Mixed Analog-Digital VLSI Mini-Project III: Folded Cascode Differential Amplifier

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Project Links

Project Github: https://github.com/QingmuDeng/MADVLSI/tree/main/mini2

1 Circuit Analysis and Bias Voltage Generation

1.1 Question a

 V_1 is the non-inverting input while V_2 is the inverting input.

This result can be reasoned as the following in conjunction with Figure 1. M_b acts as if like a current source to dump I_b into the node V. By Kirchoff's current law, the current through M_1 and M_2 must be in such a way that $I_b = I_1 + I_2$. Now let's examine the drain of the transistor M_3 . At this node, the current coming from M_1 is I_1 , and the current flowing through M_3 is I_b . By Kirchoff's current law, the current I_5 flowing into this node from M_5 must be $I_b - I_1$. Assuming all the cascode transistors are bias properly, this current will be mirrored to $I_b - I_2$ flowing into node V_{out} from M_10 . By the same token, the current I_6 flowing out of V_{out} into M_6 must be $I_b - I_2$. By Kirchoff's current law, $I_{out} = I_b - I_1 - I_b + I_2 = I_2 - I_1$.

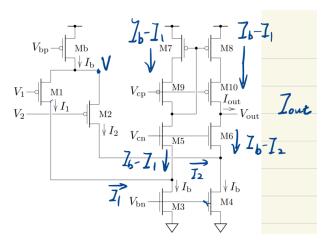


Figure 1: Illustration of current flow in folded-cascode differential amplifier for Question a.

As we increase V_2 while holding V_1 fixed, I_2 decreases whereas I_1 increases. Thus, $I_2 - I_1$ decreases. On the other hand, as we increase V_1 while holding V_2 fixed, I_1 decreases whereas I_2 increases. Thus, $I_2 - I_1$ increases. Therefore, V_1 is the non-inverting input, and V_2 is the inverting input.

1.2 Question b

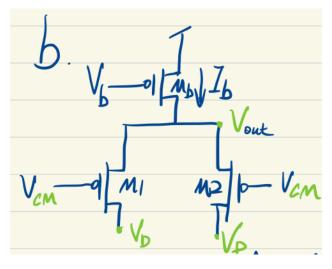


Figure 2: Illustration of differential pair for Question b.

To understand the limit on the common-mode input voltage, we treat the differential pair, M_1 and M_2 , and their bias transistor, M_b , as a source-follower. For M_b to act like a current source, it must remain in saturation. That is, $I = I_F + I_R \approx I_F$. Thus, we can describe our transistor operation with only the forward current of the source-drain-symmetric EKV model for the transistor operation.

$$I_b = SI_s \log^2 \left(1 + e^{\left[\kappa(V_{dd} - V_b) - V_{T_0} - V_{dd} + V_{dd}\right]/2U_T} \right)$$
 (1)

Similarly, for either M_1 or M_2 , we have

$$I_{\text{diff}} = SI_s \log^2 \left(1 + e^{\left[\kappa (V_{dd} - V_{cm}) - V_{T_0} - V_{dd} + V_{out}\right]/2U_T} \right)$$
 (2)

Setting the two equation equal and cancelling terms, we get

$$I_{b} = I_{\text{diff}}$$

$$\kappa(V_{dd} - V_{cm}) - V_{T_{0}} - V_{dd} + V_{out} = \kappa(V_{dd} - V_{b}) - V_{T_{0}}$$

$$V_{dd} - V_{out} = \kappa(V_{dd} - V_{cm} - V_{dd} + V_{b})$$

$$V_{out} = V_{dd} - \kappa(V_{b} - V_{cm})$$
(3)

At the same time, we know that for M_b to be in saturation, its source-drain voltage must be at least a certain V_{SDSAT} .

$$V_{dd} - V_{out} \ge V_{SDSAT}$$

$$V_b - V_{cm} \ge V_{SDSAT}/\kappa$$

$$V_{cm} \le V_b - V_{SDSAT}/\kappa$$
(4)

1.3 Question c

If the output node is held fixed near the middle of the rails and the Early effect is negligible, the output current is given by

$$I_{out} = I_b - I_1 - I_b + I_2 = I_2 - I_1 \tag{5}$$

For its derivation, see answer to Question a.

