Capacitors and Humidity Measurement

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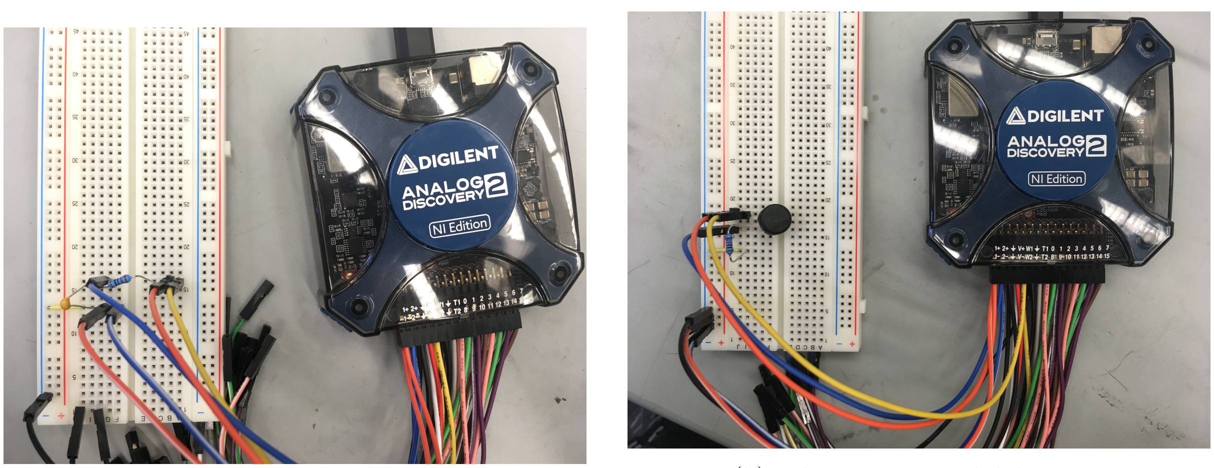
Abstract

The purpose of this lab is to develop intuitions about how RC circuits would react to low and high frequency inputs, observe different behaviors of the RC circuits with different capacitance, and, ultimately, take measurement of the relative humidity by applying the relationship found in the earlier steps of the experiment.

1 Description

In this lab, several RC, or resistor-capacitor, circuits were built to help understand their behaviors, and a humidity sensor that utilizes the idea behind the RC circuit was also constructed in the end to measure the relative humidity of the room. Two general circuit structures were used. Throughout the lab, the yellow wire W1 remained the source of square waves, Channel 1 ± measure the voltage across the whole circuit, and Channel2 ± kept tracked of the voltage drop across the second component in the circuit. However, the first setup is a RC circuit where the resistor preceded the capacitors in the first setup; the second, a CR circuit where the capacitors preceded the resistors. The varying frequencies of the square waves were achieved through WaveForms and Analog Discovery, and some of the measurement taken included voltage drops, amplitudes, and root-mean square amplitudes.



(a) The first general lab setup where the resistor preceded the capacitor.

(b) The two general lab setup where the capacitor preceded the resistor. The ”capacitor” in the picture is the humidity.

Figure 1: The RC circuit setup.

2 Evidence

In the lab, six sets of data from two parts of the experiment were taken to understand the behaviors behind RC circuits. The first circuit built was $1k\Omega$ resistor in series with a 0.1 $\mu\Omega$ capacitor. The voltage measurement was taken across the capacitor while a $1V$ square wave was sent into the circuit at 500Hz.

