In[•]:=
```
NSolve[Sin[Sqrt[x]] + Sqrt[x] Cos[Sqrt[x]] == 0, x]
```

NSolve: This system cannot be solved with the methods available to NSolve.

Out[•]= NSolve$\left[\sqrt{x}\ \text{Cos}\left[\sqrt{x}\right] + \text{Sin}\left[\sqrt{x}\right] == 0,\ x\right]$

In[•]:=
```
DSolve[{x''[t] + λ * x[t] == 0, x[0] == 0, x[1] + x'[1] == 0}, x[t], t]
```

Out[•]= $\left\{\left\{x[t] \to \begin{cases} C[1]\ \text{Sin}\left[t\sqrt{\lambda}\right] & \sqrt{\lambda}\ \text{Cos}\left[\sqrt{\lambda}\right] + \text{Sin}\left[\sqrt{\lambda}\right] == 0 \\ 0 & \text{True} \end{cases}\right\}\right\}$

In[•]:= x[t_] := Piecewise$\left[\left\{\left\{C[1]\ \text{Sin}\left[t\sqrt{\lambda}\right],\ \sqrt{\lambda}\ \text{Cos}\left[\sqrt{\lambda}\right] + \text{Sin}\left[\sqrt{\lambda}\right] == 0\right\}\right\},\ 0\right]$

Out[•]= {{x[t] → 0}}

In[•]:=
```
NDSolve[{x''''[t] + λ * x[t] == 0, x[0] == 0,
  x'[0] == 0, x'''[1] == 0, x''''[1] == 0}, x[t], {t, 0, 1}]
```

NDSolve: The initial values for all the dependent variables are not explicitly specified. NDSolve will attempt to find consistent initial conditions for all the variables.

NDSolve: The initial values for all the dependent variables are not explicitly specified. NDSolve will attempt to find consistent initial conditions for all the variables.

NDSolve: Encountered non-numerical value for a derivative at t == 0.`.

Out[•]= NDSolve$\left[\left\{\lambda\, x[t] + x^{(4)}[t] == 0,\ x[0] == 0,\ x'[0] == 0,\ x^{(3)}[1] == 0,\ x^{(4)}[1] == 0\right\},\ x[t],\ \{t, 0, 1\}\right]$

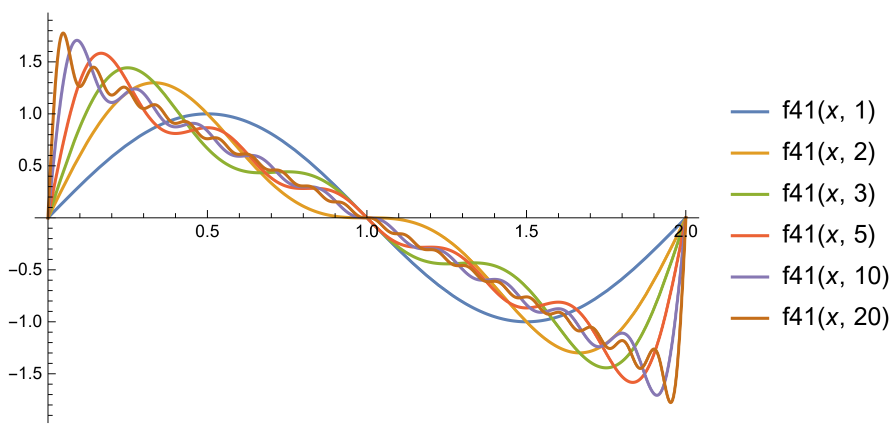
4. Visualizing Fourier Series

(1)

In[3]:= f41[x_, n_] := $\sum_{k=1}^{n} \frac{1}{k}$ Sin[k * Pi * x]

In[6]:=
```
Plot[{f41[x, 1], f41[x, 2], f41[x, 3], f41[x, 5], f41[x, 10], f41[x, 20]},
 {x, 0, 2}, PlotLegends → "Expressions"]
```

