Final_SM24

August 17, 2024

[1]:
```
import numpy as np
import cvxpy as cp
import matplotlib.pyplot as plt
import sys, os
import seaborn as sns
sys.path.insert(0, os.path.abspath('/home/qdeng/Github/
↪cvxbook_additional_exercises/python'))
```

[2]:
```
cp.version.full_version
```

[2]: '1.5.2'

0.1 Problem 1 – Completed

[3]:
```
from seasonal_shading_data import *
```

[4]:
```
T = s.shape[0]
n = 365 # Periodicity
omega = 300 # hyperparameter
T
```

[4]: 1095

[5]:
```
gamma = cp.Variable(T)

obj = (
    -1 * cp.sum(
        [cp.log(gamma[t])+gamma[t]*np.log(s[t]) for t in range(T)]
    )
    + omega * cp.sum(
        [cp.square(gamma[t+1]-gamma[t]) for t in range(n)]
    )
    + omega * cp.square(gamma[365]-gamma[0])
)

obj
```

[5]: Expression(CONVEX, UNKNOWN, ())

[6]:
```
constraints = []
constraints += [gamma[t] == gamma[t+n] for t in range(T-n)]
```

[7]:
```
prob = cp.Problem(
    cp.Minimize(obj),
    constraints
)
prob.solve()
```

```
/home/qdeng/.pyenv/versions/3.12.1/envs/cvx/lib/python3.12/site-
packages/cvxpy/problems/problem.py:158: UserWarning: Objective contains too many
subexpressions. Consider vectorizing your CVXPY code to speed up compilation.
  warnings.warn("Objective contains too many subexpressions. "
```

[7]: np.float64(-4.579298006620498)

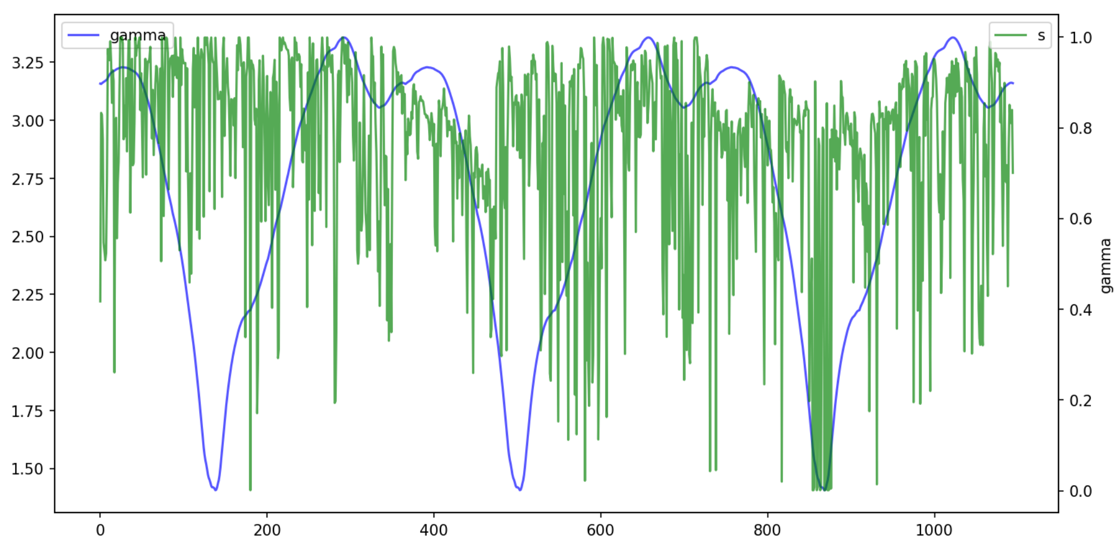
[8]:
```
fig, ax = plt.subplots(figsize=(12,6), dpi=150)
ax.plot(range(T),gamma.value,'b',label='gamma',alpha=2/3)



ax2 = ax.twinx()
ax2.plot(range(T),s,'g',label='s',alpha=2/3)
ax2.set_ylabel('s')

ax.legend(loc=2)
ax2.legend(loc=1)
ax2.set_ylabel('gamma')
# ax.grid()
```

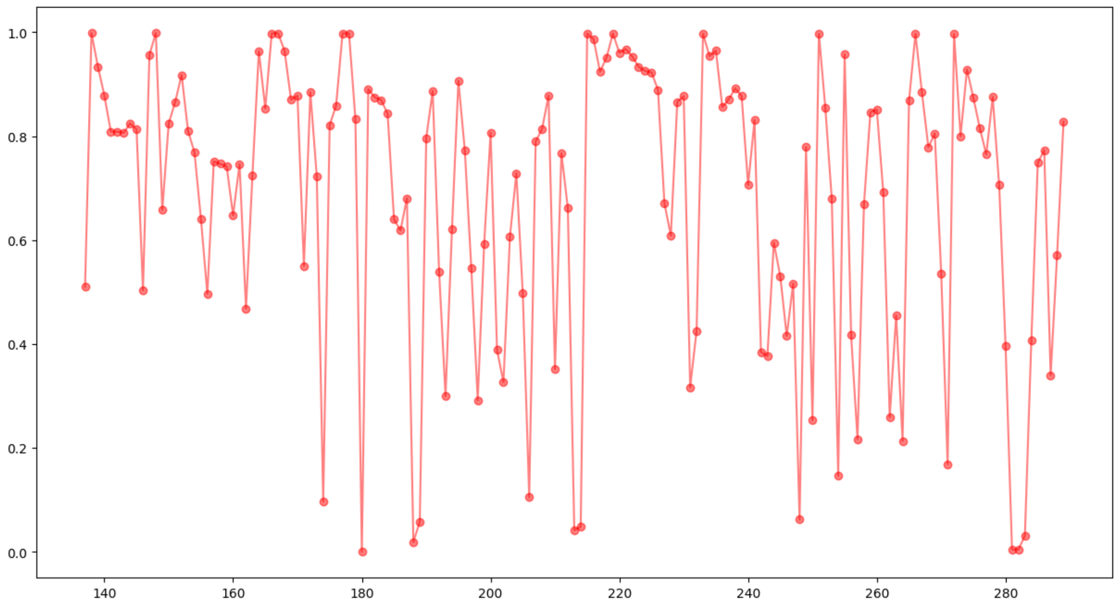
[8]: Text(0, 0.5, 'gamma')



[9]:
```
T_plt = np.arange(138-1, 290, 1) # Zero indexing in Python and 1-indexing in␣
↪Problem
T_plt

fig, ax = plt.subplots(figsize=(15,8))
ax.plot(T_plt,
        [s[t]**gamma.value[t] for t in T_plt],
         'ro-',label='gamma',alpha=1/2)
```