1.4 Question d

The current at the bias transistors M_3 and M_4 need not be biased to exactly I_b as in the bias transistor M_b . However, two constraints exist:

- 1. The bias current in M_3 , I_{b3} , and the bias current in M_4 , I_{b4} , must be equal to each other. Their equal current nature is necessary for their cancellation in the output current of the V_{out} node. Otherwise, there will be a constant term in output current that is not directly to either of the input voltages.
- 2. The bias current in both M_3 and M_4 must be at least I_b or larger. The current through M_5 and M_6 is given by $I_5 = I_{b3} I_1$ and $I_6 = I_{b4} I_2$, respectively. In situations where $I_1 = I_b$, $I_2 = 0$, for example, we have $I_5 = I_{b3} I_b < 0$ and $I_6 = I_{b4}$. This is problematic as it seems to suggest that the current should flow into the pmos current mirror from the nmos instead of the other way around. What could realistically happen is that the current I_1 drives the drain voltage of M_3 so high such that it is passing through I_b equivalent amount of current through Early effect.

1.5 Question e

The bias voltage generation circuit is designed as shown in Figure 3. A diode-connected unit nmos accepts the current and have it mirrored to a group of bias voltage generation transistors for either pmos or nmos. The group of bias-voltage generation transistors for V_{cn} generation can be laid out

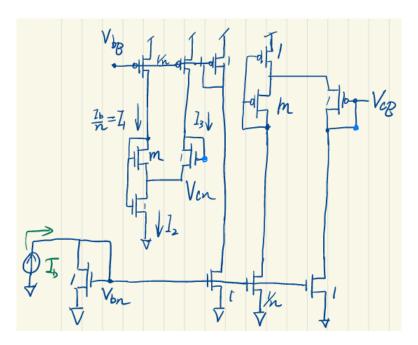


Figure 3: Cascode-bias voltage generation circuit design.

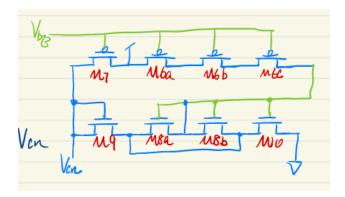


Figure 4: V_{cn} voltage generation circuit layout driven schematics.

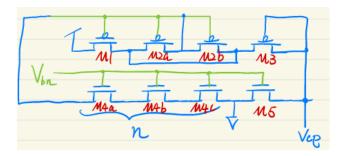


Figure 5: V_{cp} voltage generation circuit layout driven schematics.

in the way shown in Figure 4, and the group of bias-voltage generation transistor for V_{cp} generation is similarly shown in Figure 5.

To achieve the common centroid design, both group of the transistors were mirrored along their edges. The $12 \times 0.5 \mu m$ transistor is therefore divided up into two $6times0.5 \mu m$ unit transistors. The

auxiliary current mirrors forms a standalone block in the layout driven schematic. The schematic for the entire folded cascode differential amplifier is shown in Figure 6.

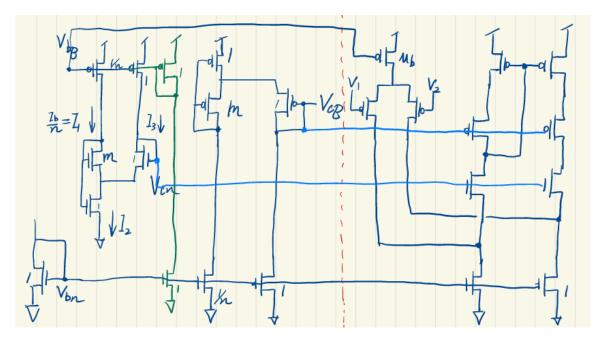


Figure 6: The circuit design for the entire folded cascode differential amplifier with bias voltage generation.

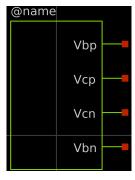
2 Schematic Capture and Simulation

2.1 Schematic Capture

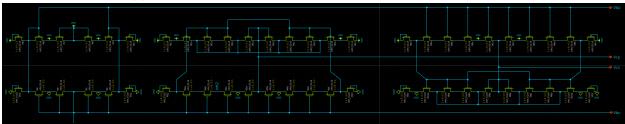
The layout driven schematics for both the cascode bias generation circuit and the folded cascode differential amplifier is created separately, as shown in Figure 7 and 8. Both circuits independently adopt a common centroid configuration. All the bias and cascode transistors were all matched to Size $6\mu m \times 0.5\mu m$ and combined in series and in parallel to ultimately achieve the effective size of $12\mu m \times 0.5\mu m$. The differential pair were matched to each other through series and parallel combinations of Size $12\mu m \times 0.5\mu m$. Appropriately sized dummy transistors have been added to the end of transistor rows wherever applicable.

2.2 Test Harness

For the test harness configuration, please refer to Figure 9 to 14. For the simulation results and their interpretations, please refer to Section 5.

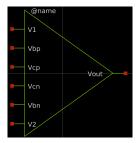


(a) Cascode bias voltage generation symbol created in Xschem.

