# The Vertical Line Test

All of the **graphs** we have seen so far, have been graphs of **functions**.

But here's a question . . . do ***all*** graphs represent functions?

Put another way . . . could you have a graph that was **not** a function?

Remember, functions have a **SPECIAL RULE**:

**There can only be one output**

**for any input**

We saw that there were some equations that could not be functions . . .

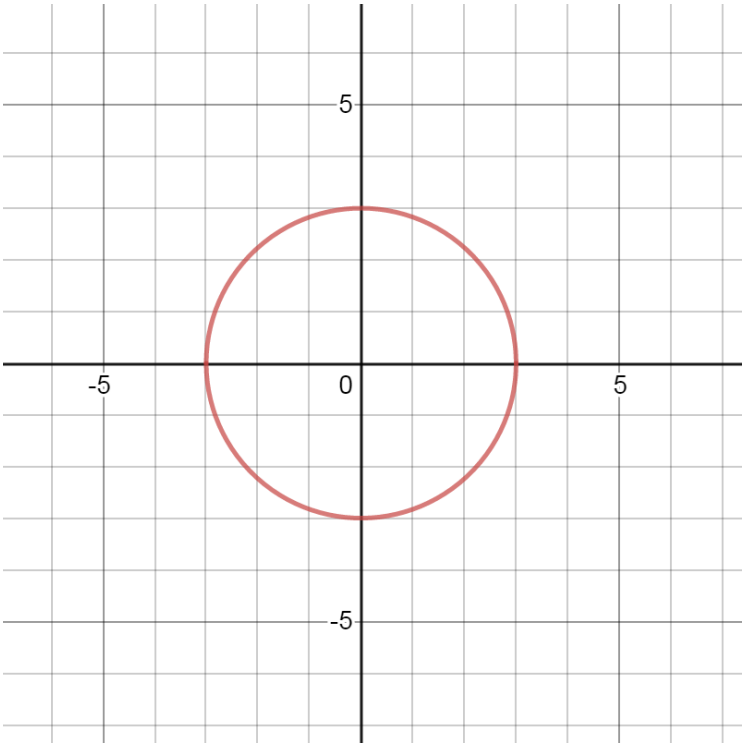
. . . because they did not obey that rule.

The example where that happened was the equation

$$x^2 + y^2 = 9$$

Do you know what the graph of this equation looks like?

It looks like this:




That's right, it's the graph of a circle.

It's not the graph of a function, because for the input $x = 0$,

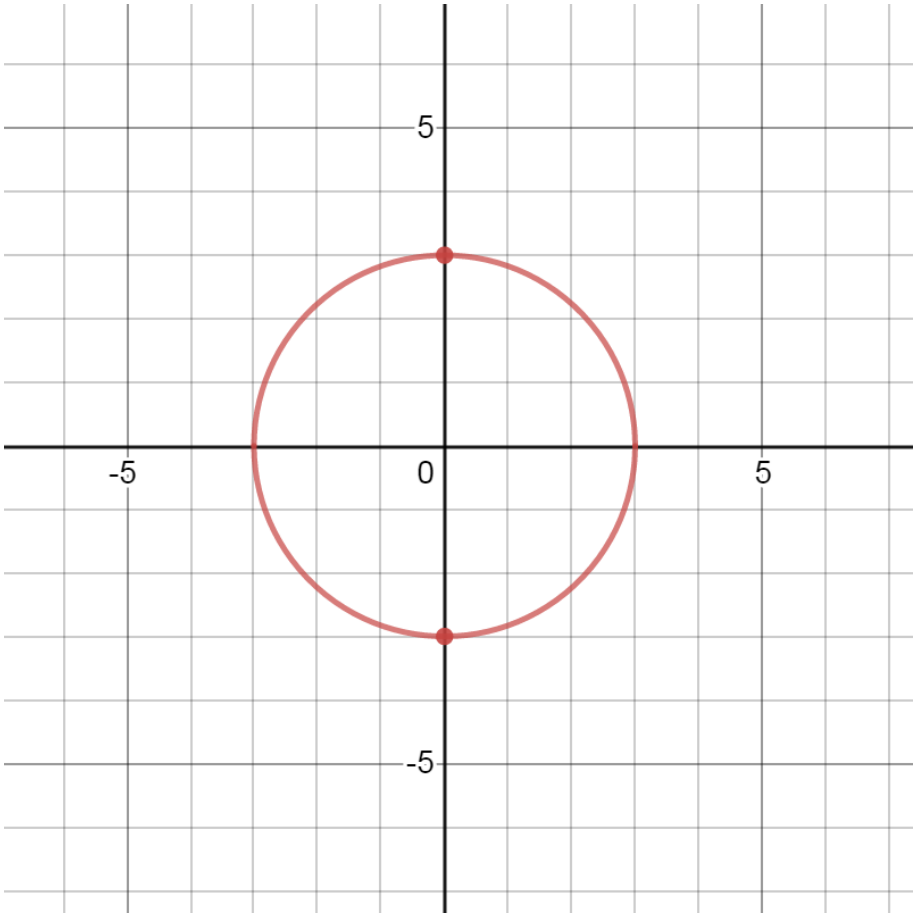
We found that there would be two outputs:

