Coffee Problem

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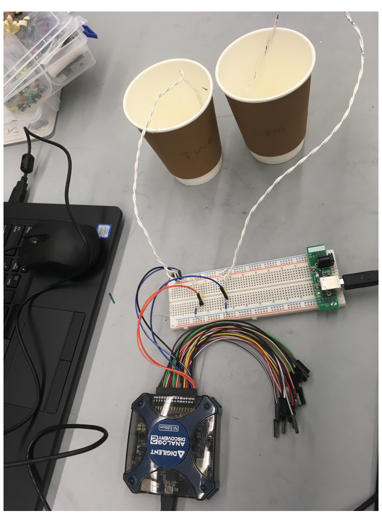
Abstract

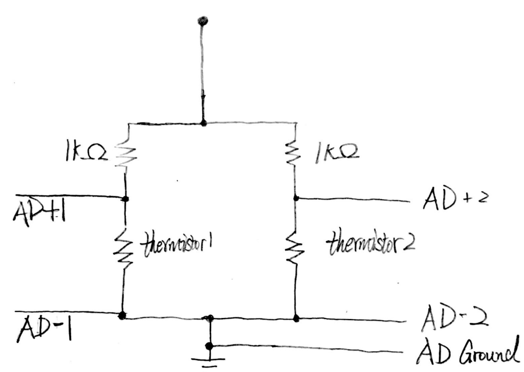
The purpose of this pendulum lab is to determine the best time–right away or some time later–to put cold milk into hot coffee so that the combined coffee is as warm as possible after a short period of time. Also, as a warm-up for this lab, a pre-lab where voltages across different combinations of resistors are measured and the internal resistance of Analog Discovery is also included.

1 Description

The lab began with a pre-lab calculating expected voltages across certain resistors in the circuits that were provided in the lab description and measuring the actual voltages. By taking measurements of three sets of two resistors of the same resistance, the internal resistance of Analog Discovery was also inferred during the second part of the pre-lab.



(a) Real world lab setup



(b) The schematics of the temperature measuring circuit. AD is short for Analog Discovery.

Figure 1: The coffee lab setup.

As shown in Figure 1(a) and 1(b), a circuit was constructed to record the temperature changes in the two coffee cups. In parallel with each other, the two thermistors, essentially temperature sensitive resistors, connected each in series with a 1 $k\Omega$ resistor and formed two voltage dividers that can be measured simultaneously. Channel 1 of Analog Discovery kept track of the voltage variations of Thermistor 1; Channel 2, that of Thermistor 2.

The mathematical equation that desribes the relationship between a thermistor's resistance and temperature is as follows:

$$R = 1000\Omega \times e^{-3528(\frac{1}{298}-\frac{1}{T})} \tag{1}$$

where T is the temperature in Kelvin. To find out about the resistance of the thermistors, another equation is used:

$$V_{out} = V_{in} \times \frac{R}{R + 1000\Omega} \tag{2}$$

where V_{in} is the voltage supplied by the power module, V_{out} is the voltage drop across t, and is the resistance of the thermistor.

2 Evidence

2.1 Resistor in series and parallel

To begin with, the following chart sums the calculations and measured voltages in the warm-up.

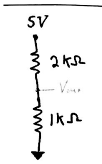
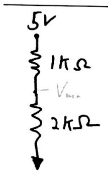
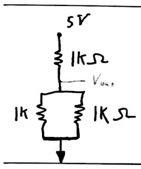
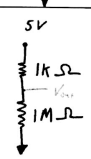
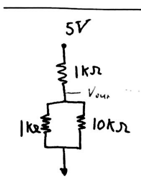
Circuit	Expected voltage (include a short calculation)	Measured voltage
	$V_{out}=V_{in}\times\frac{1k\Omega}{2k\Omega+1k\Omega}$ $=5V\times\frac{1}{3}$ $\approx 1.67V$	$V_{out}=1.677V$
	$V_{out}=V_{in}\times\frac{2k\Omega}{1k\Omega+2k\Omega}$ $=5V\times\frac{2}{3}$ $\approx 3.33V$	$V_{out}=3.367V$
	$R_p=\frac{1}{\frac{1}{1k\Omega}+\frac{1}{1k\Omega}}=.5k\Omega$ $V_{out}=V_{in}\times\frac{.5k\Omega}{.5k\Omega+1k\Omega}$ $=5V\times\frac{1}{3}$ $=1.67V$	$V_{out}=1.676V$
	$V_{out}=V_{in}\times\frac{1M\Omega}{1k\Omega+1M\Omega}$ $=5V\times\frac{1000}{1001}$ $\approx 4.995V$	$V_{out}=5.050V$
	~~Vout~~ $R_p=\frac{1}{\frac{10}{10000}+\frac{1}{10000}}=\frac{1}{\frac{11}{10000}}=\frac{10000}{11}\Omega$ $V_{out}=V_{in}\times\frac{\frac{10000}{11}\Omega}{1k\Omega+\frac{10000}{11}\Omega}$ $=5V\times\frac{909.1}{1909.1}$ $=2.38V$	$V_{out}=2.401V$

