# Value_Iter

May 19, 2024

[1]:
```
import numpy as np
import jax
import jax.numpy as jnp
import matplotlib.pyplot as plt
import seaborn as sns
```

[2]:
```
n = 20
nx = n
ny = n
  = 10   # spread of storm
  = 0.95 # Discount Factor
x_eye = np.array([15, 14]) #(y, x)

x_goal = np.array([9, 19]) #(y, x)



grid = np.zeros((ny, nx))

  = lambda x:  np.exp(-np.linalg.norm(np.array(x) - x_eye, 2)**2 / (2* **2))
 _grid = grid
for y in range(grid.shape[0]):
    for x in range(grid.shape[1]):
         _grid[y,x]  =  ([y,x])
 _grid =  _grid # the indices need to be transposed because moving to the right␣
↪is actually moving through columns
```

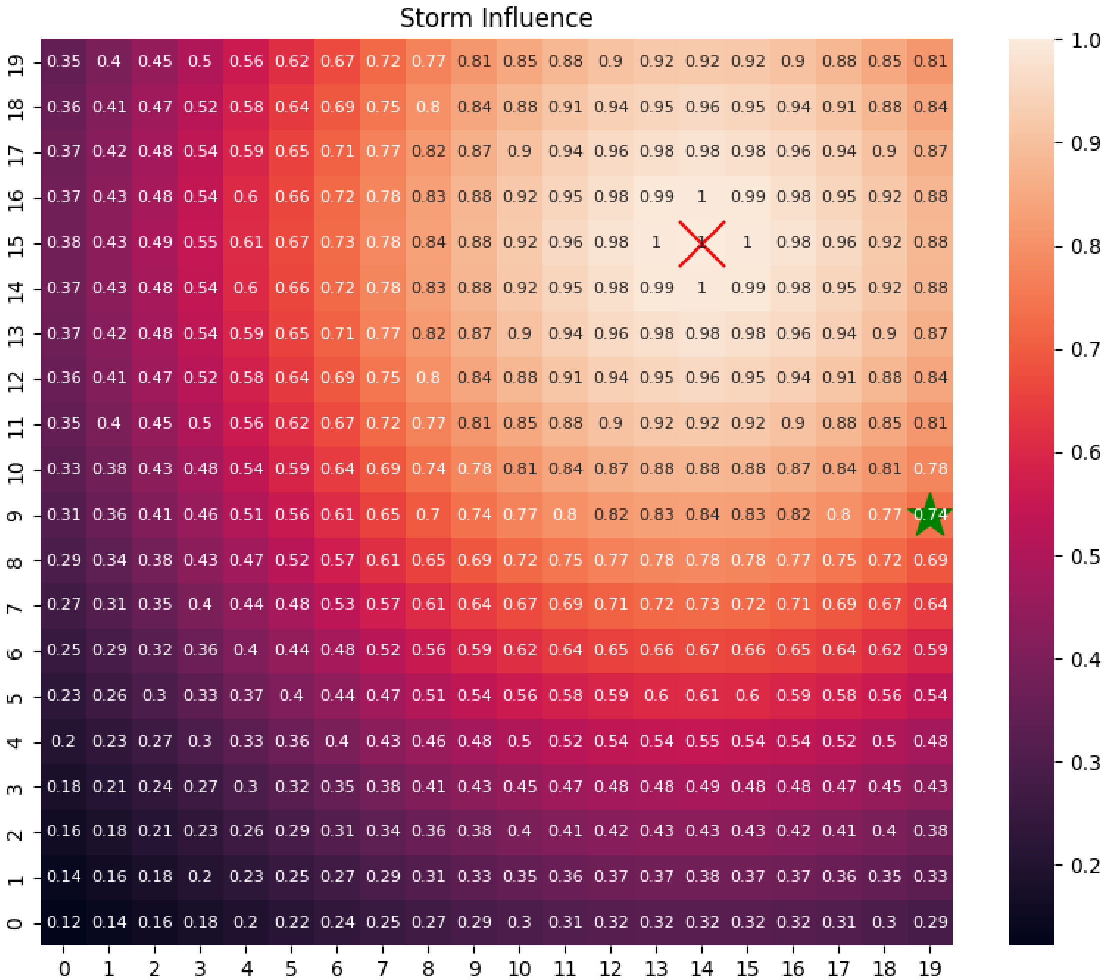
[3]:
```
 _grid[9, 19]
```

[3]: 0.7371233743916278

