

```

In[25]:= tri[x_] := UnitTriangle[x];
boxcar[x_] := UnitStep[x] - UnitStep[x - 1];
gaussian[x_] :=  $\left(\frac{1}{2 \pi}\right)^{1/2} \text{Exp}\left[-\frac{x^2}{2}\right]$ ;
erf[x_] :=  $\int_{-\infty}^x \text{gaussian}[y] \, dy$ ;
step[x_] := UnitStep[x];
estep[x_] := Exp[-x] step[x];
h[t_, x_] :=  $\left(\frac{1}{(2 t)^{1/2}}\right) * \text{gaussian}\left[x * \frac{1}{(2 t)^{1/2}}\right]$ ;

```

```

In[ ]:= h[t, x]

```

$$\text{Out[]} = \frac{e^{-\frac{x^2}{4t}}}{2 \sqrt{\pi} \sqrt{t}}$$

```

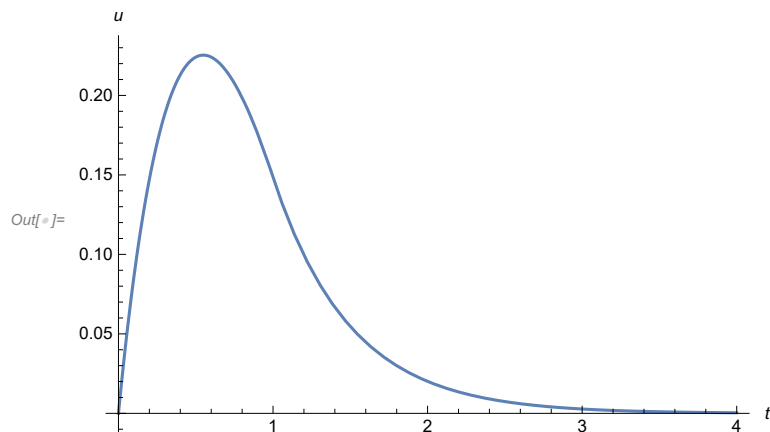
In[ ]:= ode21[t_] :=  $\int_0^t e^{-2(t-\tau)} \text{tri}[\tau] \, d\tau$ 

```

```

In[ ]:= Plot[ode21[t], {t, 0, 4}, AxesLabel -> {t, u}]

```

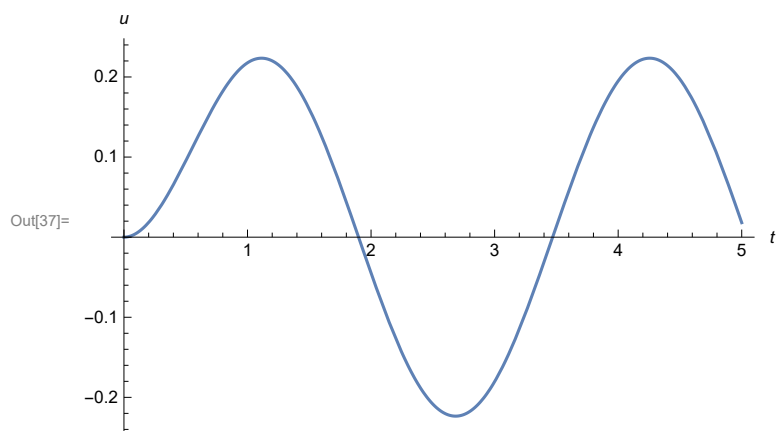


```

In[35]:= ode22[t_] :=  $\frac{1}{2} \text{NIntegrate}[\text{Sin}[2(t-\tau)] \text{tri}[\tau], \{\tau, 0, t\}]$ 

