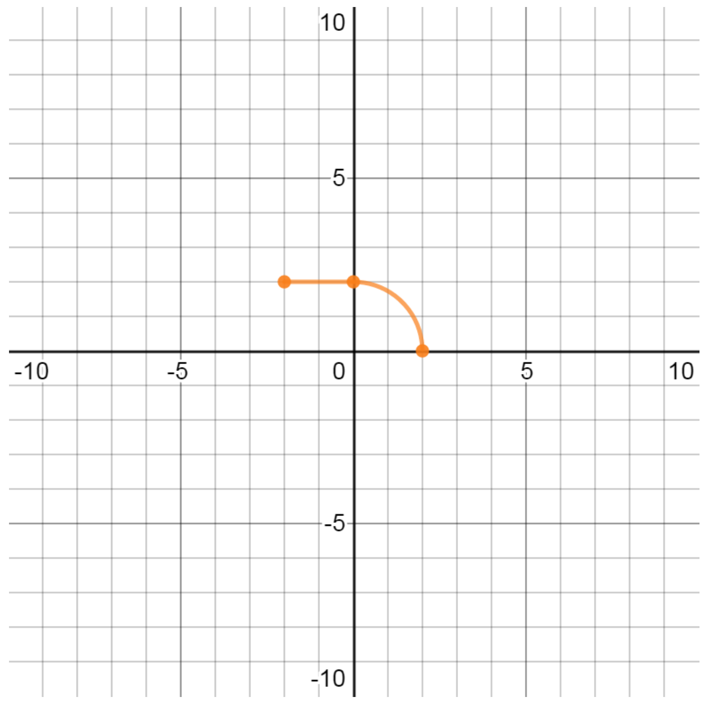
Transformations of functions defined by their graphs

Consider the following graph:

$h(x)$



There is no known formula to this graph, or if there is, we don't know it.

We will just call it $h(x)$.

We do know some **points** on $h(x)$; for example, $h(-2) = 2$ and $h(2) = 0$.

Here's the problem: draw the graph of the following transformed function:

$$f(x) = -2h(x) + 3$$

To draw the graph of $f(x)$, we note that $f(x)$ is just $h(x)$ . . .

. . . put through **three** transformations:

$$-2h(x) + 3$$

vertical reflection

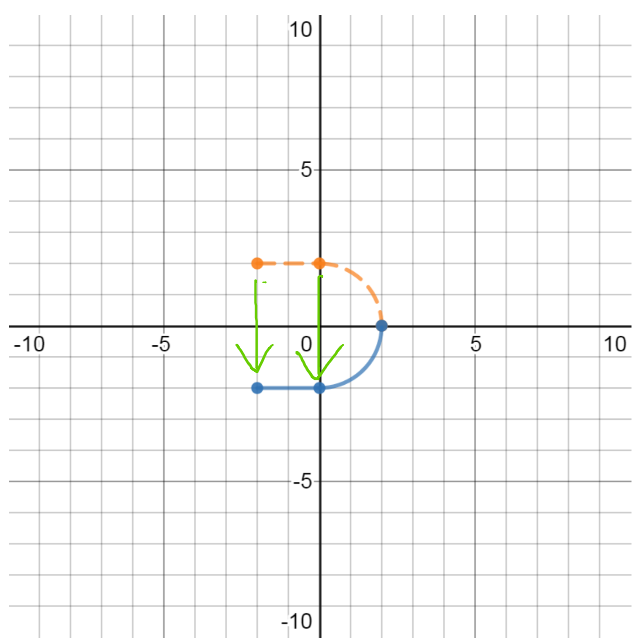
vertical stretch

vertical shift

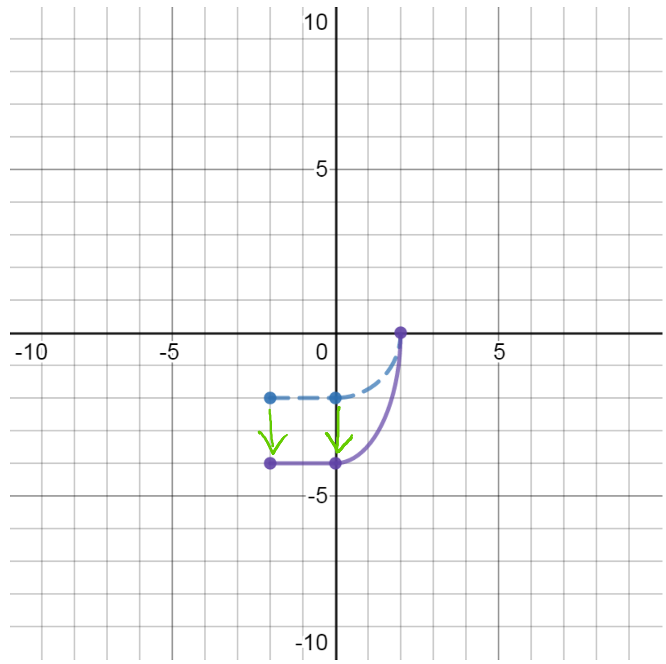
Those are:

- ✓ vertical reflection
- ✓ vertical stretch (times 2)
- ✓ vertical shift (up three)

First do the vertical reflection:

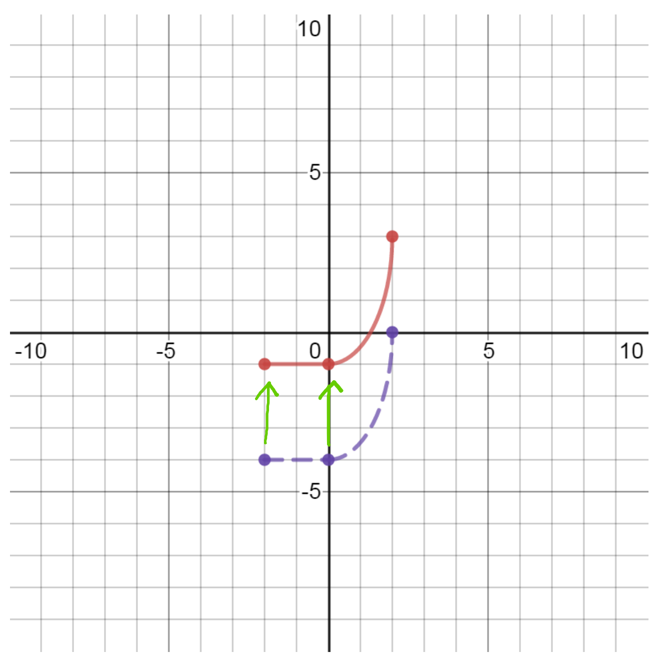


Then do the vertical stretch:



And finally, do the vertical shift!

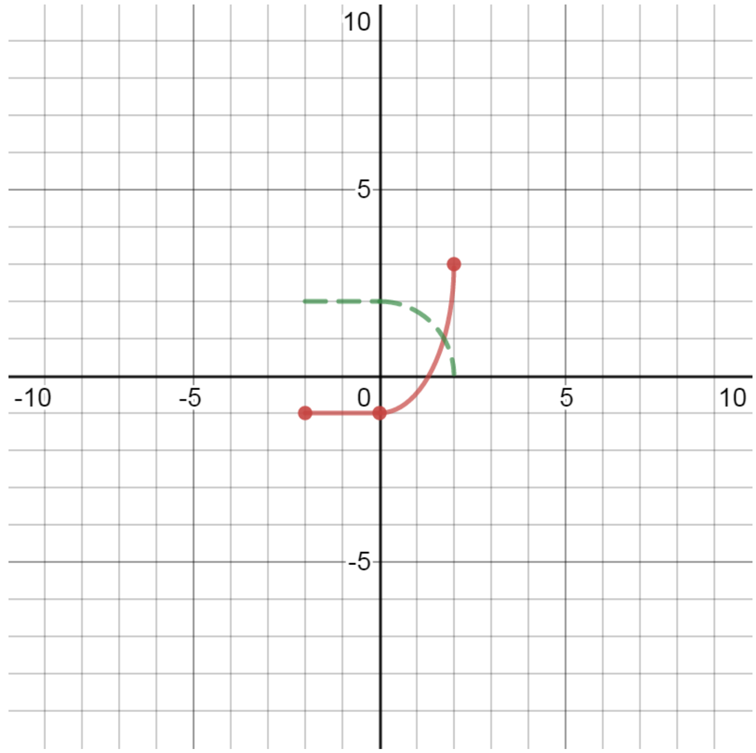
(remember, shifts are always last)



The final answer (shown with the original function):

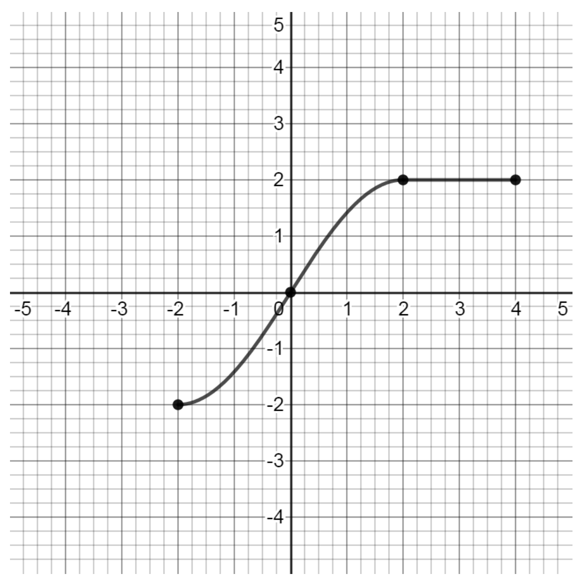
$h(x)$

$f(x) = -2h(x) + 3$



Let's do one more:

$p(x)$



Here's the problem: draw the graph of $q(x) = \frac{1}{2}p(-(x+3))$

As always, we need to figure out, for $q(x)$ . . . .

. . . what are the transformations of $p(x)$?

Look at the formula:

horizontal reflection

$$q(x) = \frac{1}{2}p(-(x+3))$$

horizontal shift

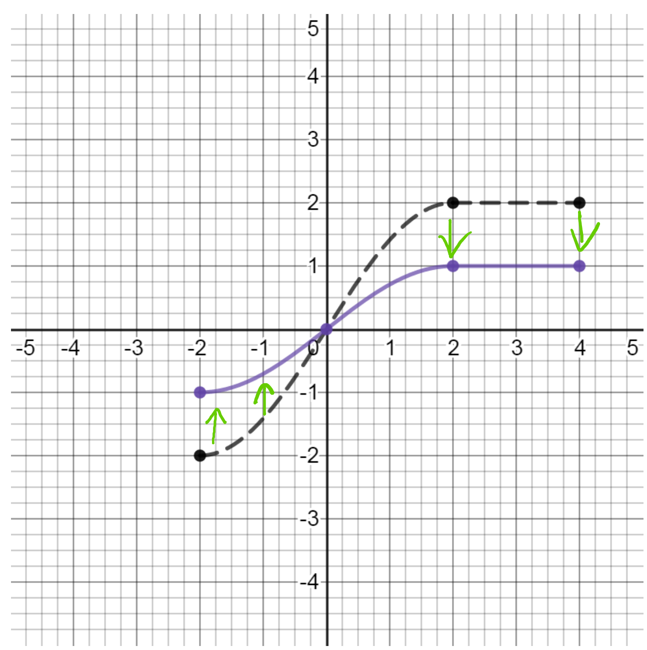
vertical shrink

We see the following transformations:

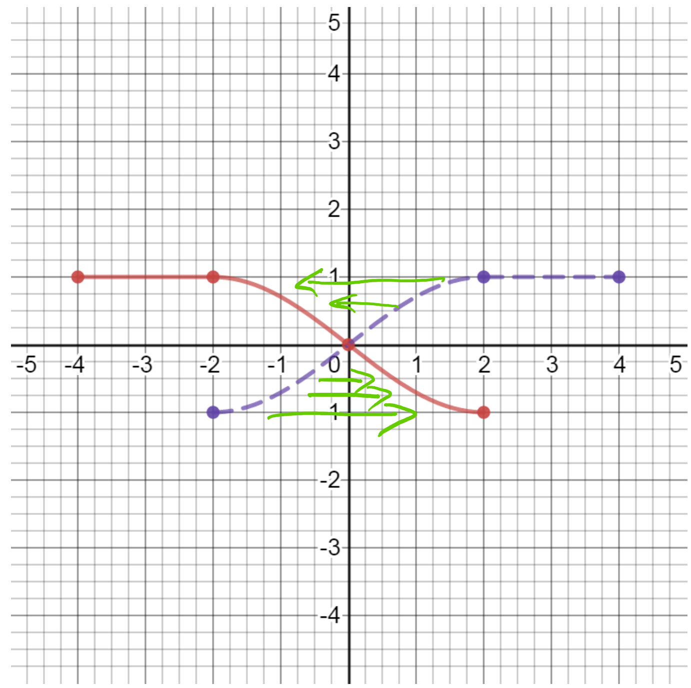
- ✓ vertical shrink
- ✓ horizontal reflection
- ✓ horizontal shift

We could do either the **vertical shrink** **or** the **horizontal reflection** first.

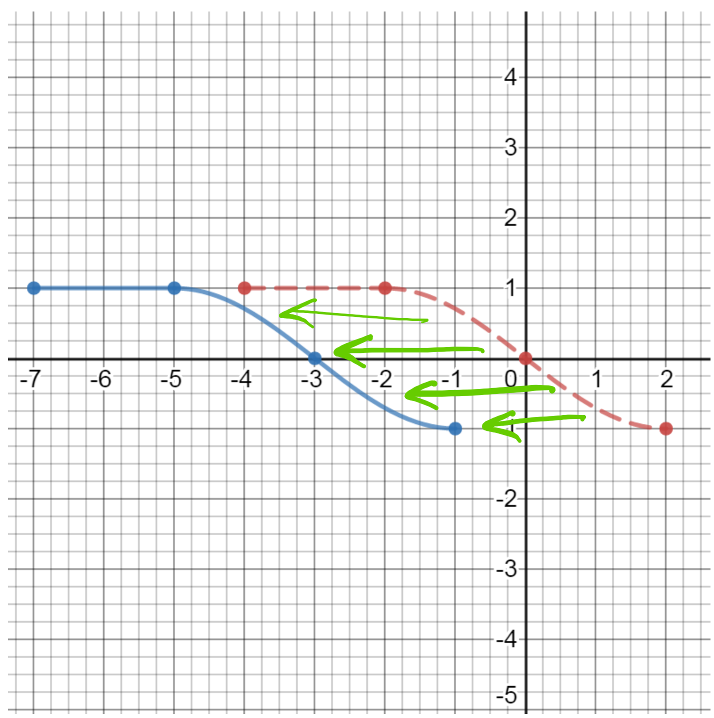
Let's do the vertical shrink:



Then we do the horizontal reflection:



And finally the horizontal shift:



The final answer (shown with the original function):

$p(x)$

$$q(x) = \frac{1}{2}p(-(x+3))$$