[9]:
```
[<matplotlib.lines.Line2D at 0x73d04ecd9640>]
```

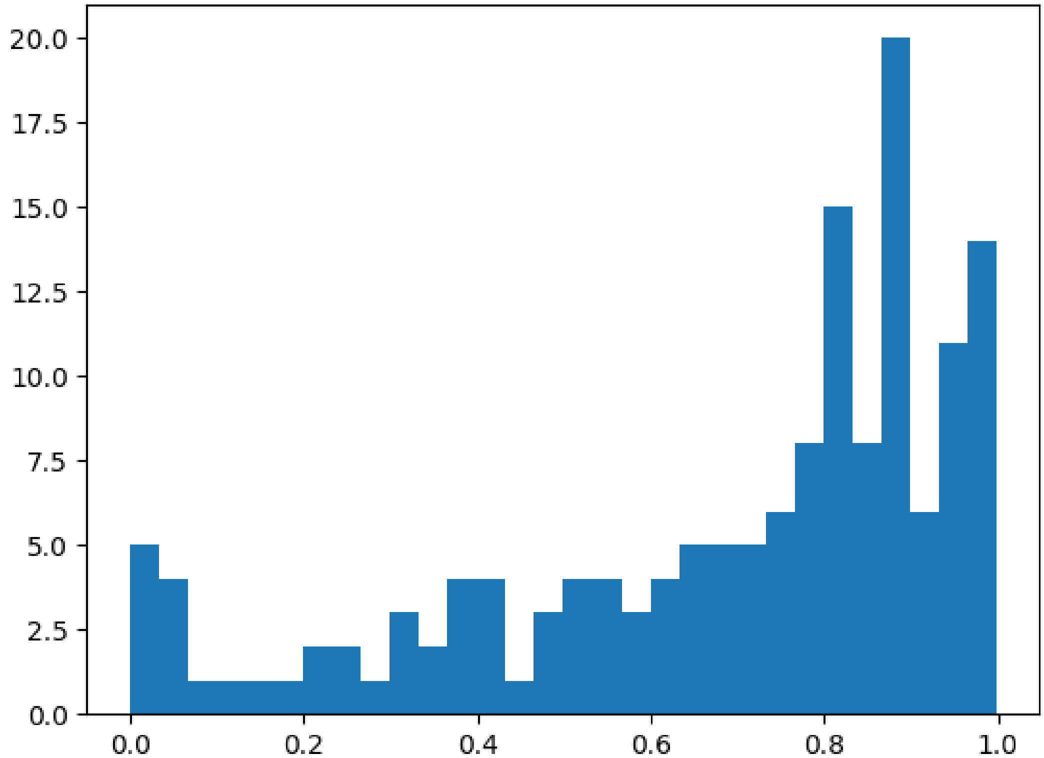


[10]:
```
fig, ax = plt.subplots()

ax.hist([s[t]**gamma.value[t] for t in T_plt],bins=30)
```

[10]:
```
(array([ 5.,  4.,  1.,  1.,  1.,  1.,  2.,  2.,  1.,  3.,  2.,  4.,  4.,
         1.,  3.,  4.,  4.,  3.,  4.,  5.,  5.,  5.,  6.,  8., 15.,  8.,
        20.,  6., 11., 14.]),
 array([2.84897304e-07, 3.32782113e-02, 6.65561377e-02, 9.98340640e-02,
        1.33111990e-01, 1.66389917e-01, 1.99667843e-01, 2.32945770e-01,
        2.66223696e-01, 2.99501622e-01, 3.32779549e-01, 3.66057475e-01,
        3.99335401e-01, 4.32613328e-01, 4.65891254e-01, 4.99169181e-01,
        5.32447107e-01, 5.65725033e-01, 5.99002960e-01, 6.32280886e-01,
        6.65558813e-01, 6.98836739e-01, 7.32114665e-01, 7.65392592e-01,
        7.98670518e-01, 8.31948444e-01, 8.65226371e-01, 8.98504297e-01,
```

```
         9.31782224e-01, 9.65060150e-01, 9.98338076e-01]),
 <BarContainer object of 30 artists>)
```



Comments: - the optimal objective value of the problem is -4.579 - Over the course of three years, we observe higher gamma around the summer months (around July), and lower around winter month (around December, that is July + ~150 day) - The variation in gamma following the pattern in s with periodicity constraints. While the first year did not see a low shade fraction in the winter months, the drop in winter months for the second and third years have a significant impact on our estimation of gamma and result in low gamma, which correponds to our interpretation that a lower gamma means the shading factor is getting more likely than higher shade factors.

0.2 Problem 2 – Completed

[3]:
```
from smooth_ride_plot import *
```

[4]:
```
T = 301  # Total time periods (in seconds)
K = 5  # Number of green lights
L = 3000  # Total length of the route (in meters)
g = [10,50,100,200,240]# Time green light turns on
r = [40,80,130,230,270]# Time green light turns off
l = [300,825,1620,1900,2800]# Positions of the lights (in meters)
S_min = 4.0  # Minimum speed (m/s)
```

```
S_max = 16.0  # Maximum speed (m/s)
```

[5]:
```
p = cp.Variable(T)
s = cp.Variable(T-1)
a = cp.Variable(T-2)
j = cp.Variable(T-3)

# obj = 1/(T-2)*cp.sum_squares(j)
obj = 1/(T-2)*cp.sum([cp.square(j_i) for j_i in j])

obj
```

[5]:
```
Expression(CONVEX, NONNEGATIVE, ())
```

[6]:
```
constraints = []
constraints += [j[t]==a[t+1]-a[t] for t in range(T-3)]
constraints += [a[t]==s[t+1]-s[t] for t in range(T-2)]
constraints += [s[t]==p[t+1]-p[t] for t in range(T-1)]
constraints += [p[0]==0]
constraints += [p[-1]==L]
constraints += [s[0]==0]
constraints += [s[i+1]-S_min>=0 for i in range(T-2)]
constraints += [s-S_max<=0]

# Red Light constraints
for k in range(K):
    constraints += [
        p[t]>=l[k] for t in np.arange(r[k], T)
    ]

# Green Light constraints
for k in range(K):
    constraints += [
        p[t]<=l[k] for t in np.arange(0,g[k])
    ]
```