[4]:
```
fig, ax = plt.subplots(figsize=(10,8))
sns.heatmap( _grid, annot=True, annot_kws={'fontsize':8}, ax=ax)
ax.scatter(x_goal[1]+0.5,x_goal[0]+0.5,s=500,c='g', marker='*', label='Goal')
ax.scatter(x_eye[1]+0.5,x_eye[0]+0.5,s=500,c='r', marker='x', label='Storm Eye')
ax.invert_yaxis()
ax.set_title('Storm Influence')
```

```
# ax.legend()
```

[4]: Text(0.5, 1.0, 'Storm Influence')




[5]:
```
# define actions
act = {
    3: np.array([0, 1]),     #  right (y, x)
    0: np.array([1, 0]),     #  up (y, x)
    2: np.array([0, -1]),    #  left (y, x)
    1: np.array([-1, 0])     #  down (y, x)
}

def action_dynamics(x:np.array, a:int):
    # x: (y, x)

    if x[0] >= ny or x[1] >= nx:
```

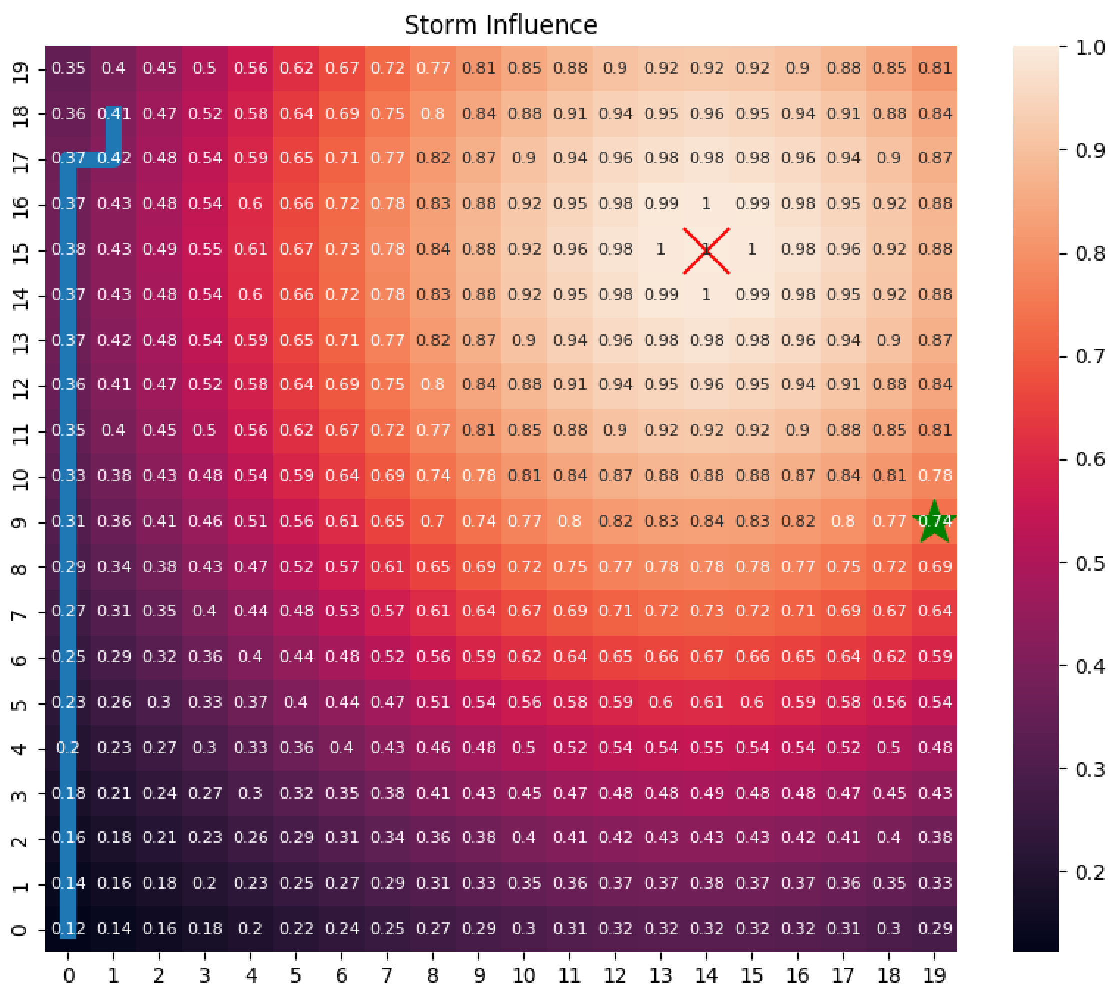
```python
        print(x)
        raise ValueError('Outside of Boundary')

    x_next = x + act[a]

    if x_next[0] < 0:
        # print('edge')
        x_next[0] = 0
    elif x_next[0] > ny-1:
        # print('edge')
        x_next[0] = ny-1

    if x_next[1] < 0:
        # print('edge')
        x_next[1] = 0
    elif x_next[1] > nx-1:
        # print('edge')
        x_next[1] = nx-1

    return x_next

# define dynamic rule for each step
def dynamics(x:np.array, a:int,  _grid:np.array, n:int):
    prob = np.random.rand(1)

    # print(x)
    if prob <  _grid[*x]:
        # print('storm')
        a = np.random.randint(0, 4,)

    x_next = action_dynamics(x, a)


    return x_next

# store value iter as 3d vector
```