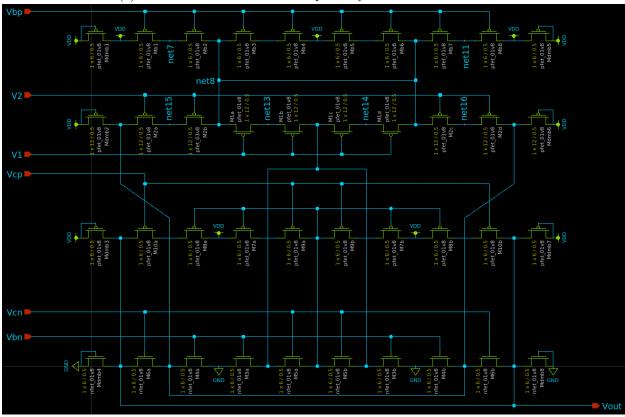


(b) Cascode bias voltage generation layout-driven schematic created in Xschem.

Figure 7: Cascode bias voltage generation schematic capture in Xschem.



(a) Folded cascode differential amplifier symbol created in Xschem.



(b) Cascode bias voltage generation layout-driven schematic created in Xschem.

Figure 8: Folded cascode differential amplifier schematic capture in Xschem.

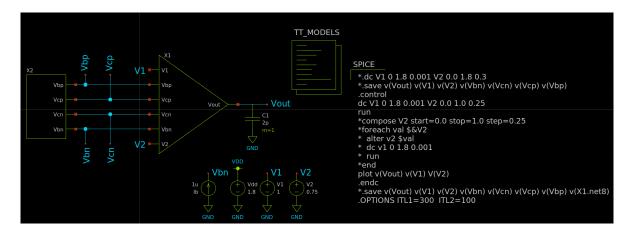


Figure 9: DC voltage transfer characteristics test harness.

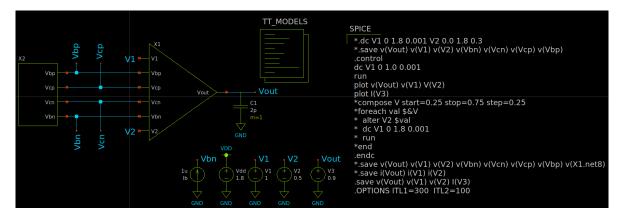


Figure 10: Voltage-to-current transfer characteristics test harness.

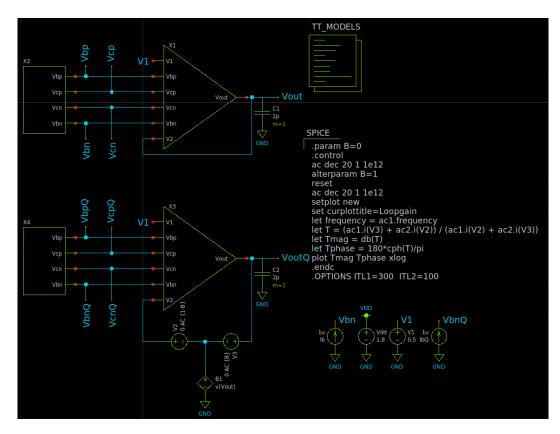


Figure 11: Loopgain test harness.

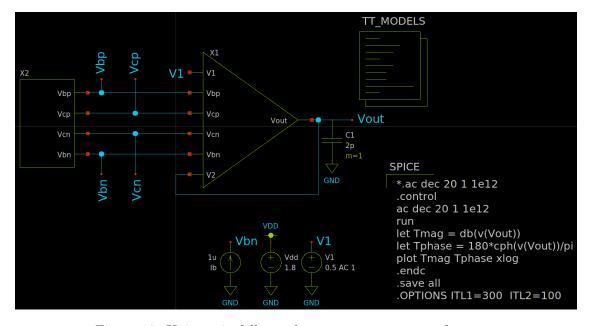


Figure 12: Unity-gain follower frequency response test harness.

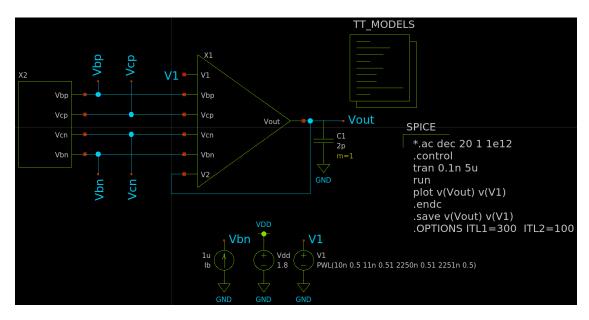


Figure 13: Small signal test harness.

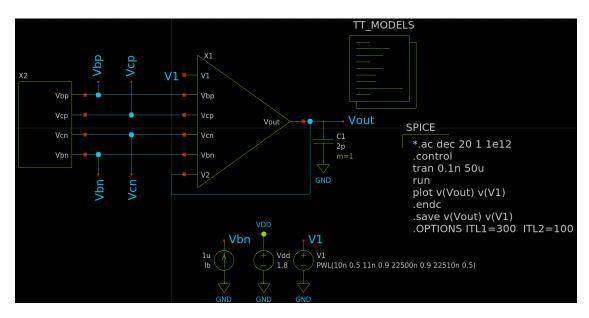


Figure 14: Large signal test harness.

3 Layout Design

There are no design rule violations. TAll the bias and cascode transistors were all matched to Size $6\mu m \times 0.5\mu m$ and combined in series and in parallel to ultimately achieve the effective size of $12\mu m \times 0.5\mu m$. The differential pair were matched to each other through series and parallel combinations of Size $12\mu m \times 0.5\mu m$. Appropriately sized dummy transistors have been added to the end of transistor rows wherever applicable.

The area of the bias circuit is 515.14 μm^2 ; the area of the folded cascode differential amplifier is 476 μm^2 .

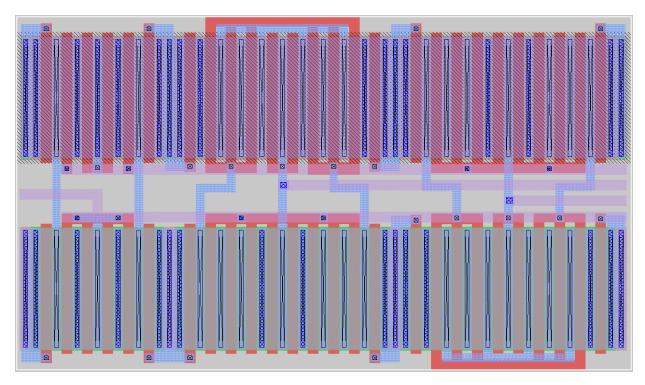


Figure 15: Magic layout of the cascode bias voltage generation circuit.

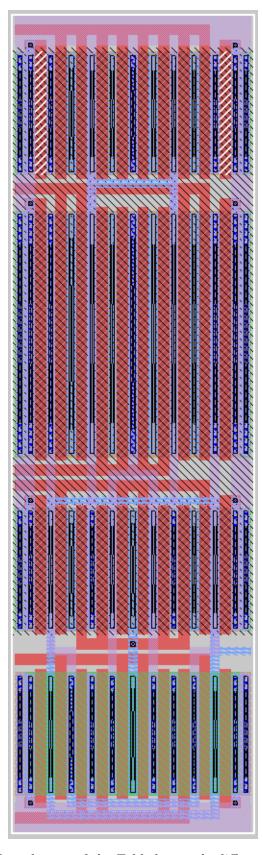


Figure 16: Magic layout of the Folded cascode differential amplifier.

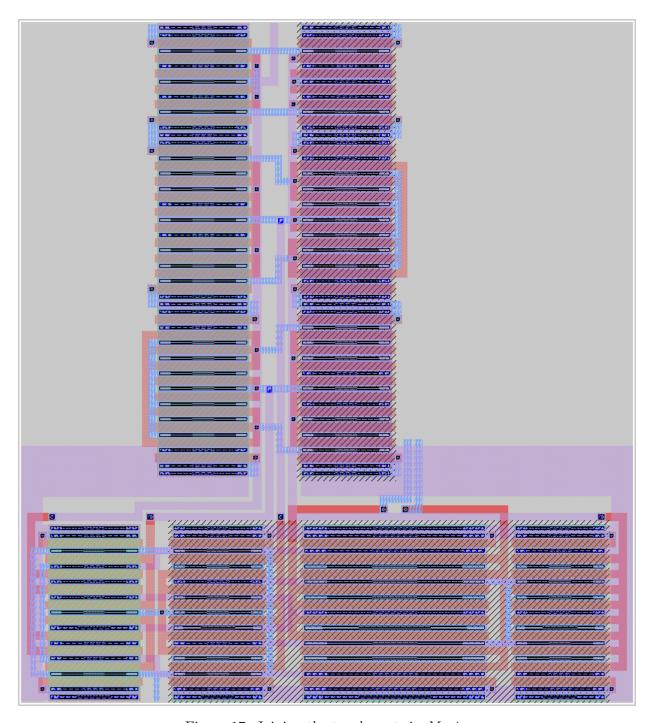


Figure 17: Joining the two layouts in ${\it Magic}.$

4 Layout versus Schematic

Finally, I performed Layout-versus-Schematic comparison at all levels of the shift register. There are three listings below:

- 1. Layout versus Schematic for Bias-voltage generation circuit
- 2. Layout versus Schematic for Cascode Differential Amplifier

Listing 1: Layout versus Schematic for Bias-voltage generation circuit

Equate elements: no current cell. Equate elements: no current cell.

Class schem/bias_lvs.spice: Merged 21 devices. Class layout/bias_lvs.spice: Merged 21 devices.

Subcircuit summary:

Circuit 1: schem/bias_lvs.spice	Circuit 2: layout/bias_lvs.spice
sky130_fd_prnfet_01v8 (16)	sky130_fd_prnfet_01v8 (16)
sky130_fd_prpfet_01v8 (15)	sky130_fd_prpfet_01v8 (15)
Number of devices: 31	Number of devices: 31
Number of nets: 22	Number of nets: 22

Resolving automorphisms by property value.

Resolving automorphisms by pin name.

Netlists match with 9 symmetries.

Circuits match correctly.

Cells have no pins; pin matching not needed.

Device classes schem/bias_lvs.spice and

layout/bias_lvs.spice are equivalent.

Circuits match uniquely.

Listing 2: Layout versus Schematic for Cascode Differential Amplifier

Equate elements: no current cell. Equate elements: no current cell.

Class schem/cas_diff_lvs.spice: Merged 8 devices. Class layout/cas_diff_lvs.spice: Merged 8 devices.

Subcircuit summary:

Circuit 1: schem/cas_diff_lvs.spice	Circuit 2: layout/cas_diff_lvs.spice
	sky130_fd_prnfet_01v8 (5) sky130_fd_prpfet_01v8 (27) Number of devices: 32 Number of nets: 25

Resolving automorphisms by property value.

Resolving automorphisms by pin name.

Netlists match with 6 symmetries. Circuits match correctly. Cells have no pins; pin matching not needed. Device classes schem/cas_diff_lvs.spice and layout/cas_diff_lvs.spice are equivalent. Circuits match uniquely.

5 Simulation Result Processing

Please see the PDF starting on the next page.

data_process

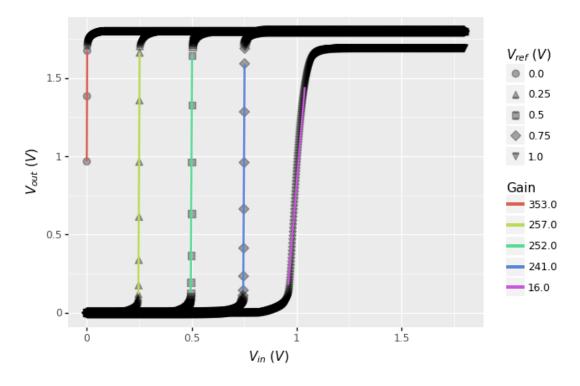
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```
[1]: import pandas as pd
import matplotlib.pyplot as plt
import plotnine as pn
import numpy as np
from scipy.optimize import minimize
[56]: %matplotlib inline
```

1 Voltage Transfer Characteristics

```
[45]: _, _, df0_plt = gain("0.0", 0, 3)
_, _, df025_plt = gain("0.25", 246, 253)
```

```
_, df05_plt = gain("0.5", 496, 503)
  _, df075_plt = gain("0.75", 746, 753)
  _, df1_plt = gain("1.0", 960, 1040)
    pn.ggplot(df_a, pn.aes(x="Vin",y="Vout", shape="Vref"))
    + pn.geom_point(size=3, alpha=0.33)
      + pn.geom_line(pn.aes(x="Vin",y="Vin"), color="b")
#
      + pn.geom_line(pn.aes(x="Vin",y="Vref"), color="salmon")
    + pn.geom_line(df0_plt, pn.aes(color="m"), size=1)
    + pn.geom_line(df025_plt, pn.aes(color="m"), size=1)
    + pn.geom_line(df05_plt, pn.aes(color="m"), size=1)
    + pn.geom_line(df075_plt, pn.aes(color="m"), size=1)
    + pn.geom_line(df1_plt, pn.aes(color="m"), size=1)
    + pn.labs(
        x="$V_{in}\ (V)$",
        y="$V_{out}\ (V)$",
        shape="$V_{ref}\ (V)$",
        color="Gain"
      )
)
```



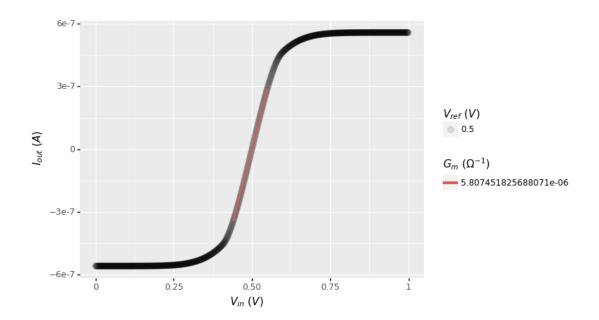
[45]: <ggplot: (8757275397458)>

The DC gain of this circuit is roughly in between 350 and 240.

2 Voltage-to-Current Transfer Characteristics

```
[7]: df_b = pd.read_csv("./schem/data/v2itc_noninv.csv")
       df_b = df_b.rename(columns={
           "v(V1)":"Vin",
           "v(V2)":"Vref",
           "v(Vout)":"Vout",
           "I(V3)":"Iout"
       df_b["Vref"] = df_b["Vref"].astype("str")
  [8]: def igain(i, j):
           df_tmp = df_b.iloc[i:j].reset_index(drop=True)
           m = ((df_tmp.iloc[-1].Iout - df_tmp.iloc[0].Iout)/(df_tmp.iloc[-1].Vin -_U)

df_tmp.iloc[0].Vin))
           b = df_tmp.iloc[-1].Iout - m*df_tmp.iloc[-1].Vin
           df_plt = pd.DataFrame({
               "Vin": df_tmp.Vin,
               "Iout": m*df_tmp.Vin+b,
               "Vref": df_tmp.Vref,
               "m": m*np.ones(len(df_tmp.Vin))
           })
           df_plt["m"] = df_plt["m"].astype("str")
           return m, b, df_plt
[117]: m, _, dfb_gain = igain(440, 550)
           pn.ggplot(df_b, pn.aes(x="Vin",y="Iout", shape="Vref"))
           + pn.geom point(size=3, alpha=0.1)
           + pn.geom_line(dfb_gain, pn.aes(color="m"), size=1)
           + pn.labs(
               x="$V_{in}\ (V)$",
               y="$I_{out}\ (A)$",
               shape="$V_{ref}\ (V)$",
               color="$G_m\ (\Omega^{-1})\n"
             )
       )
```



[117]: <ggplot: (8757275261299)>

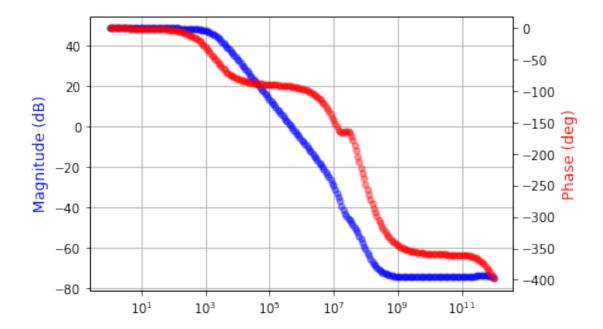
The incremental transconductance gain of the circuit, G_m , is roughly $5.81 \times 10^{-6} \Omega^{-1}$. The limiting value of the current is roughly $\pm 5.58 \times 10^{-7}$ A.

3 Loopgain

```
[54]: df_c = pd.read_csv("./schem/data/loopgain_noninv.csv")
[46]: df_c.iloc[0]
[46]: frequency
                    1.000000
                   48.655403
      Tmag
      Tphase
                   -0.032994
      Name: 0, dtype: float64
[47]:
     10**(4.8/2)
[47]: 251.18864315095797
[48]: df_csort= df_c
      df_csort["Tmag"] = abs(df_csort["Tmag"])
      df_csort.sort_values("Tmag")
[48]:
              frequency
                              Tmag
                                         Tphase
      113 4.466836e+05
                          0.444998
                                     -92.524984
      114 5.011872e+05
                          0.556461
                                    -92.884368
```

```
112 3.981072e+05 1.446151 -92.199061
     115 5.623413e+05 1.558305 -93.281962
     111 3.548134e+05 2.447059 -91.902285
     206 1.995262e+10 74.680655 -359.519020
     208 2.511886e+10 74.681201 -359.695304
     207 2.238721e+10 74.681222 -359.610403
     239 8.912509e+11 74.684153 -393.643956
     240 1.000000e+12 75.123178 -397.332897
     [241 rows x 3 columns]
[51]: m_x = (-0.556461 - 0.444998)/(5.011872e5-4.466836e5)
     b_x = -0.556461 - m_x * 5.011872e5
     freq_x = -b_x/m_x
     freq_x
[51]: 470902.25797082053
[57]: fig, ax = plt.subplots()
     c1 = "b"
     ax.semilogx(df_c.frequency, df_c.Tmag, "o", color=c1, alpha=0.4, markersize=5)
     ax.set_ylabel("Magnitude (dB)", size=12, color=c1)
     ax.grid()
     ax2 = ax.twinx()
     c2 = "r"
     ax2.semilogx(df_c.frequency, df_c.Tphase, "o", color=c2, alpha=0.4,_
      →markersize=5)
     ax2.set_ylabel("Phase (deg)", size=12, color=c2)
```

[57]: Text(0, 0.5, 'Phase (deg)')



At low frequencies, the loopgain of the folded cascode differential amplifier is 48dB, which translate to

$$Gain = 10^{4.8/2} \approx 251$$

It is very similar to most of the DC gains that we extracted in the first part of the simulation. The unity-gain crossover frequency is roughly $4.7092 \times 10^5 \ Hz$.

Theoretically, the time constant of the circuit is given by

$$\tau = \frac{C}{G_m} = \frac{2 \times 10^{-12} F}{5.81 \times 10^{-6} \Omega^{-1}}$$

and the cutoff frequency is given by

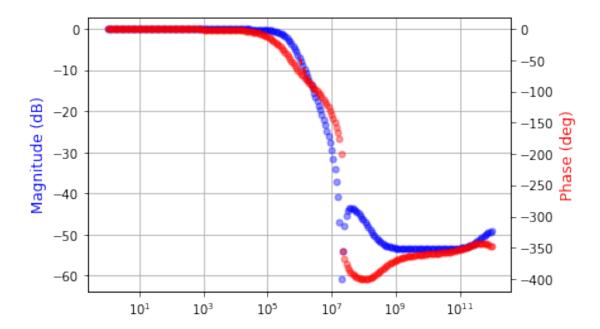
$$f_c = \frac{1}{2\pi\tau} = 4.62345 \times 10^5 \ Hz$$

which closely resembles what we have extracted from the simulation data.

[120]: 462345.109681956

4 Unity-Gain Follower Frequency Response

```
df_tmp["frequency"] = np.log10(df_tmp["frequency"])
           m = (df_tmp.iloc[-1].tmag - df_tmp.iloc[0].tmag)/(df_tmp.iloc[-1].frequency_
        → df_tmp.iloc[0].frequency)
           b = df_tmp.iloc[-1].tmag - m*df_tmp.iloc[-1].frequency
           df_plt = pd.DataFrame({
               "frequency": df tmp.frequency,
               "tmag": m*df_tmp.frequency+b,
               "m": m*np.ones(len(df_tmp.frequency))
           })
           df_plt["m"] = df_plt["m"].astype("str")
           return m, b, df_plt
 [97]: m, b, df_freqroll = freq_cutoff(df_d, 120, 130)
[104]: 10**(-b/m)
[104]: 416959.66847466904
[109]: fig, ax = plt.subplots()
       c1 = "b"
       ax.semilogx(df_d.frequency, df_d.tmag, "o", color=c1, alpha=0.4, markersize=5)
       ax.semilogx(10**df_freqroll.frequency, df_freqroll.tmag, "r--")
       ax.set_ylabel("Magnitude (dB)", size=12, color=c1)
       ax.grid()
       ax2 = ax.twinx()
       c2 = "r"
       ax2.semilogx(df_d.frequency, df_d.tphase, "o", color=c2, alpha=0.4,
       →markersize=5)
       ax2.set_ylabel("Phase (deg)", size=12, color=c2)
[109]: Text(0, 0.5, 'Phase (deg)')
```



From fitting a straight line near the initial rolloff, I believe the corner frequency in the circuit's frequency response is roughly $4.16960 \times 10^5~Hz$, which makes it veyr similar to the crossover frequency in the loopgain simulation.

5 Small Signal

```
[33]: df_e = pd.read_csv("./schem/data/trans_ssig.csv").rename(columns={
          "v(V1)":"Vin",
          "v(Vout)":"Vout"})
[30]: def exp_plot_r(X):
          return 0.01*(1 - np.exp(-df_e.time.values/X[0]))
      def exp_plot_f(X):
          return 0.01*np.exp(-df_e.time.values/X[0])
      def find_tau_r(df):
          df_tmp = df
          res = [j - i for i, j in zip(df_tmp.Vout[: -1], df_tmp.Vout[1 :])]
          df_{tmp}["d"] = np.array([0.0]+res)
          df_tmp["Vout"] = df_tmp["Vout"]-df_tmp.iloc[0].Vout
          df_tmp = df_tmp[df_tmp.d>0].reset_index(drop=True)
          t off = df tmp.iloc[0].time
          df_tmp["time"] = df_tmp["time"]-t_off
          df_res = df_tmp[df_tmp.Vout>0.01*(1-np.exp(-1))].reset_index(drop=True)
```

