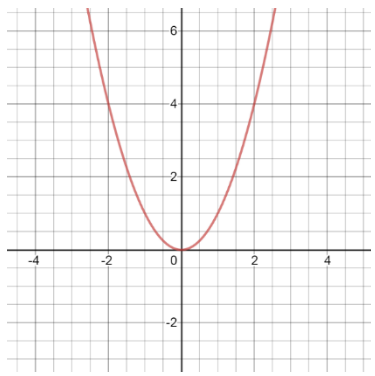
Odd and Even Functions

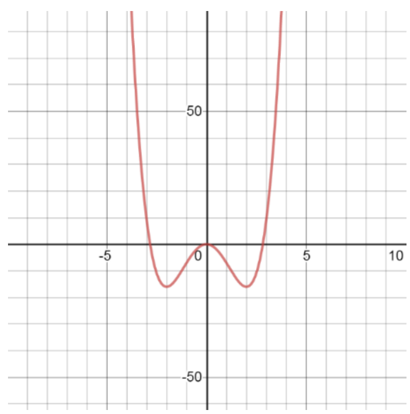
Some graphs of functions have something . . . a little special about them.

Consider the following graphs. What do they have in common?

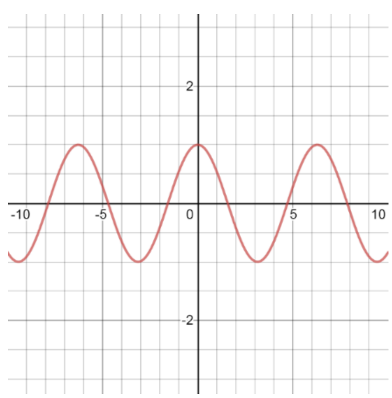
$$f(x) = x^2$$



$$g(x) = x^4 - 8x^2$$



$$h(x) = \cos(x)$$



Answer: they all have something called **symmetry**.

That is, the graph has two equal parts that are **reflections** of each other.

And not only that, they are **reflections across the y-axis**.

This often happens when the functions are polynomials ….

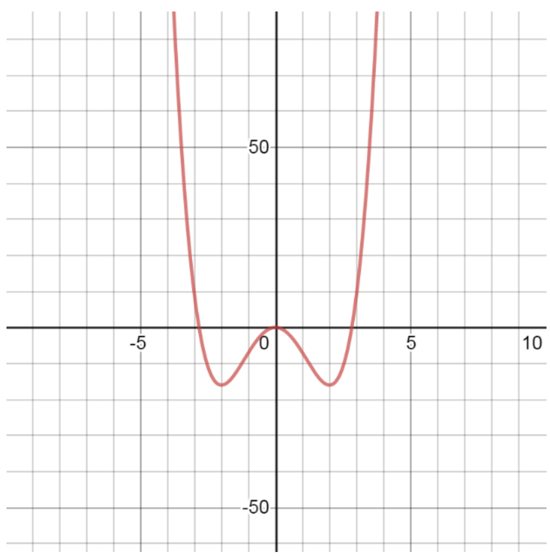
with only even exponents!

Why?

Because negative inputs (on the **left** side of the y-axis)

and positive inputs (on the **right** side of the y-axis)

produce the same outputs (the negative gets squared away):

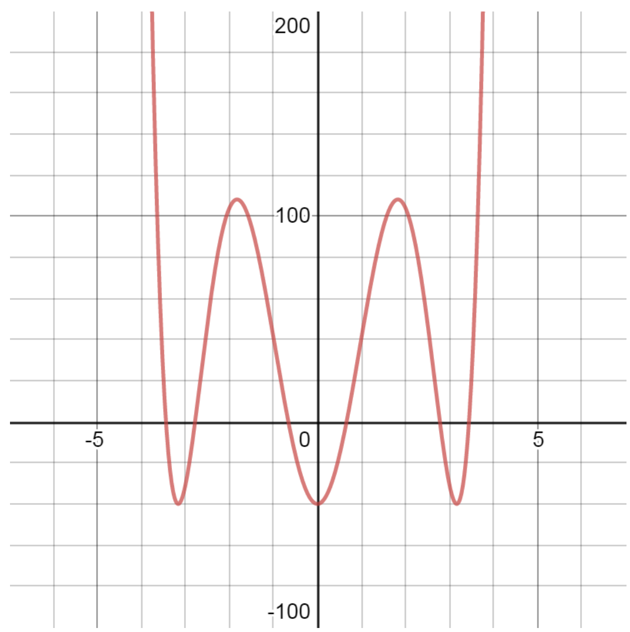


Here is another polynomial function that would **have** to be symmetric…

with respect to the y-axis:

$$p(x) = x^6 - 20x^4 + 100x^2 - 40$$

And here is its graph:



This is an even function!!

Now we need to actually **define** an even function . . .