Figure 2: Voltage measurements of different resistor setups. The calculation assumed V_{in} is $5V$, but it is actually measured to be $5.063V$ at that moment.

2.2 The internal resistance of Analog Discovery

When it comes to calculating the internal resistance of Analog Discovery, the three circuits below are used. Even though all three should have the same theoratical results, the measurements, written beneath each schematics, varied.

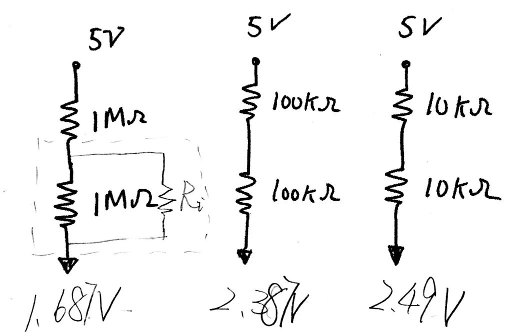


Figure 3: Three sets of two resistors of the same resistance and their respective measurement. The actual power input is still around $5.063V$.

To infer the internal resistance of Analog Discovery, a formula from Page 18 of the iSIM textbook is employed:

$$\frac{V_{out}}{V_{in}} = \frac{1}{2 + \frac{R}{R_I}} \tag{3}$$

where R is the resistance of the resistor and R_I is the internal resistance of Analog Discovery. The formula is then transformed to give R_I.

$$R_I = \frac{V_{out}R}{V_{in} - 2V_{out}} \tag{4}$$

$$R_I = \frac{1.687 \times 10^6}{5.063 - 2 \times 1.687} = 9.988 \times 10^5 \approx 1M\Omega \tag{5}$$

While making inferences with the $1M\Omega$ setup gave a decently accurate result, attempts with the other two setups had really bizarre results. The reasons why it is believed that the above calculation is the most accurate of the three and why the other two behave strangely are explored in the last section of the Interpretation.

2.3 The coffee problem

Two cups labelled One and Two and two thermistors were used in this part of the lab. Cup Two received "cold milk" in the form of $0°C$ water immediately after hot water was poured into the two cups, and Cup One only received cold water roughly ten minutes later. The voltage changes across the two thermistors were recorded on Analog Discovery and plotted in Figure 4.

(Please see the next page)

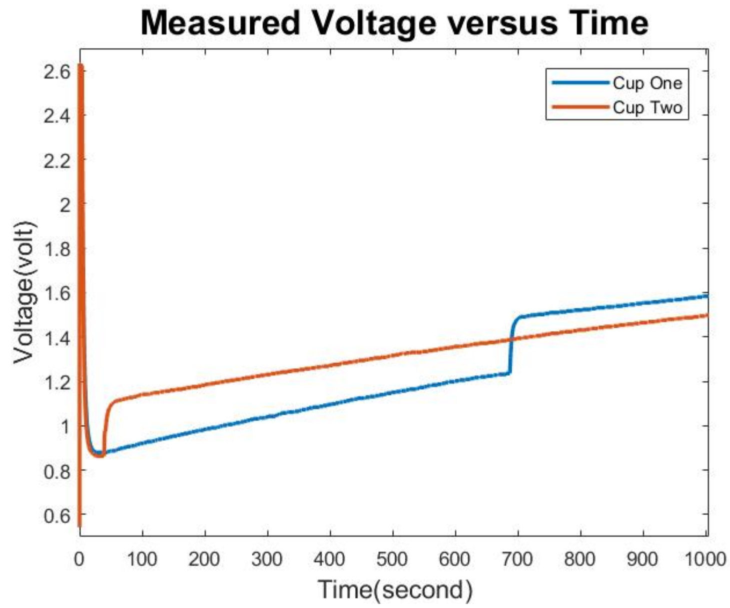


Figure 4: The voltage variations across the two thermistors over the course of the lab. The sharp peak and drop at the beginning was caused by the sudden plunging of the thermistors into hot water.