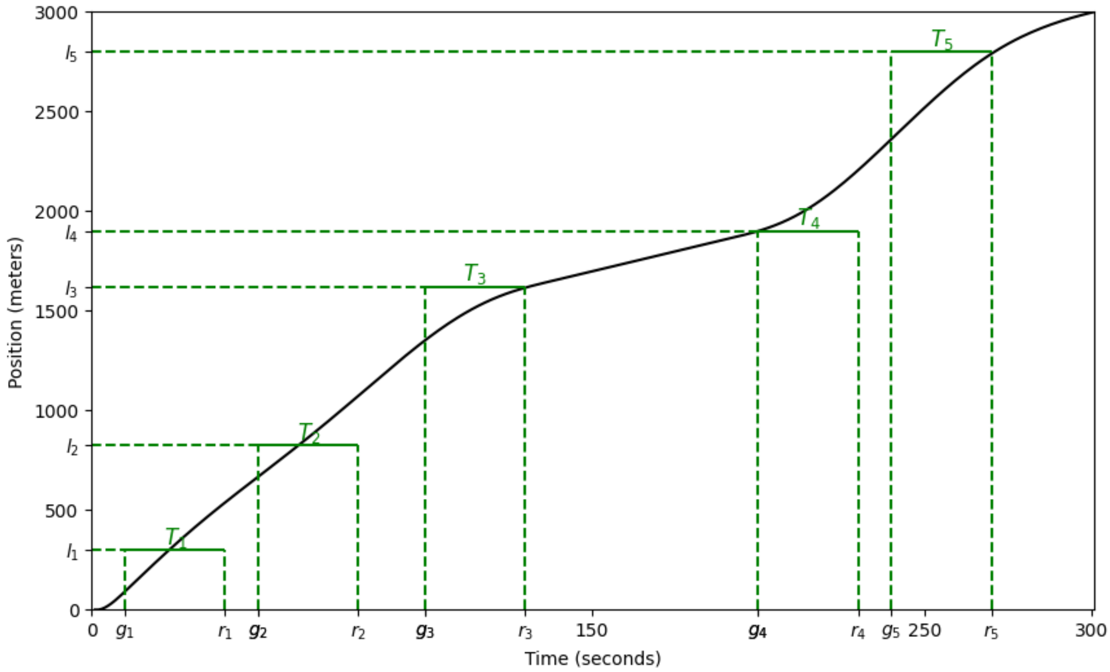
[7]:
```
prob = cp.Problem(
    cp.Minimize(obj),
    constraints
)
prob.solve('CLARABEL')
```

[7]:
```
np.float64(0.0069189503127390925)
```

[8]:
```
prob.is_dcp()
```

[8]:
```
True
```

[9]:
```
plot_trajectory(p.value, l, g, r)
```



Comment: - Compare to the piecewise linear trajectory shown in the problem prompt which violates Maximum Speed and ignore the jerkiness of the ride, the optimal trajectory that minimizes the jerk looks very much like a smoothened version of it – in the sense that, while we cross Light 1 and 2 in the middle of the green, we really waited until the last possible second to cross Light 3 and 5, and crossed Light 4 at the earliest possible instant.

0.3 Problem 3 – Completed

[3]:
```
from cccv_charging_plot import *
```

[4]:
```
Q_max = 6300
Q_min = 960
R = 0.4
a = 3.4
b = 500
Q_crit = 6925
I_max = 1.5
V_max = 4.22
E = 20975
T_fast = 120
T_normal = 180
T_slow = 240
h = 60
```

[5]:
```python
results = []

for T in [T_fast, T_normal, T_slow]:
    i = cp.Variable(T)
    q = cp.Variable(T+1)

    v_oc = cp.Variable(T)
    v = cp.Variable(T)

    obj = h*R*cp.sum([cp.square(i_i) for i_i in i])

    constraints = []
    constraints += [q[0]==Q_min]
    constraints += [q[-1]==Q_max]
    constraints += [q[t+1] == q[t] + h*i[t] for t in range(T)]
    constraints += [v[t] == v_oc[t] + R*i[t] for t in range(T)]
    constraints += [v - V_max<=0]
    constraints += [i - I_max<=0]
    constraints += [i >= 0]
    constraints += [v_oc[t] >= a + b * cp.inv_pos(Q_crit-q[t]) for t in␣
 ↪range(T)] #??? make this DCP

    prob = cp.Problem(
        cp.Minimize(obj),
        constraints
    )

    prob.solve()

    results.append(i.value)
    results.append(v.value)
    results.append(q.value)
```

```
/home/qdeng/.pyenv/versions/3.12.1/envs/cvx/lib/python3.12/site-
packages/cvxpy/problems/problem.py:1407: UserWarning: Solution may be
inaccurate. Try another solver, adjusting the solver settings, or solve with
verbose=True for more information.
  warnings.warn(
```

[6]:
```python
constraints[0].dual_value
```

[6]: 0.3036175337862522

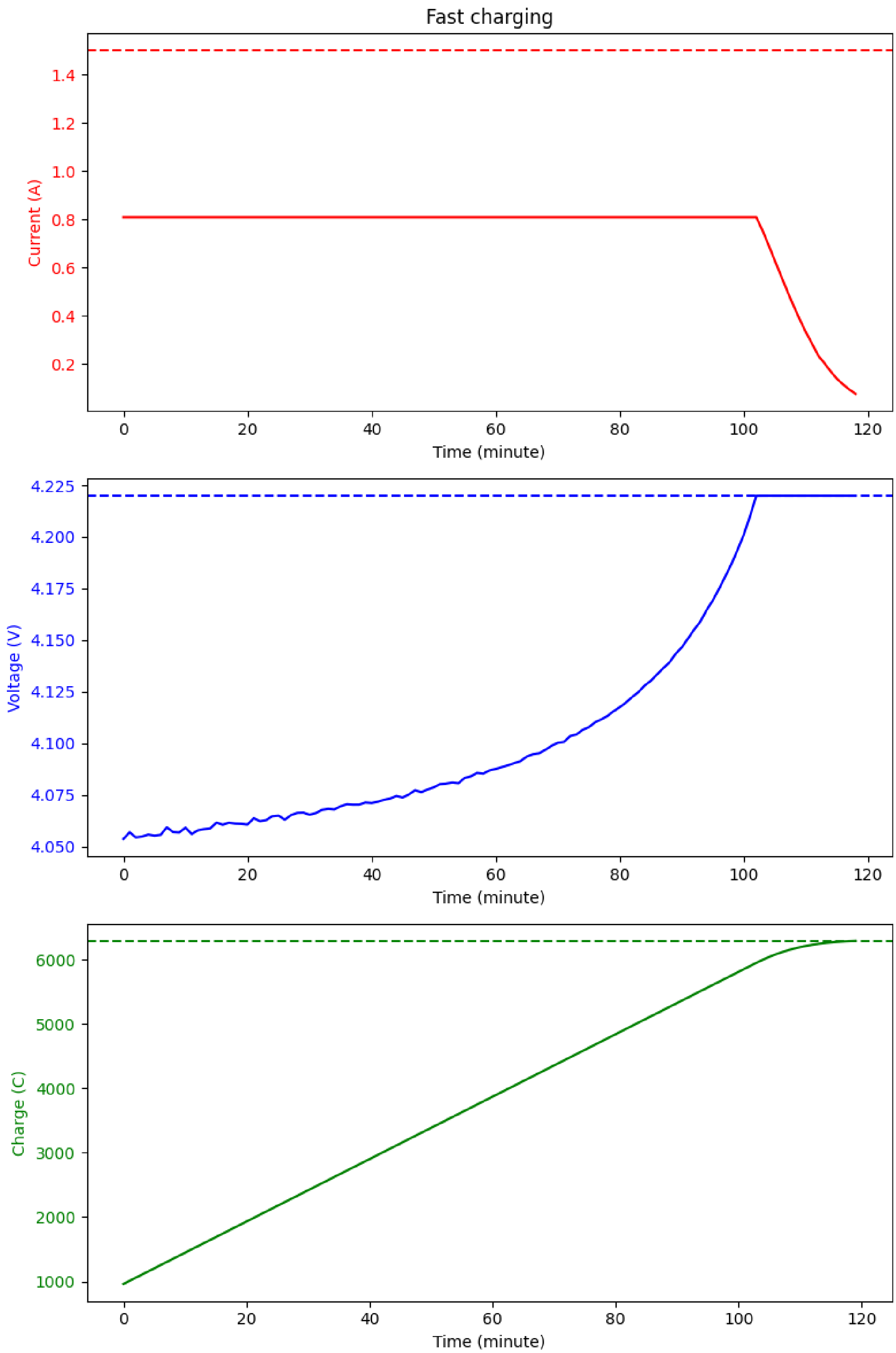
[7]:
```python
cp.inv_pos(Q_crit-q[0])
```

