Singular Value Decomposition Practice

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1 Image Compression

In this section, we use Singular Value Decomposition(SVD) to extract principal features of an image to achieve image compression. By definition, a matrix M can be decomposed into these three matrices U, Σ, and V as shown in Equation 1:

$$M = U\Sigma V^T \tag{1}$$

where U is a matrix whose columns are the orthonormal eigenvectors of the matrix MM^T, Σ is a diagonal matrix with the square root of the eigenvalues of MM^T, and V is a matrix whose rows are the orthonormal eigenvectors of the matrix M^TM. When the SVD is expanded out as a sum of Rank 1 matrices, it can be written as the following:

$$M_r = u_1\sigma_1 v_1^T + u_2\sigma_2 v_2^T + u_3\sigma_3 v_3^T + ... + u_r\sigma_r v_r^T \tag{2}$$

with r being the rth approximation we are aimming for.

Below, I found an image of an eye from *Blade Runner 2049* and did SVD image compression of different ranks. The size of the original image was 811 $\times$ 1920 pixels. The images start to get indistinguishable starting at around the 92nd rank when the σ drops below zero.

Figure 1: The original image

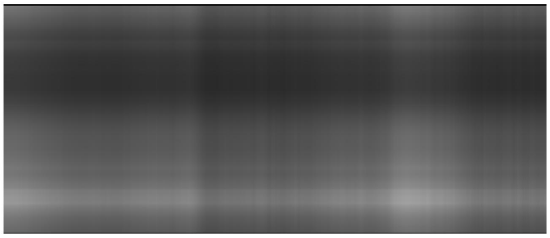

Figure 2: The Rank 1 approximation of the original image.

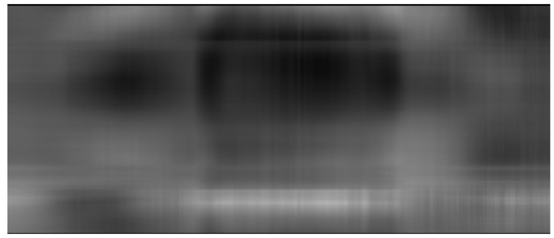

Figure 3: The rank 3 approximation of the original image.

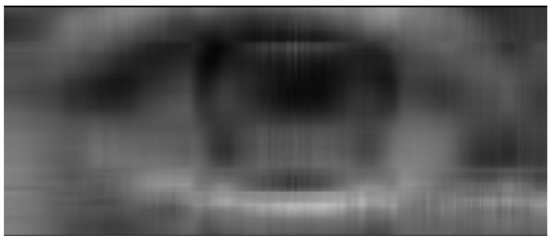

Figure 4: The rank 5 approximation of the original image.

Figure 5: The rank 10 approximation of the original image.

Figure 6: The Rank 25 approximation of the original image.

Figure 7: The Rank 50 approximation of the original image.

Figure 8: The Rank 100 approximation of the original image.

Figure 9: The Rank 200 approximation of the original image.

2 Pattern Recognition (of Made Up Data) with SVD

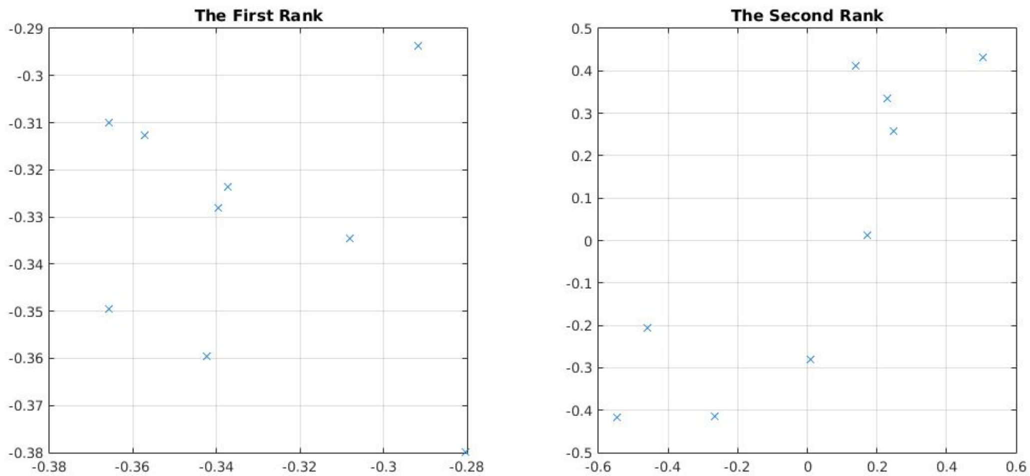

Figure 10: SVD pattern recognition of ”judges” sample code.