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Assignment 1
VaR and ES forecasts

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Abstract

Value at Risk(VaR) and Expected shortfall(ES) are vital parameters to evaluate risk factor in finance. This report introduces four models to evaluate portfolio VaR and ES : Variance-Covariance method(Var-Cov), historical simulation method, Constant Conditional Correlation(CCC) method And Filtered Historical simulation(FHS) method. For Var-Cov method and historical simulation methods, we see that stress period significantly influence the VaR and ES. Besides, assumption of student-t with 5 degree is suitable for our portfolio. And we use backtest to evaluate all models and FHS method is the best to evaluate the portfolio VaR. And all models are failled to predict ES when compared with actual ES. And then stress testing shows shock in stock price bring significantly change in portfolio values.

1 Introduction

VaR and ES are crucial standard of risk capital for investors.Four simulation models for VaR and ES from historical daily return data are discussed in this report. They are Variance-Covariance method(Var-Cov), historical simulation method, Constant Conditional Correlation(CCC) method And Filtered Historical simulation(FHS) method. The first two are unconditional or static method totally based on past, the last two methods are conditional or dynamic methods with fitting the current state from recent data. Compare the prediction with actual return in each method to choose the most applicable one for our portfolio, which is a random combination of assets chosen by teammates. Additionally, the sensitivity of estimation period and stress period distinguishing are explored to study the effect of prediction. And the difference of one-day VaR and N-day VaR is obvious. At last, backtest and stress test are useful tools to evaluate the adaptability of various methods.

2 Method and Theory

In order to investigate the VaR and ES, our portfolio is combination of one domestic stock, one foreign stock, one domestic index and one floating-rate bonds.They correspond to Louis Vuitton Malletier, Apple company, AEX - which is Amsterdam Exchange index, and the floating-rate bonds based on Netherlands 10 years government bonds. And the historical data is taken from $01/04/2010$ to $01/04/2020$. And the whole project's statistic confidence levels are 0.975 and 0.99 for all methods.

*And in our portfolio, the total capital is **5000000 Euro**. And relative weight for each asset: **LV-0.2, AEX-0.2, Apple-0.2, Bonds-0.4.***

2.1 Unconditional method

We can not directly calculate the volatility of portfolio since the return of assets in portfolio are correlated in the market. **Variance-Covariance method** is designed to calculate volatility of portfolio and also to VaR and ES. The main idea is calculating the return of each asset and then calculate variance-covariance matrix. Then using equation 1 to get the portfolio variance and volatility.

$$Var(r_p) = \sum_{i=1}^{n} w_i^2 var(r_i) + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} w_i w_j cov(r_i, r_j) \tag{1}$$

Then we calculate the VaR and ES assuming that the return of portfolio obeys normal distribution or student-t distribution (with various degree). The $\widetilde{Z_a}$ is the quantile of the distribution under a certain confidence level. And we use QQ-plot to check whether the assumptions are correct.

$$VaR = \mu + \widetilde{Z_a} \times \sigma \tag{2}$$

Another method to evaluate the volatility of portfolio is **historical simulation method**. The main idea is calculating the return of portfolio and ordering the return in increasing order. Then the VaR(1 days, 0.975) is 0.975 quantile of portfolio return. And ES is average of portfolio return which is bigger than 0.975 quantile.

[*For Unconditional VaR-Cov method and historical method, we use 1 year(250 days), 5 years, and 10 years data length to evaluate the sensitivity of length*]

2.2 Conditional method

Other two methods are **Constant Conditional Correlation method(CCC)** and **Filtered Historical Sdvolatility. Then, the VaR and ES calculate as variance-covariance method.**

$$\sigma^2 = \omega + \alpha\mu_{n-1}^2 + \beta\sigma_{n-1}^2 \tag{3}$$

Filtered Historical Simulation (FHS) currently is a popular method to evaluate the VaR and ES. In this method, we use EWMA Model to calculate non-constant volatility. The equation show as 4. Then, the new step's VaR and ES calculate as historical method, which take the 97.5% or 99% quantile of the simulation returns. For FHS, first we use 100 days to initialize the σ and then the time window are two years(500 days).

$$\sigma^2 = (1-\lambda)\mu_{n-1}^2 + \lambda\sigma_{n-1}^2 \tag{4}$$

2.3 Backtest parameters and validation

We use whole dataset(2010-2020) to do the backtest and we only compare results of each model from $03/04/2014 - 01/04/2020$(1495 days) since conditional method need some data to do initialization.

For VaR-Cov and historical simulation methods, the rolling window are all 500 days to calculate next day's VaR and ES.