[8]:

```python
x0 = np.array([0, 0])
list_a = [0]*30

x_trj = np.zeros((len(list_a)+1, x0.shape[0]), dtype=int)
x_trj[0] = x0
for i, a in enumerate(list_a):
    x_trj[i+1,:] = dynamics(x_trj[i,:], 0,  _grid, n)
# x_trj
```

[9]:
```
fig, ax = plt.subplots(figsize=(10,8))
sns.heatmap( _grid, annot=True, annot_kws={'fontsize':8}, ax=ax)
ax.scatter(x_goal[1]+0.5,x_goal[0]+0.5,s=500,c='g', marker='*', label='Goal')
ax.scatter(x_eye[1]+0.5,x_eye[0]+0.5,s=500,c='r', marker='x', label='Storm Eye')
ax.plot(x_trj[:,1]+0.5, x_trj[:,0]+0.5, linewidth=8)
ax.invert_yaxis()
ax.set_title('Storm Influence')
# ax.legend()
```

[9]: Text(0.5, 1.0, 'Storm Influence')

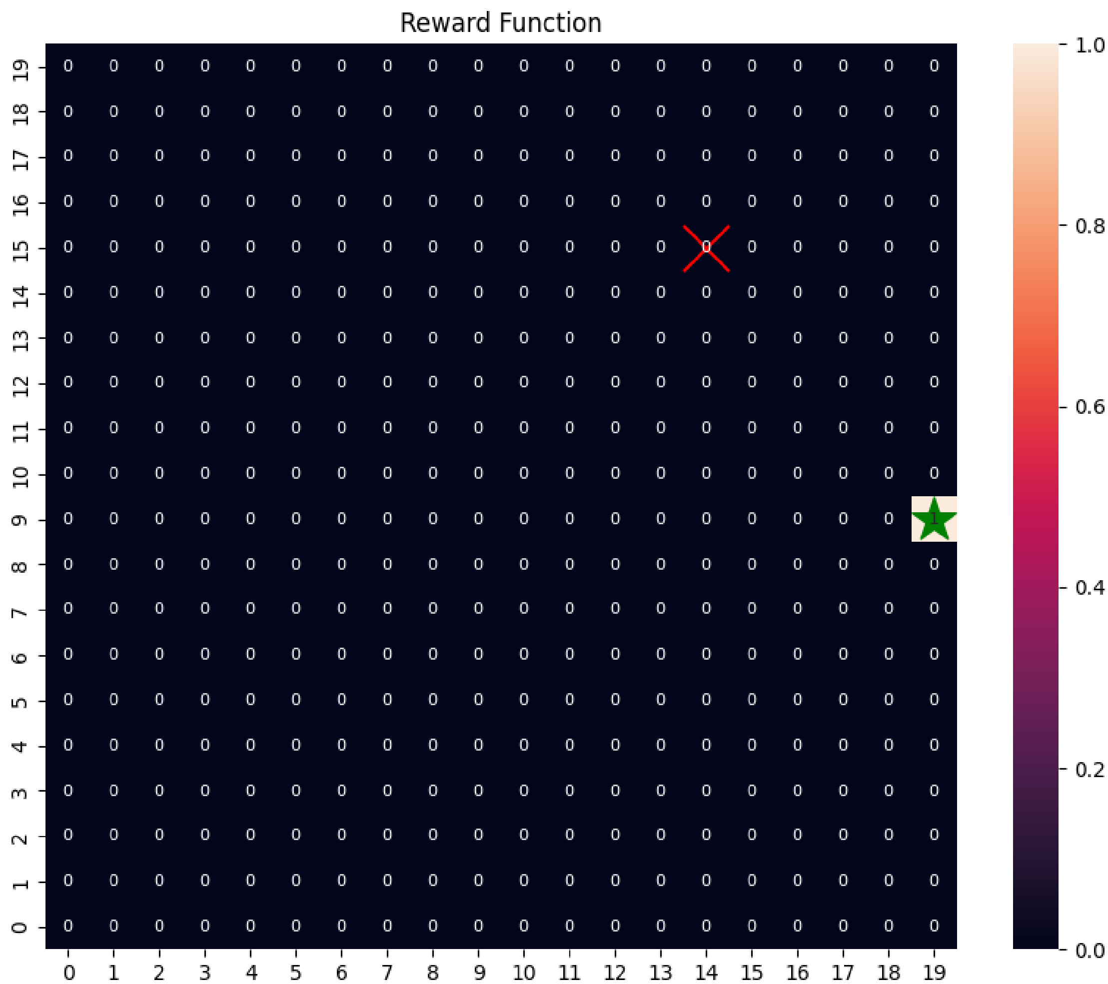



[10]:
```
# _grid[0, 20]
```

[11]:
```
R_grid = np.zeros((ny, nx))
R_grid[*x_goal] = 1
```

```python
# R_grid = R_grid.T # the indices need to be transposed because moving to the
↪right is actually moving through columns
fig, ax = plt.subplots(figsize=(10,8))
sns.heatmap(R_grid, annot=True, annot_kws={'fontsize':8}, ax=ax)
ax.scatter(x_goal[1]+0.5,x_goal[0]+0.5,s=500,c='g', marker='*', label='Goal')
ax.scatter(x_eye[1]+0.5,x_eye[0]+0.5,s=500,c='r', marker='x', label='Storm Eye')
# ax.plot(x_trj[:,0]+0.5, x_trj[:,1]+0.5, linewidth=8)
ax.invert_yaxis()
ax.set_title('Reward Function')
```

[11]: Text(0.5, 1.0, 'Reward Function')

