Profit

Let's return to a problem we considered a few weeks ago . . .

> We have a personal business making and selling hand-made designer dresses. In this example, we had that our loft rent per month was $1200, and each dress cost $40 to make in materials. From this we had a cost function that looked like this:

$$C(x) = the\ monthly\ cost\ of\ making\ x\ dresses$$
$$= 1200 + 40x$$

Now let's consider this situation from the **revenue** side:

> Suppose that we do market research on our dresses, trying to sell them at two different prices:

At $60 per dress we can sell 200 per month

At $100 per dress we can sell 120 per month

> Find the (monthly) profit for our business . . .
>
> . . . as a function of price.

To figure this out, we first consider what it means to have a **profit**.

What is profit?

Profit is the money you take in, minus the money you pay out.

Put more simply:

Profit = Revenue – Cost

We already have our cost function: $C(x) = 1200 + 40x$

we need p to be the independent variable

It's in terms of dresses instead of in terms of price

. . . so we will have to fix that eventually . . .

But for now, let's turn to the problem of ***getting a formula for revenue***.

What is revenue?

Revenue is the money a business takes in.

If that business is selling just one product, then we have that

Revenue = price x quantity sold

To get a formula for revenue, let's define variables for these quantities:

$$R = revenue$$

$$p = price$$

$$x = quantity\ sold$$

Therefore our function for Revenue becomes:

$$R = p * x$$

Here's our problem!!!!

We need to get our formula for revenue *in terms of* p!

(we don't want x in the formula)

Because this is what the problem asked us to do:

Find the (monthly) profit for our business as a function of price

We can do this! To do it, we need to get x in terms of p!

And we have done this before!!!

Remember? We did a problem involving selling designer t-shirts . . .

. . . and found a linear equation between p and x.

Let's use the same strategy!

Remember, we had some market research for our hand-made dresses:

At \$60 per dress we can sell 200 per month

At \$100 per dress we can sell 120 per month

Let's turn this information into points on a graph:

$$(p_1, x_1) = (60, 200)$$
$$(p_2, x_2) = (100, 120)$$

IMPORTANT NOTE:::

Why are we putting p into the x-position rather than the other way around???

Remember, our function is supposed to be **in terms of p**!

That means p must be the input! ← this means it's the x-variable

. . . which makes sense because we get to choose our price!!

(price is the independent variable)

Whew!! OK!

So our points

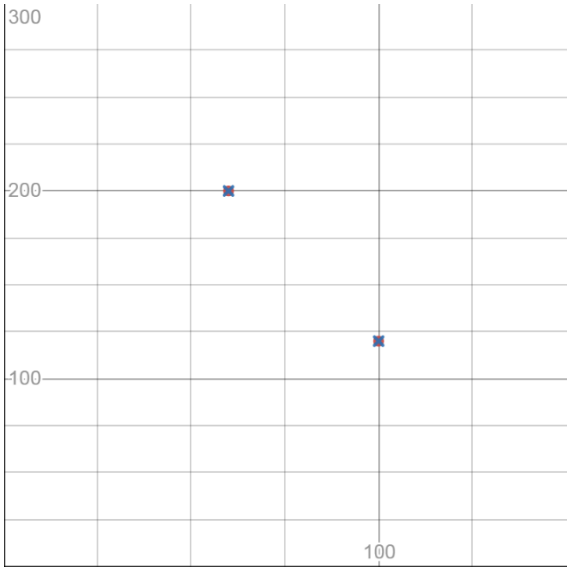
$$(p_1, x_1) = (60, 200)$$

$$(p_2, x_2) = (100, 120)$$

$quantity\ sold$

(x)

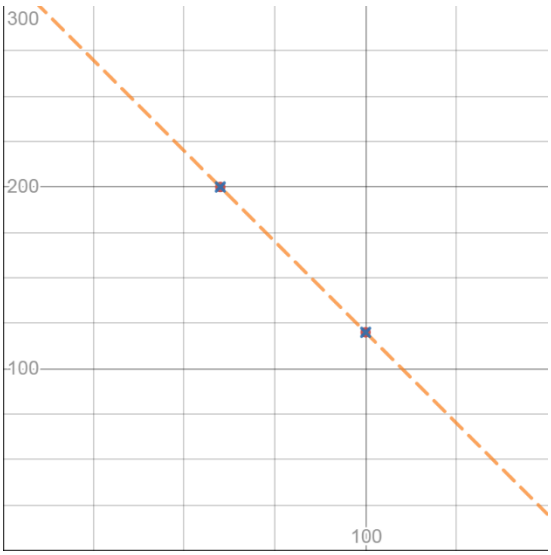
$in\ dresses$



$price\ (p)\ in\ dollars$

Can be connected through a linear equation . . .

. . . that will predict other sales at other prices . . .



To find this equation, we must first find the slope:

Important:

$$y = mx + b$$

$$x = mp + b$$

remember p is the input, so it goes in the x spot

$$m = \frac{x_2 - x_1}{p_2 - p_1}$$

$$= \frac{120 - 200}{100 - 60}$$

$$= -2$$

And then find b:

$$x = -2p + b$$

$$200 = -2(60) + b$$

$$200 = -120 + b$$

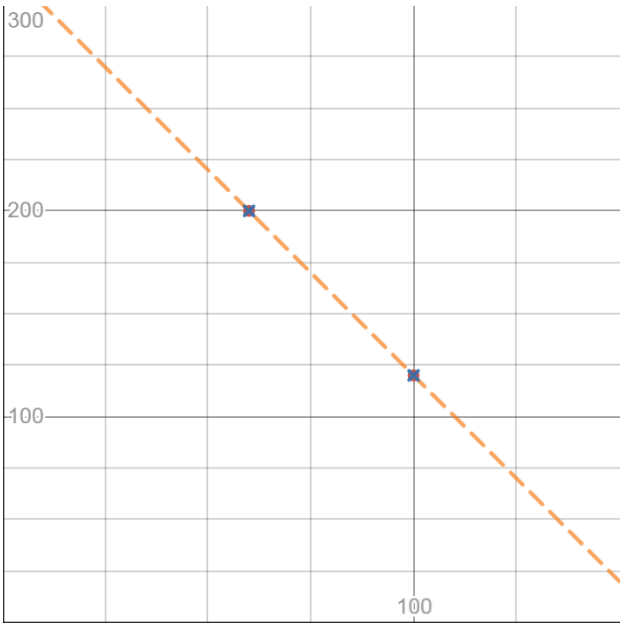
$$320 = b$$

So we get

$$x = -2p + 320$$

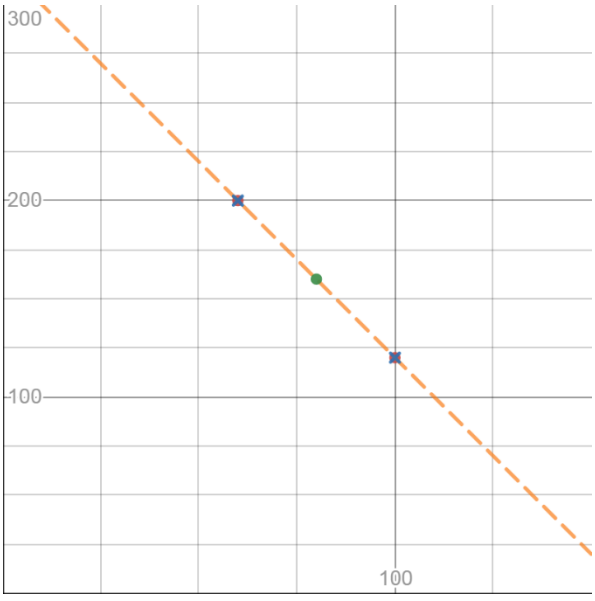
$x = -2p + 320$

$x = quantity\ sold$



$p = price$

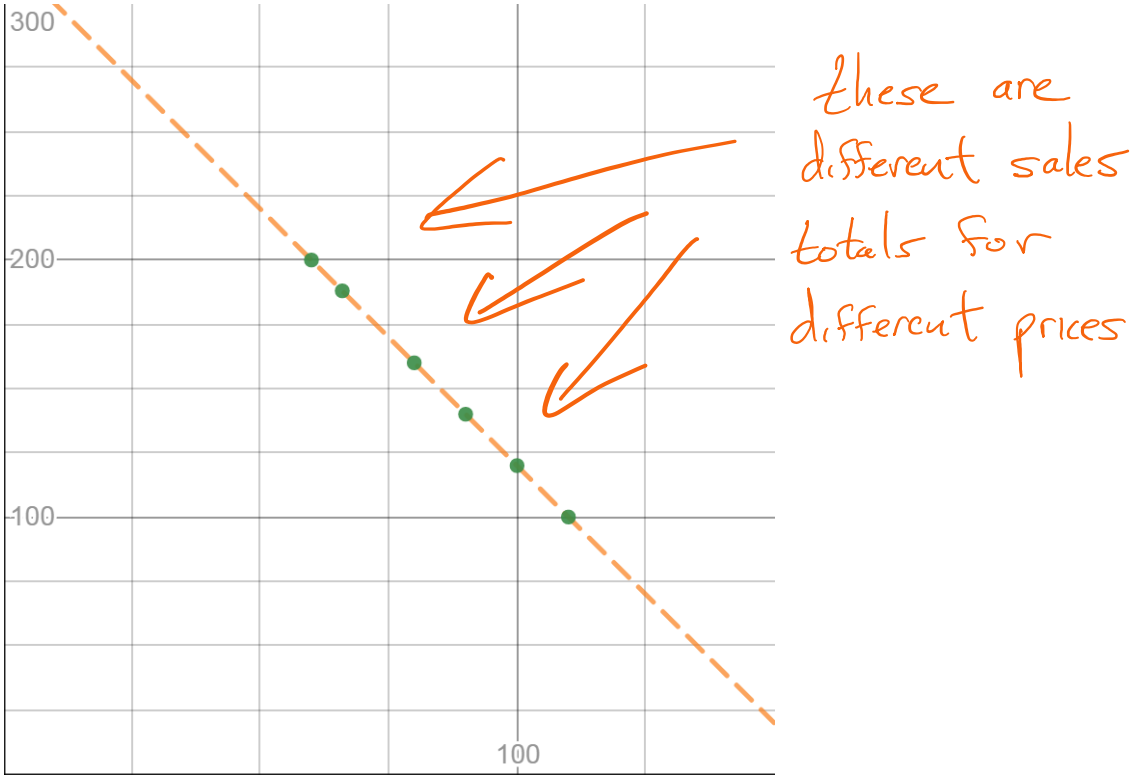
For example, we can see that at $80 per dress, we predict sales of $160:



Now, this equation that we have created . . .

contains many other possibilities for prices too . . .

. . . with their corresponding number sold . . .



Here's the question that we would like to answer:

Which price would give us the biggest profit??

This is why we are trying to get

profit

as a function of

price!