Out[6]=

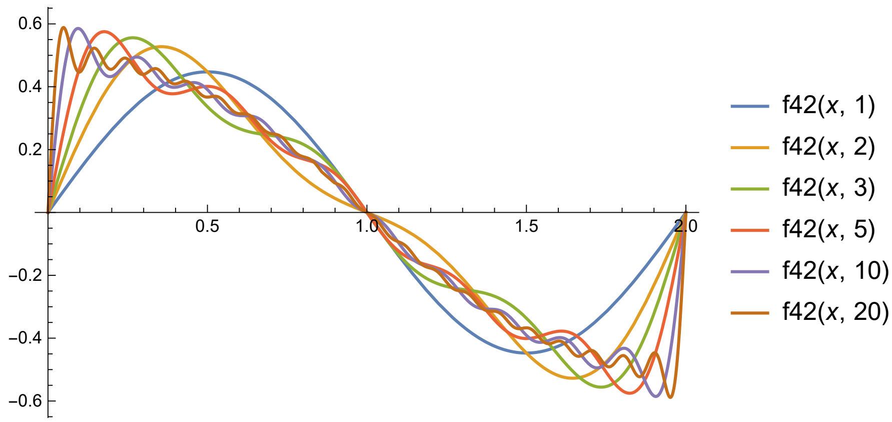


(2)

In[7]:=
```
f42[x_, n_] := Sum[(1/(Pi*k) + 2/(Pi*k)^3 (1 - Cos[Pi*k])) Sin[k*Pi*x], {k, 1, n}]
```

$$f42[x\_, n\_] := \sum_{k=1}^{n} \left(\frac{1}{Pi * k} + \frac{2}{(Pi * k)\,^{\wedge}3} (1 - Cos[Pi * k])\right) Sin[k * Pi * x]$$

In[8]:=
```
Plot[{f42[x, 1], f42[x, 2], f42[x, 3], f42[x, 5], f42[x, 10], f42[x, 20]},
  {x, 0, 2}, PlotLegends → "Expressions"]
```

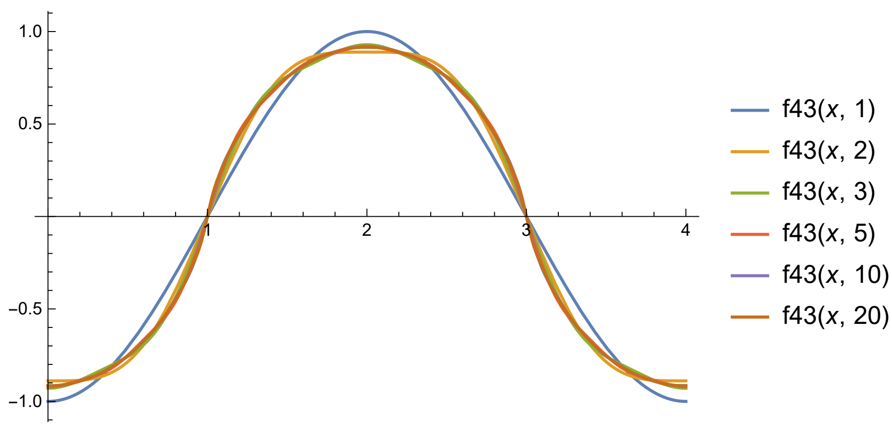
Out[8]=


(3)

In[9]:=

$$f43[x\_, n\_] := \sum_{k=1}^{n} \left(\frac{(-1)\,^{\wedge}k}{(2\,k - 1)\,^{\wedge}2}\right) Cos\left[\frac{2\,k - 1}{2} * Pi * x\right]$$

In[10]:=
```
Plot[{f43[x, 1], f43[x, 2], f43[x, 3], f43[x, 5], f43[x, 10], f43[x, 20]},
  {x, 0, 4}, PlotLegends → "Expressions"]
```

