Graphs of functions

In the previous example, we used the graph of a function to understand something about it.

A graph tells the "story" of a function in a visual way.

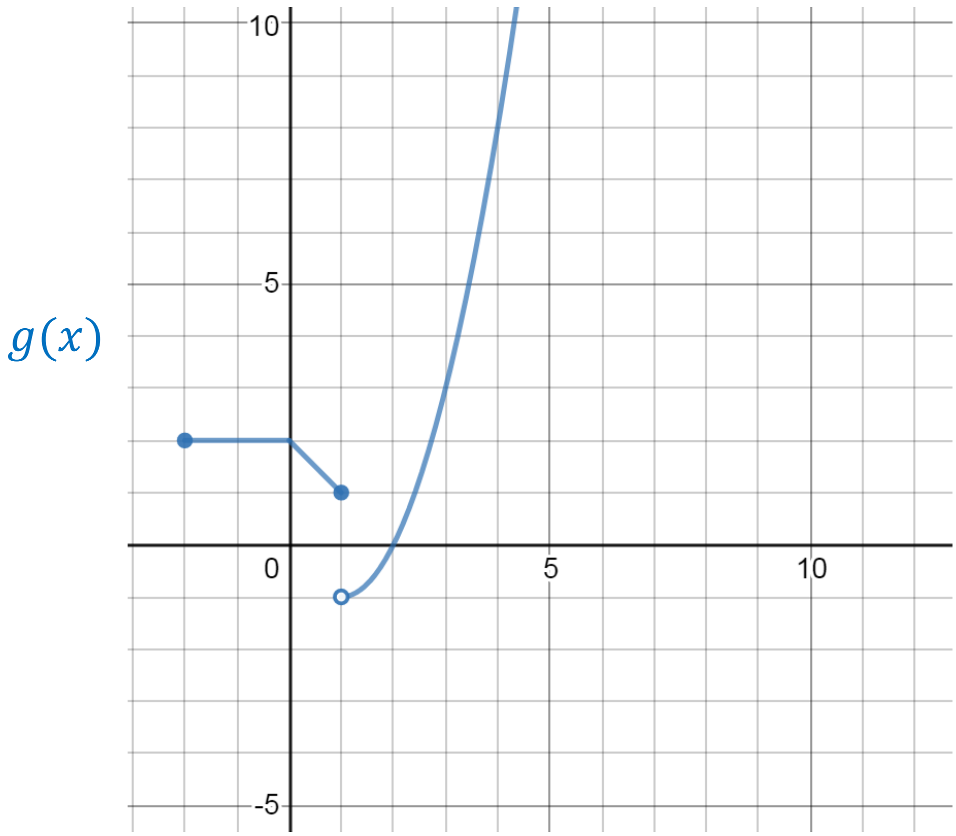
So it's like a *picture of a function*.

Therefore, for every type of function we study in this class . . .

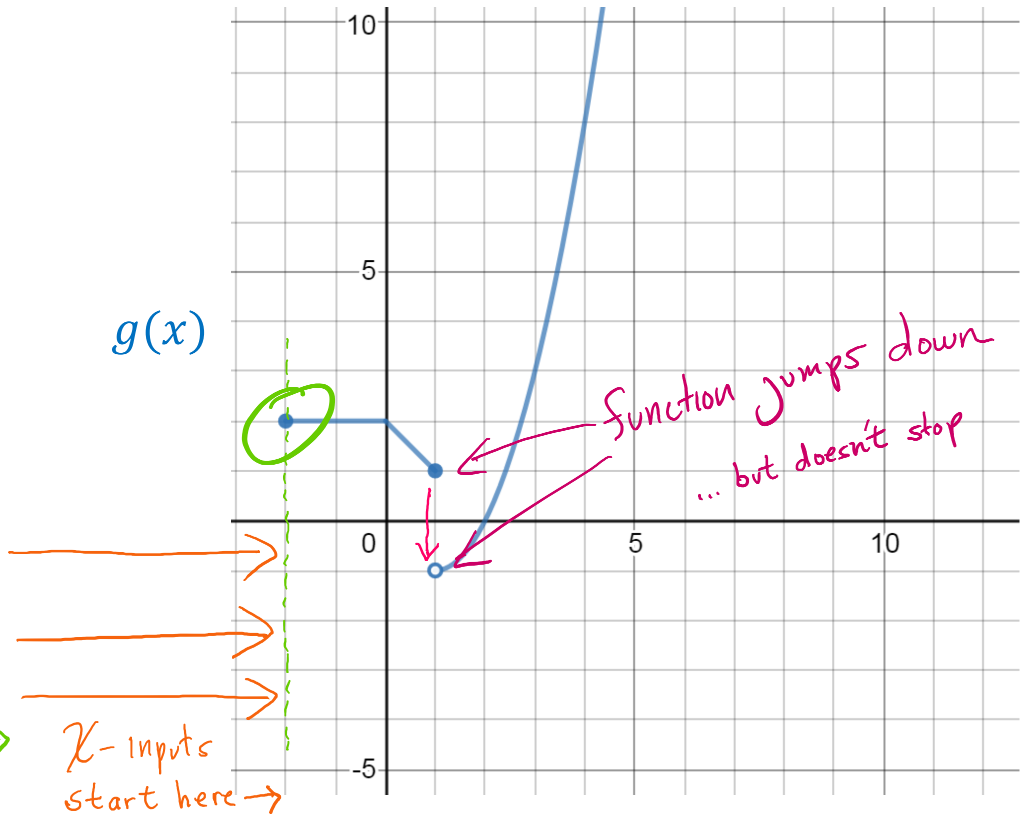
knowing how to **create** and **interpret** its graph is very important.

