AOPS AMC12 class Note 2

functions

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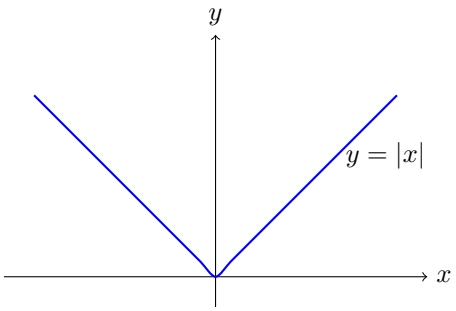
1 Lesson 4: Functions and Polynomials

Today we will look at the properties of certain functions, such as the "floor" function and logarithm, as well as polynomials, which form a very special class of functions

1.1 ABSOLUTE VALUE

The absolute value signs make equation difficult to work with. How might we deal with those pesky bars (|a|)

Example. if $x < 0$, then what happens to the equation $|x| + x + y = 10$, if $x > 0$, then what happens to the equation $[x] + x + y = 10$



1.2 Floor Function

In case you have not seen it before, $\lfloor x \rfloor$ is the greatest integer less than or equal to x, also called the floor of x. In other words, $\lfloor x \rfloor$ is x rounded down to the nearest integer.

(In general
$\lfloor x + n \rfloor = \lfloor x \rfloor + n$
for any integer n)

1.3 Logarithms

Logarithms identity:

$$\log_b x + \log_b y = \log_b xy \textbf{ (Product law)}$$
$$\log_b x - \log_b y = \log_b \tfrac{x}{y} \textbf{ (Quotient Law)}$$
$$\log_b x^n = n \log_b x \textbf{ (Power law)}$$
$$\log_b x = \tfrac{\log_a x}{\log_a b} \textbf{ (Power Law)}$$
$$\log_{b^n} x^n = \log_b x$$

1.4 POLYNOMIALS

Let $F(x)$ and $G(x)$ be polynomials. If we divide $G(x)$ into $F(x)$, then we will obtain a quotient $Q(x)$ and a remainder $R(x)$, where the degree of $R(x)$ is less than the degree of $G(x)$. The quotient $Q(x)$ and $R(x)$ are unique.

Also, if a polynomial has real coefficients, then its nonreal roots must come in conjugate pairs.