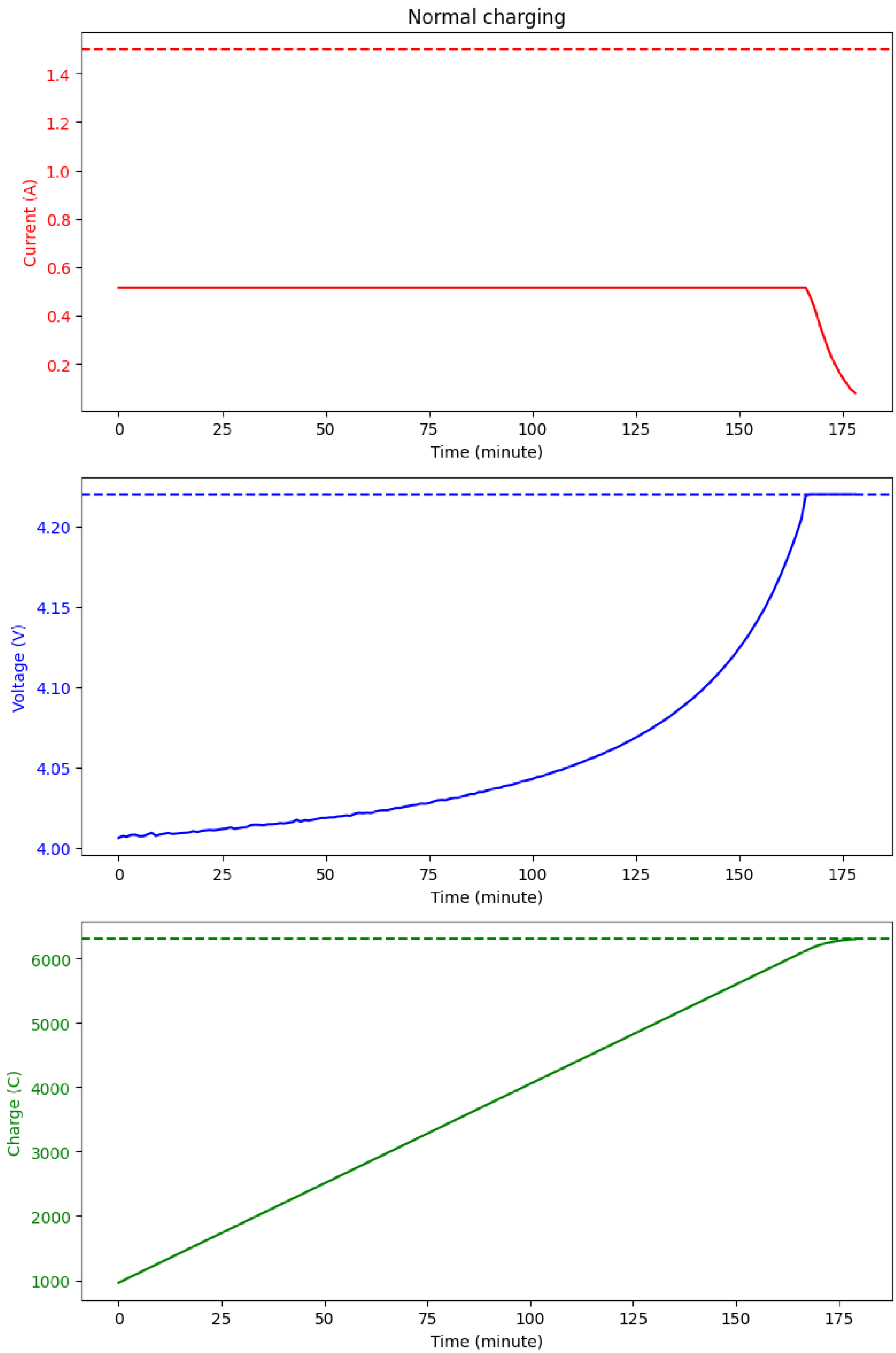
[7]: Expression(CONVEX, NONNEGATIVE, ())