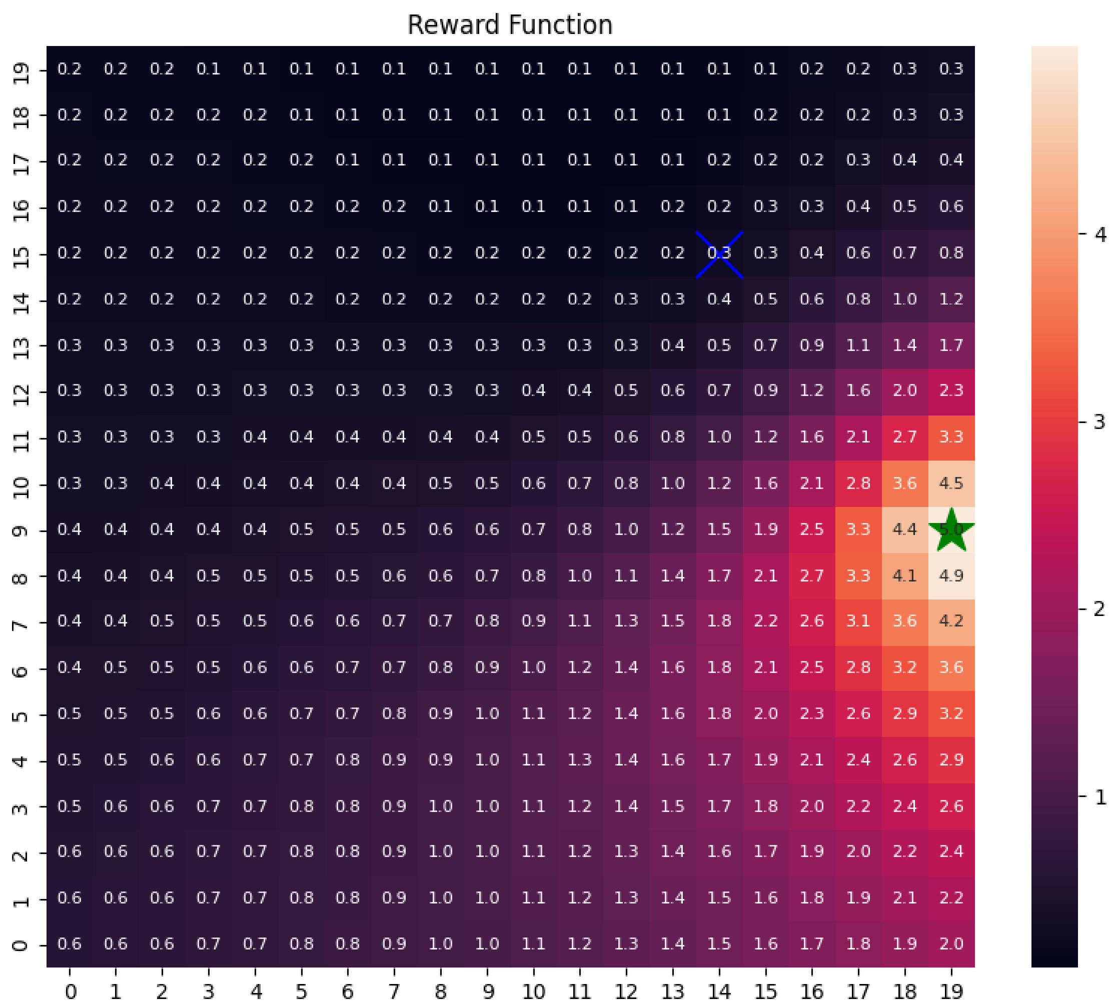



[12]:
```python
V_grid = np.zeros(R_grid.shape)
V_grid_next = np.zeros(R_grid.shape)

num_iter = 500
```

```python
for _ in range(num_iter):

    V_grid = V_grid_next.copy()

    V_grid_next = np.zeros(R_grid.shape)

    # One iteration of grid update
    for y in range(V_grid.shape[0]):
        for x in range(V_grid.shape[1]):

            list_V = []

            x_neighbors = [action_dynamics(np.array([y, x]), a) for a in act]

            for a in act:

                # update for center locations
                x_next = action_dynamics(np.array([y, x]), a)

                R_plus_V = R_grid[*x_next] +   * V_grid[*x_next]

                prob = (1 -  _grid[y, x])

                sum_val = prob * R_plus_V

                # Account for random movement values
                for x_ngbr in x_neighbors:
                    R_plus_V = R_grid[*x_ngbr] +   * V_grid[*x_ngbr]

                    # equal probability for each of the neighbors based on the current position
                    prob =  _grid[y, x] / 4

                    sum_val += prob * R_plus_V


                list_V.append(sum_val)
                # print(sum_val)
                # break
            V_grid_next[y, x] = max(list_V)

    if np.max(np.abs(V_grid_next - V_grid)) < 1e-6:
        break
```

```python
fig, ax = plt.subplots(figsize=(10,8))
sns.heatmap(V_grid_next, annot=True, annot_kws={'fontsize':8}, ax=ax, fmt='.1f')
ax.scatter(x_goal[1]+0.5,x_goal[0]+0.5,s=500,c='g', marker='*', label='Goal')
ax.scatter(x_eye[1]+0.5,x_eye[0]+0.5,s=500,c='b', marker='x', label='Storm Eye')
# ax.plot(x_trj[:,0]+0.5, x_trj[:,1]+0.5, linewidth=8)
ax.invert_yaxis()
ax.set_title('Reward Function')
```

[12]: Text(0.5, 1.0, 'Reward Function')

