Synthetic Substitution

Plugging x-values into functions efficiently is important . . .

... for finding zeroes

... and plotting points to graph.

In the case of polynomials of degree 3 or greater . . .

... the standard method *takes too long*:

$$f(2) = (2)^{3} - 5(2)^{2} + 2(2) + 8$$

$$= 8 - 5(4) + 4 + 8$$

$$= 8 - 20 + 4 + 8$$

$$= 0$$
a calculation error

You NEED to learn a BETTER way to do this: synthetic substitution

To learn this method, let's do another problem:

Find the zeros of

$$g(x) = 2x^3 + 3x^2 - 23x - 12$$

Since we don't know how to **factor** a cubic polynomial, we must look for the zeroes by **trial and error**.

That means *plugging in x*-values . . .

Here's how synthetic substitution works:

Write down the **coefficients** of the function:

$$g(x) = 2x^{3} + 3x^{2} - 23x - 12$$

$$2 \quad 3 \quad -23 \quad -12$$

Set up a **synthetic substitution bar** for organization:

And now let's plug in the value x = 1:

We are ready to begin! First, we bring down the leading coefficient:

And then we **multiply this value** by our input, $x = 1 \dots$

... and put it under the next coefficient:

Now we add:

And repeat the process:

Adding:

Multiplying:

And finally adding:

We have that

$$g(1) = -30$$

Just to make sure this is right, let's check:

$$g(1) = 2(1)^{3} + 3(1)^{2} - 23(1) - 12$$

$$= 2(1) + 3(1) - 23 - 12$$

$$= 2 + 3 - 35$$

$$= -30$$

So our method of synthetic substitution works!

You might be thinking that it **Seems MORE COMPLICATED** than the standard method . . .

Trust me: this will be the way to do the problems . . .

. . . and it will get faster!!

Now let's plug in x = 2:

By coincidence, g(2) also equals -30!

Still no zeroes!!!

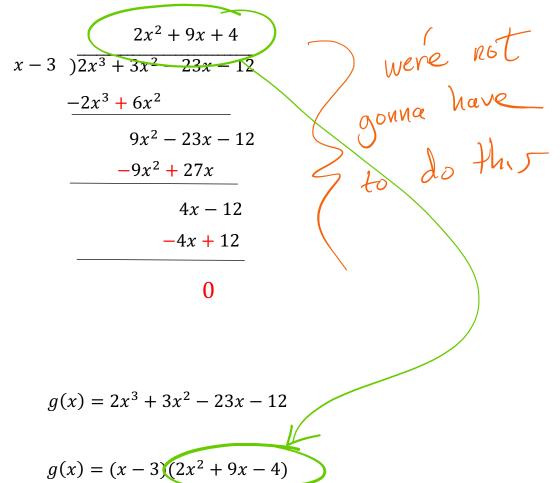
Next let's try x = 3:

We found a zero!!!!

Zero:
$$x = 3$$

Now that we know that x = 3 is a zero, we know that (x - 3) is a factor!

So we can divide (x - 3) into g(x) to find the other factors . . .



factors into

And find that

g(x) = (x - 3)(2x + 9x - 4)

We can find the remaining two zeros by factoring $2x^2 + 9x - 4$:

$$2x^2 + 9x + 4$$
$$= (2x + 3)(x + 4)$$

Which will give us our remaining zeroes:

$$g(x) = (x-3)(2x+3)(x+4)$$

$$x-3 = 0 2x+3 = 0 x+4 = 0$$

x = 3 $x = -\frac{3}{2}$ x = -4

Problem SOLVED!

But WAIT!!

To get the answer, we had to use polynomial long division . . .

... which takes a **long** time!

As I promised, there is another way!