For CCC method, we recalculate the coefficients(ω, α, β) every one years(Note: the first year is not 250, it is 244). And then rolling window are also two years(500 days) and correlation matrix also updates every time we do rolling window. Then, after we get the σ we calculate the VaR. To note that we calculate 500 days' mean μ (500 days before the day you calculate) to calculate VaR.

For FHS method, we use first 100 days to initialize σ, μ. And using equation 4 to calculate the σ and λ is 0.94. And rolling window are still 500 days.

In order to statistically test VaR, we use binomial test statistics and the process is counting number of violations of VaR and compared it with expected number. And using equation to get the P-value.

3 Result and discussion

3.1 Variance-Covariance method

After preprocessing and combining the assets' historical data with unconditional Var-cov method, the dynamic changes of returns of portfolio in 10 years is shown in left figure 1. It is easy to notice that the returns in 2011 ,2015 and 2020 years suffered great violations, due to these two years are stress period in financial market.

In the right hist plot, the yellow and red filling color corresponds to 97.5% and 99% loss in 10 years' historical data separately.

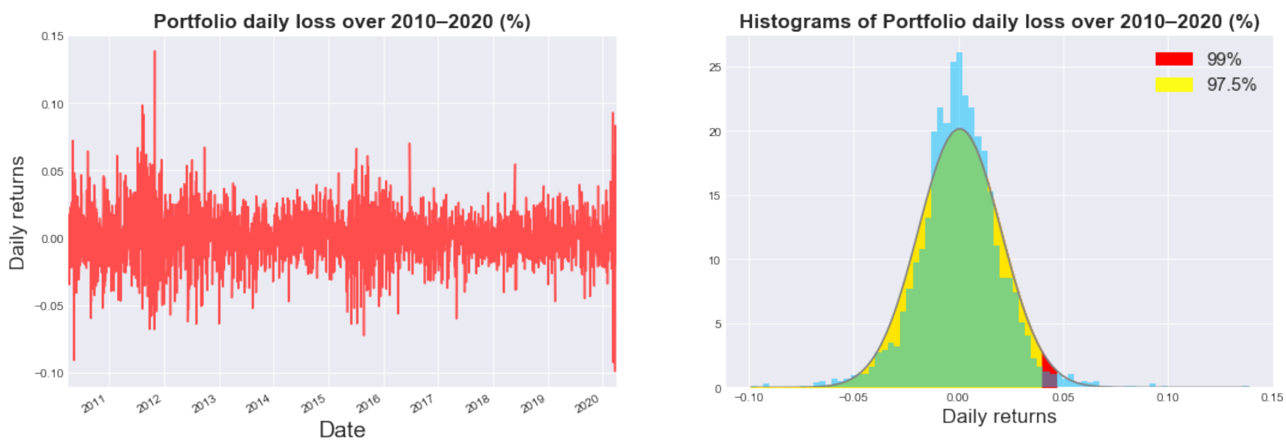


Figure 1: Left plot is Portfolio daily loss. To note that we let the loss is positive. The date is from $01/04/2010$ to $01/04/2020$. And right plot is distribution (normal assumption) of portfolio loss. Yellow shade part is 97.5% and red shade part is 99%.

3.1.1 Check normality or student-t property with QQ-plot

The assumption of Var-Cov method is that the return of portfolio obeys normal distribution or student-t distribution. And we test normality and student-t with degree 3,4,5,6 with all assets and portfolio. Here we just show some plots. After check the normality or student-t properties by QQ-plot in the figure 2, it is obvious that the student-t with 5 degree is a better assumption. The plot of portfolio return seems to be close to quantile straight line in this case. It is the same case for bonds that its daily return obeys the 5 degree student-t distribution better.