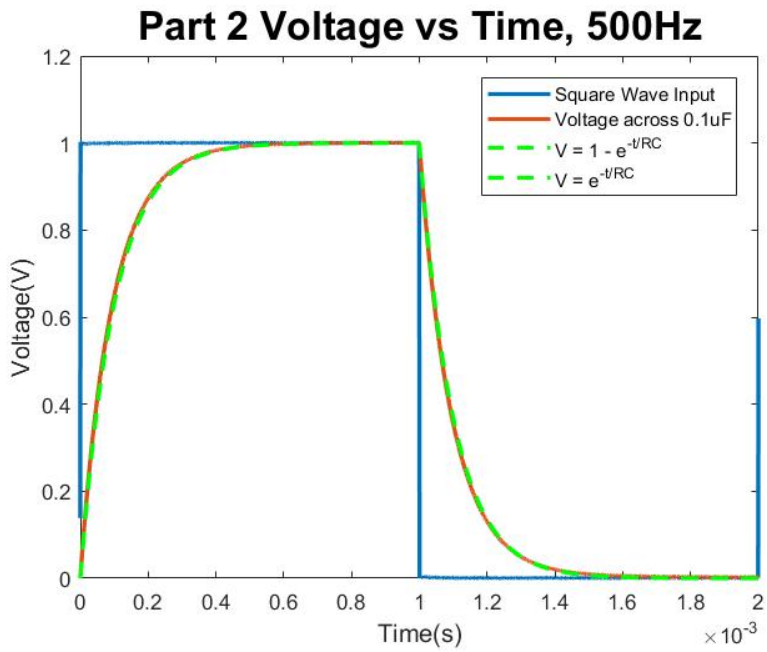


Figure 2: One period of 1 Volt square wave driving of a RC circuit at 500Hz.

The green dash line in the figure a mathematical description of the charging and discharging of a RC circuit based on the equation $V = e^{\frac{-t}{RC}}$, which is derived by combining Ohm's Law and Capacitor Law. While the circuit remain unchanged, the driving frequency of the square wave was raised to 1500Hz, and Figure 3 on the next page showed the result.

Compared to Figure 2, Figure 3 has a significant change in shape as the voltage never reached 1 V whereas it did in the former. In other words, the change can also be described as a change in amplitude of the output voltage. To explore this change further, 9 measurements of the output amplitudes were taken at 9 different frequencies.

Table 1: Output Voltage Amplitude vs Square Wave Frequency

Frequency(kHz)	.5	1	2	5	10	20	50	100	200
Amplitude(mV)	497.8	486.9	424.8	235.4	116.3	64.9	18.9	10.8	0

The recorded data is then plotted in log scale for both x and y axes in Figure 4.

At this point, all the data collection is done with the RC circuit, and the next part of the lab began with the CR circuit whose the voltage across the resistor was measured. The square wave frequency was reverted back to 500Hz. The resulting measurement is plotted in Figure 5.

When the capacitor was changed from $0.1\mu F$ to $0.047\mu F$, the corresponding amplitude root mean square measurement also decreased. Such a behavior can be used to measure the relative humidity with a humidity sensor which has a variable capacitance once a calibration curve is fitted

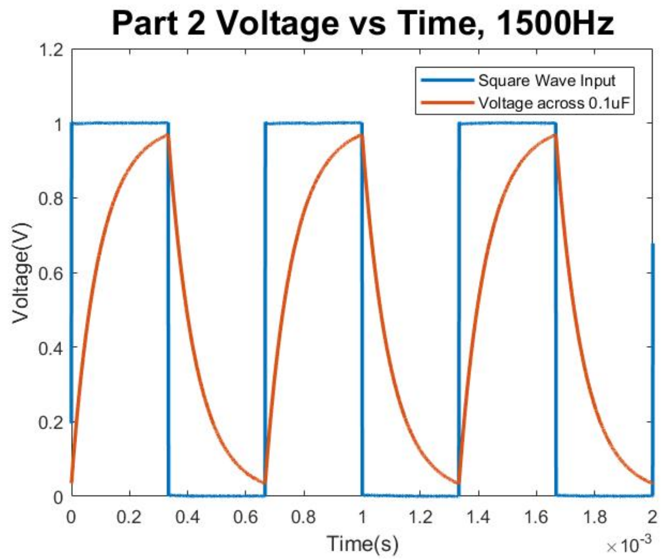


Figure 3: One period of 1 Volt square wave driving of the RC circuit at 1500Hz.

