Solving Quadratic Equations by factoring

Solve the equation:

$$x^2 - 5x - 6 = 0$$

This is a classic type of problem from algebra . . . maybe the most classic!

If we tried solving this equation by isolating the variable . . .

. . . we wouldn't get very far:

$$5x = x^2 - 6$$

$$x = \frac{x^2 - 6}{5}$$

We will solve it by factoring.

This method is based on a very important rule for math:

The Zero Product Rule

If $A \cdot B = 0$

Then $A = 0 \quad or \quad B = 0$

First, note that this method ***only works for expressions that equal zero***.

So if we can write

$$x^2 - 5x - 6$$

as two factors multiplied, we can set each of them equal to zero and solve:

$$x^2 - 5x - 6 = 0$$

$$(x - 6)(x + 1) = 0$$

$$x - 6 = 0 \qquad x + 1 = 0$$

$$x = 6 \qquad x = -1$$

The solution set to this equation is $\{6, -1\}$

Note: we could directly factor the quadratic expression here because it was **already equal to zero**:

$$x^2 - 5x - 6 = \mathbf{0}$$

When the quadratic expression is equal to zero, we say that it is in

Standard Form

In general, a quadratic equation is in standard form if it is written:

$$ax^2 + bx + c = 0$$

standard form of a quadratic equation

Consider this next equation:

$$2(x^2 + 3x) = 36$$

Because this equation contains an x^2 term . . .

. . . we see it's a quadratic equation.

But we are ***not ready to factor***!

The equation must first be put into *standard form*.

$$2(x^2 + 3x) = 36$$

First we bring the constant term over, so we have the **expression = 0**:

$$2(x^2 + 3x) - 36 = 0$$

Next we need to multiply out the parenthesis:

$$2(x^2 + 3x) - 36 = 0$$

$$2x^2 + 6x - 36 = 0$$

And only now do we factor!

To begin, we must remember the first rule of factoring:

The **first** step in factoring is . . .

. . . factor out the greatest common factor.

$$2x^2 + 6x - 36 = 0$$

$$2(x^2 + 3x - 18) = 0$$

Now we look for two numbers that multiply to be -18 . . . and add to be 6:

$$2(x+6)(x-3)=0$$

From here we can use the **zero product rule** . . . for **three** factors:

$$2(x+6)(x-3)=0$$

$$2=0 \qquad x+6=0 \qquad x-3=0$$

The first equation is never true! That doesn't give us any solution!

$$2=0$$

The next two equations do produce solutions:

$$x+6=0 \qquad\qquad x-3=0$$

$$x=-6 \qquad\qquad x=3$$

So our solution set to our original equation is

$$\{-6,3\}$$