We understand the work done by a force is basically the area under the curve one a Force vs displacement graph

So:

$$\begin{aligned}
\text{Work done} &= \int_{r=R}^{r=\infty} F_g dr \\
&= \int_{r=R}^{r=\infty} \frac{GM_E m_s}{R^2} dr \\
&= GM_E m_s * \int_{r=R}^{r=\infty} \frac{1}{R^2} dr \\
&\dots \\
W &= \frac{GM_E m_s}{R}
\end{aligned} \tag{3.33}$$

The work done is equal to the increase in gravitational potential energy prossessed by the satellite

$$\begin{aligned}
W = \Delta E_g = E_{g\infty} - E_{gR} &= \frac{GM_E m_s}{R} \\
E_{gR} &= \frac{-GM_E m_s}{R}
\end{aligned} \tag{3.34}$$

3.10.2 Satellite

The Kineatic Energy of a satellite orbit around the earth could be calculated using thine formula:

$$E_k = \frac{GM_E m_s}{2R} \tag{3.35}$$

Then we can calculate the total mechancial energy of the system:

$$E_T = E_k + E_g \tag{3.36}$$

$$E_T = \frac{-GM_E m_s}{R} + \frac{GM_E m_s}{2R} \tag{3.37}$$

$$= -\frac{GM_E m_s}{2R} \tag{3.38}$$

The 3.7 is the *Total Mechancial Energy* of the satellite

3.11 Escape from a Gravational Field

Definition 3.11.1. *(Binding Energy): The **Minimum** E_k required for an object to escape a second object's gravational field. For the minimum E_k, required by the first object, the first object must reach an **infinite distance away** from the second object by the time that the second object's force of gravity **stop** the first object*

Definition 3.11.2. *(Escape Energy): This is a specific type of binding energy. It is an object's binding energy when the object is originally **on the surface** of the second object*

Example. The **escape energy** of an object from the Earth's surface is the minimum **additional** kinetic energy the object needs to have to escape the Earth's gravational field

Definition 3.11.3. *(Escape Velocity): The **Minimum speed** an object would need to have (when launched vertically) when it is on an object's surface to escape its gravitational field*

Solve for Escape Velocity

Let's assume position 2 is the postion where the object escape the planet's gravitational field and position 1 is on the surface of that planet

$$E_{M2} = E_{M1} \tag{3.39}$$
$$0 = E_{T1} \tag{3.40}$$
$$0 = E_{k1} + E_{g1} \tag{3.41}$$
$$E_{k1} = -E_{g1} \tag{3.42}$$
$$\frac{1}{2}mv_1^2 = -\frac{GM_E m_s}{R_1} \tag{3.43}$$
$$v_1 = \sqrt{\frac{2GM_E}{R_1}} \tag{3.44}$$

The 3.13 is the formula for Escape Velocity

Potential Communication Question

Why do most rocket launches occur from locations close to the equator?

Because on the Earth's surface, the rocket's speed is not actually 0. The rocket has whatever speed the Earth's surface has and the Earth is moving fastest at the equator.

Let's solve for the speed of the rocket at the equator:

$$v = \frac{2\pi R}{T} \tag{3.45}$$
$$v = \frac{2\pi 6.38 * 10^3}{24.0h} \tag{3.46}$$
$$v = \frac{2 * \pi * 6.38 * 10^3}{86400s} \tag{3.47}$$
$$v = 463.96 \cdots m/s \tag{3.48}$$

How does this help?
Assume position is surface, position 2 is on the orbit

$$E_{M2} = E_{M1} + E_{added}$$
$$E_{M2} = E_{k1} + E_{g1} + E_{added}$$
$$E_{added} = E_{M2} - E_{k1} - E_{g1}$$

Because we have initial E_{K1}, less E_{added} is needed. As a result, less fuel will be used. Cost less money

Remark. Assume that for an "Object on a surface", E_K is always 0 unless the questions explicitly says "At the equator"