Synthetic Substitution

Plugging x-values into functions efficiently is important . . .

. . . for finding zeroes

. . . and plotting points to graph.

In the case of polynomials of degree 3 or greater . . .

. . . the standard method ***takes too long***:

$$\begin{aligned} f(2) &= (2)^3 - 5(2)^2 + 2(2) + 8 \\ &= 8 - 5(4) + 4 + 8 \\ &= 8 - 20 + 4 + 8 \\ &= \mathbf{0} \end{aligned}$$

plus you're likely to make a calculation error

You NEED to learn a BETTER way to do this: **synthetic substitution**

To learn this method, let's do another problem:

Find the zeros of

$$g(x) = 2x^3 + 3x^2 - 23x - 12$$

Since we don't know how to **factor** a cubic polynomial, we must look for the zeroes by **trial and error**.

That means *plugging in* x-values . . .

Here's how synthetic substitution works:

Write down the **coefficients** of the function:

$$g(x) = 2x^3 + 3x^2 - 23x - 12$$

$$2 \qquad 3 \qquad -23 \qquad -12$$

Set up a **synthetic substitution bar** for organization:

$$\begin{array}{|cccc} 2 & 3 & -23 & -12 \\ & & & \\ \hline \end{array}$$

And now let's plug in the value $x = 1$:

$$\begin{array}{c|cccc} 1 & 2 & 3 & -23 & -12 \\ & & & & \\ \cline{2-5} \end{array}$$

We are ready to begin! First, we bring down the leading coefficient:

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -23 & -12 \\ & & & & \\ \hline & 2 & & & \end{array}$$

And then we **multiply this value** by our input, $x = 1$ . . .

. . . and **put it under** the next coefficient:

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -23 & -12 \\ & & 2 & & \\ \hline & 2 & & & \end{array}$$

Now we add:

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -23 & -12 \\ & & 2 & & \\ \hline & 2 & 5 & & \end{array}$$

And repeat the process:

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -23 & -12 \\ & & 2 & 5 & \\ \hline & 2 & 5 & & \end{array}$$

Adding:

$$
\begin{array}{r|rrrr}
1 & 2 & 3 & -23 & -12 \\
 & & 2 & 5 & \\
\hline
 & 2 & 5 & \color{red}{-18} &
\end{array}
$$

Multiplying:

$$
\begin{array}{r|rrrr}
1 & 2 & 3 & -23 & -12 \\
 & & 2 & 5 & \color{red}{-18} \\
\hline
 & 2 & 5 & -18 &
\end{array}
$$

And finally adding:

$$
\begin{array}{r|rrrr}
1 & 2 & 3 & -23 & -12 \\
 & & 2 & 5 & -18 \\
\hline
 & 2 & 5 & -18 & \color{red}{-30}
\end{array}
$$

We have that

$$g(1) = -30$$

Just to make sure this is right, let's check:

$$
\begin{aligned}
g(1) &= 2(1)^3 + 3(1)^2 - 23(1) - 12 \\
&= 2(1) + 3(1) - 23 - 12 \\
&= 2 + 3 - 35 \\
&= -30
\end{aligned}
$$

So our method of synthetic substitution works!

You might be thinking that it **seems** MORE COMPLICATED than the standard method . . .

Trust me: this will be the way to do the problems . . .

. . . and it will get faster!!

Now let's plug in $x = 2$:

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -23 & -12 \\ & & 4 & 14 & -18 \\ \hline & 2 & 7 & -9 & -30 \end{array}$$

By coincidence, $g(2)$ also equals -30!

Still no zeroes!!!

Next let's try $x = 3$:

$$\begin{array}{r|rrrr} 3 & 2 & 3 & -23 & -12 \\ & & 6 & 27 & 12 \\ \hline & 2 & 9 & 4 & 0 \end{array}$$

$g(3) = 0$

We found a zero!!!!

Zero: $x = 3$

Now that we know that $x = 3$ is a zero, we know that $(x - 3)$ is a factor!

So we can divide $(x - 3)$ into $g(x)$ to find the other factors . . .

$$
\begin{array}{r l}
 & \quad 2x^2 + 9x + 4 \\
x - 3 & \overline{)2x^3 + 3x^2 - 23x - 12} \\
 & \underline{-2x^3 + 6x^2} \\
 & \qquad 9x^2 - 23x - 12 \\
 & \qquad \underline{-9x^2 + 27x} \\
 & \qquad\qquad 4x - 12 \\
 & \qquad\qquad \underline{-4x + 12} \\
 & \qquad\qquad\quad 0
\end{array}
$$

we're not gonna have to do this

And find that

$$g(x) = 2x^3 + 3x^2 - 23x - 12$$

factors into

$$g(x) = (x - 3)(2x^2 + 9x - 4)$$

We can find the remaining two zeros by factoring $2x^2 + 9x - 4$:

$$2x^2 + 9x + 4$$
$$= (2x + 3)(x + 4)$$

Which will give us our remaining zeroes:

$$g(x) = (x - 3)(2x + 3)(x + 4)$$

$$x - 3 = 0 \qquad 2x + 3 = 0 \qquad x + 4 = 0$$

$$x = 3 \quad x = -\frac{3}{2} \quad x = -4$$

Problem SOLVED!

But WAIT!!

To get the answer, we had to use **polynomial long division** . . .

. . . which takes a **long** time!

As I promised, there is another way!