To turn the voltages into resistances, Equation (2) is transformed into the following new equation. A new plot was generated to display the resistance changes.

$$R = \frac{1000V_{out}}{V_{in} - V_{out}} \tag{6}$$

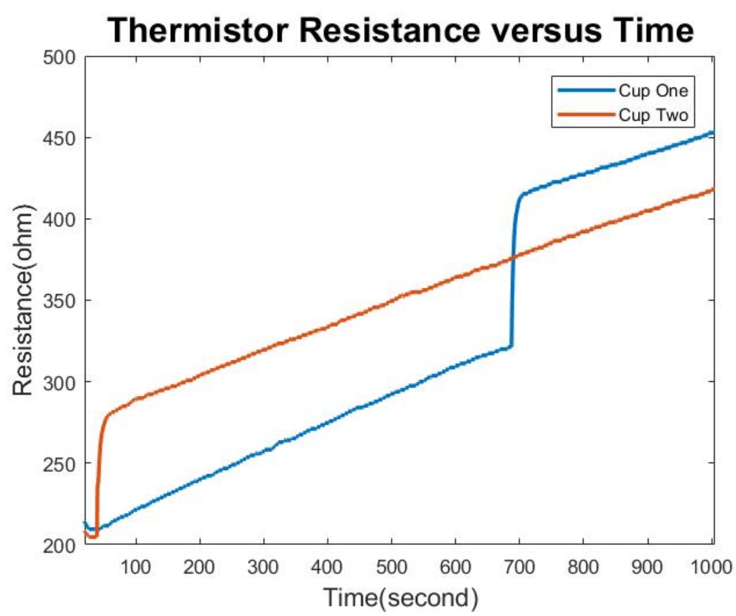


Figure 5: Thermistor resistance variations over time. The dramatic beginning of the data has been removed for clarity.

Finally, plotting temperature variations over time would require invert Equation (1) for T.

$$\frac{1}{T} = \frac{1}{298} + \frac{\ln \frac{R}{1000}}{3528} \tag{7}$$

Plugging in Equation (6) also gives another way to calculate the temperatures:

$$\frac{1}{T} = \frac{1}{298} + \frac{\ln \frac{V_{out}}{V_{in}-V_{out}}}{3528} \tag{8}$$

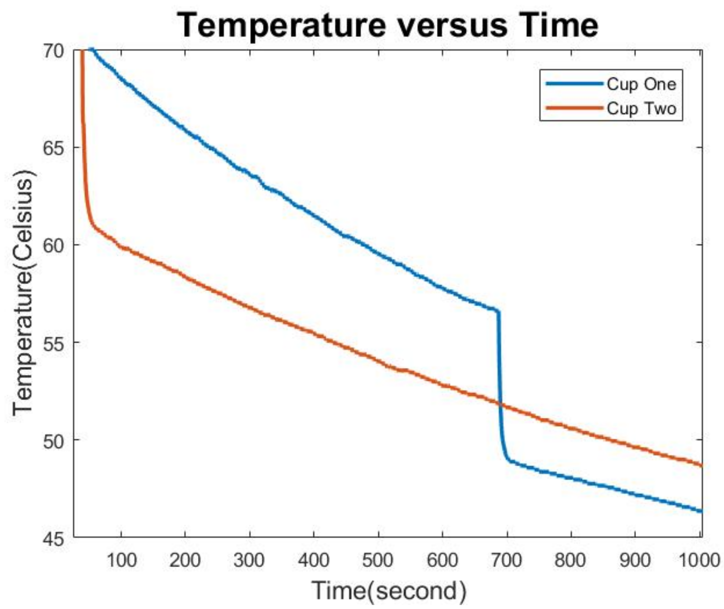


Figure 6: Temperature variations of the two cups over time. The plot is made after the temperatures were converted into Celsius by subtracting 273 from T.

3 Interpretation

3.1 The coffee problem

As it can be seen in Figure 6, "coffee" in Cup One started at 70 ^{circ}C while that in Cup Two, with "cold milk" added, started at slightly above 60 $°C$. Before cold water was poured into Cup One, it is quite noticeable that the temperature in Cup One assumed a steeper slope than that in Cup two. This conforms to Newton's law of cooling which states that the rate at which an object is cooling is proportional to the difference between the temperature of the object and that of its environment. Once cold water was added, the temperature in Cup One dropped below that in Cup Two. The trend where higher temperature was measured in Cup Two continued all the way to the end of the lab.

Therefore, it is concluded that in order to get the warmest possible coffee after a certain period of time, **a person should put in the cold milk as soon as the hot coffee itself is ready**.

3.2 The internal resistance of Analog Discovery

Based on the calculation with two 1 $M\Omega$ resistors, the internal resistance of Analog Discovery is about 1 $M\Omega$. Verifying the measured voltages for the other two circuits with this result shows little consistenecy. For example, with two 100 $k\Omega$:

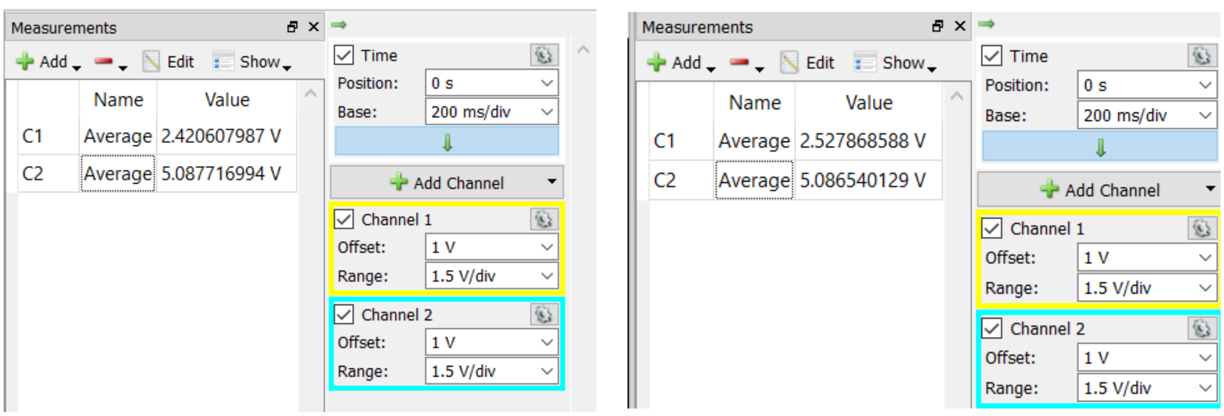
$$R_{||} = \frac{1}{\frac{1}{10^5} + \frac{1}{10^6}} \tag{9}$$

$$V_{out} = 5.063 \times \frac{R_{||}}{R_{||} + 10^5} = 2.411V \tag{10}$$

where the error from the measured value is only 1%. However, should we choose the 100 $k\Omega$ resistors to do the calculations with, the outcome deviate significantly from 1 $M\Omega$.

$$R_I = \frac{2.387 \times 10^5}{5.063 - 2 \times 2.387} = 826k\Omega \tag{11}$$

The result only deviates further with the two 10 $k\Omega$ resistors. There are many places where errors could arise, including the real-time voltage supplied by the power module, the resistor error, and the measurement accuracy. The resistor error is very hard to be accounted for because the information provided is a range of values rather than one specific value. As a result, I conducted the experiment again for the 10 $k\Omega$ resistors and the 100 $k\Omega$ resistors, not only measuring the real-time voltage supplied but also taking as many decimal values as possible. Here are the data.



(a) 100$k\Omega$ resistors

(b) 10$k\Omega$ resistors

Figure 7: The results of a second experiment on the internal resistance of Analog Discovery

This time, with more decimal places and the real-time input voltage, the outcome of 100$k\Omega$ comes very close to 1$M\Omega$.

$$R_I = \frac{2.420607987 \times 10^5}{5.087716994 - 2 \times 2.420607987} = 981987\Omega \tag{12}$$

The error from 1 $M\Omega$ is only 1.8%. With the 10$k\Omega$ resisors, the error goes up to 17.9%.

$$R_I = \frac{2.527868588 \times 10^4}{5.086540129 - 2 \times 2.527868588} = 820657\Omega \tag{13}$$

My hypothesis is that slight inaccuracy (e.g., rounding, decimal places, etc.) has led to the apparently false result. 10 $k\Omega$ is very small compared to 1$M\Omega$, so V_{out} would come immensely close to

$\frac{1}{2}V_{in}$. Subsequently, the denominator of Equation (4), $V_{in} - 2V_{out}$, would be an extremely small number that is capable of scales any tiny error by folds. This can be easily proven by tweaking $V_{out} = 2.527...$ to $V_{out} = 2.525...$

$$R_I = \frac{2.525868588 \times 10^4}{5.086540129 - 2 \times 2.525868588} = 540925\Omega \tag{14}$$

This is a 54.1% error! Thus, it is concluded that it is not that Analog Discovery has a variable resistance, but that we are not measuring accurately enough to all the results to come out the same.