```

```
In[37]:= Plot[ode22[t], {t, 0, 5}, AxesLabel -> {t, u}]
```



```
In[ ]:=  $\tau_2[t_, x_] = x + t;$   

 $\xi_2[t_, x_] = x - t;$ 
```

```
In[ ]:= pde23[t_, x_] :=  $\frac{1}{2} \text{Exp}\left[\frac{-\tau_2}{2}\right] \int_0^{\tau_2[t,x]} \text{Exp}[s] \text{tri}[\tau_2[t, x]] \text{tri}[\xi_2[t, x]] ds$ 
```

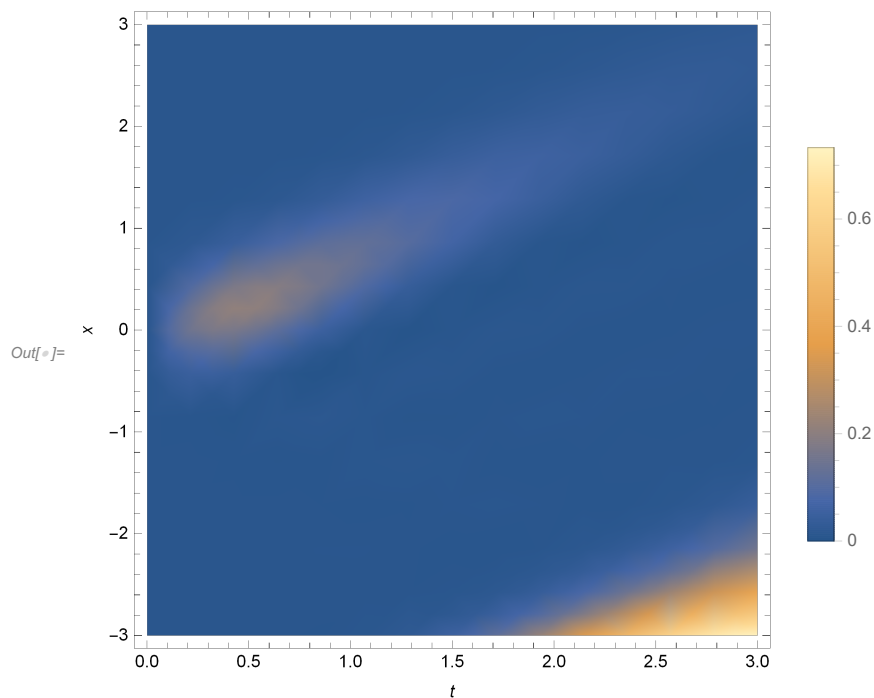
```
In[ ]:= s = NDSolve[{D[u[t, x], t] + u[t, x] + D[u[t, x], x] == tri[x + t] tri[x - t], u[0, x] == 0},  

u, {t, 0, 3}, {x, -3, 3}];
```

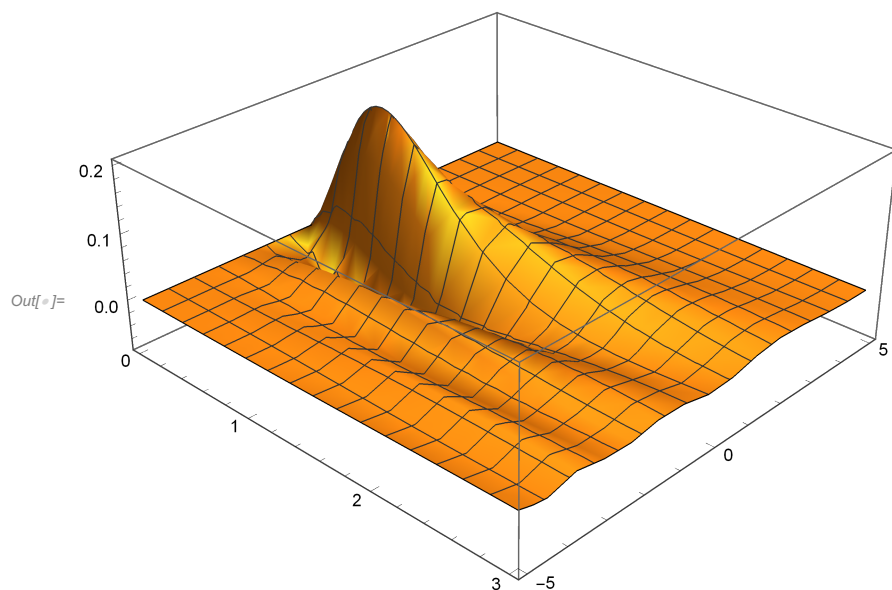
NDSolve: Warning: an insufficient number of boundary conditions have been specified for the direction of independent variable x. Artificial boundary effects may be present in the solution.

```
In[ ]:= DensityPlot[Evaluate[u[t, x] /. s], {t, 0, 3}, {x, -3, 3}, PlotRange -> All,  

FrameLabel -> {t, x}, PlotLegends -> Automatic, PerformanceGoal -> "Quality"]
```

