# Simplifying Radical Expressions

Many fractions represent the same number:

$$\frac{20}{40} = \frac{10}{20} = \frac{1}{2}$$

All of the above fractions represent 0.5, but only $\frac{1}{2}$ represents the fully simplified version of those expressions.

We have a rule in math that all fractions . . .

. . . should be expressed in *simplifed form*

The same is true of radicals!

Consider the following equation:

$$x^2 = 8$$

This equation can be solved by **taking the square root of both sides**:

every positive number has 2 square roots

$$\sqrt{x^2} = \pm\sqrt{8}$$

$$x = \pm\sqrt{8}$$

But we are still not finished . . .

. . . because $\sqrt{8}$ is **not simplified**!

> To **simplify radicals**, we will . . .
>
> . . . ***factor out from the number under the radical*** . . .
>
> *. . . the largest perfect square*

We have that

$$\sqrt{8}$$

Can be rewritten as

$$\sqrt{4 \cdot 2}$$

Now we will use the following multiplication rule for radicals:

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$$

The multiplication rule **for radicals**

We will be using this rule **in reverse:**

$$\sqrt{8}$$

$$= \sqrt{4 \cdot 2}$$

$$= \sqrt{4} \cdot \sqrt{2}$$

because $\sqrt{4} = 2$

which then becomes

$$= 2\sqrt{2}$$

Let's try another example.

Simplify:

$$\sqrt{32}$$

We can factor 32 in **two different ways** that have a ***perfect square***:

$$\begin{aligned} &\sqrt{32} \\ &= \sqrt{4 \cdot 8} \end{aligned} \qquad\qquad \begin{aligned} &\sqrt{32} \\ &= \sqrt{16 \cdot 2} \end{aligned}$$

*Which one should we use?*

If we do the first one, we get this:

$$\begin{aligned} &\sqrt{32} \\ &= \sqrt{4 \cdot 8} \\ &= 2\sqrt{8} \end{aligned}$$

But here we still have more work to do, because as we've already seen,

$\sqrt{8}$ is not completely simplified

Whereas if we do the second one, we get this:

$$\begin{aligned} &\sqrt{32} \\ &= \sqrt{16 \cdot 2} \\ &= 4\sqrt{2} \end{aligned}$$

$4\sqrt{2}$ is the fully simplifed answer.

The lesson here is that we . . .

. . . ***always factor out the greatest perfect square***.

Simplify:

$$\sqrt{200}$$

There are a number of different ways to find a perfect square:

$$\begin{aligned} &\sqrt{200} \\ &= \sqrt{4 \cdot 50} \end{aligned} \qquad \begin{aligned} &\sqrt{200} \\ &= \sqrt{25 \cdot 8} \end{aligned} \qquad \begin{aligned} &\sqrt{200} \\ &= \sqrt{100 \cdot 2} \end{aligned}$$

The last version has the ***greatest perfect square*** . . .
. . . so it will take the **fewest steps to finish**:

$$
\begin{aligned}
&\sqrt{200} \\
&= \sqrt{100 \cdot 2} \\
&= \ 10\sqrt{2}
\end{aligned}
$$

We can also simplify other kinds of roots, such as cube roots:

$$\sqrt[3]{16}$$

For these radicals, the same strategy will apply with one exception:

To **simplify radicals**, we will . . .

. . . ***factor out from the number under the radical*** . . .

***. . . the largest perfect square***

So we get

$$
\begin{aligned}
&\sqrt[3]{16} \\
&= \ \sqrt[3]{8 \cdot 2}
\end{aligned}
$$

We get this because 8 is a **perfect cube**:

$$8 = 2^3$$

So we get

$$\begin{aligned} &\sqrt[3]{16} \\ =\ &\sqrt[3]{8 \cdot 2} \\ =\ &\sqrt[3]{8} \cdot \sqrt[3]{2} \\ =\ &2\sqrt[3]{2} \end{aligned}$$

$\sqrt[3]{8} = 2$