Grade 12 Math of Data Management

MDM4U

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Contents

Chapter 1

Probability Distributions

1.1 Probability Distributions

1.1.1 Definitions for Probability Distributions

Definition 1.1.1 (Random Variable)
A variable whose values are numerical outcomes of a random phenomenon, such as a probability experiment.

Definition 1.1.2 (Discrete random variable)
A random variable that hs a FINITE number of possible values in a given interval. Examples include number of books, shoe sizes , and report card marks

Definition 1.1.3 (Continuous variable)
A random variable that can have an INFINITE number of possible values in a given interval. Examples include height, time, distance and money

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Definition 1.1.4
Probability Distribution The use of it illustrate the PROBABILITY of all possible outcomes of an experiment. The illustration may be in the form of a table of values, a graph, or an equation

Definition 1.1.5
The probability that a discrete random variable X have a particular value x is expressed as $P(X = x)$ or $P(x)$

Definition 1.1.6
The expected value, $E(x)$ of a random variable X is the predicted MEAN of all possible outcomes of a probability experiment. If X is discrete, then

$$E(x) = \Sigma x_i P(x_i)$$

and

$$\sigma = \sqrt{\Sigma(x - E(x))^2 P(x)}$$

1.2 Uniform Distributions

1.2.1 Different Distributions

Distributions of data can be classified by considering the general shape of its graph. This picture is a table of distributions which is copied from the teacher's note

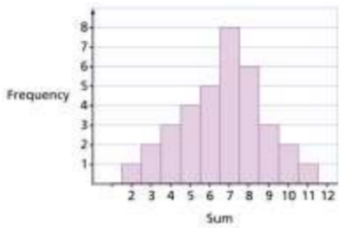
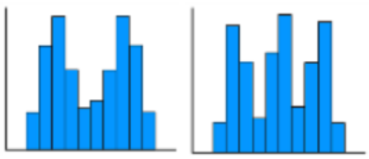
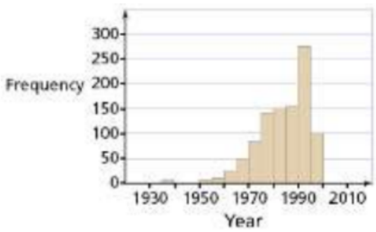
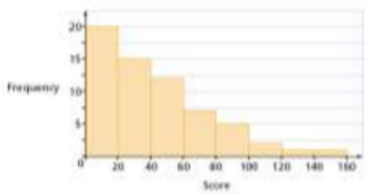
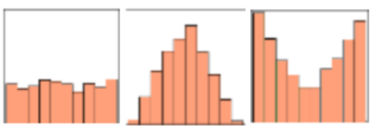
Distribution Type	Description	Example(s)
Uniform	height of EACH bar in the distribution histogram is roughly equal	
Mound-shaped	an interval with the HIGHEST frequency, frequencies of other intervals decrease as you move along either side	
Multimodal	Bimodal distributions has two PEAKS and trimodal distributions has three peaks	
Left-skewed (Negatively skewed)	mean is skewed to the LEFT hence the distribution is asymmetrical with a left-direction (i.e. a "left-tail" exists)	
Right-skewed (Positively skewed)	mean is skewed to the RIGHT hence the distribution is asymmetrical with a right-direction (i.e. a "right-tail" exists)	
Symmetric	shows mirror SYMMETRY about the centre of the distribution	

Figure 1.1: Thanks to **Mr Tang**

1.2.2 Characteristics of Uniform Distribution

If an distribution is considered Uniform, it will have following characteristics:

1. Each outcome is EQUALLY likely in any single trial of experiment

2. If X is discrete and n is the number of possible outcomes in the probability experiment, then

$$P(X = x) = P(x) = \frac{1}{n}$$
$$E(x) = \frac{\Sigma x_i}{n}$$
$$\sigma = \sqrt{\frac{1}{n}\sum(x - E(x))^2}$$

3. If X is continous with values in range from a to b, then the expected value $E(x)$ will be $\frac{a+b}{2}$

1.3 Binnomial Distributions

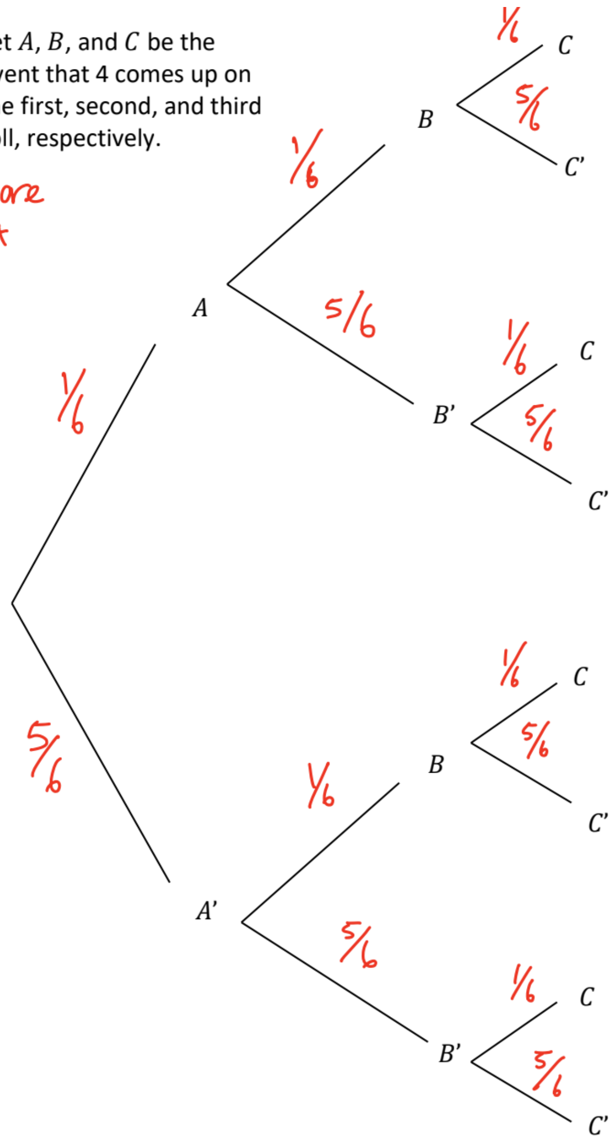
1.3.1 Example

Example 1

A probability experiment involves rolling a regular 6-sided die three times. Let the random variable X represent all possible number of 4's rolled. Complete the tree diagram, fill in the blanks, then show the distribution of X in a table.

Let A, B, and C be the event that 4 comes up on the first, second, and third roll, respectively.

A, B, C are independent events



$P(A \text{ and } B \text{ and } C) = (\frac{1}{6})$ 1

$P(A \text{ and } B \text{ and } C') = (\frac{1}{6})^2(\frac{5}{6})^1$ 3

$P(A \text{ and } B' \text{ and } C) = (\frac{1}{6})^2(\frac{5}{6})^1$

$P(A \text{ and } B' \text{ and } C') = (\frac{1}{6})^1(\frac{5}{6})^2$ 3

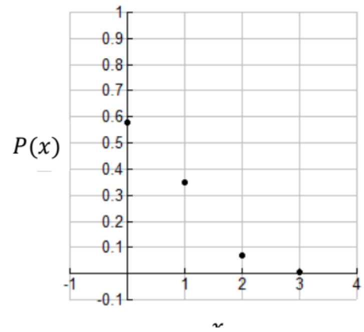
$P(A' \text{ and } B \text{ and } C) = (\frac{1}{6})^2(\frac{5}{6})^1$

$P(A' \text{ and } B \text{ and } C') = (\frac{1}{6})^1(\frac{5}{6})^2$

$P(A' \text{ and } B' \text{ and } C) = (\frac{1}{6})^1(\frac{5}{6})^2$

$P(A' \text{ and } B' \text{ and } C') = (\frac{5}{6})^3$ 1

x	0	1	2	3
$P(x)$	$\binom{3}{0}(\frac{1}{6})^0(\frac{5}{6})^3 = \frac{125}{216}$	$\binom{3}{1}(\frac{1}{6})^1(\frac{5}{6})^2 = \frac{75}{216}$	$\binom{3}{2}(\frac{1}{6})^2(\frac{5}{6})^1 = \frac{15}{216}$	$\binom{3}{3}(\frac{1}{6})^3(\frac{5}{6})^0 = \frac{1}{216}$



$\binom{n}{x} p^x (1-p)^{n-x}$

This question is a typical example of ***Binominal Distribution***

1.3.2 Definitions

Definition 1.3.1
The distribution of a discrete random variable X is a binomial probability distribution if:

1. *The probability experiment is repeated a FIXED number of times (In the example, $n = 3$)*
2. *The outcome of each trial can be CATEGORIZED into successful or failed outcomes*
3. *The random varialbe X represents the number of* ***successes***
4. *Each trial of the experiment is* ***Independent***

Theorem 1.3.2
If the distribution of X is a binominal probability, N is the number of independent trials of the experiment and p is the probability of success of each independent trail, then:

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x} \tag{1.1}$$

$$E(x) = np \tag{1.2}$$

$$\sigma = \sqrt{np(1-p)} \tag{1.3}$$

Remark. The binominal distribution is only work for Discrete random variable!