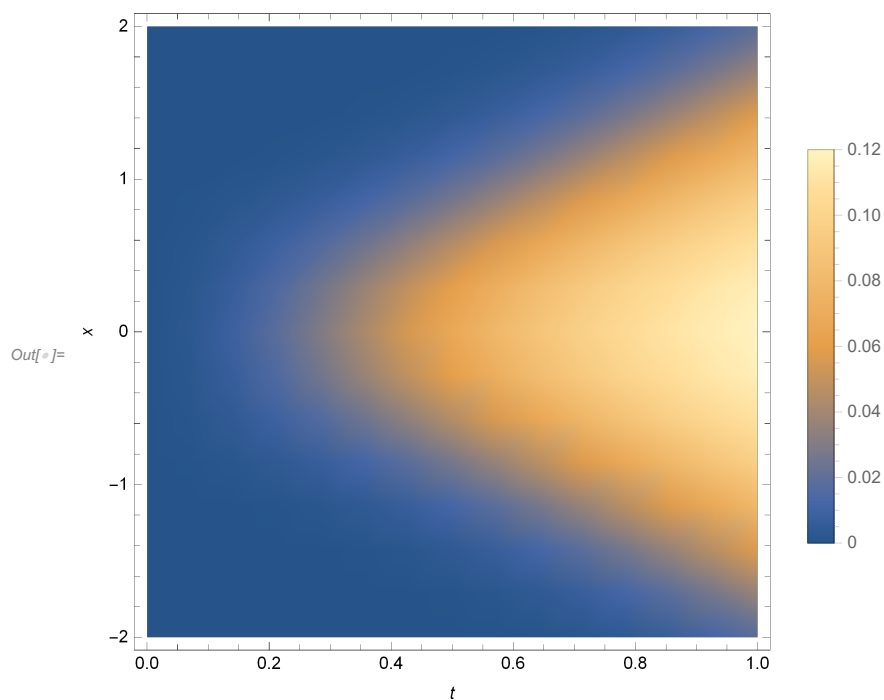


```
In[ ]:= Plot3D[Evaluate[u[t, x] /. s], {t, 0, 3},
  {x, -5, 5}, PlotRange -> All, PlotLegends -> Automatic]
```



```
In[ ]:= pde24[t_, x_] :=  $\frac{1}{4} \int_0^t \int_{x-2(t-\tau)}^{x+2(t-\tau)} \text{tri}[y] \text{tri}[\tau] dy d\tau$ 
```

```
DensityPlot[pde24[t, x], {t, 0, 3}, {x, -3, 3},
  PerformanceGoal -> "Quality", FrameLabel -> {t, x}, PlotLegends -> Automatic]
```



```
In[38]:= pde25[t_, x_] :=
  NIntegrate[NIntegrate[(tri[y] tri[tau - 2]) h[t - tau, x - y], {y, -infinity, infinity}], {tau, 0, t}]
```

```
DensityPlot[pde25[t, x], {t, 0, 4}, {x, -1, 1},  
PerformanceGoal -> "Quality", FrameLabel -> {t, x}, PlotLegends -> Automatic]
```