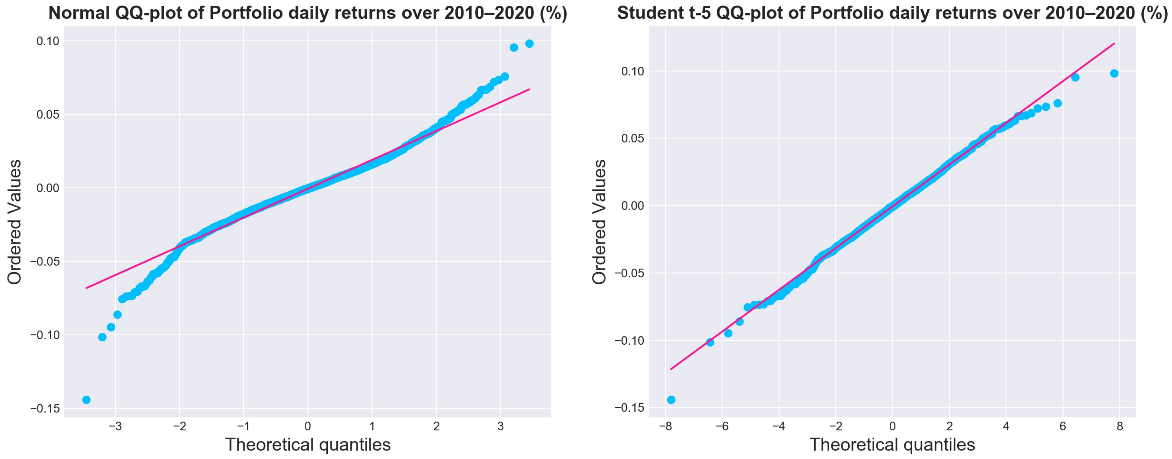


Figure 2: Normal QQplot and student-t with 5degree QQplot of portfolio return.

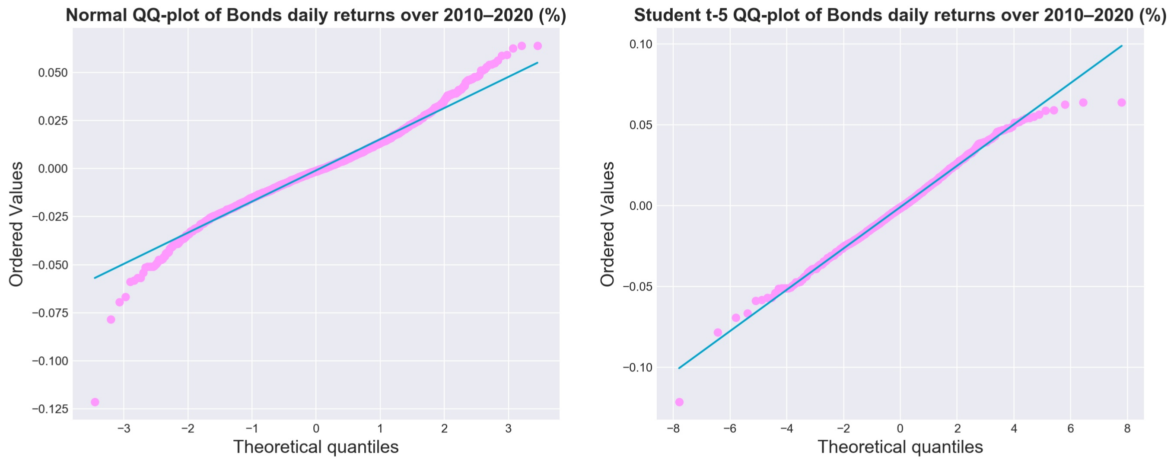


Figure 3: Normal QQplot and student-t with 5degree QQplot of bonds return.

3.1.2 Sensitivity of length of estimation period

In the unconditional Var-Cov method, the forecasting VaR or ES vary with the length of estimation period taken from historical data. Here, we take last one year, last five year and last ten year histotical data to predict the risk capital in the table 1 and table 2. Some reasonable and easy rules in the columns are: 0.975VaR $<$ 0.99VaR; 0.975ES $<$ 0.99ES; 0.975VaR $<$ 0.975ES and 0.99VaR $<$ 0.99ES.

For different length estimation period with stress period in data, all VaR and ES predicted by Normal assumption and Student-t assumption have: (1) One year's prediction $>$ Ten year's prediction $>$ Five year's prediction. (2) VaR and ES predicted by student-t distribution are generally bigger than that predicted by normal distribution.

Table 1: VaR and ES of different time length for Variance-Covariance method

Time	0.975VaR	0.99VaR	0.975ES	0.99ES
One years-normal	199079.701	235658.369	236801.993	269489.767
Five years-normal	169793.344	201417.963	202406.700	230667.391
Ten years-normal	197444.117	233820.176	234957.466	267464.182
One years-t_5	202194.630	263624.279	275738.350	347724.289
Five years-t_5	172486.401	225596.297	236069.694	298306.184
Ten years-t_5	200541.792	261631.183	273678.154	345265.363

(1) One year's prediction $>$ Ten year's prediction: Since the beginning of 2020 is hugely impacted by the covid-19, there are big crash in the market, which makes the VaR or ES bigger. Compared with 10 year's historical data, there are few crashes happened except the 2011 and early 2020, this estimation period is relatively stable and data has less connection, which makes smaller average loss and volatility when increasing the estimation length. One year's prediction $>$ Five year's prediction: even thought there seems to be violations in 2015-2016 and great loss at the end of 2015, one-year's data are more correlated internally than five-years' data, which makes higher volatility. And Ten year's prediction $>$ Five year's prediction. That's mainly because there is huge crush at 2010 according to 1.