1.4 Geometric Distributions

1.4.1 Example

Example 2

Here is an example from our teacher's note:

A standard 6-sided die is rolled. Let X represent the number of unsuccessful rolls (i.e. the waiting time) before a 4 appears.

a) Complete the given table.

x	0	1	2	3	4
$P(x)$	$1/6$	$(5/6)(1/6) = \frac{5}{36}$	$(5/6)^2(1/6) = \frac{25}{216}$	$(5/6)^3(1/6) = \frac{125}{1296}$	$(5/6)^4(1/6) = \frac{625}{7776}$

b) What is the probability that 4 appears within the first 3 rolls?

$P(0)+P(1)+P(2)$

$= \frac{1}{6} + \frac{5}{36} + \frac{25}{216} = \frac{96}{216}$

1.4.2 Definitions

Definition 1.4.1

The distribution of a discrete random variable X is a geometric probability distribution if:

1. *each trail of this experiment is **INDEPENDENT***
2. *the outcome of each trial can be categorized into **Successful** or failed outcomes*
3. *the random varaible X represents the number of failed outcomes **before** a success occurs*

Theorem 1.4.2

If the distribution of X is a geometric probability distribution then:

$$P(x) = (1-p)^x p \tag{1.4}$$

$$E(x) = \frac{1-p}{p} \tag{1.5}$$

$$\sigma = \frac{\sqrt{1-p}}{p} \tag{1.6}$$

1.5 Hypergeometric Distributions

1.5.1 Example

Example 3

An example from Teacher's note!

1. There are ten red and eight green dice in a container. Six dice are drawn at random (without replacement). Let the random variable X represent the number of red dice drawn. Display the probability distribution of X in a table then determine $E(X)$.

x	0	1	2	3	4	5	6
$P(x)$	$\frac{\binom{10}{0}\binom{8}{6}}{\binom{18}{6}} = \frac{1}{663}$	$\frac{\binom{10}{1}\binom{8}{5}}{\binom{18}{6}} = \frac{20}{663}$	$\frac{\binom{10}{2}\binom{8}{4}}{\binom{18}{6}} = \frac{75}{442}$	$\frac{\binom{10}{3}\binom{8}{3}}{\binom{18}{6}} = \frac{80}{221}$	$\frac{\binom{10}{4}\binom{8}{2}}{\binom{18}{6}} = \frac{70}{221}$	$\frac{\binom{10}{5}\binom{8}{1}}{\binom{18}{6}} = \frac{24}{221}$	$\frac{\binom{10}{6}\binom{8}{0}}{\binom{18}{6}} = \frac{5}{442}$

$E(x) = \sum x_i P(x_i)$

$= 0\left(\frac{1}{663}\right) + 1\left(\frac{20}{663}\right) + \cdots + 6\left(\frac{5}{442}\right)$

$= \frac{10}{3}$ red dice

OR $=$

$E(x) =$

$= \left(\frac{10}{18}\right)(6)$

$= \frac{10}{3}$ red dice

2. Fill in the blanks.

1.5.2 Definitions

Definition 1.5.1

The distribution of a discrete random variable X is a hypergeometric probability distribution if:

1. *the probability experiment is repeated a **fixed** number of times*
2. *the outcome of each trial can be categorized into **success** or failed outcomes*
3. *the random varaible X represents the number of **successes***
4. *each trial of experiment is **dependant***

Theorem 1.5.2

If the distribution of X is hypergeometric with n dependent trials, and a represents the number of successful outcomes initially among a total of N possible outcomes in the beginning, then

$$P(x) = \frac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}} \tag{1.7}$$

$$E(x) = \frac{aN}{n} \tag{1.8}$$

$$\sigma = \sqrt{n(\frac{a}{N})(\frac{N-a}{N})(\frac{N-n}{N-1})} \tag{1.9}$$