Remember, to get profit as a function . . .

. . . we need to get REVENUE as a function . . .

And we know that

$$R = Revenue = (price) * (quantity\ sold) = p * x$$

We also now figured out that:

$$x = -2p + 320$$

We can plug this formula for x into our formula for R to get:

$$\begin{aligned} R &= p * x \\ &= p * (-2p + 320) \\ &= -2p^2 + 320p \end{aligned}$$

So

$$R(p) = -2p^2 + 320p$$

we have revenue in terms of price

We're almost done!

Remember that

$$Profit = Revenue - Cost$$

And

$$C(x) = 1200 + 40x$$

Given that

$$x = -2p + 320$$

We get that

$$\begin{aligned} C(p) &= 1200 + 40(-2p + 320) \\ &= 1200 - 80p + 12800 \\ &= 14000 - 80p \end{aligned}$$

PUTTING EVERYTHING TOGETHER . . .

$$\begin{aligned} P(p) &= monthly\ profit\ if\ dresses\ are\ sold\ at\ price\ p \\ &= R(p) - C(p) \\ &= -2p^2 + 320p - (14000 - 80p) \\ &= -2p^2 + 320p - 14000 + 80p \\ &= -2p^2 + 400p - 14000 \end{aligned}$$

We are done! We have a formula for monthly profit based on price per dress!

$$P(p) = -2p^2 + 400p - 14000$$