Some functions can be completely **defined** by their graphs.

Consider the function shown below, which we will call $g(x)$:



Can you find the domain of the function? And the range? Think about it before turning to the next page.



To see the domain, we are thinking about what **inputs** are possible.

That means we are looking at the **x-values**.

So we are looking from left to right.

Starting from the left end of the graph, we see that the function **begins** at

$$x = -2.$$

From there, as we move right, there is a point for every x-value . . .

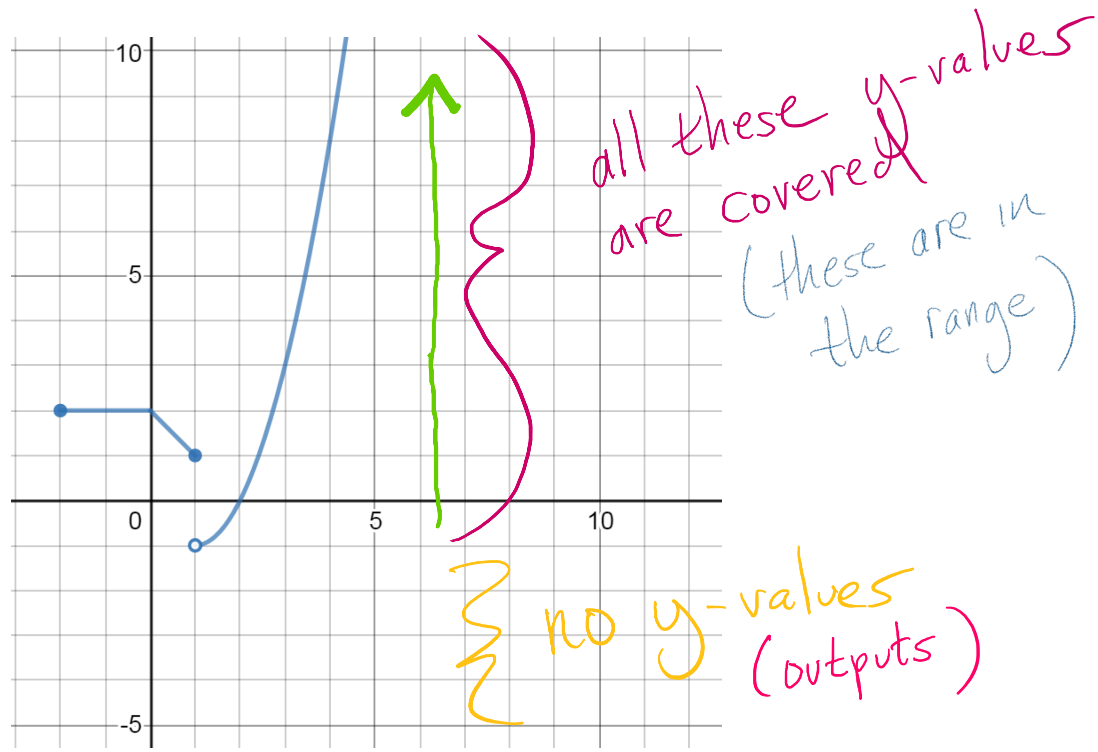
. . . all the way to infinity.

Anywhere the function exists, that x-value is in the domain.

At $x = 1$, the graph is discontinuous, but it still exists, jumping smoothly from $y = 1$ to $y = -1$. In fact, $g(1) = 1$, as shown by the dot at (1,1).

The domain of this function is $[-2, \infty)$.

How about the range?



To see the range, we are looking at the **resulting outputs**.

So we are looking at the y-values.

They are seen by looking vertically, from bottom to top.

What is the least number that **comes out** of the function?

It seems like $y = -1$. And this is the lower bound.

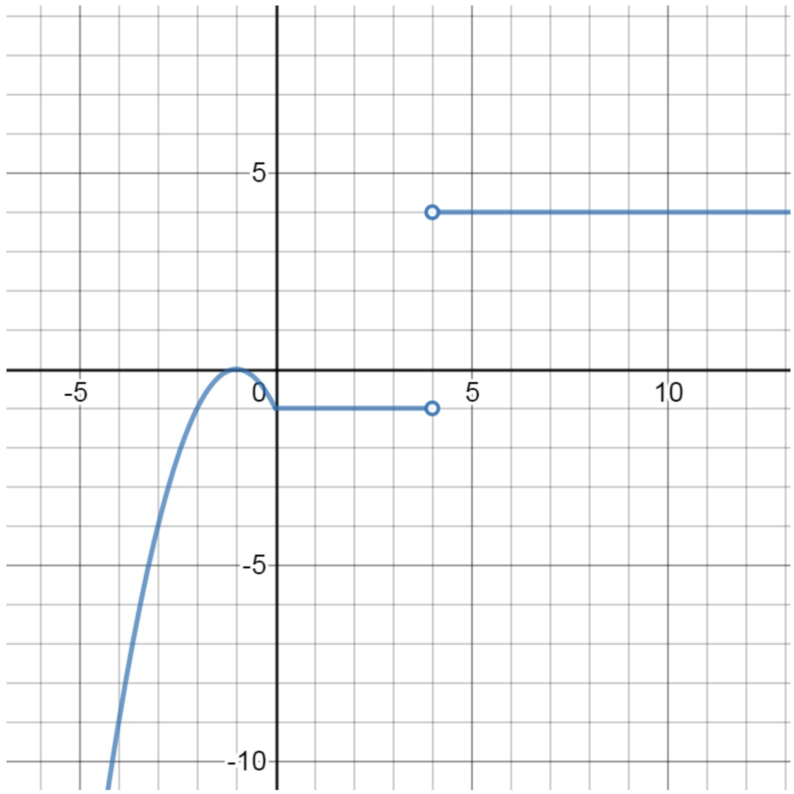
But it is not included in the range. The **open** point indicates that the graph includes the points greater than $y = -1$, but not equal to $y = 1$.

From there, going up, we see that the function produces y-values all the way to infinity. In some cases, it produces the same y-values twice!

The range of the function is $(-1, \infty)$.

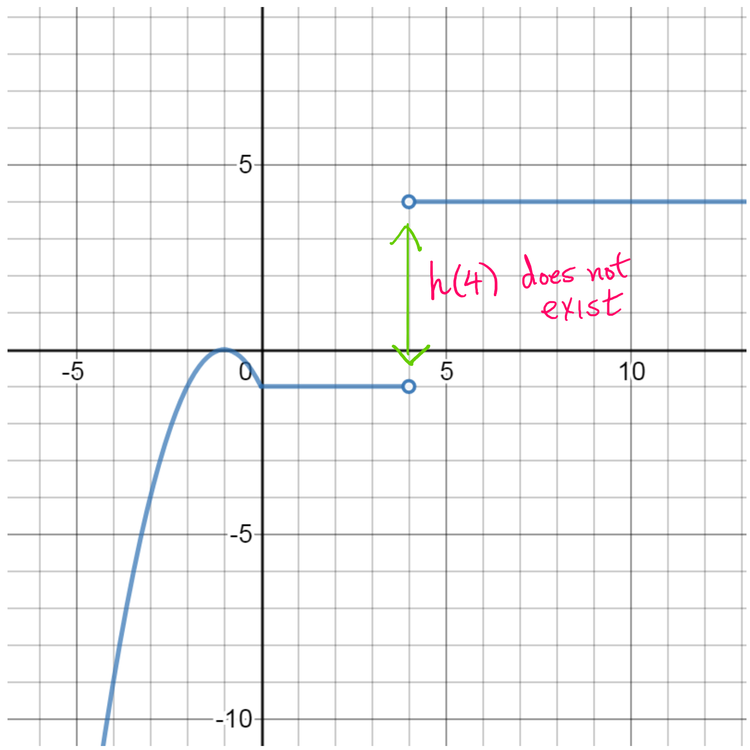
Let’s look at another example. This function we will call $h(x)$.

$h(x)$



Go ahead and try to figure out the domain and range!

$h(x)$



First let's do the domain. Looking from left to right . . .

There really IS NO STARTING PLACE.

The graph goes off of the grid at the bottom.

We can assume that the graph of $h(x)$ just goes on forever . . .

So the **lower bound** of the domain is **negative infinity**.

Proceeding rightward, there is a point associated with every x-value . . .

. . . all the way to **positive infinity** . . .

With one exception! There is no y-value at $x = 4$!

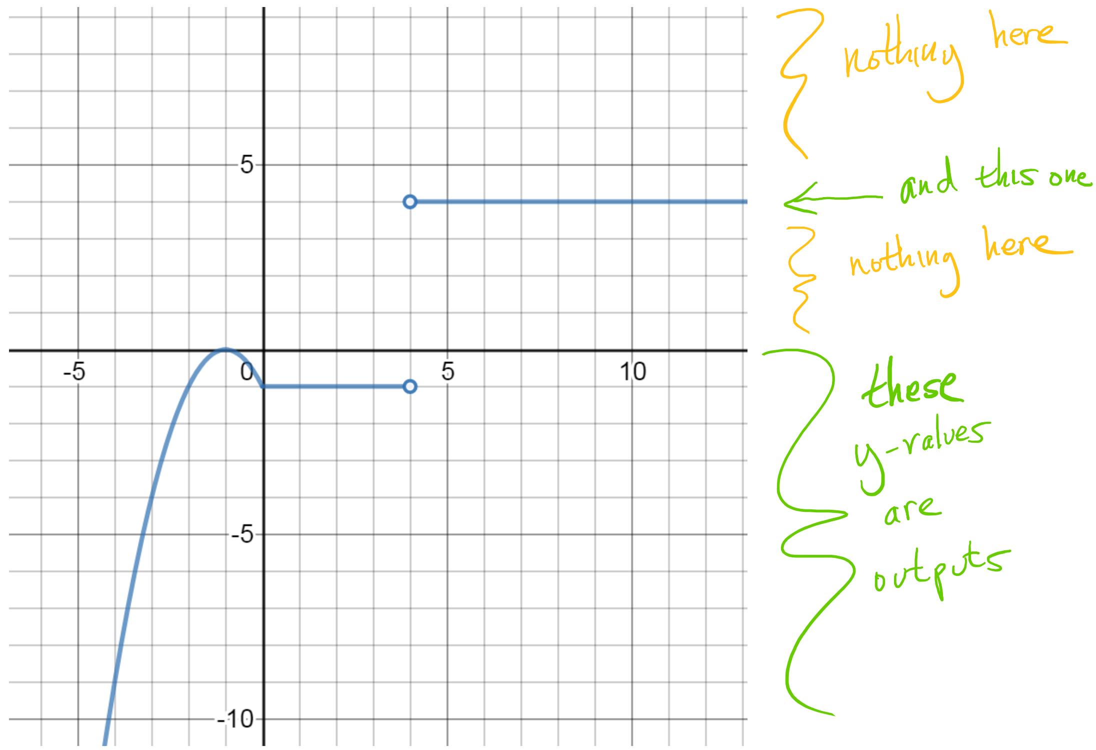
So the domain is $(-\infty, 4) \cup (4, \infty)$

or $\{x|x \neq 4\}$

"such that"

How about the range?

$h(x)$



Again, we are looking at ***resulting outputs***, so looking at y-values

From lowest to highest . . .

So from the bottom part of the graph to the top.

Where do the y-values start?

As with the domain, the graph of the function must be assumed to continue forever at the lower-left part of the graph.

The ***lower bound*** of the range is $-\infty$.

There are y-values up to $y = 0$ (with some overlapping y-values)

And then there is a range of y-values for which there are no points!

Nothing . . . until we get to $y = 4$!

And then nothing after that!

So the range is $(-\infty, 0] \cup 4$.