# Modeling visual working memory with the MemToolbox

The authors<sup>1</sup>

Department of Psychology, Harvard University, Cambridge, MA 02138

#### Abstract

The MemToolbox is a collection of MATLAB functions for modeling visual working memory. In support of its goal to provide a full suite of data analysis tools, the toolbox includes implementations of popular models of visual working memory, real and simulated data sets, Bayesian and maximum likelihood estimation procedures for fitting models to data, visualizations of data and fit, validation tools, model comparison metrics, and experiment scripts. The MemToolbox is released under a BSD license and is freely available at memtoolbox.org.

Keywords: visual working memory, mixture models, model comparison

#### Introduction

Working memory is a storage system that actively holds information in mind and allows for its manipulation, providing a workspace for thought (Baddeley, 1986). Its strikingly limited capacity has inspired a slew of research aimed at characterizing those limits in terms of both the spatiotemporal properties of the stimulus and the age, gender, intelligence, sleepiness, and mental health of the individual.

A handful of experimental paradigms are popular in the study of working memory. These include the delayed match-to-sample task used in studies of animal cognition (Blough, 1959) and the span tasks used in studies of verbal working memory (Daneman & Carpenter, 1980). In the case of working memory for visual information, research has primarily relied on the partial report task and the change detection task (Fig. 1). In a partial report task, the participant is shown a set of letters, shapes, or colorful dots, and then after a brief delay is asked to report the properties of one or a subset of the

items (Sperling, 1960). In a change detection task, the participant is shown a pair of displays, one after the other, and is then asked a question that requires comparing them, e.g., whether they match (Phillips, 1974; Pashler, 1988).

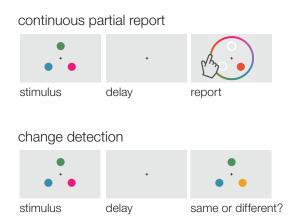


Figure 1: Figure caption

Performance on these visual tasks is used to draw conclusions about working memory. For example, using data from a change detection task, Luck & Vogel (1997) showed that the same number of features as conjunctions of features can be stored in working memory, and from this inferred that the storage format in working memory is integrated objects, not individual features. A number of mathematical models have been proposed that link performance in change detection and partial report tasks to specific properties of the working memory system, e.g. its capacity. These include the item-limit model (Pashler, 1988), the slots, slots+averaging, and slots+resources model (Zhang & Luck, 2008), the continuous resource model (Wilken & Ma, 2004), the swap model (Bays et al., 2009), and the variable-precision model (van den Berg et al., 2012). Each model defines a likelihood function that captures the model's predictions for each possible setting of its parameters.

Having defined a model, estimation procedures such as the EM algorithm (Dempster et al., 1977) or simplex search (Lagarias et al., 1998) are used to find the combination of parameter values that maximize that model's likelihood function given the data. This procedure is typically performed separately for each participant and experimental condition, resulting in a set of estimates that are then compared using traditional statistical tests.

#### The MemToolbox

We created the MemToolbox, a collection of MATLAB routines for modeling visual working memory. Though the toolbox provides everything needed to perform the analyses outlined above (including model implementations, maximum likelihood routines, and data validation checks), it also provides tools that offer a deeper look into the data. In the following sections, we highlight some of the toolbox's core functionality.

#### The Bayesian approach

The MemToolbox defaults to a Bayesian workflow that encourages full exploration of the data. This approach begins with a set of prior beliefs about the value of each of the model's parameters in the form of a probability density function (pdf) or probability mass function (pmf) on the space of all possible parameter values. For the purposes of exploratory data analysis, it is common to use a noninformative or weakly informative prior that spreads the probability thinly over a swath of plausible parameter values. Following this tradition, the toolbox uses the Jeffreys prior, a class of noninformative priors that is invariant under reparameterization of the model (Jeffreys, 1946; Jaynes, 1968). The prior beliefs are then updated according to Bayes' rule to take into account the experimental data. This leads to a posterior distribution that can be used to make decisions. For some models it is possible to derive closed-form expressions for the posterior distribution, but for others this is intractable and so sampling-based algorithms are used to approximate it. One such algorithm, the Metropolis-Hastings variant of Markov Chain Monte Carlo (MCMC), constructs a random walker that visits parameter settings with frequency proportional to their probability under the posterior (Metropolis et al., 1953; Hastings, 1970).

With the posterior distribution in hand, there are a number of ways to analyze and visualize the results. First, the maximum of the posterior distribution (the so-called "maximum a posteriori" or MAP estimate) can be used as a point-estimate that is analogous to the maximum likelihood estimate, differing only in the MAP's consideration of the prior. Visualizing the full posterior distribution provides information about how well the data constrains the parameter values and whether there are tradeoffs (Fig. X).

#### Posterior predictive checks

Sometimes a whole class of models performs poorly, such that there are no parameter settings that will produce a good fit. In this case, maximum likelihood and maximum a posteriori estimates are misleading: they dutifully pick out the best, even if the best is quite bad. A good practice is to check the quality of the fit, examining which aspects of the data it captures, and which aspects of the data it fails to capture. This can be accomplished through a posterior predictive check, which simulates new data from the best-fit model, and then compares the histograms of the actual and simulated data (Fig. 2).

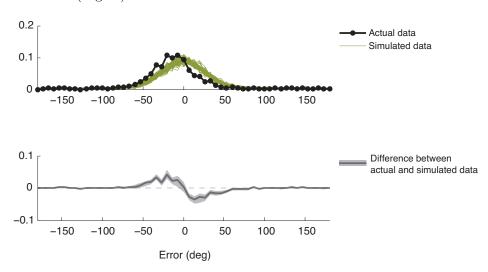


Figure 2: Figure caption

#### Model comparison

Often there is a question about which model best describes the data. Answering it requires considering both the resemblance between the model and data and also the flexibility of the model. Flexible models can fit many data sets, and so a good fit provides only weak evidence of a good match between model and data. In contrast, finding the same fit between an inflexible model and the data provides stronger evidence of a good match. To account for this, many approaches penalize more flexible models; these include the Akaike Information Criterion with correction for finite data (AICc; Akaike, 1974; Burnham & Anderson, 2004), the Bayesian Information Crite-

rion (BIC; Schwarz, 1978), the Deviance Information Criterion (DIC, Spiegelhalter et al., 2002), and the Bayes factor (Kass & Raftery, 1995). All of them are included in the toolbox.

### Conclusion

We created the MemToolbox for modeling visual working memory. This introduction gave a high-level overview of its approach and core features. To learn to use the toolbox, we recommend the tutorial, available in the supplementary materials or at memtoolbox.org.

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Corresponding author:

Email:

Address: 33 Kirkland Street, Cambridge, MA, 02138.

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