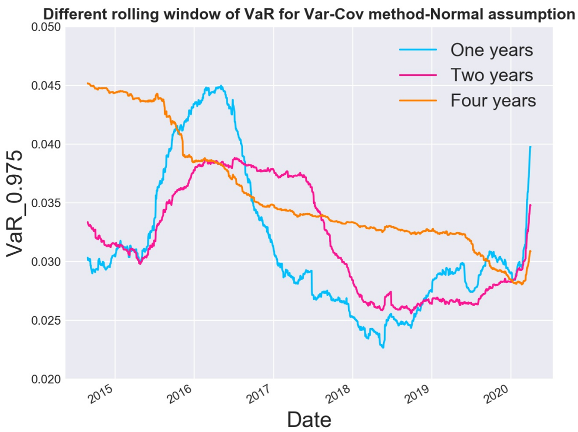
(2) Compared with normal distribution, student-t distribution has fatter tails when probability approaching zero at two sides. This means for the same 0.975 or 0.99 quantile, student-t distribution has larger T_a than Z_a, which makes the VaR and ES calculated in the equation 1 are bigger.

Table 2: VaR and ES of with or without stress years for Variance-Covariance method

Time	0.975VaR	0.99VaR	0.975ES	0.99ES
With stress years-normal	197444.117	233820.176	234957.466	267464.182
Without stress years-normal	176073.249	208669.741	209688.864	238818.049
With stress years-t_5	200541.792	261631.183	273678.154	345265.363
Without stress years-t_5	178849.068	233591.113	244386.373	308535.486

For different length estimation period distinguished stress period in data, all the VaR and ES calculated with stress years' data are larger than these calculated without stress years' data. The reason is obvious that stress years have more violations in returns which make the average loss and volatility larger.

In the figure 4 below, the rules discussed above about sensitivity of length of estimation period is shown more vividly. For points in the 0.975-VaR curve is predicted by the corresponding previous estimation period with a certain length by rolling window one today each time. In summary, when there is crisis recently, one year's prediction is supposed to be the largest one. This conclusion indirectly improved that there is turmoil in this portfolio in 2015, because one-year prediction arrived a peak in 2016. Through analysis to each asset, both AEX and floating rate bonds based on 10 year Netherlands government bonds showed fluctuation in 2015-2016, these two counts 0.6 percent of the total weight.



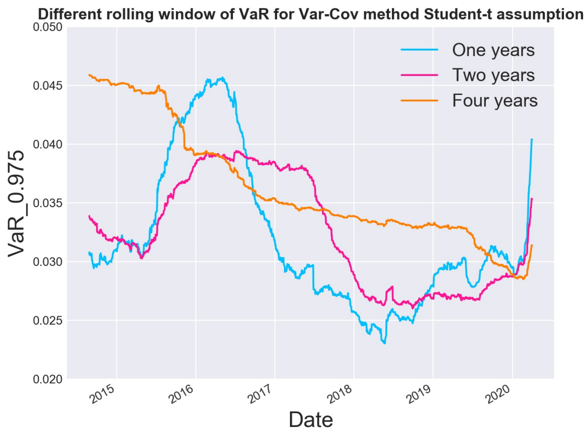


Figure 4: Different rolling window for 0.975VaR of Variance-Covariance method with normal and student-t assumption

3.2 Historical simulation method.

This method is one of the most basic method to calculate VaR and ES, which just according to the quantile value of the ascending returns.

3.2.1 Sensitivity of length of estimation period

For different length estimation period with stress period in data, the conclusion keeps same as the previous part for VaR-Cov method that - One year's prediction > Ten year's prediction > Five year's prediction for all VaR and ES. This is because there are three crisis in our portfolio in 2011, 2015-2016 and early 2020. So that the ten-year's prediction is higher than five-year prediction. and the one-year's prediction is greatest because it include crisis of 2020.

Table 3: VaR and ES of different time length for historical method

Time	0.975VaR	0.99VaR	0.975ES	0.99ES
One years	195999.865	282507.832	274909.884	274909.884
Five years	171347.935	214853.163	229147.733	229147.733
Ten years	195724.911	281320.013	284279.593	284279.593

For different length estimation period distinguished stress period in data, it is the same that estimation period without stress period leads to lower risk capital in historical simulation method, due to almost violations have been dropped in the dataset.

