Solving Rational Equations

Solve:

$$\frac{1}{4} + \frac{x}{2} = \frac{5}{8}$$

This is an example of a **rational equation**.

Rational equations involve fractions. Fractions are tough to deal with!!

Here's the good news: with any rational equation . . .

We can get rid of the fractions!

Want to know how? Maybe you already know!

Rule for equations with fractions:

We can cancel the denominators . . .

. . . by multiplying both sides by the LCD!

$$(8)\frac{1}{4}+(8)\frac{x}{2}=(8)\frac{5}{8}$$

$$2+4x=5$$

The remainder of the problem is so easy I don't even need to do it!

NOTE:

I have noticed that most students prefer a variation on this method . . .

. . . preferring to convert the denominators first:

$$\frac{(2)}{(2)}\frac{1}{4}+\frac{(4)}{(4)}\frac{x}{2}=\frac{5}{8}$$

$$\frac{2}{8}+\frac{4x}{8}=\frac{5}{8}$$

$$(8)\frac{2}{8}+(8)\frac{4x}{8}=(8)\frac{5}{8}$$

This method involves an extra step that's not strictly necessary . . .

. . . but since it's so popular I will use it!

Solve:

$$\frac{1}{2x^2 - 4x} + \frac{3}{x-2} = \frac{5}{6x}$$

Here is another equation with fractions . . .

. . . so we must **multiply both sides by the LCD!**

And **to find the LCD . . .**

. . . **we need to factor!**

$$\frac{1}{2x^2 - 4x} + \frac{3}{x-2} = \frac{5}{6x}$$

$$= \frac{1}{2x(x-2)} + \frac{3}{x-2} = \frac{5}{6x}$$

Now that we see what's in those denominators . . .

. . . **let's convert to make them the same!**

$$\frac{(3)}{(3)}\frac{1}{2x(x-2)} + \frac{(6x)}{(6x)}\frac{3}{x-2} = \frac{(x-2)}{(x-2)}\frac{5}{6x}$$

$$\frac{3}{6x(x-2)} + \frac{18x}{6x(x-2)} = \frac{5(x-2)}{6x(x-2)}$$

Now we can *multiply both sides by the LCD* to **cancel the denominators**:

$$\cancel{(6x(x-2))}\frac{3}{\cancel{6x(x-2)}}+\cancel{(6x(x-2))}\frac{18x}{\cancel{6x(x-2)}}=\cancel{(6x(x-2))}\frac{5(x-2)}{\cancel{6x(x-2)}}$$

$$3+18x=5(x-2)$$

$$3+18x=5x-10$$

$$18x-5x=-10-3$$

$$13x=-13$$

$$x=-1$$

Let's do one more to show how sometimes . . .

. . . things get more interesting at the end:

Solve:

$$\frac{x}{x^2+x-2}=\frac{x}{x^2+3x+2}-\frac{x}{x^2-1}$$

First we factor:

$$\frac{x}{x^2+x-2} = \frac{x}{x^2+3x+2} - \frac{x}{x^2-1}$$

$$\frac{x}{(x+2)(x-1)} = \frac{x}{(x+1)(x+2)} - \frac{x}{(x-1)(x+1)}$$ s!

We will do what we have to to make them the same:

$$\frac{(x+1)x}{(x+1)(x+2)(x-1)} = \frac{(x-1)x}{(x-1)(x+1)(x+2)} - \frac{(x+2)x}{(x+2)(x-1)(x+1)}$$

Now all the denominators are the same!

Let's multiply both sides by those denominators . . .

. . . and do this step in our head!

We get:

$$(x+1)x = (x-1)x - (x+2)x$$

$$x^2 + x = x^2 - x - x^2 - 2x$$

$$x^2 + x = -x - 2x$$

This is a quadratic equation! We must put into standard form and factor:

$$x^2 + x = -3x$$

$$x^2 + 4x = 0$$

$$x(x + 4) = 0$$

Let’s set both factors equal to zero and solve:

$$x = 0 \qquad x + 4 = 0$$

$$x = 0 \qquad x = -4$$