```
return t_off, df_res.iloc[0].time#t_off, res_fit
      def find_tau_f(df):
          df_tmp = df
          res = [j - i for i, j in zip(df_tmp.Vout[: -1], df_tmp.Vout[1 :])]
          df_{tmp}["d"] = np.array([0.0]+res)
          df_tmp["Vout"] = df_tmp["Vout"]-df_tmp.iloc[0].Vout
          df tmp = df tmp[df tmp.time>2e-6].reset index(drop=True)
          df_tmp = df_tmp[df_tmp.d<0].reset_index(drop=True)</pre>
          t_off = df_tmp.iloc[0].time
          df_tmp["time"] = df_tmp["time"]-t_off
          df_res = df_tmp[df_tmp.Vout<0.01*(np.exp(-1))].reset_index(drop=True)</pre>
          return t_off, df_res.iloc[0].time
[31]: t_off_r, tau_r = find_tau_r(df_e)
      t_off_f, tau_f = find_tau_f(df_e)
[35]: fig, ax = plt.subplots()
      c1 = "b"
      ax.plot(df_e.time, df_e.Vin, "-", color=c1, alpha=1, markersize=5,_
      \rightarrowlabel="$V_{in}$")
      ax.plot(df_e.time, df_e.Vout, ".", color="r", alpha=1, markersize=5,_
      \rightarrowlabel="$V_{out}$")
      ax.plot(df_e.time+t_off_r, exp_plot_r([tau_r])+0.5014, "--", color="g", __
      →alpha=1, markersize=5, label="Rise $t$:%.4E"%tau_r)
      ax.plot(df_e.time+t_off_f, exp_plot_f([tau_f])+0.5014, ":", color="g", alpha=1,__
      →markersize=5, label="Fall $t$:%.4E"%tau_f)
```

```
<IPython.core.display.Javascript object>
```

ax.set ylabel("Voltage (V)", size=12, color=c1)

<IPython.core.display.HTML object>

ax.set_xlabel("Time (s)", size=12)
ax.set_title("Small Signal Analysis")

ax.grid()
ax.legend()

```
[35]: <matplotlib.legend.Legend at 0x7f6f66188be0>
```

```
[113]: 1/(2*np.pi*tau_r), 1/(2*np.pi*tau_f)
```

```
[113]: (473901.09305590566, 478747.87357687176)
```

The input step needs to be small enough that the output current of the folded-cascode differential amplifier is not near its limiting value. (Referencing the figure in the voltage-to-current transfer

characteristics.) The chosen step of 0.01V is small enough that it does not go near the limiting current value.

The response is not exactly symmetrical. The rise time constant is longer by something on the order of 10^{-9} second.

The natural frequency after being converted from their corresponding time constants with the following equation, is $4.74 \times 10^5~Hz$ for the rise and $4.79 \times 10^5~Hz$ for the fall. They closely resemble the cutoff frequency in the frequency sweep and the crossover frequency in the loopgain simulation.

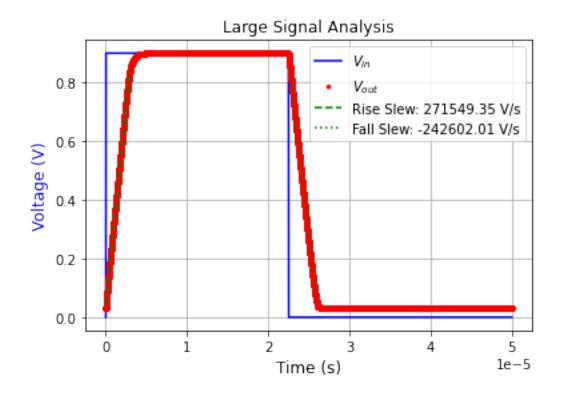
$$f_c = \frac{1}{2\pi\tau}$$

6 Large Signal

```
[20]: m_r,_,slew_r = slew(df_f, 500, 30000)
m_f,_,slew_f = slew(df_f, 22.7e4, 26e4)
```

```
ax.set_xlabel("Time (s)", size=12)
ax.set_title("Large Signal Analysis")
ax.grid()
ax.legend()
```

[114]: <matplotlib.legend.Legend at 0x7f6f6479f4c0>



The response of the circuit to the large-amplitude steps is not exactly symmetrical. The rise slew rate is slightly larger in magnitude than the fall slew rate.

The slew rate calculated from the below equation is $2.79 \times 10^5 \ V/s$.

$$\frac{I_{limit}}{C_{load}} \approx \frac{5.58 \times 10^{-7} \; A}{2 \times 10^{-12} \; F} = 279000 \; V/s$$

which is very close to the value that we have extracted from the large signal analysis of the response.

[40]: 5.58e-7/2e-12

[40]: 279000.0

[]: