Piecewise Functions

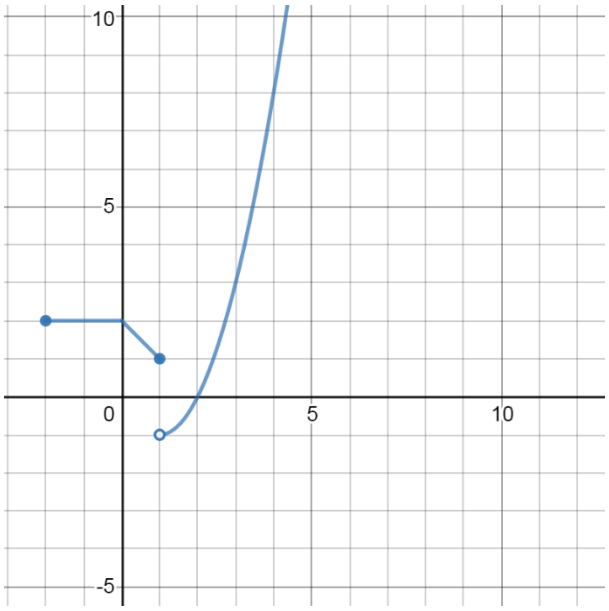
We have just looked at some graphs that are . . . a bit unpredictable.

They seemed to have different parts, or sections.

Put differently, these functions had different formulas . . .

. . . for different intervals of the domain.

For example, consider again the function $g(x)$:



This function would be written using functional notation as . . .

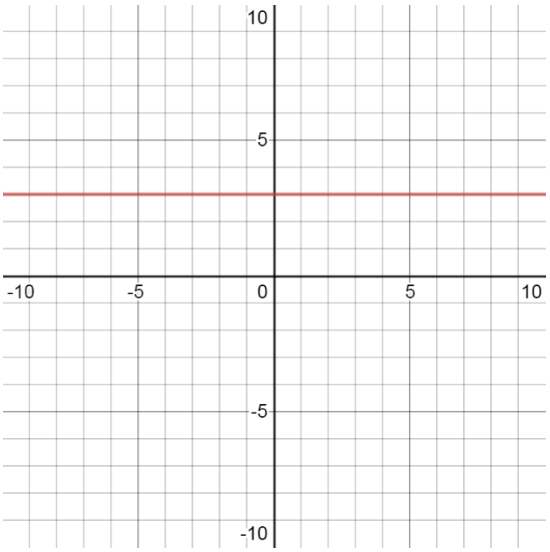
$$g(x) = \begin{cases} 2 & -2 \le x < 0 \\ 2 - x & 0 \le x \le 1 \\ (x-1)^2 - 1 & -1 < x \end{cases}$$

For a simpler example, let's start with the following piecewise function:

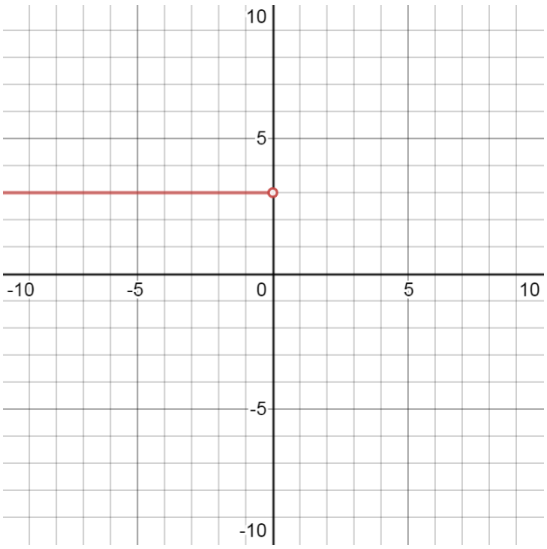
$$f(x) = \begin{cases} 3 & x < 0 \\ x & 0 \leq x \end{cases}$$

This function has two formulas, both very simple.

The graph of $f(x) = 3$ would look like this:

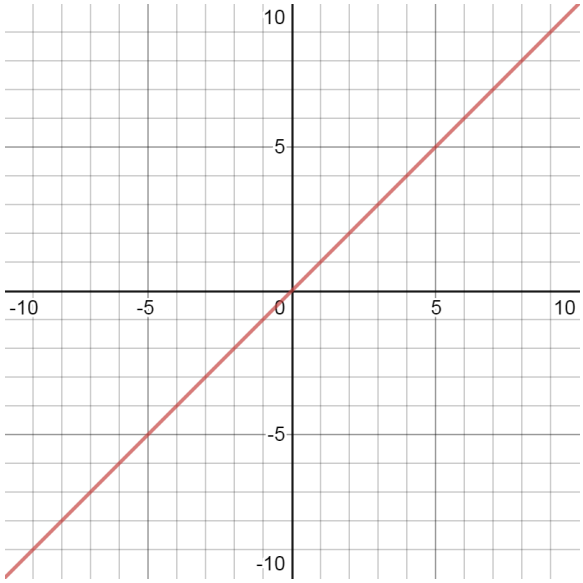


But we should erase all the points except the ones that have $x < 0$:

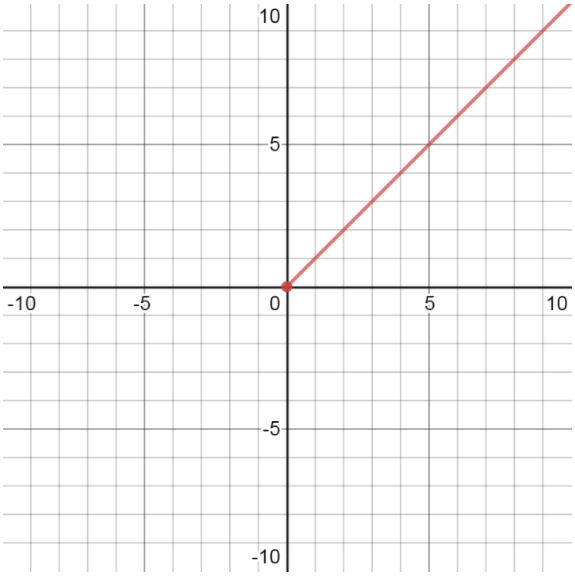


We're not done! There's another formula in our function: $f(x) = x$.

Normally, that graph would look like:

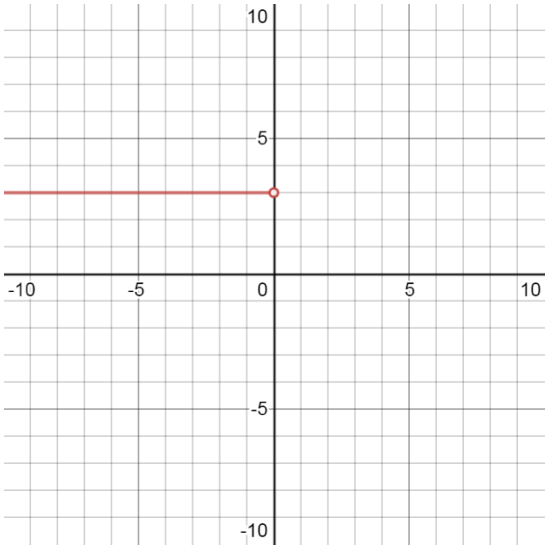


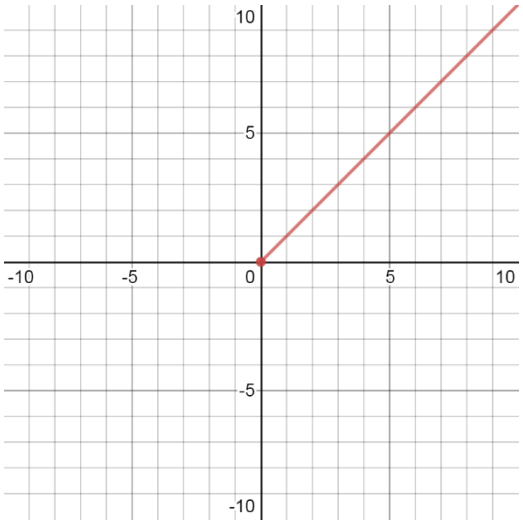
But we should erase all the points except the ones that have $x \geq 0$:



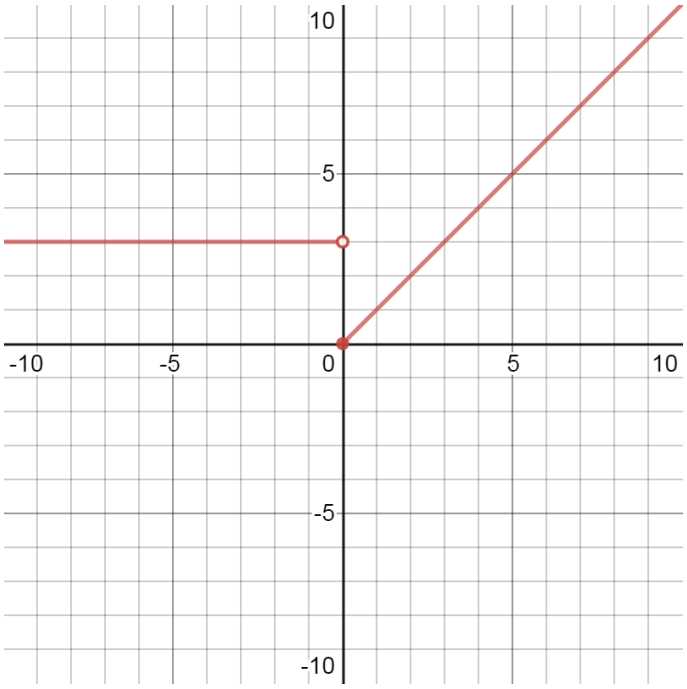
We're still not done!

Both graphs are part of the same function, so they should be combined!

Combining 

with 

We get:

$f(x)$ 

Let's see how a ***piecewise function*** could emerge from a real-life situation:

> Joe works as an airplane mechanic. Sometimes he has to work overtime when things get very busy at the airport.
>
> Normally, he makes $22 per hour.
>
> But if he works more than 40 hours in the week, he gets paid time-and-a-half for his overtime hours.
>
> Find the function giving his total weekly pay based on the total number of hours he works.

We are asked to find the following function:

$$P(x) = total\ weekly\ pay\ given\ x\ hours\ of\ work$$

If he works anything less than or equal to a normal week, the formula to get his pay is simple:

$$P(x) = 22x$$

It gets complicated when he works more than 40 hours. Then he makes time-and-a-half, which would be **$33 per hour**.

but only for the time over 40 hours!

For the other hours he's still making **$22/hr**!

It's complicated! We need a piecewise function!

The original formula, $P(x) = 22x$, still applies for all $x \leq 40$.

But if he works more than 40 hours, **we must use a different formula**.

He would still get paid for the first 40 hours at $22/hour, for a total of

$$\$22 * 40 = \$880.$$

In addition to this $880 from his regular pay rate, he would make $33/hr …

for all the hours over 40!

To get the hours over 40, subtract 40 from the total number of hours:

$$x - 40$$

This represents his overtime hours, for which he is paid at $33/hr.

Putting it all together, we get:

$$P(x) = \begin{cases} 22x & x < 40 \\ 880 + 33(x - 40) & x \geq 40 \end{cases}$$

Remember, we only use the top **or** the bottom formula, depending on x!