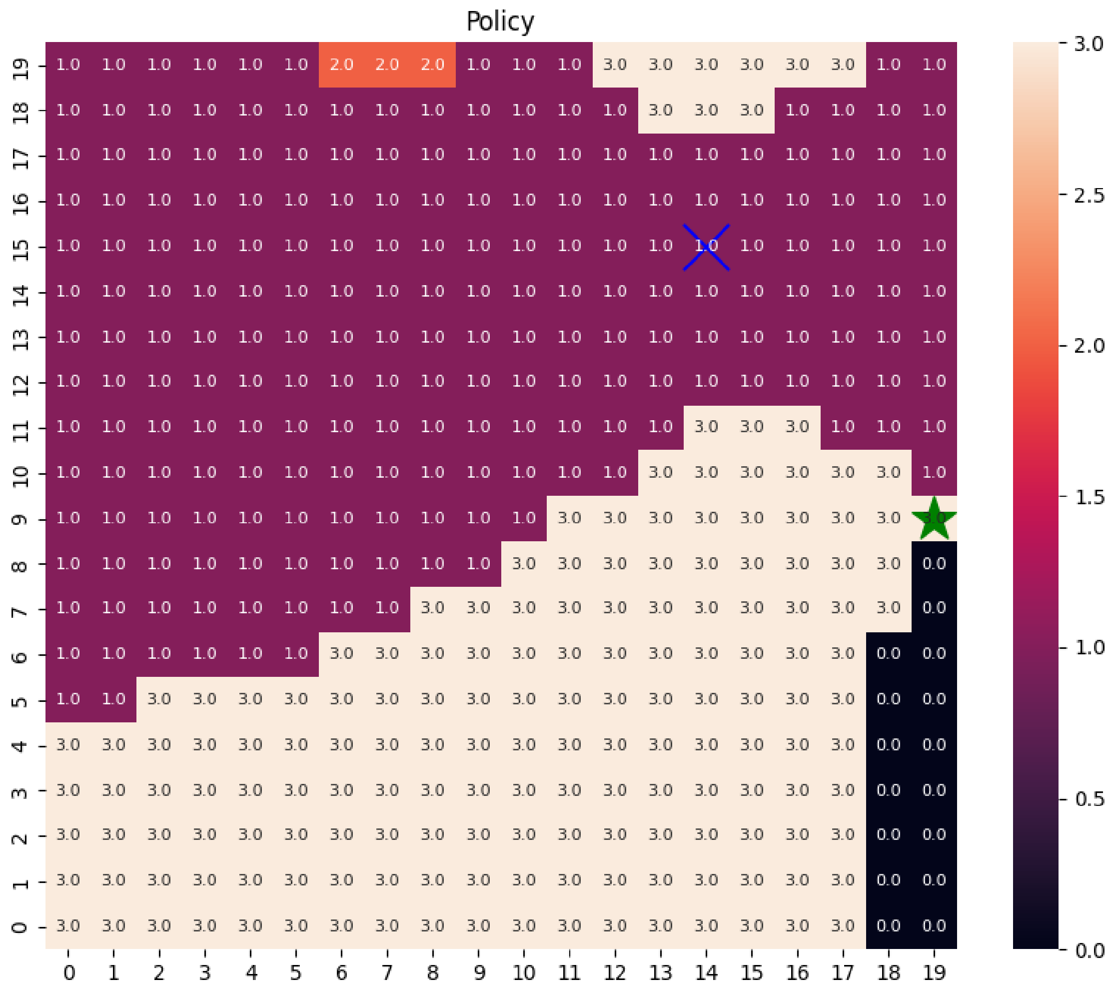



[13]:
```python
# Find Optimal Policy based on Value
act_grid = np.ones(R_grid.shape) * -1

for i in range(V_grid.shape[0]):
    for j in range(V_grid.shape[1]):
        # i = 9
```

```
        # j = 15

        a_x_pair = [(a, action_dynamics(np.array([i, j]), a)) for a in act]

        list_a = []
        list_v_diff = []
        for a, x_next in a_x_pair:
            # print(a, x_next)
            V_x = V_grid[i, j]

            V_x_next = V_grid[*x_next]

            list_v_diff.append(V_x_next-V_x)
            list_a.append(a)

        act_grid[i, j] = list_a[np.argmax(list_v_diff)]
        # print(act_grid[i, j], list_v_diff)
    #     break
    # break
    # act_grid[i][j] = np.where(list_v_diff == np.max(list_v_diff))[0]
    # print(np.where(list_v_diff == np.max(list_v_diff))[0])


# act_grid
```

[14]:
```
fig, ax = plt.subplots(figsize=(10,8))
sns.heatmap(act_grid, annot=True, annot_kws={'fontsize':8}, ax=ax, fmt='.1f')
ax.scatter(x_goal[1]+0.5,x_goal[0]+0.5,s=500,c='g', marker='*', label='Goal')
ax.scatter(x_eye[1]+0.5,x_eye[0]+0.5,s=500,c='b', marker='x', label='Storm Eye')
# ax.plot(x_trj[:,0]+0.5, x_trj[:,1]+0.5, linewidth=8)
ax.invert_yaxis()
ax.set_title('Policy')
```