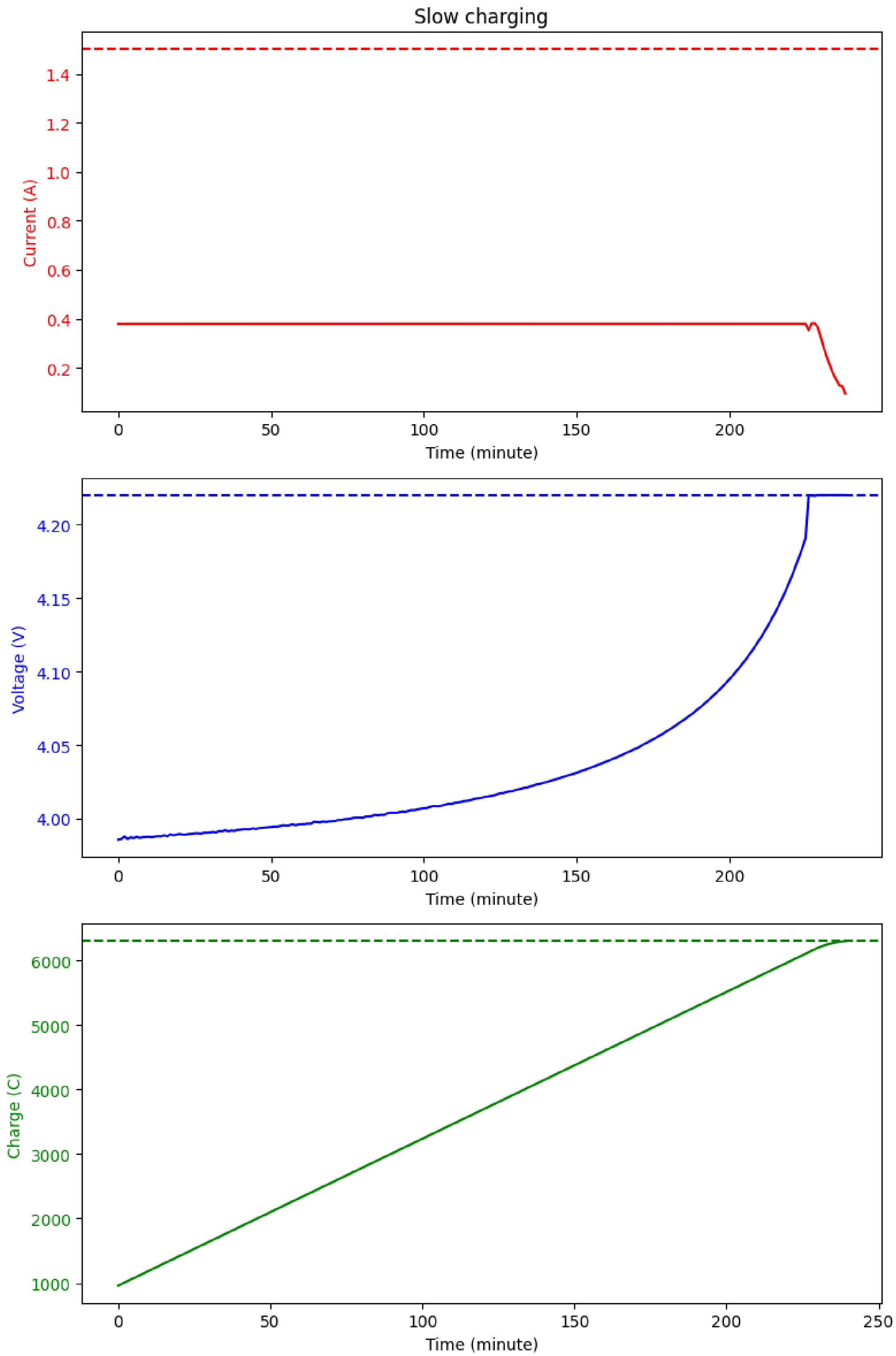
[8]:
```
len(results)
```

[8]: 9

[9]:
```
#Fast charging
# i_fast = i.value#np.zeros((T_fast,))
# v_fast = v.value#np.zeros((T_fast,))
# q_fast = q.value#np.zeros((T_fast,))
# #Normal charging
# i_normal = np.zeros((T_normal,))
# v_normal = np.zeros((T_normal,))
# q_normal = np.zeros((T_normal,))
# #Slow charging
# i_slow = np.zeros((T_slow,))
# v_slow = np.zeros((T_slow,))
# q_slow = np.zeros((T_slow,))
i_fast, v_fast, q_fast, i_normal, v_normal, q_normal, i_slow, v_slow, q_slow =␣
↪results
#Example usage
plot_charging(i_fast, v_fast, q_fast, i_normal, v_normal, q_normal, i_slow,␣
↪v_slow, q_slow, I_max, V_max, Q_max)
```


Fast charging
Current (A)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
20
40
60
80
100
120
Time (minute)
Voltage (V)
4.225
4.200
4.175
4.150
4.125
4.100
4.075
4.050
0
20
40
60
80
100
120
Time (minute)
Charge (C)
6000
5000
4000
3000
2000
1000
0
20
40
60
80
100
120
Time (minute)


Normal charging
Current (A)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
25
50
75
100
125
150
175
Time (minute)
Voltage (V)
4.20
4.15
4.10
4.05
4.00
Time (minute)
Charge (C)
6000
5000
4000
3000
2000
1000
Time (minute)



Slow charging
Current (A)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
50
100
150
200
Time (minute)
Voltage (V)
4.20
4.15
4.10
4.05
4.00
0
50
100
150
200
Time (minute)
Charge (C)
6000
5000
4000
3000
2000
1000
0
50
100
150
200
250
Time (minute)

Part (c) Comment: - The industry Standard CCCV charging is exactly what the solved optimal charging strategy is above. Across all three charging speed, we see a charging profile that consists of a constant current charging for the first majority amount of charging and switching to constant voltage but variable current charging once the maximum allowed voltage has been reached.

0.4 Problem 4 – Completed

[3]:
```
from cond_gauss_reg_data import *
```

[4]:
```
T, n = x.shape
x.shape, y.shape
```

[4]:
```
((1000, 10), (1000,))
```

[5]:
```
omega = cp.Variable(T)
kappa = cp.Variable(T)
alpha = cp.Variable(n)
beta = cp.Variable(n)
u = cp.Variable()
v = cp.Variable()
```

[6]:
```
omega == alpha@x.T+u
```

[6]:
```
Equality(Variable((np.int32(1000),), var1), Expression(AFFINE, UNKNOWN,
(np.int32(1000),)))
```

[7]:
```
kappa == beta@x.T+v
```

[7]:
```
Equality(Variable((np.int32(1000),), var2), Expression(AFFINE, UNKNOWN,
(np.int32(1000),)))
```

[8]:
```
cp.log(omega)-1/2*(cp.multiply(y, omega)-kappa)
```

[8]:
```
Expression(CONCAVE, UNKNOWN, (np.int32(1000),))
```

[9]:
```
cp.multiply(y, omega)
```

[9]:
```
Expression(AFFINE, UNKNOWN, (np.int32(1000),))
```

[10]:
```
cp.square(cp.multiply(y, omega)-kappa)
```

[10]:
```
Expression(CONVEX, NONNEGATIVE, (np.int32(1000),))
```

[11]:
```
obj = cp.sum(
    cp.log(omega)-1/2*cp.square(cp.multiply(y, omega)-kappa)
```

```
)
obj
```

