Special theory of Relativity and Quantum Mechanics

SPH4U

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Contents

Chapter 2

Quantum Physics

2.1 Introduction to Quantum Theory

Why we need quantum theory

Physicists such as J.J. Thomson discovered that Newton's laws failed to explain the behaviour of electrons and atoms. Similarly, although Maxwell correctly described electromagnetic phenomena in the every day world, his equations failed to describe the microscopic world. This microscopic world is called the quantum world, where *quantum* refers to a very small increment of energy. The study of the behaviour of these very small bundles of energy, called *quantum theory*.

Particles and Waves

According to Newton's laws and Maxwell's equations, energy can be carried from one point to another in two ways: by particles and by waves. However, this is not accurate in the quantum world.

Definition 2.1.1 (Interference Effect)
The net effect of the combination of two or more wave trains moving on intersecting or coincident paths.

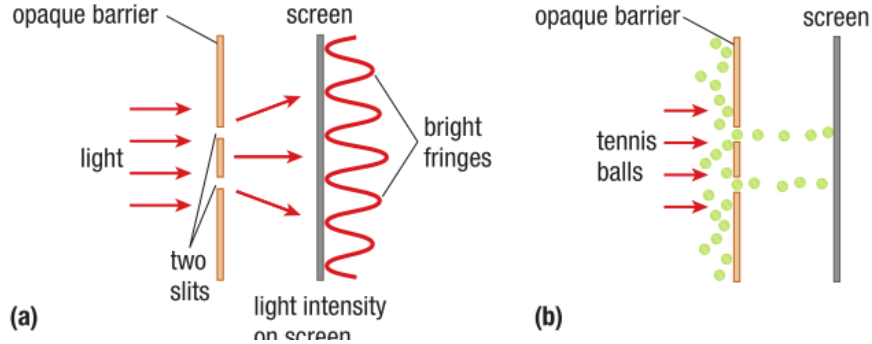
Differences between particle and wave

Wave

- Do show interference effects
- Waves do not deliver energy in discrete quantities. Wave deliver their energy continuously over time and spread out over the screen. (a) is a wave.

Particle

- Do not show interference effects
- Deliver energy in discrete quantities
- Double slit experiment, (b) is a particle.



The energy carried by a wave is described by its intensity, which equals the amount of energy the wave transports per unit time across a surface of unit area. For the light wave in figure 1(a), the amount of energy absorbed by the screen depends on the intensity of the wave and the absorption time. The amount of absorbed energy can take on any non-negative value.

An interference Experiment with Electrons

The separation of particles and waves is not found in the quantum world. There are only two types of behaviour are possible: waves exhibit interference; particles do not.

However, in reality, electron can exhibit interference, a property that classical theory says is possible only for waves.

Definition 2.1.2 (wave-particle duality)
The property of matter that defines is dual nature of displaying both wave-like and particle-like characteristics.

The following properties is followed in the quantum world:

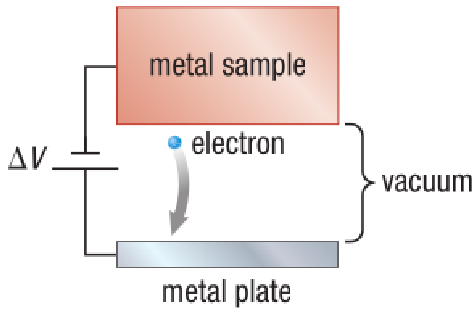
- All quantum objects, including electromagnetic radiation and electrons, can exhibit interference.
- All quantum objects, including electromagnetic radiation and electrons, transfer energy in distinct, or discrete, amounts. There discrete "parcels" of energy are quanta.

2.2 Photons and the Quantum Theory of Light

The Work Function

Definition 2.2.1 (Work Function)
The minimum energy required to remove a single electron from a piece of metal.

The work function can be calculated through this experiment:



We can manipulate the Δv in a circuit.

When the electron is pulled out of the metal piece, we can record the Δv, and calculate the work function through this formula:

$$W = e\Delta v$$

The Photoelectric Effect

Another way to extract electrons from a metal is by shining light onto it. Light striking a metal surface is absorbed by the electrons.

Definition 2.2.2 (The Photoelectric Effect)
If an electron absorbs an amount of light energy above the metal's work function, it ejects from the metal in a phenomenon.

Experimental studies of the photoelectric effect carried out around 1900 revealed that no electrons are emitted unless the light's frequency is greater than the threshold frequency

Definition 2.2.3 (Threshold frequency (f_0))
the minimum frequency at which electrons are ejected from a metal.

Through the experiment, scientists discovered that thershold frequency is independent of the intensity of the light, which is conflict to the classical physics (energy carried by a light wave is proportional to the intensity of light). Experiments found that when the frequency is below the threshold frequency, however, no electrons are ejected, no matter how great the light intensity.

Einstein's Quantum Theory of Light

Definition 2.2.4 (Photon)
A thought particle which two important properties

- *Do not have any mass*

- *Exhibit interference effects*

According to Einstein, each photon carries a parcel of maximum kinetic energy according to the following equation:

$$E_{photon} = hf$$

Where f is the frequency of the wave and h is the planck constant.

$h = 6.63 \times 10^{-34} J$
High intensity of light only means high amount of protons collide with metal

The absorption of light by an electron is just like a collision between two particles, a photon and an electron. Each photon is only responsible for removing one electron from the piece of matel.

If

$$\text{Energy of the Proton} < W$$

even with high intensity, no electron will be eject

In any other cases, electron will be ejected. As a result, we get the formula:

$$W = hf_0$$

and the E_k of the electron can be calculated as this:

$$E_k = E_{photon} - W$$
$$E_k = hf - hf_0$$
$$E_k = h(f - f_0)$$

This is a straight line. Hence, the kinetic energy of an ejected electrons should be linearly proportional to f.

Photons Possess Energy and Momentum

Einstein's quantum theory states that light energy can only be absorbed or emitted in discrete parcels, that is, as single photons. The classical theory of electromagnetic waves predicts that a light wave with Energy E also carries a certain amount of momentum:

$$P = \frac{E}{c}$$

Einstein's quantum theory predicts that the momentum of a single photon is:

$$P_{Photon} = \frac{hf}{c} \tag{2.1}$$

The wavelength of a light wave is related to is frequency as:

$$f\lambda = c$$

We can sub this equation for c in the photon momentum equation:

$$P_{Photon} = \frac{hf}{c}$$
$$P_{Photon} = \frac{hf}{f\lambda}$$
$$P_{Photon} = \frac{h}{\lambda} \tag{2.2}$$

Evidence of Photon Momentum

Instead of using visible light, Compton directed a beam of high-energy X ray photons at a thin metal foil. The foil ejected both electrons and lower-energy X-ray photons. This effect, in which incident X-ray photons lose energy and scatter of a metal foil along with free electrons, is called the **Compton effect**

X-ray photon acts like a particle in an elastic collision with an electron in the metal.

The electron absorb low-frequency x-ray and collide with high-frequency x-ray.

After the collision, the photon emerges from the collision with lower energy and a different momentum. The electron deflects with the kinetic energy and momentum lost by the photon.

Definition 2.2.5 (Compton effect)
The scattering of a photon by a free or weakly bound electron, in which total energy and momentum are conserved, leading to a change in the photon's wavelength.

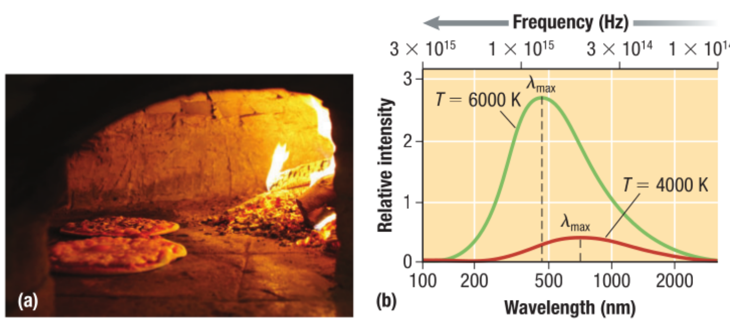
Compton's data indicated that the effect conserves both energy and momentum. Compton had to use equations of special relativity to analyze the collsion, including the equation for relativistic momentum. He would not have obtained the correct results without using special relativity. In this way, Einstein's ideas on relativity and the speed of light influenced work that confirmed Einstein's ideas about the behaviour and characteristics of photons.

Blackbody Radiation

Definition 2.2.6 (Blackbody)
An object that absorbs all radiation reaching it.

Definition 2.2.7 (Blackbody Radiation)
Radiation emitted by a blackbody.

The specific problem that puzzled Planck is represented by the glowing oven. This oven emits radiation over a range of wave-lengths and frequencies. To the eye, the colour of the oven is determined by the wavelength of the largest radiation intensity.

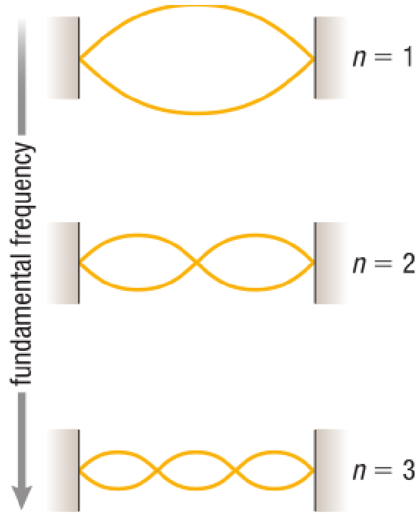


Experiments prior to Planck's work showed that the intensity curve has the same shape for a wide variety of objects. The blackbody intensity falls to zero at both long and short wavelengths, corresponding to low and high frequencies, respectively, with a peak in the middle. Planck tried to explain this behaviour.

At this time, physicists knew that electromagnetic waves from standing waves as they reflect back and forth inside an oven. These standing waves are just like the standing waves on a string. Standing waves on a string have frequencies that follow the pattern:

$$f_n = nf_0$$

, where f_0 is the fundamental frequency and $n \in \mathbb{Z}$



According to the classical physics, each of these standing waves carries energy, and as their frequency increases, so does the total energy. As a result, the classical theory predicts that the blackbody intensity should become infinite as the frequency approaches infinite values.

Planck's Hypothesis

The classical theory would be in conflict with the experimental intensity curves in the graph. The true intensity falls to zero at high frequencies.

Planck resolved this disagreement by hypothesizing that the energy in a blackbody comes in discrete parcels (quanta). He believed that each parcel has energy equal to hf_n, where f_n is one of the standing wave frequencies and h is a universal constant. This explanation can correctly predit the behaviour of wave in the experiment. However, he could give no reason or justification for his assumption about standing-wave quanta.

Wien's Law

The wavelength at which the radiation intensity of a blackbody is largest is donoted by λ_{max} and is determined by the temperature, T, of the blackbody through ane expression called Wien's law.

$$\lambda_{max} = \frac{2.90 \times 10^{-3} m * K}{T} \tag{2.3}$$

2.3 Wave Properties of Classical Particles

Wave-like Properties of Classical Particles

In 1924, Louis de Broglie first suggested that all classical particles have wave-like Properties. In the previous section, you read that a photon has a momentum given by

$$P_{photon} = \frac{h}{\lambda}$$

De Broglie turned this result around and hypothesized that a particle with momentum p has a wavelength of

$$\lambda = \frac{h}{p} \tag{2.4}$$

Definition 2.3.1 (de Broglie wavelength)
The wavelength associated with the motion of a particle possessing momentum of magnitude p

If a particle has a wavelength, the particle should exhibit interference just as wave do.

According to his equation, a longer wavelength means that the value for momentum has to be small.

Definition 2.3.2 (Matter wave)
The wave-like behaviour of particles with mass

It is possible to determine the de Broglie wavelength of larger objects, such as baseballs. However, the momentum of large objects tends to be so large that it implies an incredibly small wavelength. That is why we are unable to see the interference of these objects.

Interpreting the Double-Slit Experiment

Collapse Interpretation

The electron leaves its source behaving as a particle, but then it spreads out and travels as a wave until it is measured at the screen.

This law claims that an electron physically changes from a particle to a wave and back again. These two behaviours and the physical laws that go with them alternative in a way that is not predicted by quantum mechanics.

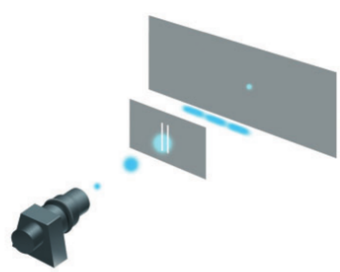


Figure 2 In the collapse interpretation, each electron in a double-slit experiment travels as a spread-out wave.

Pilot Wave Interpretation

The electron is just a simple particle whose motion is described by a single law. The motion of the electron depends on a mysterious pilot wave. To obtain the interference pattern in the double-slit experiment, the behaviour of the pilot wave must depend on everything everywhere in the universe, including future events. For example, the pilot wave "knows" whether one or two slits are open, and whether or not a detector is turned on at the screen.

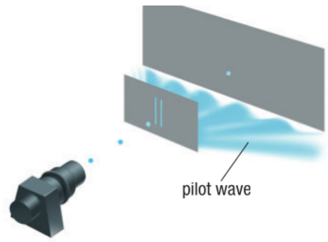


Figure 3 In the pilot wave interpretation, the electrons in a double-slit experiment are particles whose motion depends on a pilot wave.

Many Worlds Interpretation

A parallel universe exists for each of the electron's possible states. The Universe constantly splits into many versions of itself.

Copenhagen Interpretation

This interpretation view interprets the physical laws in terms of information about actual measurement made on a quantum-mechanical system. Certain questions do not have answers, such as what electrons are "doing" as they travel to the detection screen. You can only ask what the results will be if you do a certain experiment.

The Wave Function: A mathematical Description of Wave-Particle Duality

A wave function gives the probability for a particle to take any possible path, or for the particle to show up at any possible location on the detection screen in the double-slit experiment.

Wave fuction can be calculated by an equation from Erwin Schrodinger. Researchers use the Schrodinger equation to determine the wave function and how it varies with time.

Consider an electron confined to a particular region of space. A classical particle moving around inside the box would simply travel back and forth, bouncing from one wall to another. The wave function for a particle-wave inside this box is described by standing waves, similar to those you would see on a string.

The figure 4(b) shows two possible wave function solutions corresponding to electrons with different kinetic energies. The wavelengths of these standing waves are different, since the wavelength of an electron depends its kinetic energy. In the case of mechanical waves in classical mechanics, these two solutions correspond to two standing waves with different wavelength.

After get the wave function for a particular situation, such as for the electron in (b), you can try to calculate the position and speed of the electron. However, the results do not give a simple single value for x. The probability of finding the electron at certain value of x is large in some regions and small in others. The probability distribution is different for each wave function.

The Heisenberg Uncertainty Principle

Theorem 2.3.3 (Heisenberg Uncertainty Principle)
A mathematical statement that says that if Δx is the uncertainty in a particle's position, and Δp is the uncertainty in its momentum, then

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

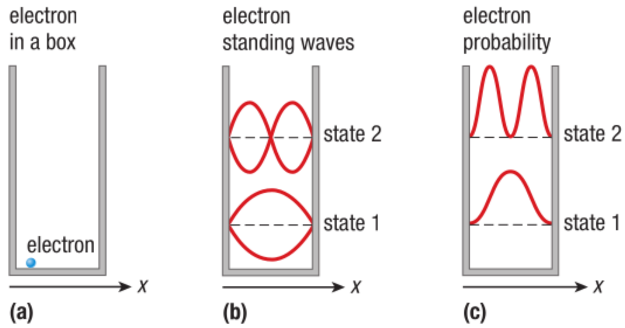


Figure 4 (a) A thought experiment of an electron that is trapped in a box. (b) The electron wave probability function forms a standing particle–wave similar to the standing waves on a string fastened to the walls of the box. The electron wavelength must therefore "fit" into the box as it would for a standing wave. (c) The quantum-mechanical probabilities of finding the electron at different locations in the box correspond to each of the two wave functions in (b).

where h is Planck's constant

This principle says that there is a limit to how accurately simultaneous measurements of the position and momentum of a quantum object can be.

If you measure the position of a quantum object with great accuracy, then you can only measure its momentum with little accuracy.

The act of measuring the system itself disturbs the system.

2.4 The Standard Model of Elementary Particles

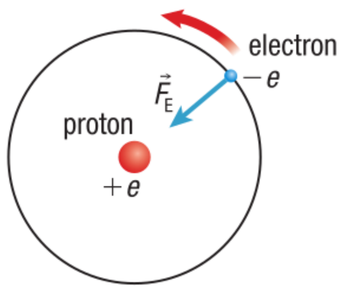
Ernest Rutherford

In 1909, Ernest Rutherford and his students did an experiment. They aimed high-speed, positively charged particles at a thin sheet of gold foil. Under the traditional theory, Rutherford and his team expected most of the particles to pass through the foil.

Instead, they discovered that a small number of particles were deflected. Rutherford realized that this result meant that all the positive charge in an atom must be concerntrated in a very small volume.

Rutherford suggested that the atom is like a miniature solar system, with electrons orbiting the nucleus just as planets orbit the sun. The model asl proposed that they could move in any orbit.

The strong force hold the protons together.

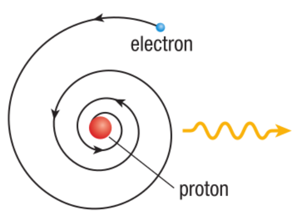


The electrons must move in orbits to avoid "*falling*" into the nucleus as a result of the electric force.

Problems with the Planetary Model

Maxwell's classical theory of electromagnetic predicts that an electron emits electromagnetic radiation when it orbits a proton. The radiation carries away energy. If the electron in a hydrogen atom loses energy in this way, Newtonian mechanics predicts that it will spiral inward to the nucleus.

If this model were correct, all atoms would collapse, which is not the case. Physicists were unable to modify the planetary model to make the atoms stable.



The Bohr Model of the Atom

Bohr proposed a quantum-mechanical approach to the motion of electrons within the atom. He was inspired by the Planck-Einstein introduction of quanta into the theory of electromagnetic radiation. Bohr's theory went against the well-established classical laws of mechanics and electromagnetism.

Bohr proposed that an electron in an atom can have only certain orbits with particular values for the radius of each orbit. The special values of the orbital radius meant that the electron could only have special values of potential energy and kinetic energy. The total energy could take on only certain discrete, quantized values. Each value of energy corresponds to what is now called an energy level.

Bohr's model was partially successful. It provided a physical model of the hydrogen atom. The model matched the internal energy levels to the levels observed in a hydrogen spectrum. At the same time, the model accounted for the stability of the hydrogen atom.

However, Bohr's model, was incomplete. When applied to atoms with many electrons, the model broke down.

An explanation for the BOhr model came from de Broglie's matter waves, which were developed 10 years after Bohr's work. As mentioned in previous chapter, the electrons in a given energy level have special values of kinetic energy. Therefore, those electrons have certain values of momentum.

Using de Broglie's model, the allowed electron orbits in hydrogen correspond exactly to those orbits in which electron waves from circular standing waves around the nucleus.

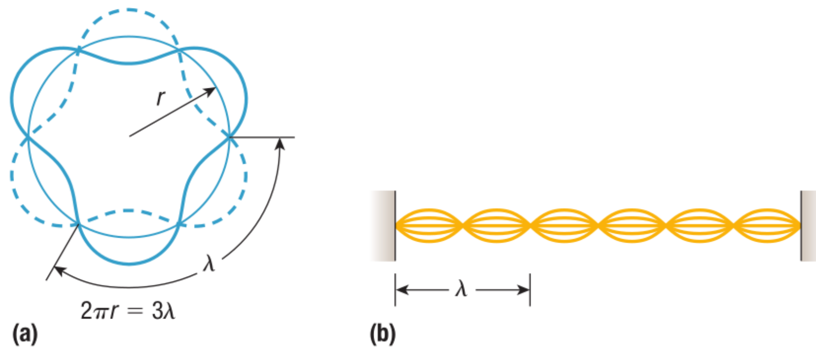


Figure 3 (a) The standing-wave pattern for an electron wave in a stable orbit of hydrogen requires that the orbital circumference equal a whole number of wavelengths. (b) The standing-wave pattern for a string fixed at both ends requires that the distance between supports equal a whole number of wavelengths.

Twenty-First-Century Physics and Antimatter

Definition 2.4.1 (Antimatter)
a form of matter in which each particle has the same mass and an opposite charge as its counterpart in ordinary matter

Two examples of antimatter are

Anti-protons
Anti-neutrons

Although the neutron and anti-neutron are both neutral, they are different particles. Neutron is made up of a certain combination of quarks, and the anti-neutron is made up of the corresponding combination of anti-quarks

Table 1 Some Properties of Electrons, Protons, Neutrons, and Their Anti-particles

Particle	Symbol	Mass (kg)	Mass (MeV/c^2)	Charge
electron	e^-	9.109×10^{-31}	0.511	-1
positron	e^+	9.109×10^{-31}	0.511	$+1$
proton	p	1.673×10^{-27}	938	$+1$
anti-proton	$\bar{p}$	1.673×10^{-27}	938	-1
neutron	n	1.675×10^{-27}	940	0
anti-neutron	$\bar{n}$	1.675×10^{-27}	940	0

Anti-particles offer researchers a chance to see special relativity at work at the microscopic level. For example, when an electron encounters its anti-particle, the positron, the two undergo a reaction that destroys both particles.

The Standard Model

Definition 2.4.2 (Quark)
an elementary particle that makes up protons, neutrons, and other hadrons.
Was first discovered in collision experiments involving protons

Definition 2.4.3 (Hadrons)
A class of particles that contains the neutron, the proton, and the pion; composed of combinations of quarks and anti-quarks

- *Hadrons composed of three quarks are called baryons.*
- *A quark and an anti-quark can also combine to form a particle. Hadrons composed of just two quarks are called mesons.*

Definition 2.4.4 (Leptons)
A class of particles that includes the electron, the muon, the tauon, and the three types of neutrinos; not composed of smaller particles

Definition 2.4.5 (Standard model)
the modern theory of fundamental particles and their interactions

When a high-energy electron collides with a proton, the way that the electron scatters (that is, its outgoing direction and energy) gives information about how mass and charge are distributed inside the proton.

All hadrons are composed of quarks, so the interactions between quarks determine the properties of hadrons and how they relate to one another. The two most important hadrons are the proton and the neutron, so the behaviour of quarks also determines the properties of nuclei.

Quarks are charged, so they act on each other through the electric force. They also interact through the strong force mentioned earlier. Quarks bind together to form protons and neutrons (nucleons), and the strong force is responsible for holding protons and neutrons together to make nuclei.

Leptons

Leptons are naturally grouped into three pairs:

- Electron and Electron neutrino
- Muon and Muon neutrino
- Tau and Tau Neutrino

The muon and the tau are not stable. Electrons are stable, but the behaviour of neutrinos is more complicated. Neutrinos travel through space, they change from one to another of the three types of neutrinos, an effect called neutrino oscillation.

Bosons: Force Mediating Particles

The fundamental forces of nature are the ways in which individual particles interact with each other. Every interaction in the universe can be described using only three forces.

- Strong nuclear force
- Weak nuclear force
- electromagnetism

The strong nuclear force holds the subatomic particles of the nucleus together.

The weak nuclear force causes radioactive decay and starts the process of hydrogen fusion and other nuclear process in star

The electromagnetic force is responsible for the attraction and repulsion among electrical charges.

Definition 2.4.6 (Fermion)
A fundamental particle that forms matter
Including quark and leptons

Definition 2.4.7 (Bosons)
The particle responsible for transmitting electromagnetic, strong, and weak force.

From this view,

- The electromagnetism force is "mediated" by the photon, which means that photons transmit the electromagnetic force acting on charged particles.
- The strong nuclear force is mediated between quarks by particles called Gluons. Eight different types of gluons exist, and the force exerted between two quarks depends on type of each quark.
- The weak nuclear force is mediated by a family of three particles called the W^+, W^- and Z bosons. Unlike the photon and gluons, which have zero mass, the W^+, W^- and Z bosons have mass.

According to the standard model, another boson, the Higgs boson, exists.

Definition 2.4.8 (Higgs Bosons)
the theoretical particle thought to play a role in giving mass to other particles.

Chapter 3

Appendax

3.1 Special Theory of relativity

Two posulates