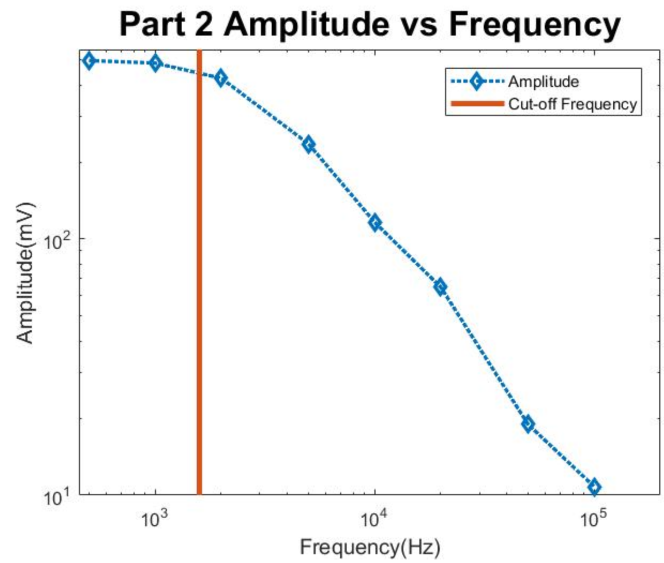


Figure 4: The measured amplitude across the capacitor in the RC circuit over a wide range of square wave frequencies.

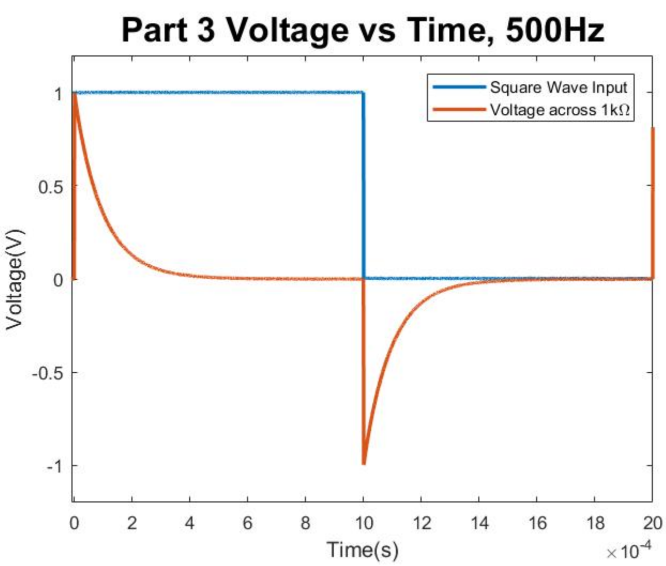


Figure 5: One period of 1 Volt square wave driving of the CR circuit at 500Hz.

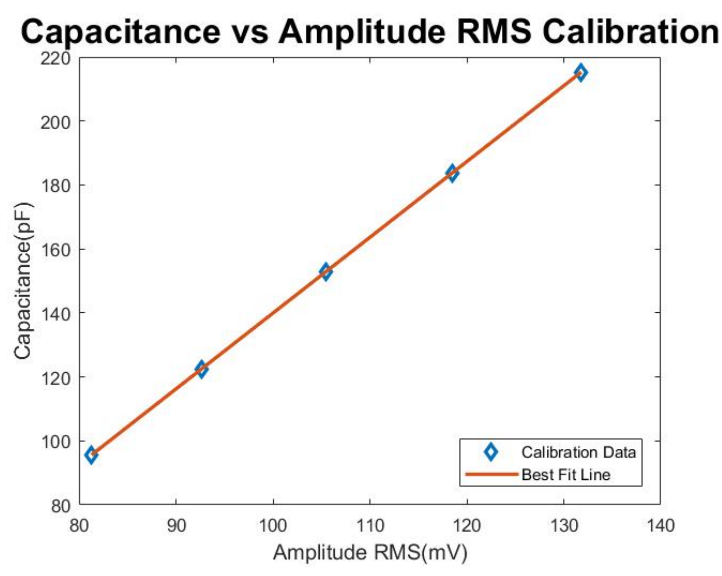


Figure 6: The amplitude root mean square(on x-axis) collected at different capacitance(on y-axis) for calibration and the best fit calibration line.

with the circuit design. In order to cater to the design of the humidity sensor, the 1 $k\Omega$ resitor was replaced with a 10 $k\Omega$ resistor. The data collected are as follows.

Table 2: Amplitude RMS vs Capacitance Calibration

Capacitance(pF)	100	120	150	180	220
Amplitude RMS(mV)	81.3	92.6	105.5	118.5	131.8

Based on the calculation, the calibration curve can be described with the following equation with $R^2 = .9923$:

$$Capacitance = 2.3696 \times AmpRMS - 97.032 \tag{1}$$

The amplitude RMS taken for the humidity sensor during the lab was **115.8mV**, which can be converted to relative humanity at the time with the formula provided in the datasheet.

