# Grade 12 Math of Data Management

MDM4U

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# Contents

# Chapter 1

# Relativity

## 1.1 The Special Theory of Relativity

At the beginning of 1900s, few famous physics experiments came to a similar statement. The speed of light is $3.0 \times 10^8 \frac{m}{s}$ at whatever frame of reference. Einstein developed the *Special Theory of Relativity* to explain why the speed of light is constant at different frames of reference.

The *Special Theory of Relativity* has two postulate:

**Postulate 1** (The Principle of Relativity)
*The laws of physics are the same in all inertial frames of reference. No physics experiment can ever determine whether you are at rest or moving at a constant velocity.*

**Postulate 2** (The Speed of Light Principle)
*There is at least one inertial frame of reference in which, for an observer at rest in this frame of reference, the speed of light, c, in a vacuum is independent of the motion of the source of the light.*

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The consequence of this is that the speed of light must be constant and the same in all inertial frame of reference because the laws of physics do not prefer one frame of reference over another.

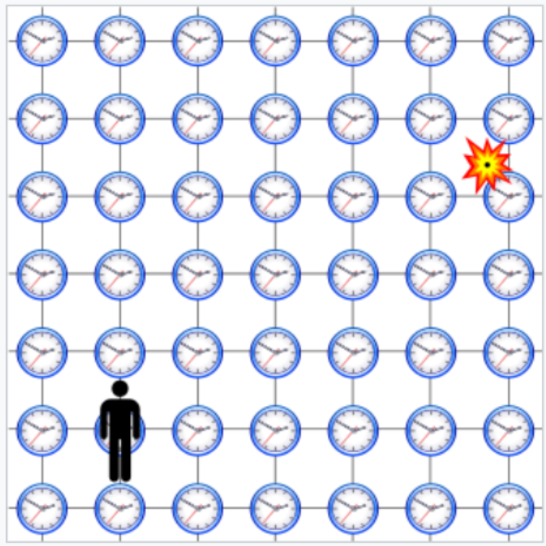
**Theorem 1.1.1** (Special Theory of Relativity)
*All physical laws are the same in all inertial frames of reference, and the speed of light is independent of the motion of the light source or its observer in all inertial frames of reference.*

## 1.2 Time Dilation

*Remark.* Please go to Mr. Yang's video here! Here is just a brief personal review note about *time dilation* in that video. Please check the textbook first, this note is only a summary!

*Time Dilation*, the model explains the slowing down of time in one reference frame moving relative to an observer in another reference frame.

**Proper time vs non-proper time measurement**



In special relativity, the observer measures events against an infinite latticework of synchronized clocks.

**Definition 1.2.1** (Synchroinzed Clock Measurement/Proper time)
*A people measures a time at the same location/coordinate from his frame of reference!*

**Definition 1.2.2** (Moving Clock Measurement/Non-proper time measurement)
*A people measures a time at the different location/coordinate from his frame of reference!*

**$\Delta t$ vs $\Delta t_s$**

$\Delta t_s$ is a Synchronized clock measurement of the time from $v$'s frame of reference. $\Delta t$ is the moving clock measurement from a stationary perspective.

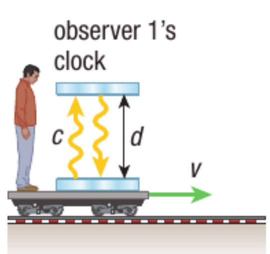



Figure 1.1: From the inertial FOR which has recorded $\Delta t_s$

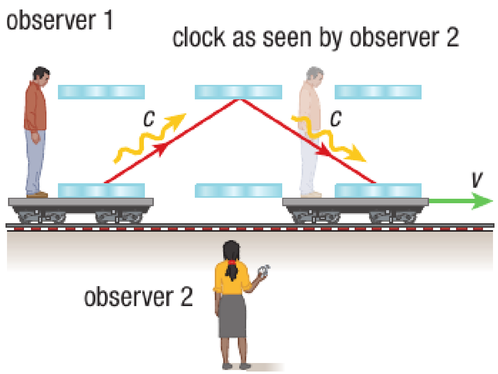



Figure 1.2: From the inertial FOR which has recorded $\Delta t$

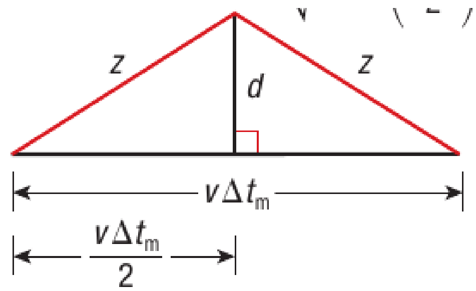



Figure 1.3: Path of light from observer 2's FOR

**Derivation of Special Relativity equation**

**Lemma 1.2.3**

$$\Delta t = \frac{distance}{speed}$$

*Proof.* To start off, we need to solve for $z$:

$$z = \sqrt{d^2 + (\frac{v\Delta t}{2})^2} \tag{1.1}$$

Then, let's solve for $\Delta t$

$$\Delta t = \frac{2z}{c}$$
$$\Delta t = \frac{2}{c} \times \sqrt{d^2 + (\frac{v\Delta t}{2})^2}$$
$$(\Delta t)^2 = \frac{4}{c^2} \times (d^2 + (\frac{v\Delta t}{2})^2)$$
$$\Delta t^2 = (\frac{2d}{c})^2 + (\frac{v\Delta t}{c})^2$$
$$\Delta t^2 = (\Delta t_s)^2 + (\frac{v\Delta t}{c})^2$$
$$\Delta t = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

□

According to the theory of special relativity, $\Delta t > \Delta t_s$. As a result,

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \geq 1$$

**Lorentz factor $\gamma$**

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If we look at the factor of the $\gamma$, it only matters when the speed of the rocket is fast enoguh!