Simplifying Radical Expressions

Many fractions represent the same number:

$$\frac{20}{40} = \frac{10}{20} = \frac{1}{2}$$

All of the above fractions represent 0.5, but only $\frac{1}{2}$ represents the fully simplified version of those expressions.

We have a rule in math that all fractions . . .

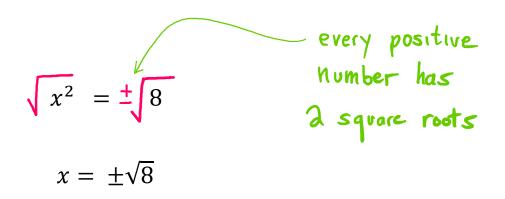
... should be expressed in *simplifed form*

The same is true of radicals!

Consider the following equation:

$$x^2 = 8$$

This equation can be solved by taking the square root of both sides:



But we are still not finished . . .

... because $\sqrt{8}$ is **not simplified!**

To simplify radicals, we will . . .

... factor out from the number under the radical ...

... the largest perfect square

We have that

$$\sqrt{8}$$

Can be rewritten as

$$\sqrt{4\cdot 2}$$

Now we will use the following multiplication rule for radicals:

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$$

The multiplication rule **for radicals**

We will be using this rule in reverse:

$$\sqrt{8}$$

$$= \sqrt{4 \cdot 2}$$

$$= \sqrt{4} \cdot \sqrt{2}$$

which then becomes

$$=2\sqrt{2}$$

because
$$\sqrt{4} = 2$$

Let's try another example.

Simplify:

$$\sqrt{32}$$

We can factor 32 in two different ways that have a perfect square:

$$\sqrt{32} \qquad \sqrt{32} \\
= \sqrt{4 \cdot 8} \qquad = \sqrt{16 \cdot 2}$$

Which one should we use?

If we do the first one, we get this:

$$\sqrt{32} = \sqrt{4 \cdot 8} \\
= 2\sqrt{8}$$

But here we still have more work to do, because as we've already seen,

 $\sqrt{8}$ is not completely simplified

Whereas if we do the second one, we get this:

$$\sqrt{32}$$

$$= \sqrt{16 \cdot 2}$$

$$= 4\sqrt{2}$$

 $4\sqrt{2}$ is the fully simplified answer.

The lesson here is that we . . .

... always factor out the greatest perfect square.

Simplify:

$$\sqrt{200}$$

There are a number of different ways to find a perfect square:

$$\sqrt{200}$$
 $\sqrt{200}$
 $\sqrt{200}$
 $= \sqrt{4 \cdot 50}$
 $\sqrt{25 \cdot 8}$
 $\sqrt{200}$
 $= \sqrt{100 \cdot 2}$

The last version has the **greatest perfect square** so it will take the **fewest steps to finish**:

$$\sqrt{200}$$

$$= \sqrt{100 \cdot 2}$$

$$= 10\sqrt{2}$$

We can also simplify other kinds of roots, such as cube roots:

$$\sqrt[3]{16}$$

For these radicals, the same strategy will apply with one exception:

To simplify radicals, we will . . .

... factor out from the number under the radical ...

... the largest perfect square

So we get

$$= \sqrt[3]{16}$$
$$= \sqrt[3]{8 \cdot 2}$$

We get this because 8 is a **perfect cube**:

$$8 = \frac{2^3}{2}$$

So we get

$$\sqrt[3]{16}$$

$$= \sqrt[3]{8 \cdot 2}$$

$$= \sqrt[3]{8} \cdot \sqrt[3]{2}$$

$$= 2\sqrt[3]{2}$$