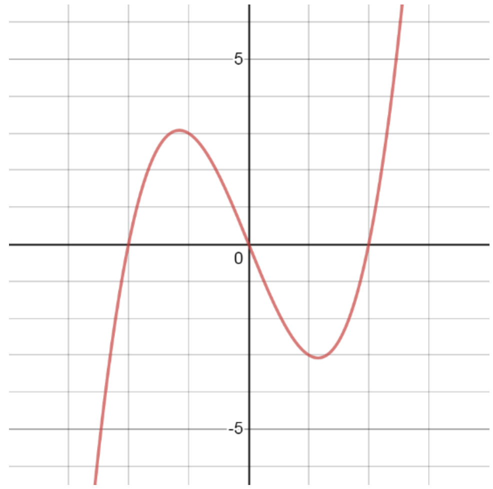
Which is to say what it is **as a rule**.

An even function is a function that produces the same results on the left side of the y-axis as the right side of the y-axis.

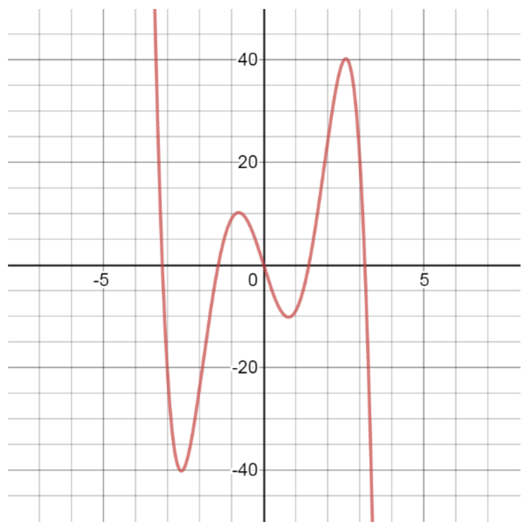
$$f(x) = f(-x)$$

Now, take a look at **these** functions and see what they have in common:

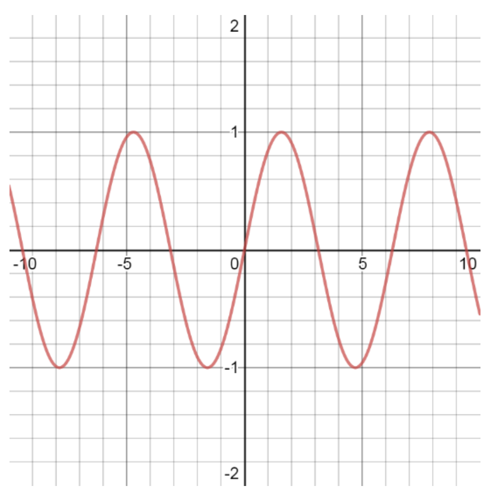
$$f(x) = x^3 - 4x$$



$$g(x) = -x^5 + 12x^3 - 20x$$



$$h(x) = \sin(x)$$

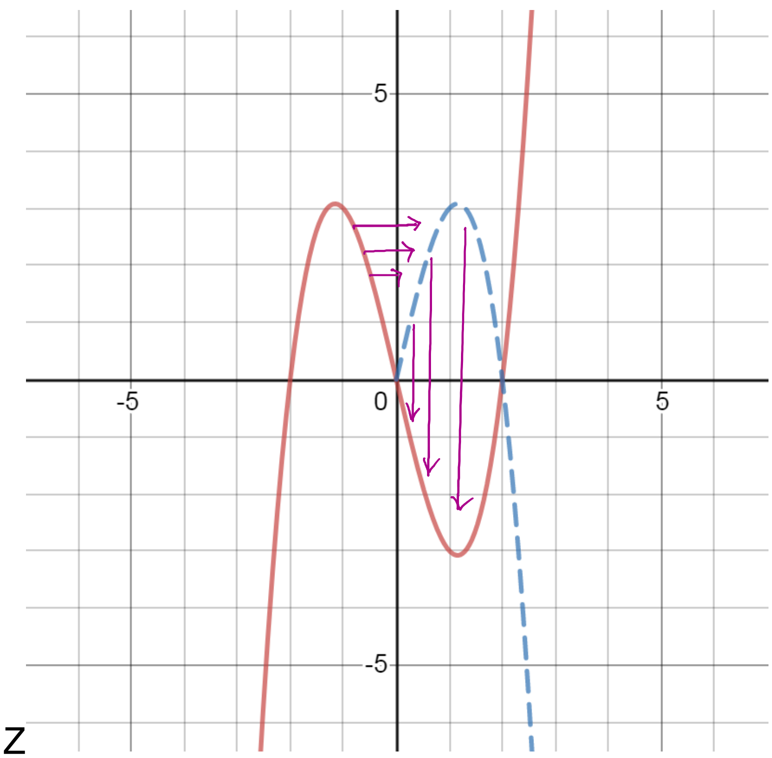


Do you also see **symmetry** with these graphs?

But they have a different kind.

They are not only **reflected across the *y-axis***

But **across the *x-axis*** too!



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Therefore, the **rule** for **odd functions** is

$$f(x) = -f(-x)$$