Complex Fractions

Here's a situation that comes up a lot in the world of algebra:

$$\frac{\frac{5}{9}}{\frac{7}{12}}$$

This is called a **complex fraction**.

A complex fraction has **little fractions** . . .

. . . inside of a **big fraction**.

We need to simplify complex fractions.

Here is the way I will suggest you do that:

To simplify a complex fraction . . .

Cancel the denominators . . .

. . . by multiplying the top & bottom . . .

. . . by the LCD

$$\frac{\frac{5}{9}\cdot \mathbf{36}}{\frac{7}{12}\cdot \mathbf{36}} = \frac{5\cdot 4}{7\cdot 3} = \frac{20}{21}$$

How about this one?

Simplify:

$$\frac{\frac{1}{6}-\frac{1}{3}}{\frac{1}{4}-\frac{1}{8}}$$

Again, the best way to simplify this complex fraction is to . . .

. . . multiply the top & bottom by the LCD.

$$\frac{\frac{1}{6}-\frac{1}{3}}{\frac{1}{4}-\frac{1}{8}} \quad \begin{matrix}\mathbf{x\ 24} \\ \mathbf{x\ 24}\end{matrix}$$

$$= \frac{\frac{1}{6}(24)-\frac{1}{3}(24)}{\frac{1}{4}(24)-\frac{1}{8}(24)} = \frac{4-8}{6-3} = -\frac{4}{3}$$

Let's do another:

$$\frac{3-\frac{1}{x+3}}{3+\frac{1}{x+3}}$$

The LCD here is pretty clear. We need to multiply it to the **whole** top and bottom:

$$\frac{\left(3-\frac{1}{x+3}\right)(x+3)}{\left(3+\frac{1}{x+3}\right)(x+3)}$$

Now we can distribute to the **whole** top and bottom:

$$\frac{3(x+3)-\frac{1}{x+3}(x+3)}{3(x+3)+\frac{1}{x+3}(x+3)}$$

$$=\frac{3(x+3)-1}{3(x+3)+1}$$

$$=\frac{3x+9-1}{3x+9+1}$$

$$=\frac{3x+8}{3x+10}$$

Let's do one more:

Simplify:

$$\frac{1-\frac{9}{x^2}}{1-\frac{1}{x}-\frac{6}{x^2}}$$

Clearly the LCD is x^2. Let's multiply to the top and bottom:

$$\frac{1-\frac{9}{x^2}}{1-\frac{1}{x}-\frac{6}{x^2}}$$

$$=\frac{1(x^2)-\frac{9}{x^2}(x^2)}{1(x^2)-\frac{1}{x}(x^2)-\frac{6}{x^2}(x^2)}$$

$$=\frac{x^2-9}{x^2-x-6}$$

We're still not done!

$$\frac{x^2 - 9}{x^2 - x - 6}$$

$$= \frac{(x + 3)(x - 3)}{(x + 2)(x - 3)}$$

$$= \frac{x + 3}{x + 2}$$

Not so complex after all! ☺