$$0^2 + y^2 = 9$$
$$y^2 = \quad 9$$
$$y \quad = \pm\sqrt{9}$$
$$y = \quad \pm 3$$

Is there a way we could have seen that from the graph?

**I think so!**

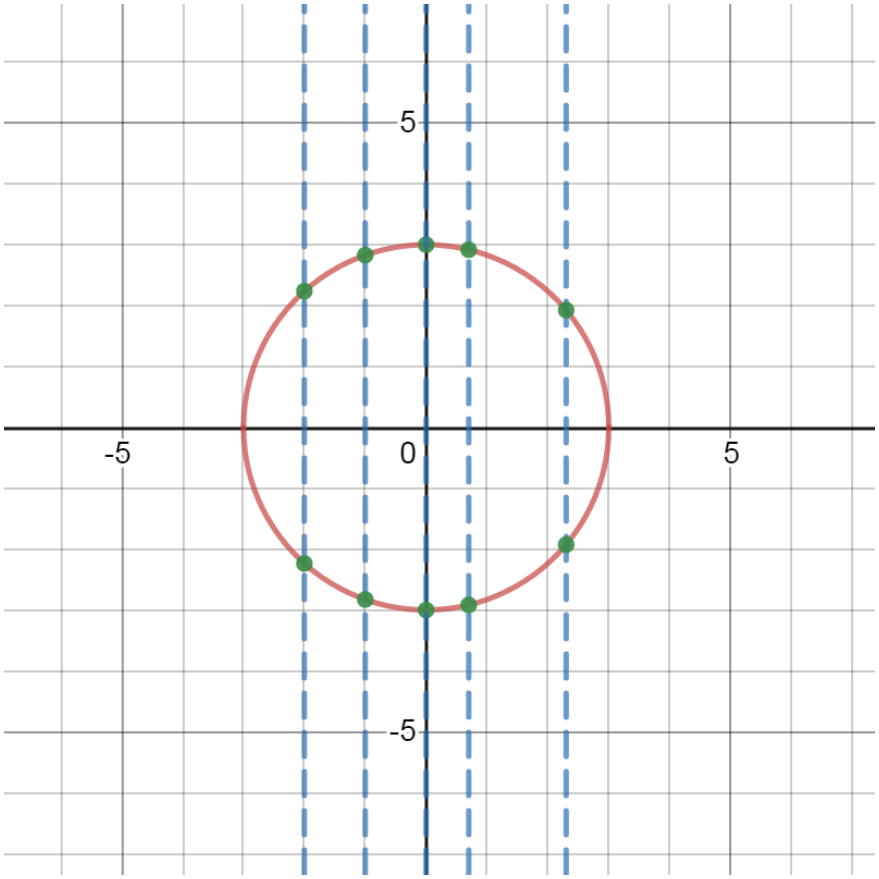
Look:




These two points are directly above (or below) each other!

Which is to say they are ***on the same vertical line***.

Note that this happens in many places on this graph:




Which is to say that our **SPECIAL RULE OF FUNCTIONS** is being broken in *many places* (only $x = -3$ and $x = 3$ have only one output associated with them).

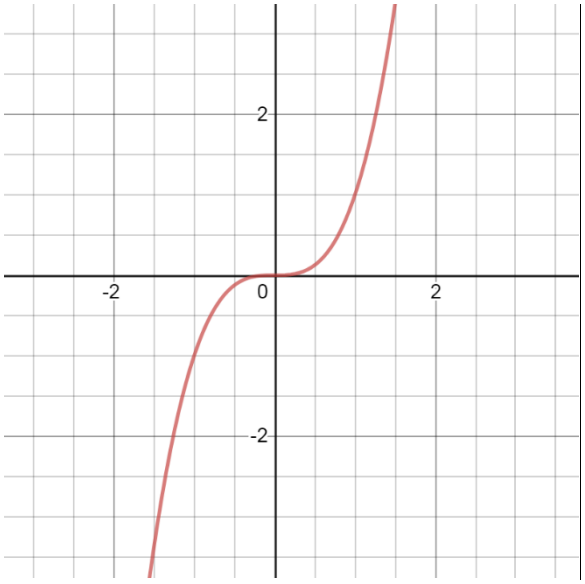
But if it's broken even *once*, that graph is not a function.

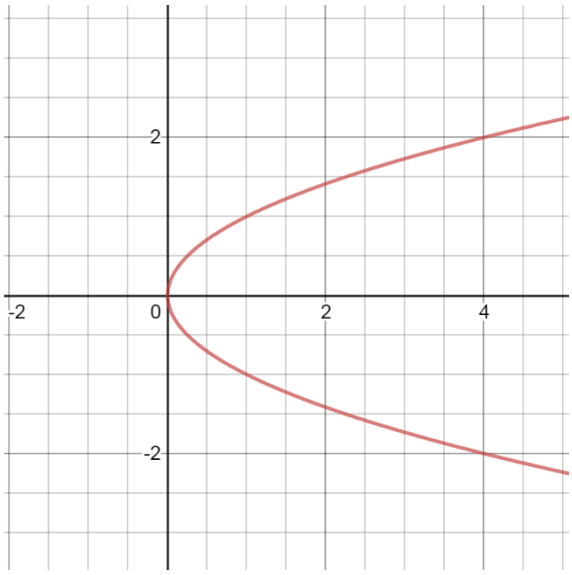
For graphs of functions, then, we have the vertical line test.

It goes like this:

*If a vertical line can be drawn that intersects the graph of a function in more than one point, the graph does not represent a function.*

Consider the following two graphs:




Which of the graphs represents a function?

The first graph comes from the equation $y = x^3$.

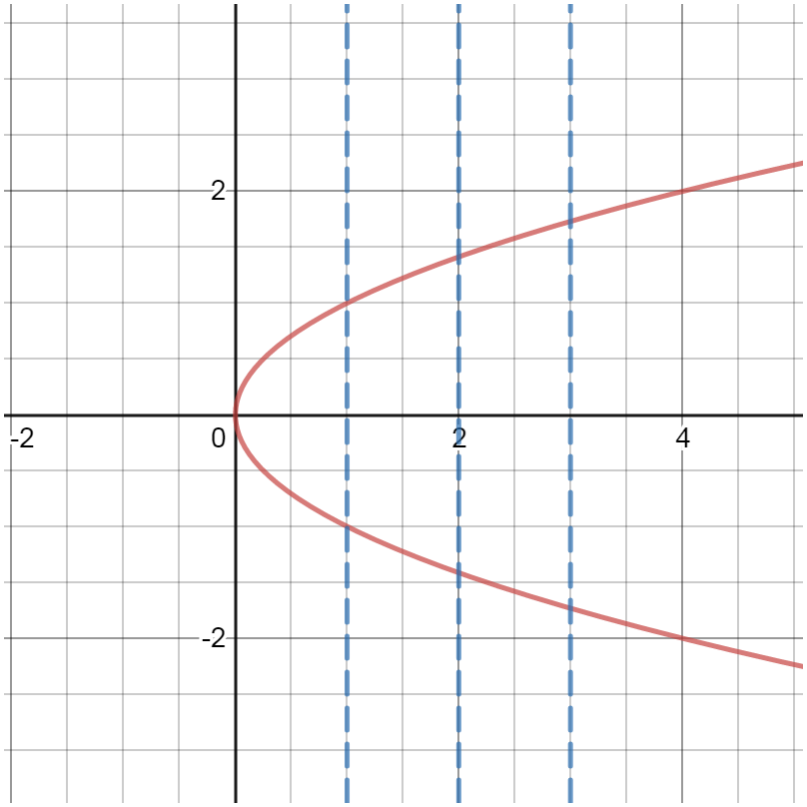
Try drawing a vertical line that crosses the graph twice.

You cannot do it. It's a function (the "cube" function)

The second graph comes from the equation $x = y^2$.

Try drawing a vertical line that crosses the graph twice.

There are many!




This graph **fails** the Vertical Line Test!

It does not represent a function!!!