[11]: Expression(CONCAVE, UNKNOWN, ())

[12]:
```
constraints = []
constraints += [omega == alpha@x.T+u]
constraints += [kappa == beta@x.T+v]
constraints += [-cp.norm2(alpha)+u>=0]
constraints += [cp.norm_inf(alpha)<=1]
# constraints += [alpha@x[i,:]+u>=0 for i in range(T)]
# constraints += [omega>=0]
```

[16]:
```
prob = cp.Problem(
    cp.Maximize(obj),
    constraints
)
prob.solve()
```

[16]: np.float64(400.59972701237064)

[17]:
```
sigma = 1/omega.value
mu = np.multiply(omega.value, kappa.value)
```

[18]:
```
print('Distribution Mean of y1 given x1: %.2f'%mu[0]**2)
print('Distribution Variance of y1 given x1: %.2f'%sigma[0]**2)
```

```
Distribution Mean of y1 given x1: 1.51
Distribution Variance of y1 given x1: 0.16
```

0.5 Problem 5 – Completed

[3]:
```
n = 5
lamb = 10
C = cp.Variable((n,n))

Cg = np.array(
[
    [1, -0.5, 0, 0, 0.7],
    [-0.5, 1, 0, -0.6, 0.8],
    [0, 0, 1, 0.3, 0],
    [0, -0.6, 0.3, 1, 0],
    [0.7, 0.8, 0, 0, 1],
]
)
```

```
Cg==Cg.T
```

[3]:
```
array([[ True,  True,  True,  True,  True],
       [ True,  True,  True,  True,  True],
       [ True,  True,  True,  True,  True],
       [ True,  True,  True,  True,  True],
       [ True,  True,  True,  True,  True]])
```

[4]:
```
Cg!=0
```

[4]:
```
array([[ True,  True, False, False,  True],
       [ True,  True, False,  True,  True],
       [False, False,  True,  True, False],
       [False,  True,  True,  True, False],
       [ True,  True, False, False,  True]])
```

[5]:
```
Cg[Cg!=0]
```

[5]:
```
array([ 1. , -0.5,  0.7, -0.5,  1. , -0.6,  0.8,  1. ,  0.3, -0.6,  0.3,
        1. ,  0.7,  0.8,  1. ])
```

[6]:
```
obj = -cp.log_det(C) + lamb*cp.norm1(cp.hstack([C[Cg!=0]-Cg[Cg!=0]]))
obj
```

[6]:
```
Expression(CONVEX, UNKNOWN, ())
```

[7]:
```
constraints = []
constraints += [C>>0]
constraints += [C[i,i]==1 for i in range(n)]
```

[8]:
```
prob = cp.Problem(
    cp.Minimize(obj),
    constraints
)
prob.solve()
```

[8]:
```
np.float64(14.152968271939406)
```

[9]:
```
constraints[0].dual_value
```

[9]:
```
array([[ 1.65250690e-15,  0.00000000e+00, -1.46062354e-16,
         1.16849883e-15,  7.78999222e-16],
       [ 0.00000000e+00,  1.65250690e-15, -3.89499611e-16,
        -3.89499611e-16,  1.16849883e-15],
       [-1.46062354e-16, -3.89499611e-16,  6.05919196e-15,
         1.94749805e-15, -2.28831021e-15],
       [ 1.16849883e-15, -3.89499611e-16,  1.94749805e-15,
```

```
 -5.50835632e-16, -7.78999222e-16],
[ 7.78999222e-16,  1.16849883e-15, -2.28831021e-15,
 -7.78999222e-16,  2.75417816e-15]])
```

[10]:
```
np.round(C.value, 2)
```

[10]:
```
array([[ 1.  , -0.48,  0.09,  0.29,  0.48],
       [-0.48,  1.  , -0.18, -0.6 ,  0.48],
       [ 0.09, -0.18,  1.  ,  0.3 , -0.09],
       [ 0.29, -0.6 ,  0.3 ,  1.  , -0.29],
       [ 0.48,  0.48, -0.09, -0.29,  1.  ]])
```

[11]:
```
Cg
```

[11]:
```
array([[ 1. , -0.5,  0. ,  0. ,  0.7],
       [-0.5,  1. ,  0. , -0.6,  0.8],
       [ 0. ,  0. ,  1. ,  0.3,  0. ],
       [ 0. , -0.6,  0.3,  1. ,  0. ],
       [ 0.7,  0.8,  0. ,  0. ,  1. ]])
```

Comment - 6 of the off diagonal entries from C_g were modified 1. Cg(1,2) = -0.5 -> C(1,2) = -0.48 2. Cg(2,1) = -0.5 -> C(2,1) = -0.48 3. Cg(1,5) = 0.7 -> C(1,5) = 0.48 2. Cg(5,1) = 0.7 -> C(5,1) = 0.48 3. Cg(2,5) = 0.8 -> C(2,5) = 0.48 2. Cg(5,2) = 0.8 -> C(5,2) = 0.48

0.6 Problem 6 – Completed

[3]:
```
T = 20
n = 3
m = 4

x0 = np.array([0,0,0])
xT = np.array([1, 2, -1])

A = np.array([
    [1, 0.1, 0],
    [0, 9,-0.1],
    [-0.1, 0, 0.9]
])

B = np.array(
    [
        [0.9, -0.4, 0, 0.6],
        [2.0, 0.7, 0.2, -0.4],
        [0.0, 0.2, -0.3, 1.7]
    ]
)
```

```
list_bin = []
for i in range(2**m):
    bin_index = f'{i:04b}'
    list_bin.append(bin_index)

bool_index = [[True if digit=='1' else False for digit in num] for num in␣
↪list_bin]

list_B = []
for i_list in bool_index:
    print(i_list)
    B_i = B.copy()
    B_i[:, i_list] = 0

    list_B.append(B_i)
```

```
[False, False, False, False]
[False, False, False, True]
[False, False, True, False]
[False, False, True, True]
[False, True, False, False]
[False, True, False, True]
[False, True, True, False]
[False, True, True, True]
[True, False, False, False]
[True, False, False, True]
[True, False, True, False]
[True, False, True, True]
[True, True, False, False]
[True, True, False, True]
[True, True, True, False]
[True, True, True, True]
```