Out[10]=


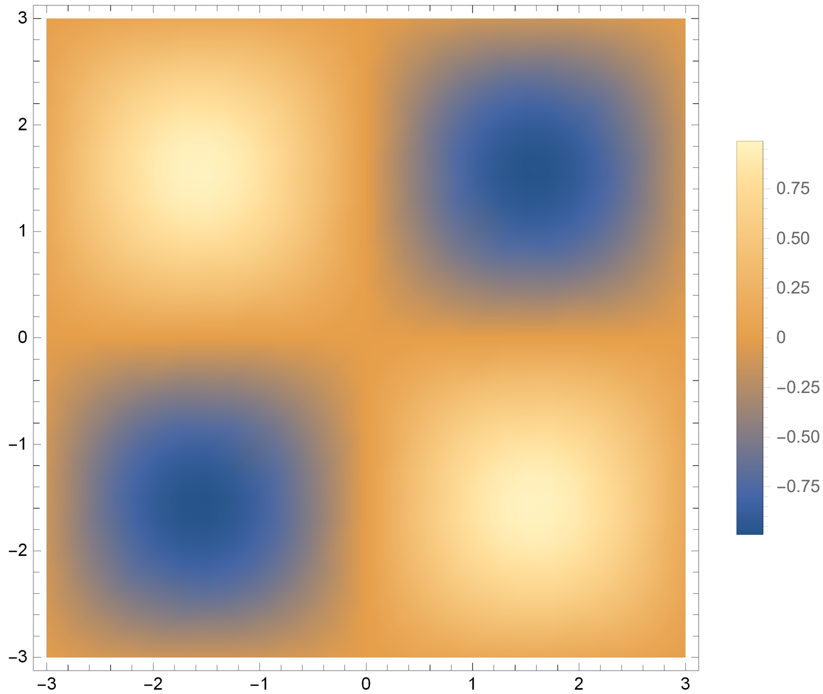
(4)

In[∘]:=

$$f44[x\_, y\_, n\_] := \sum_{k=1}^{n} \frac{(-1)\,^{\wedge}k}{k} Sin[(2\,k - 1)\,x]\,Sin[(2\,k - 1)\,y]$$

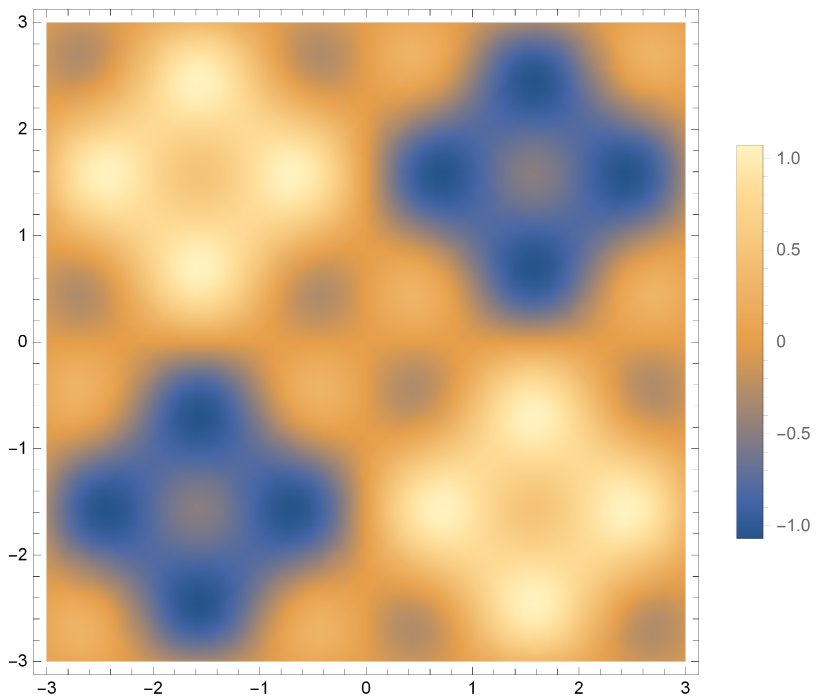
In[ ]:=
```
DensityPlot[f44[x, y, 1], {x, -3, 3}, {y, -3, 3}, PlotLegends → Automatic]
```

Out[ ]=



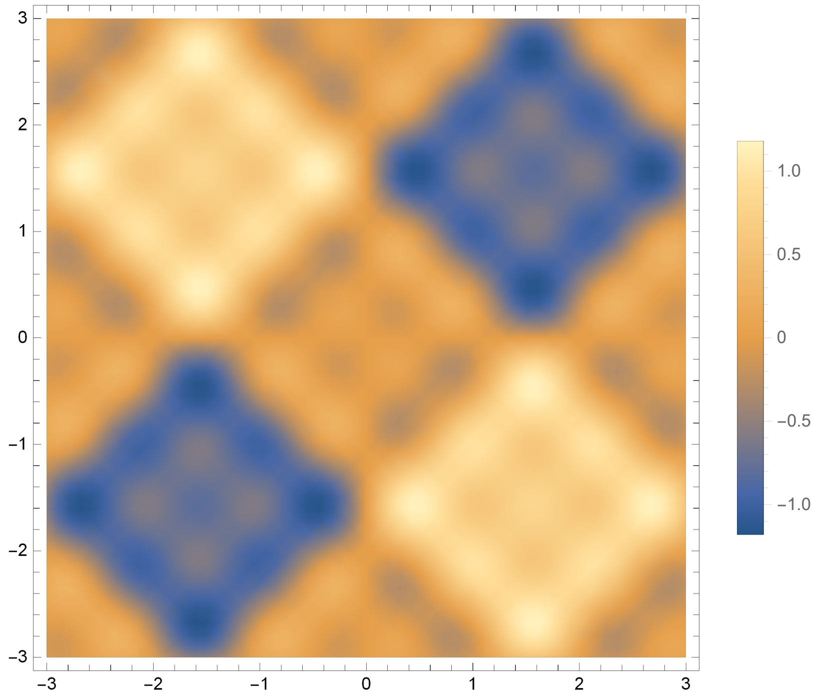
In[ ]:=
```
DensityPlot[f44[x, y, 2], {x, -3, 3}, {y, -3, 3}, PlotLegends → Automatic]
```

Out[ ]=



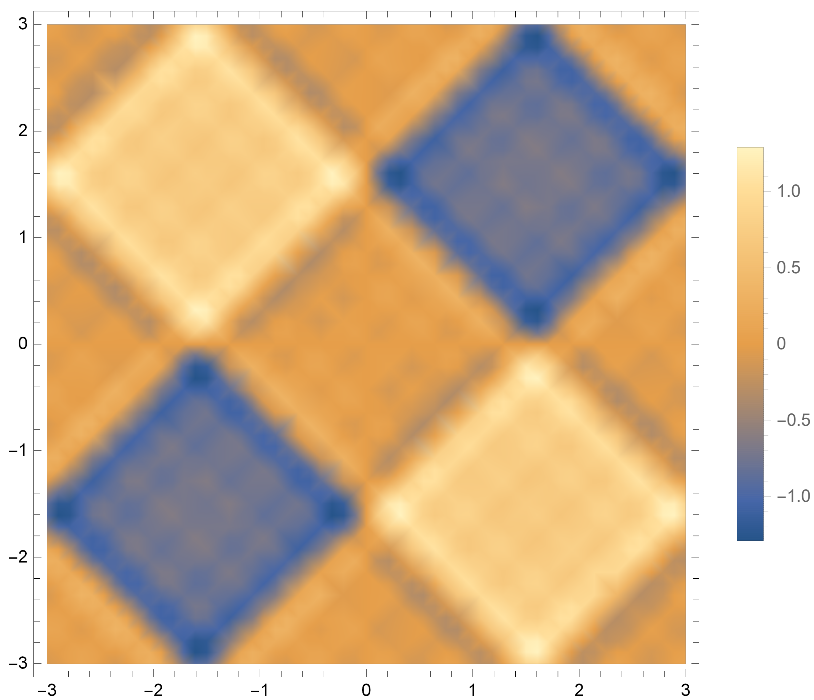
In[ ]:= `DensityPlot[f44[x, y, 3], {x, -3, 3}, {y, -3, 3}, PlotLegends → Automatic]`

Out[ ]=



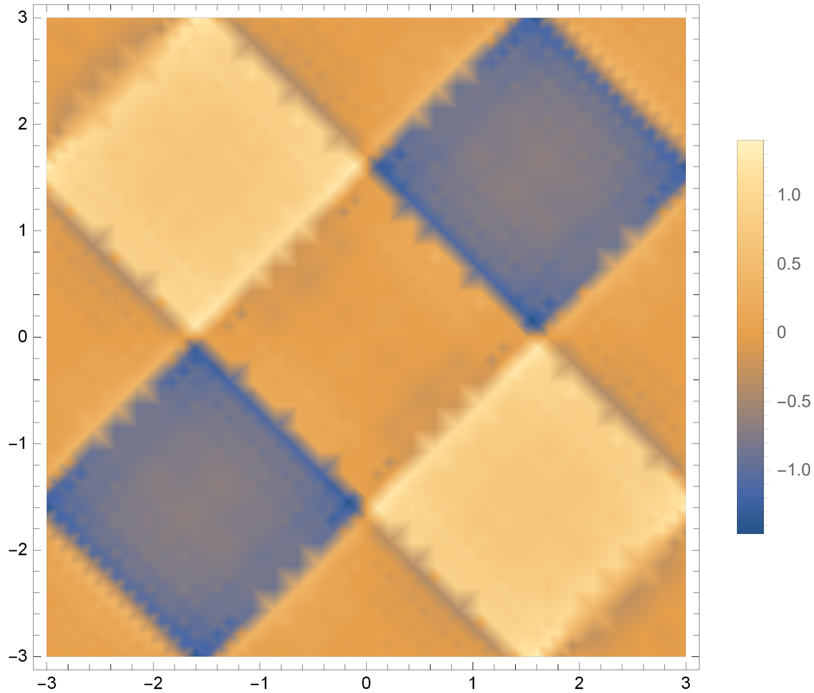
In[ ]:= `DensityPlot[f44[x, y, 5], {x, -3, 3}, {y, -3, 3}, PlotLegends → Automatic]`

Out[ ]=



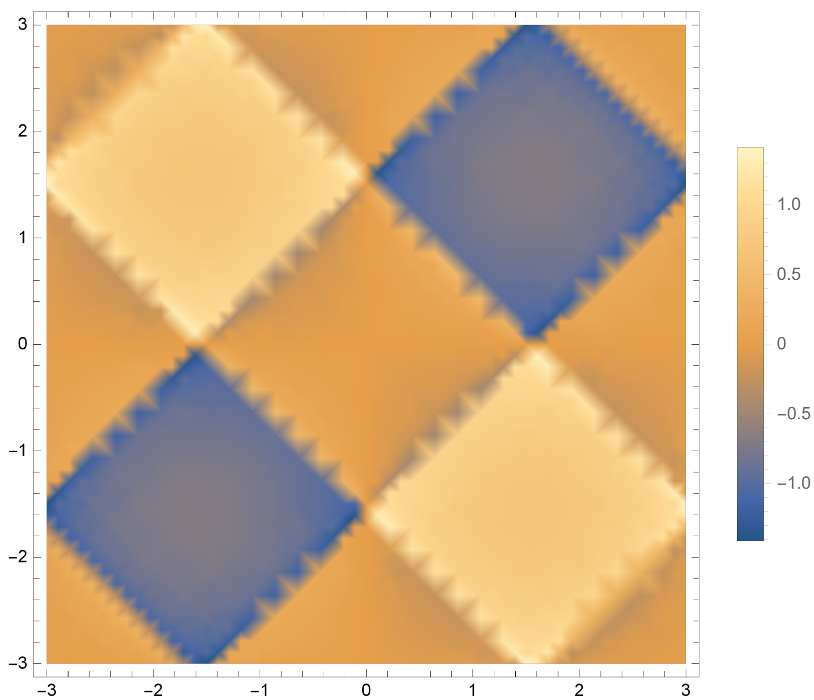
In[◦]:=
```
DensityPlot[f44[x, y, 10], {x, -3, 3}, {y, -3, 3},
 PlotLegends → Automatic, PerformanceGoal → "Quality"]
```

Out[◦]=



In[◦]:=
```
DensityPlot[f44[x, y, 20], {x, -3, 3}, {y, -3, 3}, PlotLegends → Automatic]
```

Out[◦]=



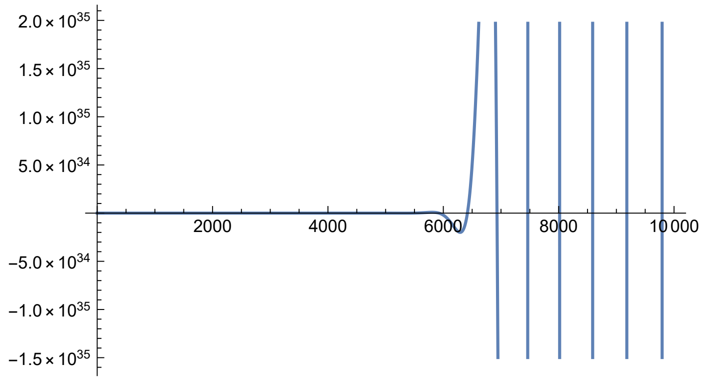
In[•]:=
```
Solve[0 == 2 + 2 Cos[Sqrt[w]] Cosh[Sqrt[w]], w]
```

... Solve: This system cannot be solved with the methods available to Solve.

