semi_definite_program

May 19, 2024

[1]:
```
import cvxpy as cvx

import matplotlib.pyplot as plt

import numpy as np

def generate_ellipsoid_points(M, num_points=100):
    """Generate points on a 2-D ellipsoid.

    The ellipsoid is described by the equation
    `{ x | x.T @ inv(M) @ x <= 1 }`,
    where `inv(M)` denotes the inverse of the matrix argument `M`.

    The returned array has shape (num_points, 2).
    """
    L = np.linalg.cholesky(M)
      = np.linspace(0, 2*np.pi, num_points)
    u = np.column_stack([np.cos( ), np.sin( )])
    x = u @ L.T
    return x
```

[2]:
```
A = np.array([[0.9, 0.6],[0, 0.8]])
B = np.array([[0],[1]])
r_x = 5
r_u = 1
Q = np.eye(2)
R = np.eye(1)
P = np.eye(2)
N = 4
A, B
```

[2]:
```
(array([[0.9, 0.6],
        [0. , 0.8]]),
 array([[0],
        [1]]))
```

[3]:
```python
M = cvx.Variable((2,2),symmetric=True)

constraints = []

constraints += [M<<r_x**2 * np.eye(2)]

constraints += [cvx.bmat([[M, A@M],[M@A.T, M]])>>0]

prob = cvx.Problem(
    cvx.Maximize(cvx.log_det(M)),
    constraints
)
```

[4]:
```python
prob.solve()
```

[4]:
```
4.832662985874696
```

[5]:
```python
M.value
```

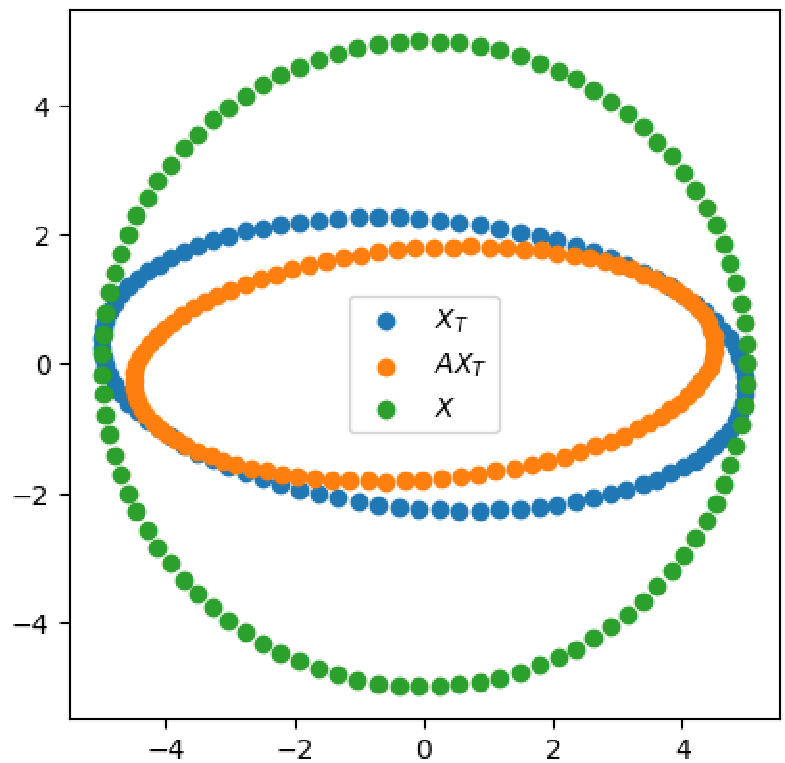
[5]:
```
array([[24.86408695, -1.64219805],
       [-1.64219805,  5.15770616]])
```

[6]:
```python
np.linalg.inv(M.value)
```

[6]:
```
array([[0.04108258, 0.01308057],
       [0.01308057, 0.19804945]])
```

[7]:
```python
ellipsoid_XT = generate_ellipsoid_points(M.value)
ellipsoid_A_XT = np.array([A@x for x in ellipsoid_XT])

  = np.linspace(0, 2*np.pi, 100)
ellipsoid_X = r_x*np.column_stack([np.cos( ), np.sin( )])
```