[4]:
```
list_feasible = []
for Bi in list_B:
    x = cp.Variable((n,T))
    u = cp.Variable((m,T-1))

    obj = 0

    constraints = []
    constraints += [x[:,0]==x0]
    constraints += [x[:,-1]==xT]
    constraints += [cp.norm_inf(u[:, t])<=1 for t in range(T-1)]
    constraints += [x[:,t+1]==A@x[:,t]+Bi@u[:,t] for t in range(T-1)]

    prob = cp.Problem(
```

```
        cp.Minimize(obj),
        constraints
    )

    if prob.solve()==0:
        list_feasible.append(1)
    else:
        list_feasible.append(0)
```

[5]:
```
list_feasible
```

[5]: [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0]

[6]:
```
np.array(list_feasible)
```

[6]: array([1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0])

[7]:
```
np.sum(np.array(bool_index), axis=1)#[list_feasible]
```

[7]: array([0, 1, 1, 2, 1, 2, 2, 3, 1, 2, 2, 3, 2, 3, 3, 4])

[8]:
```
np.array(bool_index)[(np.array(list_feasible))&(np.sum(np.array(bool_index),␣
↪axis=1)==3)==1,:]
```

[8]:
```
array([[False,  True,  True,  True],
       [ True, False,  True,  True],
       [ True,  True,  True, False]])
```

Comment: - The minimum number of actuators need to achieve a trajectory is 1 - This can be achieved by 1. Having only the first (0) actuator: u_0 used, u_1 u_2 u_3 unused 2. Having only the second (1) actuator: u_1 used, u_0 u_2 u_3 unused 3. Having only the fourth (3) actuator: u_3 used, u_0 u_1 u_2 unused

[9]:
```
# Verification for only first actuator
x = cp.Variable((n,T))
u = cp.Variable((m,T-1))

obj = 0

constraints = []
constraints += [x[:,0]==x0]
constraints += [x[:,-1]==xT]
constraints += [cp.norm_inf(u[:, t])<=1 for t in range(T-1)]
constraints += [x[:,t+1]==A@x[:,t]+B@u[:,t] for t in range(T-1)]

constraints += [u[[1,2,3],:]==0]
```

```
prob = cp.Problem(
    cp.Minimize(obj),
    constraints
)

prob.solve()
```

[9]: 0.0

[10]:

```
# Verification for only second actuator
x = cp.Variable((n,T))
u = cp.Variable((m,T-1))

obj = 0

constraints = []
constraints += [x[:,0]==x0]
constraints += [x[:,-1]==xT]
constraints += [cp.norm_inf(u[:, t])<=1 for t in range(T-1)]
constraints += [x[:,t+1]==A@x[:,t]+B@u[:,t] for t in range(T-1)]

constraints += [u[[0,2,3],:]==0]

prob = cp.Problem(
    cp.Minimize(obj),
    constraints
)

prob.solve()
```

[10]: 0.0

[11]:

```
# Verification for only fourth actuator
x = cp.Variable((n,T))
u = cp.Variable((m,T-1))

obj = 0

constraints = []
constraints += [x[:,0]==x0]
constraints += [x[:,-1]==xT]
constraints += [cp.norm_inf(u[:, t])<=1 for t in range(T-1)]
constraints += [x[:,t+1]==A@x[:,t]+B@u[:,t] for t in range(T-1)]

constraints += [u[[0,1,2],:]==0]

prob = cp.Problem(
```

```
    cp.Minimize(obj),
    constraints
)

prob.solve()
```

[11]: 0.0

0.7 Problem 7 –Attempt 1

[3]:
```
A = np.array(
    [
        [0.2, -0.2, 0.6, -0.6] ,
        [-0.2, 0.4, -1.4, 1.3],
        [0.6, -1.4, 5.2, -4.7],
        [-0.6, 1.3, -4.7, 4.4]
    ]
)

n, _ = A.shape

I = np.eye(n)
```

[4]:
```
beta = 0.5
D = cp.Variable((n,n))

obj = 0

constraints = []
constraints += [D[i,j]==0 for i in range (n) for j in range(n) if i!=j]
constraints += [D-I>>0]
constraints += [cp.lambda_max(D)-beta*cp.lambda_min(D)<=0]

prob = cp.Problem(
    cp.Minimize(obj),
    constraints
)

prob.solve()
```

[4]: inf

[5]:
```
A
```

[5]:
```
array([[ 0.2, -0.2,  0.6, -0.6],
       [-0.2,  0.4, -1.4,  1.3],
       [ 0.6, -1.4,  5.2, -4.7],
```

```
       [-0.6,  1.3, -4.7,  4.4]])
```

[6]:
```
U, S, Vh = np.linalg.svd(A)
# U==Vh.T
U.T
```

[6]:
```
array([[-0.088678  ,  0.19660857, -0.71878621,  0.6609294 ],
       [ 0.84455772, -0.1993116 , -0.41296455, -0.27650935],
       [-0.47078505,  0.09100114, -0.54757315, -0.6857433 ],
       [ 0.23920682,  0.95568758,  0.11388731, -0.12833952]])
```

[7]:
```
A_half = np.diag(np.sqrt(S))@U.T
A_half.T@A_half
```

[7]:
```
array([[ 0.2, -0.2,  0.6, -0.6],
       [-0.2,  0.4, -1.4,  1.3],
       [ 0.6, -1.4,  5.2, -4.7],
       [-0.6,  1.3, -4.7,  4.4]])
```

