$$S_n = \sum_{i=1}^{n} X_i = E[S_n] + O\left(\left(\sum_{i=1}^{n} Var\{X_i\}\right)^{1/2}\right)$$

Known.

$$X_i = g_{oi\psi}\, g^*_{i\phi}\, O(r_{oi}^{-\alpha-1})$$

and $\psi \neq \phi$

$$E[S_n] = 0$$

$$Var\{X_i\} = var\left\{ g_{oi\psi} g^*_{i\phi}\, O(r_{oi}^{-\alpha-1}) \right\}$$

$$= var\left\{ g_{oi\psi} g^*_{i\phi} \right\} O\left(r_{oi}^{-2\alpha-2}\right) = O(r_{oi}^{-2\alpha}$$

b/c of probability property

$$Var\{AX\} = A^2 var\{X\}$$

constant

$$S_n = \sum g_{oi\psi}\, g^*_{i\phi}\, O(r_{oi}^{-\alpha-1})$$

$$= O\left(\left(\sum_{i=1}^{n} (r_{oi}^{-2\alpha-2})\right)^{1/2}\right)$$

$$r = O\sqrt{K}$$

$$n = O(K)$$

$$S_n = O\left(\left(\sum_{i=1}^{O(K)} (K^{-\alpha-1})\right)^{1/2}\right)$$

$$= O\left(\left(K \cdot (K^{-\alpha-1})\right)^{1/2}\right)$$

$$= O\left(K^{-\alpha/2}\right)$$

I:e off diagonal terms of B are $O(K^{-\alpha/2})$

If $\psi = \phi$ $\qquad g_{oi\psi} = g_{i\psi}$

$$S_n = \sum_{i=1}^{n} |g_{i\psi}|^2 O(r_{oi}^{-\alpha-1})$$

$r_{oi} = O(\sqrt{K})$, $n = O(K)$

$$S_n = \sum_{i=1}^{O(K)} |g_{i\psi}|^2 O(K^{-\alpha/2 - \frac{1}{2}})$$

$$E[S_n] = O(K^{-\alpha/2 - 1/2}) \cdot O(K)$$

$$= O(K^{-\alpha/2 + \frac{1}{2}})$$

$$\mathrm{Var}\left\{ |g_i\psi|^2 O(K^{-\alpha/2 - \frac{1}{2}}) \right\}$$

$$= O(K^{-\alpha-1})$$

$$D = \begin{bmatrix} O(1) & & \\ O(K^{-1/2}) & \ddots & \\ & & O(1) \end{bmatrix}$$

$$\|D\| = \sqrt{\sum_{j=1}^{K} \sum_{i=1}^{K} (D_{ij})^2}$$

$$= \sqrt{K\,O(1) + (K^2 - K)\,O(K^{-1/2})}$$

$$= \sqrt{O(K) + O(K^{2-1/2})}\ \cancel{\#}\ O(K^{1/2})$$

$$= \sqrt{O(K^{3/2})}$$

$$\|D\|^2 = O(K^{3/2})$$

~~Theo~~ ~~Exercse~~ ~~Excersi~~ From Exercise 2.4.3

we are concerned with $\frac{1}{K^2}\|D\|^2$

$$\frac{1}{K^2}\|D\|^2 = O(K^{-1/2}) \to 0$$

$$\frac{n}{\pi r_n^2} \to \lambda \qquad \frac{cK}{\pi r_n^2} = \lambda$$

$$\cancel{\frac{O(K)}{r_n^2} \to \lambda}$$

$$K = \frac{\lambda}{\pi c} r_n^2$$

$$\psi = \psi$$

$$O(S_n) = O\left(K^{-\alpha/2+\frac{1}{2}}\right) + O\left(\left(\sum_{i=1}^{n} K^{-\alpha-1}\right)^{1/2}\right)$$

$$= O\left(K^{-\alpha/2+1/2}\right) + O\left(K^{-\alpha/2}\right)$$

$$= O\left(K^{-\alpha/2+\frac{1}{2}}\right)$$

$$B = \begin{bmatrix} O(K^{-\alpha/2+1/2}) & & & \\ O(K^{-\alpha/2}) & \ddots & & \\ \vdots & \ddots & \ddots & \\ \ldots & \ldots & \ldots & O(K^{-\alpha/2+1/2}) \end{bmatrix}$$

$$\| \underbrace{\sqrt{K}\, K^{\alpha/2-1} B}_{D} \|^2$$

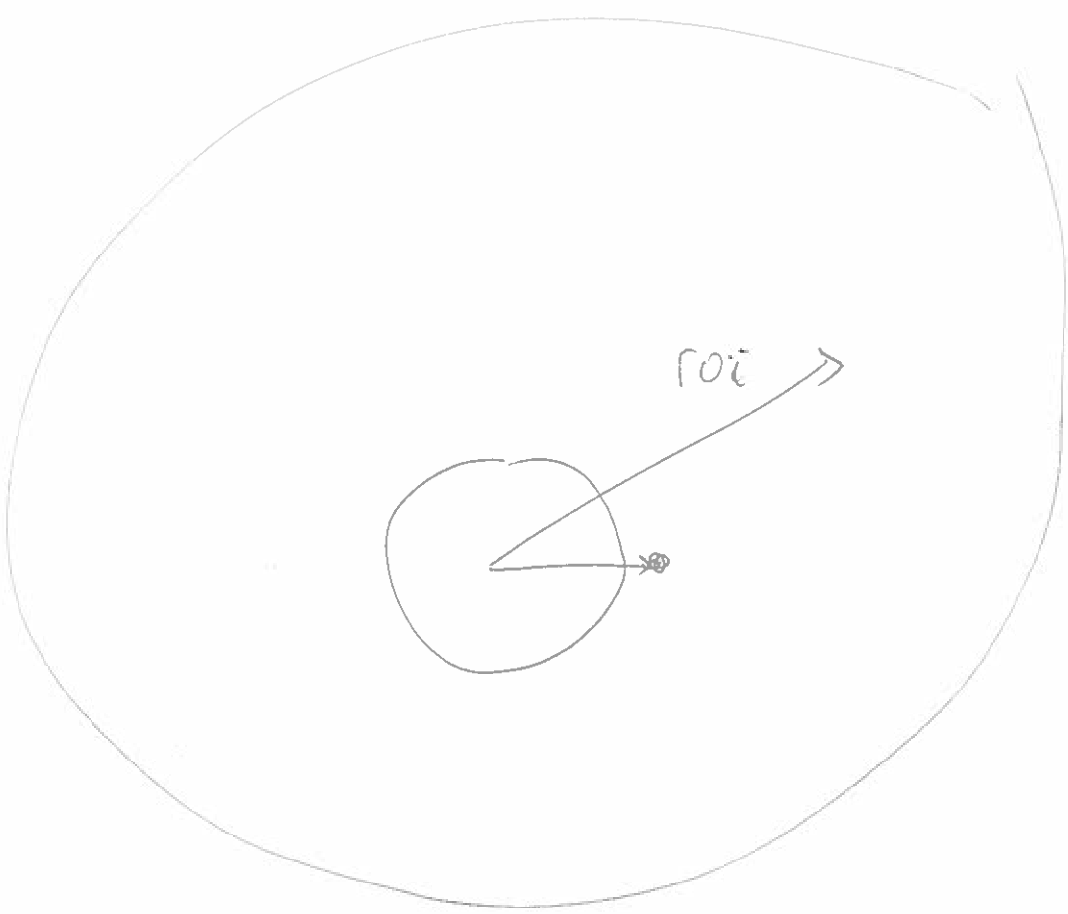
TODO:

Make ~~O(K)~~ $r_{oi} = O(\sqrt{K})$ precise.

Make connection

To probability, using

Chebyshev inequality (If necessary)



Connection of

$K^{-\alpha/2}$ SIR with eigenvalues,

Done in overleaf (Eqn 29)

To get rid of upper bound use (33) with LLN for scaled random variables. cf overleaf