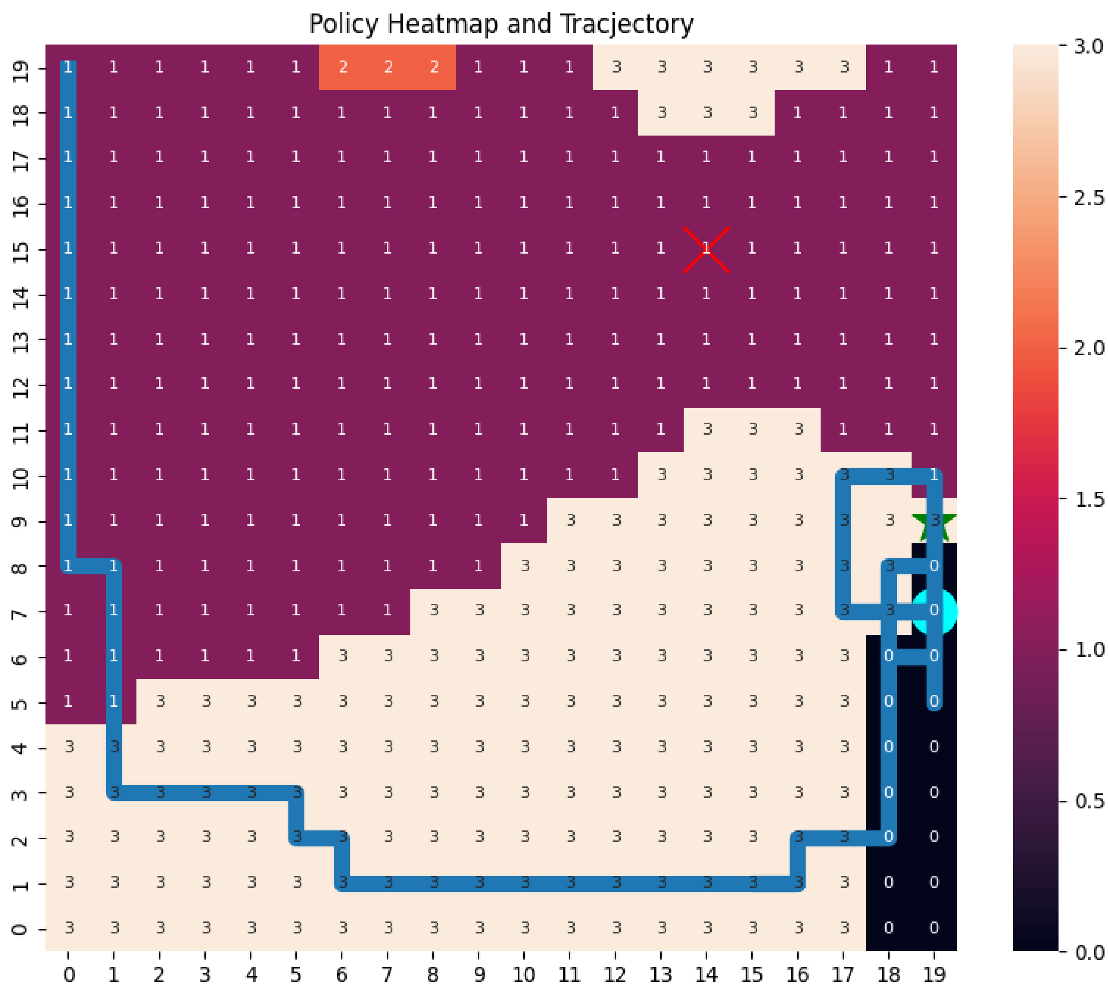
[14]: Text(0.5, 1.0, 'Policy')




```
[17]: x0 = np.array([19, 0]) #y, x

num_iter = 100

x_trj = np.zeros((num_iter+1, x0.shape[0]), dtype=int)
x_trj[0] = x0

for i in range(num_iter):
    a = int(act_grid[*x_trj[i,:]])
    x_trj[i+1,:] = dynamics(x_trj[i,:], a,  _grid, n)

fig, ax = plt.subplots(figsize=(10,8))
sns.heatmap(act_grid, annot=True, annot_kws={'fontsize':8}, ax=ax)
ax.scatter(x_goal[1]+0.5,x_goal[0]+0.5,s=500,c='g', marker='*', label='Goal')
ax.scatter(x_eye[1]+0.5,x_eye[0]+0.5,s=500,c='r', marker='x', label='Storm Eye')
```

```
ax.scatter(x_trj[-1,1]+0.5, x_trj[-1,0]+0.5,s=500,c='cyan', marker='o',␣
 ↪label='Final Destination')
ax.plot(x_trj[:,1]+0.5, x_trj[:,0]+0.5, linewidth=8)
ax.invert_yaxis()
ax.set_title('Policy Heatmap and Tracjectory')
```

[17]: Text(0.5, 1.0, 'Policy Heatmap and Tracjectory')




The policy can be largely divided into three sections.

In the bottom right corner, we are directly below our goal position. The policy simply drives our position upwards until we hit the destination.

In the bottom half of the grid, the policy tells us to keep going right – such that we are either hit right into the goal position, or just go up or down along the grid border until we hit the goal state.

In the top half of the grid, the policy tells us to keep doing down until we reach the region where the optimal policy is to go right until we hit the right edge

[ ]:

```

```