Table 4: VaR and ES of with or without stress years for historical method

Time	0.975VaR	0.99VaR	0.975ES	0.99ES
With stress years	195724.911	281320.013	284279.593	284279.593
Without stress years	175821.885	225476.722	226540.982	226540.982

By rolling the time window to predict the consecutive VaRs day by day, using historical simulation method. The figure 5 shows three predicted VaR curves wit different estimation period. The general trend of them are similar to these forecasted by VaR-Cov method in the previous part. Again this plot shows that one-year prediction acts fastest to any violations happened in history. And the two-year prediction curve seems to have same trend as the one-year one. The four-year trend is decreasing until early 2020; the high smooth trend at the beginning (2015) is affected by the crisis happened in 2011.

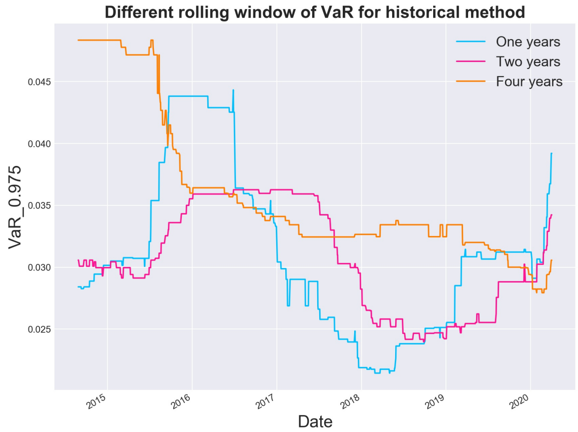


Figure 5: Different rolling window for 0.975VaR of historical method

3.2.2 N-day VaR

In this part, we want to investigate whether it is sufficient to use N-day VaR of historical method to evaluate our portfolio. 5-day VaR and 10-day VaR are combined and calculated by non-overlapping data that united from the historical data. And then using equation $VaR(n,\alpha) = \sqrt{n}VaR(1,\alpha)$ to transform into 1-day VaR. And for all VaRs the rolling time window are 500 days as estimation period.

Figure 6 shows N-day VaR violation. It is clear to see that the trend of 5-day VaR is close to the 1-day VaR. These two VaRs are more likely to simulate the extreme violations in the daily returns. In another words, their curves are more close to VaRs or big loss. In the contrast, 10-day VaR's curve seems to be close to the general trend (average) of daily loss, which is definitely not adequate to assess our portfolio.

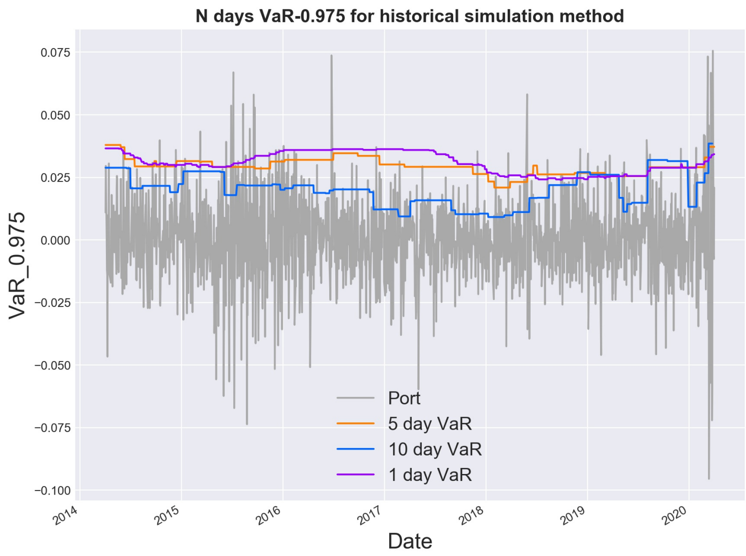


Figure 6: 0.975VaR violation plot of N-day VaR. Time length is six years. From $03/04/2014 - 01/04/2020$.

Since 10-day VaR is not ideal, we test 5-day VaR. Tabel 5 shows results of 1-day VaR and 5-day VaR. For 0.975 VaR, we expected total violations are 36. But for both 1-day VaR and 5-day VaR the violations are $51, 58$ respectively. And the p-values are $0.031, 0.001$ respectively. For 0.99 VaR, the results are even worse. It means both methods are failed to evaluate our portfolio VaR. That's because historical method returns a long-term empirical VaR and ES and it cannot successfully predict future fluctuation precisely in some cases, especially without any similar experience before. Even 1-day VaR failed to prospect it. And the situation will be even worse for N-day VaR because N-day VaR evaluates the portfolio return every N days.

Table 5: N days VaR violation

Year	2014	2015	2016	2017	2018	2019	ALL	P-value
Trading days	250	250	250	250	250	245	1495	-
Var-0.975								
Expected number	6	6	6	6	6	6	36	-
1-day Historical	8	14	3	2	10	14	51	0.031
5-days Historical	7	20	4	4	10	13	58	0.001
Var-0.99								
Expected number	3	3	2	2	3	2	15	-
1-day Historical	4	8	1	1	6	9	29	0
5-days Historical	3	11	2	2	5	5	28	0

3.3 CCC method And Filtered Historical method

From this part, we start to explore conditional methods - constant correlation conditional method based on Garch(1,1) and Filtered Historical method based on EWMA. These two are dynamic methods, which will update the volatility σ and average loss μ very step. As figure 7 shows, 0.99 VaR's curve overlaps with 0.975 ES's curve in CCC method. While for FHS methods, curves of 0.99 VaR, 0.975 ES and 0.99 ES are overlapped. Additionally it is clear to see that the trends of two VaR plot is quite similar because these two methods all use un-constant volatility. But VaR prediction in CCC method is bigger than that in FHS methods. The possible explanation could be that CCC method uses GARCH to calculate the volatility and it not only includes the previous σ, μ but also long-term variance V_L, there are some great violations of our portfolio in the history.

Another remarkable phenomenon in these two plots is that the all loss curves rise steeply in the early 2020, affected by covid-19. Compared with other methods like historical simulation and VaR-cov method, these two conditional methods predict quite larger daily potential loss (arounding 0.14), which is exactly in consist with the actual daily return (around 0.12) of our portfolio at the beginning.

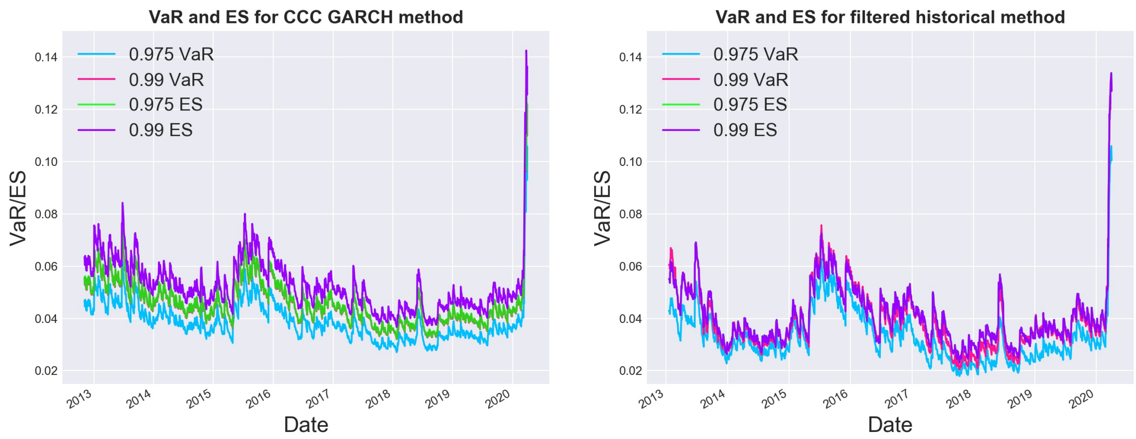


Figure 7: The left plot is VaR and ES of CCC method. Right plot is VaR and ES of FHS.Both time window are 500 days. The parameters are stated as section 2.3.

3.4 Backtest

In this part, we use backtest method to evaluate and validate the four models for our portfolio. Figure 8 shows four methods' VaR violation compared with actual return for each year.

In the stressed period or after crashes in the market, the phenomenon called violation clusters appeared for all methods on special days. During usual time, the violations seems timely even spread for each method. In general, CCC Garch method is the most conservative one with highest risk capital predicition, which is suitable for those risk aversion investor and big company. Historical simulation is the most risky one, since it has most violations. And the FHS method seems to approach the actual loss closely in every year.

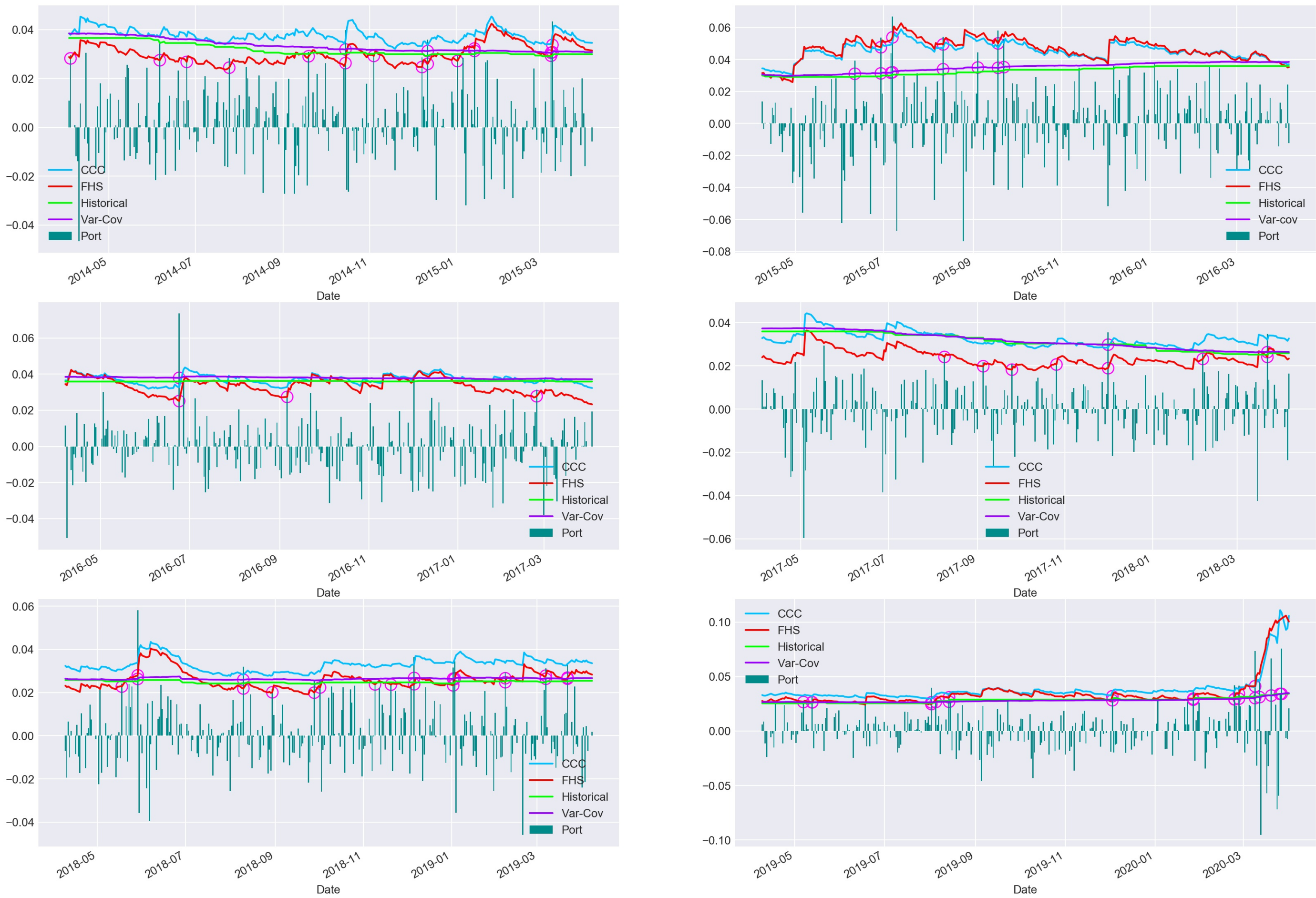


Figure 8: 0.975VaR violation plot of each years for each method. And the pink circle is every violation. The green bar is actual portfolio loss. Time length is six years. From 03/04/2014 – 01/04/2020.

Table 8 shows four methods' VaR violation for each years. We see that at $2015-2016$ and $2019-2020$ the number of violations for four methods are larger than other years, which is because the stressed period discussed before. Historical simulation has the worst performance for our portfolio. We can see the number of violations are quite large and p-value are small for both 0.975 and 0.99 VaR. Thus, this model is not applicable to our portfolio. For Var-Co method, for 0.975VaR the number of violation is close to the expected number 36. While for 0.99VaR, the violations become larger than expected number and p-value become small. Thus, this method is still not ideal for our portfolio. Furthermore, we see that for CCC method the number of violations is always smaller than expected number. But the p-value are always smaller than 0.05, which means the difference is significant and CCC method overestimates our portfolio VaR. The possible explanation is that assumption of VaR is not normal distribution but student-t and the coefficients should estimate every time when we do rolling window. Among these four method the best model for our portfolio is FHS model. Even though the violations are a little bigger than expected number, the p-values show statistically significance, which are 0.056 for 0.975VaR and 0.191 for 0.99VaR. Thus, the FHS method corresponds the assumption and is efficient to evaluate our portfolio VaR.