3 Interpretation

3.1 The RC Circuit

In Figure 2, the charging and discharging behaviors of the RC circuit align almost perfectly with the exponential increase and decrease that were expected from the mathematical equations:

$$Charging : V_{out} = 1 - e^{\frac{-t}{RC}} \tag{2}$$

$$Discharging : V_{out} = e^{\frac{10^{-3}-t}{RC}} \tag{3}$$

As the frequency tripled from 500Hz to 1500Hz, the resulting plot over the same time interval has three periods of the charging and discharging instead of one. But the charging and discharging were obviously not as complete as under 500Hz square waving pushing. According to the definition, the time constant of a RC circuit is defined as:

$$\tau = RC = \frac{1}{2\pi f_c} \tag{4}$$

where f_c is the cut-off frequency of the circuit. It's also the point of intersection of $A = 1$ and $A = \frac{1}{RC\omega}$ on a logarithmic Bode plot. Whether it cuts off high values or low values depends on how the circuit is set up. As in this case where the resistor was directly connected to the input voltage, the capacitor was connected to the ground, and we are measuring the voltage across the capacitor, this circuit was a low pass filter which would significantly attenuate frequencies higher than the cut-off frequency. To find the cut-off frequency of this particular RC circuit

$$f_c = \frac{1}{2\pi\tau} = \frac{1}{2\pi \times 1000\Omega \times 0.1\mu F} = 1591.55Hz \tag{5}$$

This indicates the RC circuit we set up for this part of the lab will noticeably smooth out frequency input higher than 1591.55Hz. Since a 1500Hz square wave, which was very close to 1591.55Hz, was driving the circuit, it could be seen clearly that the voltage across the capacitor did not charge to 1V or discharge to 0V fully like how the measured voltage was like with the 500Hz square wave.

When the frequencies were tuned up from 500Hz to 200kHz as shown in Figure 4, before reaching the cut-off frequency 1951.55Hz, the amplitude remains a constant horizontal line. Then, the measurement seems to drop drastically in a decreasing linear relationship all the way to nearly 0 mV at 200kHz.

3.2 The CR Circuit and Humidity Sensor

With the values of the 1 $k\omega$ resistor and 0.1 μF capacitor unchanged, the CR circuit was constructed by swapping the positions of the two component. The square wave was set back to 1V, 500Hz, and measurement was taken across the resistor and plotted in Figure 5. The resulting figure looks quite different from Figure 2 in that it even has negative voltage readings. However, by subtracting the voltage across the resistor from the square wave input, the same plot as Figure 2 is obtained because of the already stated reason that the voltage across the resistor instead of the capacitor was measured.

As mentioned before, varying the capacitance in the circuit from 0.1 μF to 0.047 μF significantly reduced the amplitude RMS. The humidity sensor takes advantage of this phenomena by changing its measured amplitude RMS according to the relative humidity in the surrounding. The measurement at the time was **158 mV**. With the calibration equation, the capacitance of the humidity sensor can be inferred to be:

$$Capacitance = 2.3696 \times 115.8 - 97.032 = 177.37pF \tag{6}$$

According to the datasheet of the humidity sensor, the relative humidity can be calculated through the following equations:

$$RH(\%) = -3.4656 \times 10^3 x^3 + 1.0732 \times 10^4 x^2 - 1.0457 \times 10^4 x + 3.2459 \times 10^3 \tag{7}$$

where x is defined as:

$$x = \frac{C_{read}}{180pF} = 0.985376 \tag{8}$$

which then gives the relative humidity:

$$RH = 46.46\% \tag{9}$$

In contrast with the calculated result, the relative humidity of the day that I recorded online was 37%. Nevertheless, that does not necessarily mean the humidity sensor was inaccurate. The online weather information only provide an average over a large area when the relative humidity at a particular location can differ. The facts that we were indoors rather than outdoors, that we were on the 4th floor rather than the 1st floor, and that we were in an electrical lab rather than a wood shop, are all relevant factors that can affect the actual relative humidity in the room. Still, the viability of using the humidity sensor by converting a voltage value first to capacitance and then to relative humidity has been demonstrated.