[8]:
```
cp.square(cp.norm(A_half@D))
```

[8]:
```
Expression(CONVEX, NONNEGATIVE, ())
```

[9]:
```
cp.square(cp.norm(A_half@cp.inv_pos(D)))
```

[9]:
```
Expression(UNKNOWN, NONNEGATIVE, ())
```

[10]:
```
lambda_A_max = max(S)
lambda_A_min = min(S)
```

[30]:
```
u = 1000
l = 0

while (u-l)>1e-6:
    beta = (u+l)/2
    # print(beta)
    D = cp.Variable((n,n))

    obj = 0

    constraints = []
    constraints += [D[i,j]==0 for i in range (n) for j in range(n) if i!=j]
    constraints += [D-I>>0]
    constraints += [cp.lambda_max(A@D)-beta*cp.lambda_min(A@D)<=0]
    # constraints += [cp.lambda_max(D@A@D)-beta*cp.lambda_min(D@A@D)<=0] ###␣
 ↪CVXPY does not accept Matrix Quad Form
    # constraints += [(cp.square(cp.norm(A_half@D))-beta<=0)]
```

```
    prob = cp.Problem(
        cp.Minimize(obj),
        constraints
    )

    val = prob.solve()

    if val==np.inf:
        l = beta
    else:
        u = beta


# Do a final solve with u such that we report a feasible D
beta = u
D = cp.Variable((n,n))

obj = 0

constraints = []
constraints += [D[i,j]==0 for i in range (n) for j in range(n) if i!=j]
constraints += [D-I>>0]
constraints += [cp.lambda_max(A@D)-beta*cp.lambda_min(A@D)<=0]
# obj = 0

# constraints = []
# constraints += [D[i,j]==0 for i in range (n) for j in range(n) if i!=j]
# constraints += [D-I>>0]
# # constraints += [Y==A@D]
# # constraints += [M[i,j] >= A[i,j]*cp.geo_mean(D[i,i], D[j,j]) for i in range␣
 ↪(n) for j in range(n)]
# # constraints += [cp.lambda_max(D@A@D)-beta *cp.lambda_min(D@A@D)<=0]
# # constraints += [cp.square(cp.lambda_max(D))-beta *cp.square(cp.
 ↪lambda_min(D))<=0]
# constraints += [(cp.square(cp.norm(A_half@D))-beta<=0)]

prob = cp.Problem(
    cp.Minimize(obj),
    constraints
)
prob.solve()
```

[30]: 0.0

[31]:
```
cp.square(cp.max(D))#-beta *cp.square(cp.norm_inf(-D))
```

[31]: Expression(UNKNOWN, NONNEGATIVE, ())

[32]:
```
A@cp.square(D)@A
```

[32]: Expression(UNKNOWN, UNKNOWN, (np.int32(4), np.int32(4)))

[33]:
```
u, l, beta
```

[33]: (224.08816777169704, 224.08816684037447, 224.08816777169704)

[35]:
```
eigval_A = np.linalg.eigvals(A)
cond_A = np.max(eigval_A) / np.min(eigval_A)
cond_A
```

[35]: np.float64(1170.1104124302037)

[36]:
```
eigval_DAD = np.linalg.eigvals(D.value@A@D.value)
cond_DAD = np.max(eigval_DAD) / np.min(eigval_DAD)
cond_DAD
```

[36]: np.float64(1170.1082748362933)

[37]:
```
D.value
```

[37]:
```
array([[ 9.99954321e-01,  5.14407986e-08, -3.03613969e-09,
         2.37509480e-10],
       [ 1.29185768e-07,  9.99953084e-01,  1.94706254e-08,
        -2.14011441e-08],
       [-9.56315948e-08,  8.87066798e-08,  9.99952288e-01,
         1.42542573e-08],
       [ 1.04730558e-07, -9.52184333e-08,  1.35969837e-08,
         9.99952297e-01]])
```

0.8 Problem 7 –Attempt 2

[ ]:
```
d
```

[133]:
```
A = np.array(
    [
        [0.2, -0.2, 0.6, -0.6] ,
        [-0.2, 0.4, -1.4, 1.3],
        [0.6, -1.4, 5.2, -4.7],
        [-0.6, 1.3, -4.7, 4.4]
    ]
)

n, _ = A.shape
```

```
I = np.eye(n)
```

[157]:
```
d = cp.Variable(n)

i = 0
j = 1
d[i]*A[i,j]*d[j]
```

[157]: `Expression(UNKNOWN, UNKNOWN, ())`

[150]:
```
beta = 10000
D = cp.Variable((n,n))

obj = 0

constraints = []
constraints += [D[i,j]==0 for i in range (n) for j in range(n) if i!=j]
constraints += [D>>I]
# constraints += [beta >= 0]
constraints += [A.T@D@A >= D]
constraints += [beta*D >= A.T@D@A ]

prob = cp.Problem(
    cp.Minimize(obj),
    constraints
)

prob.solve()
```

[150]: `inf`

[151]:
```
constraints
```

[151]:
```
[Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 Equality(Expression(AFFINE, UNKNOWN, ()), Constant(CONSTANT, ZERO, ())),
 PSD(Expression(AFFINE, UNKNOWN, (np.int32(4), np.int32(4)))),
```

```
 Inequality(Variable((np.int32(4), np.int32(4)), var26393)),
 Inequality(Expression(AFFINE, UNKNOWN, (np.int32(4), np.int32(4))))]
```

0.9 Problem 8 – Completed

[3]:
```python
sigma = np.array(
    [
        [4.9, -3.8, 1.4],
        [-3.8, 3.8, -1.9],
        [1.4, -1.9, 2.5]
    ]
)

n = 3

sigma==sigma.T
```

[3]:
```
array([[ True,  True,  True],
       [ True,  True,  True],
       [ True,  True,  True]])
```

[4]:
```python
np.linalg.svd(sigma)
```

[4]:
```
SVDResult(U=array([[-0.69882028, -0.46563551,  0.54298599],
       [ 0.63207126, -0.04660389,  0.7735076 ],
       [-0.33486734,  0.88374863,  0.32688257]]), S=array([9.00790176,
1.86255367, 0.32954456]), Vh=array([[-0.69882028,  0.63207126, -0.33486734],
       [-0.46563551, -0.04660389,  0.88374863],
       [ 0.54298599,  0.7735076 ,  0.32688257]]))
```

[9]:
```python
lamb = cp.Variable((2*n, 2*n))
delta = cp.Variable(n)
```

[10]:
```python
obj = cp.sum(
    cp.hstack(
        [lamb[i,n+i] for i in range(n)]
    )
)
```

[11]:
```python
constraints = []
constraints += [lamb[:n,-n:]==sigma+cp.diag(delta)]
constraints += [lamb[-n:,:n]==sigma+cp.diag(delta)]
constraints += [lamb[:n,:n]==sigma]
constraints += [lamb[-n:,-n:]==sigma]
constraints += [lamb>>0]
# constraints += [W@lambda == W@lambda]
```

[12]:
```
prob = cp.Problem(
    cp.Minimize(obj),
    constraints
)

prob.solve()
```

[12]:
```
np.float64(7.899909909236449)
```

[13]:
```
np.round(lamb.value,2)
```

[13]:
```
array([[ 4.9, -3.8,  1.4,  3.7, -3.8,  1.4],
       [-3.8,  3.8, -1.9, -3.8,  3.8, -1.9],
       [ 1.4, -1.9,  2.5,  1.4, -1.9,  0.4],
       [ 3.7, -3.8,  1.4,  4.9, -3.8,  1.4],
       [-3.8,  3.8, -1.9, -3.8,  3.8, -1.9],
       [ 1.4, -1.9,  0.4,  1.4, -1.9,  2.5]])
```

[14]:
```
for i in range(n):
    print(f'lambda[{i},{i+n}] %.2f'%lamb.value[i, i+n])
```

```
lambda[0,3] 3.70
lambda[1,4] 3.80
lambda[2,5] 0.40
```

[ ]:
```
```