Out[•]= $\text{Solve}\left[0 == 2+2\,\text{Cos}\left[\sqrt{w}\right]\,\text{Cosh}\left[\sqrt{w}\right],\ w\right]$

In[•]:=
```
Plot[2 + 2 Cos[Sqrt[w]] Cosh[Sqrt[w]], {w, 0, 10000}]
```

Out[•]=



Computing Fourier

In[33]:=
```
Δ[x_] := UnitTriangle[x]
```

In[80]:=
```
$Assumptions = {n ∈ Integers}
```

Out[80]= $\{n \in \mathbb{Z}\}$

In[86]:=
```
boxcar[x_] := UnitStep[x] - UnitStep[x - 1];
```

In[93]:= $\text{a1c}[n\_] := \dfrac{\int_{-1}^{1}\text{boxcar}[x]\,\text{Cos}\left[\frac{2*n*Pi*x}{2}\right]dx}{\int_{-1}^{1}\left(\text{Cos}\left[\frac{2*n*Pi*x}{2}\right]\right)\text{^}2\,dx}$

In[94]:=
```
a1c[0]
```

Out[94]= $\dfrac{1}{2}$

In[95]:= $\dfrac{\int_{-1}^{1}\text{boxcar}[x]\,\text{Sin}\left[\frac{2*n*Pi*x}{2}\right]dx}{\int_{-1}^{1}\left(\text{Sin}\left[\frac{2*n*Pi*x}{2}\right]\right)\text{^}2\,dx}$

Out[95]= $\dfrac{1+(-1)^{1+n}}{n\,\pi}$

(d)

$\dfrac{\int_{-1}^{1}\Delta[x]\,\text{Sin}\left[\frac{2*n*Pi*x}{2}\right]dx}{\int_{-1}^{1}\left(\text{Sin}\left[\frac{2*n*Pi*x}{2}\right]\right)\text{^}2\,dx}$

Out[12]= 0

In[100]:= $\texttt{a1d[n\_] := FullSimplify}\left[\dfrac{\int_{-1}^{1} \Delta[x]\,\text{Cos}\left[\frac{2*n*Pi*x}{2}\right] dx}{\int_{-1}^{1}\left(\text{Cos}\left[\frac{2*n*Pi*x}{2}\right]\right)\text{^}2\, dx}\right]$

In[105]:= `a1d[n]`

Out[105]= $-\dfrac{2\,(-1+(-1)^n)}{n^2\,\pi^2}$

(f)

In[109]:= $\texttt{FullSimplify}\left[\dfrac{\int_{-1}^{1} \Delta[x]\,\text{Sin}[5*Pi*x]*\text{Sin}\left[\frac{2*n*Pi*x}{2}\right] dx}{\int_{-1}^{1}\left(\text{Sin}\left[\frac{2*n*Pi*x}{2}\right]\right)\text{^}2\, dx}\right]$

Out[109]= $\dfrac{20\,(1+(-1)^n)\,n}{(-25+n^2)^2\,\pi^2}$

In[108]:= $\texttt{a1g[n\_] := FullSimplify}\left[\dfrac{\int_{-1}^{1} \Delta[x]\,\text{Sin}[5*Pi*x]*\text{Cos}\left[\frac{2*n\,Pi\,x}{2}\right] dx}{\int_{-1}^{1}\left(\text{Cos}\left[\frac{2*n*Pi*x}{2}\right]\right)\text{^}2\, dx}\right]$

In[112]:= `a1g[n]`

Out[112]= 0

In[37]:= $\texttt{FullSimplify}\left[\left(\int_{-1}^{1}\left(\Delta[x]\;*\text{Cos}\left[\frac{n\,Pi\,x}{2}\right]\right)\text{^}p\, dx\right)\text{^}(1/p)\right]$

Out[37]= $\left(\int_{-1}^{1}\left(\text{Cos}\left[\frac{n\,\pi\,x}{2}\right]\text{UnitTriangle}[x]\right)^p dx\right)^{\frac{1}{p}}$