Table 6: Numbers of violations of the 97.5% and 99% VaR estimate calculated for all four models

Year	2014	2015	2016	2017	2018	2019	ALL	P-value
Trading days	250	250	250	250	250	245	1495	-
Var-0.975								
Expected number	6	6	6	6	6	6	36	-
Var-Co method	5	8	1	2	8	14	38	0.868
Historical	8	14	3	2	10	14	51	0.031
CCC	3	5	1	2	4	8	23	0.016
FHS	13	4	3	7	14	8	49	0.056
Var-0.99								
Expected number	3	3	2	2	3	2	15	-
Var-Co method	2	8	1	2	6	10	29	0.001
Historical	4	8	1	1	6	9	29	0.001
CCC	0	2	1	1	1	2	7	0.036
FHS	6	1	1	2	5	5	20	0.191

Additionally, we compare estimate expected shortfall with actual expected shortfall for four methods. All the method are failed to evaluate ES. For 0.975ES, the CCC and FHS is close to the actual ES. And for 0.99ES only CCC method closes to actual ES.

Table 7: 97.5% and 99% ES estimate calculated for all four models

Year	2014	2015	2016	2017	2018	2019	Estimate mean	True mean
Trading days	250	250	250	250	250	245	-	-
ES-0.975								
Var-Co method	0.029	0.05	0.007	0.011	0.041	0.078	0.036	0.084
Historical	0.048	0.13	0.038	0.016	0.076	0.113	0.070	0.093
CCC	0.02	0.046	0.006	0.012	0.024	0.057	0.027	0.035
FHS	0.057	0.03	0.024	0.053	0.106	0.062	0.055	0.061
ES-0.99								
Var-Co method	0.011	0.045	0.006	0.01	0.028	0.05	0.025	0.053
Historical	0	0.006	0.002	0.003	0.006	0.015	0.005	0.054
CCC	0	0.017	0.006	0.005	0.006	0.012	0.007	0.015
FHS	0.004	0	0	0.008	0.014	0.005	0.005	0.029

3.5 Stress testing

At previous part, we compare VaR with and without stress period. In this part, we do another stress testing. This stress testing is shocking the risk factors in our portfolio and then compare the portfolio value without shock. There are three scenarios in stress testing. Firstly, stock prices or index values increase or decrease 20%, 40%. Secondly, currencies increase or decrease 10%,20%. And bonds' interest rate increases or decreases 2%, 3%.

Table 8 shows the result of portfolio value change before and after shock. Since our weight of stock is largest, the shock of stock price brings significant change in portfolio value. When stock prices increase 20% and 40%. We gain 2875742 and 5751485 Euro respectively when comparing the portfolio value before shock.

While it is intuitive to get the conclusion when stock prices decrease significantly our loss will increase, which are 2875743 and 5751486 Euro respectively.

Shock in currencies brings less significant change in portfolio value when compared with shock in stock price. When major currency increases 10%, the portfolio value increases 470814 Euro. And when foreign currency increases 20%, the portfolio increases 941627 Euro. And we lose more 470815 and 941627 Euro when major currency and foreign currency drop 10% and 20% respectively.

Portfolio values show smallest change when we apply the shock into bonds interest rate. When bonds' interest rate increases 2% and 3%, we gain 98017 and 146296 Euro more. And when bonds' interest rate decreases 2% and 3%, we lose 99998 and 150752 Euro more.

Table 8: Stress testing for different scenarios

Stress testing scenarios	Portfolio change	Stress testing scenarios	Portfolio change
Stock price +20%	+2875742	Stock price -20%	-2875743
Stock price +40%	+5751485	Stock price -40%	-5751486
Major currency +10%	+470814	Major currency -10%	-470815
Foreign currency +20%	+941627	Foreign currency -20%	-941627
Interest rate +2%	+98017	Interest rate -2%	-99998
Interest rate +3%	+146296	Interest rate -3%	-150752

4 Conclusion

In this report, we use four models to evaluate portfolio VaR and ES. For Var-Cov model and historical model, stress period increases the VaR and ES. And the assumption of distribution should be student-t with 5 degree. For CCC and FHS, it shows dynamic VaR. Backtest shows FHS is the best for our portfolio to predict VaR. And all four models are failled to predict ES. Stress testing shows stock prices' fluctuation greatly influences portfolio value.