[8]:
```python
fig, ax = plt.subplots()
ax.scatter(ellipsoid_XT[:,0],ellipsoid_XT[:,1], label='$X_T$')
ax.scatter(ellipsoid_A_XT[:,0],ellipsoid_A_XT[:,1], label='$AX_T$')
# ax.scatter(np.array([A@x for x in ellipsoid_A_XT])[:,0],np.array([A@x for x in ellipsoid_A_XT])[:,1], label='$AAX_T$')
ax.scatter(ellipsoid_X[:,0],ellipsoid_X[:,1], label='$X$')
ax.legend()
ax.set_aspect('equal')
```



0.1 The MPC Problem

[9]:
```
x0 = np.array([[0],[-4.5]])
x0_cvx = cvx.Parameter((2,1), value=x0)

n, m = Q.shape[0], R.shape[0]

W = np.linalg.inv(M.value)

# Form the CVX Problem
cost = 0.0
constraints = []

x_cvx = cvx.Variable((N + 1, n))
u_cvx = cvx.Variable((N, m))

# Define Cost
cost += cvx.QuadForm(x_cvx[-1,:], P)
for i in range(N):
    cost += cvx.QuadForm(x_cvx[i,:], Q)
    cost += cvx.QuadForm(u_cvx[i,:], R)
```

```
# Define Constraints
constraints += [cvx.QuadForm(x_cvx[-1,:], W)<=1]  # Contraint x.T * M^-1 * x < 1
constraints += [cvx.norm(x_cvx[i,:], 2)<=r_x for i in range(N)]
constraints += [cvx.norm(u_cvx[i,:], 2)<=r_u for i in range(N)]
constraints += [ x_cvx[i+1,:] == A @ x_cvx[i,:] + B @ u_cvx[i,:] for i in␣
↪range(N)]
constraints += [x_cvx[0,:]==x0_cvx[:,0]]


prob = cvx.Problem(cvx.Minimize(cost), constraints)
prob.solve()
```

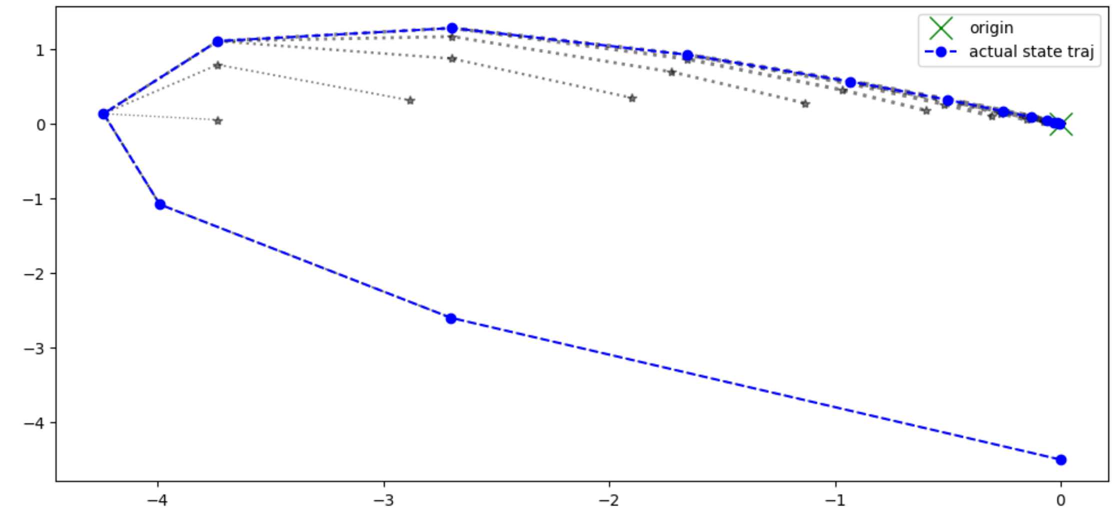
```
/home/qdeng/.pyenv/versions/3.12.1/envs/AA203/lib/python3.12/site-
packages/cvxpy/reductions/solvers/solving_chain.py:336: FutureWarning:
    Your problem is being solved with the ECOS solver by default. Starting in
    CVXPY 1.5.0, Clarabel will be used as the default solver instead. To
continue
    using ECOS, specify the ECOS solver explicitly using the ``solver=cp.ECOS``
    argument to the ``problem.solve`` method.

  warnings.warn(ECOS_DEPRECATION_MSG, FutureWarning)
```

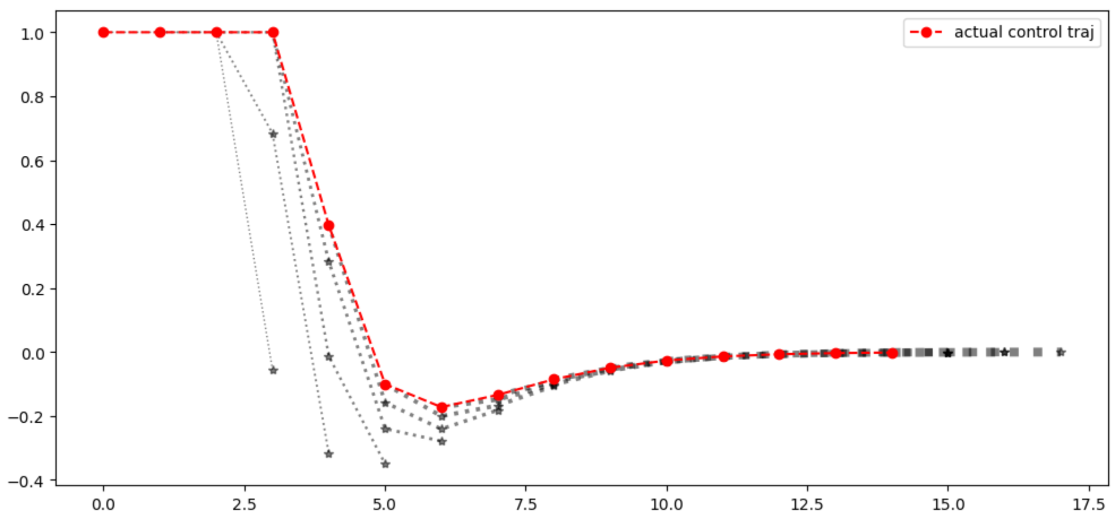
[9]: 86.31905795612

[11]:
```
T = 15
x0_cvx.value = x0
x_mpc = np.zeros((T, N + 1, n))
u_mpc = np.zeros((T, N, m))

fig, ax = plt.subplots(2,1, figsize=(12, 12))

for t in range(T):
    prob.solve()

    x_mpc[t] = x_cvx.value
    u_mpc[t] = u_cvx.value

    # set the parameter value to where the state is at the next time instance
    x0_cvx.value = A @ x0_cvx.value + B * u_mpc[t, 0, :]

    # Plot out the optimized trajectory for the horizon with length N
    ax[0].plot(x_mpc[t,:,0], x_mpc[t,:,1], 'k*:', linewidth=t*1/3+1, alpha=1/2)
    ax[1].plot(t+np.arange(0,N), u_mpc[t,:,0], 'k*:', linewidth=t*1/3+1,␣
↪alpha=1/2)

ax[0].plot(0,0,'gx',label='origin',markersize=15)
ax[0].plot(x_mpc[:,0,0], x_mpc[:,0,1], 'bo--', label='actual state traj')
```

```
ax[0].legend()


ax[1].plot(u_mpc[:,0,0], 'ro--', label='actual control traj')
ax[1].legend()
```

[11]: <matplotlib.legend.Legend at 0x7d8a5da37500>





[ ]: