# Linear Equations
## in two variables

We've learned how to solve linear equations in **one** variable:

$$2x - 1 = 5$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

The solution to the equation is 3.

We can **check** that solution by plugging it into the equation:

$$2(3) - 1 = 5$$

$$5 = 5$$

What would we do with an equation in **two** variables?

$$2x + 3y = 6$$

Can you think of what a solution would *mean*?

Since there are two variables, a solution would have to include both an

$x$-value . . .

. . . and a $y$-value.

$$2x - 3y = 6$$

What is an $x$-value and a $y$-value that makes the statement true?

Try the point $(3, 0)$:

$$2(3) - 3(0) = 6$$

$$6 = 6$$

So the point $(3, 0)$ is a solution!

Are there any more?

How about the point $(0, 2)$?

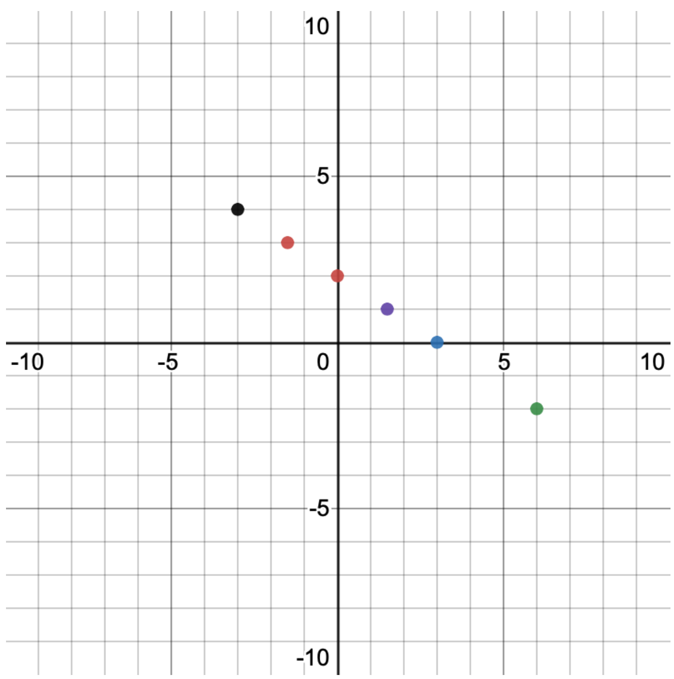
$$2(0) + 3(2) = 6$$

$$6 = 6$$

This is a solution too!

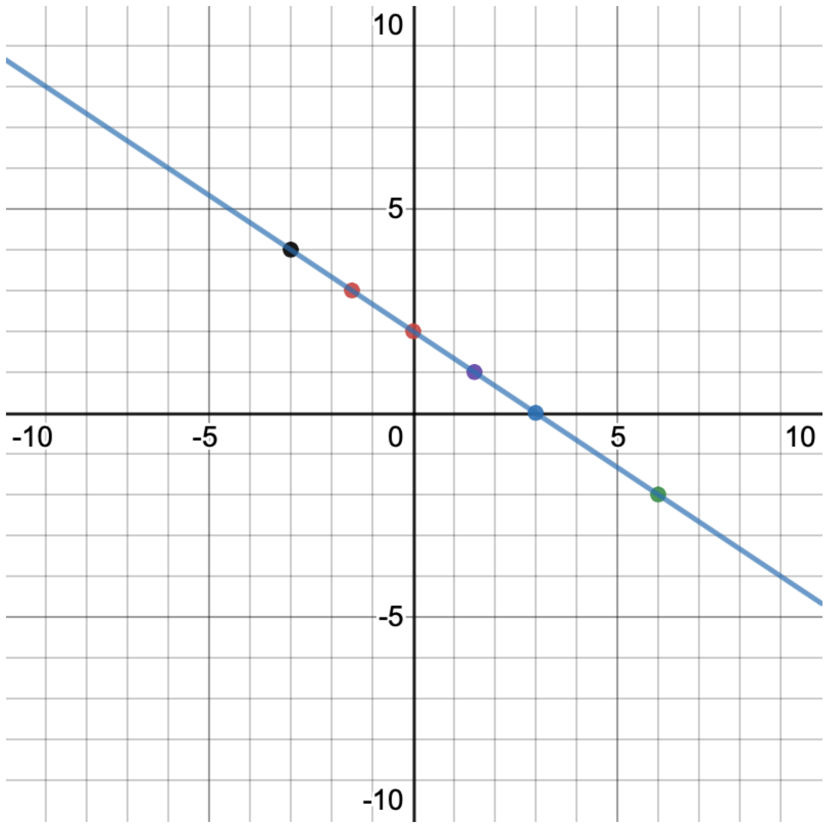
In fact, there are an *infinite number of solutions*!

The only way we can write them all down is by depicting them with a graph:



These points are contained in a line . . .

. . . which consists of **all** solutions!




So the graph of the equation . . .

. . . is a picture of the solutions.

But there’s a different way that we can think of “solving” this equation . . .

. . . which gives us another way to draw a graph.

Solve for $y$:

$$2x + 3y = 6$$

$$-2x \qquad -2x$$

$$\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3}$$

$$y = -\frac{2}{3}x + 2$$

Have you seen this version of a linear equation before?

It's called . . .

. . . slope-intercept form

$$y = mx + b$$

This form of the equation makes it easy to find points on the line:

put these in for x

| $x$ | $y$ |
|---|---|
| -3 | 4 |
| 0 | 2 |
| 3 | 0 |
| 6 | -2 |
| 9 | -4 |

these come out for y

But also, this form of the equation reveals two important numbers:

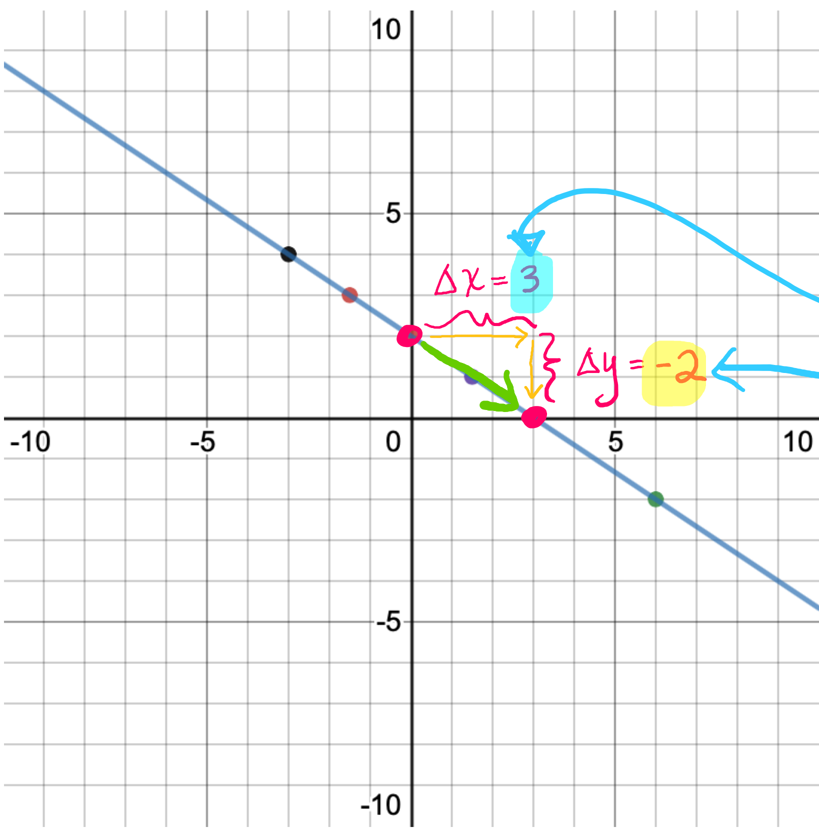
$$y = mx + b$$

slope

y-intercept

The **slope** shows the direction of the line:

$$y = -\frac{2}{3}x + 2$$




"delta"

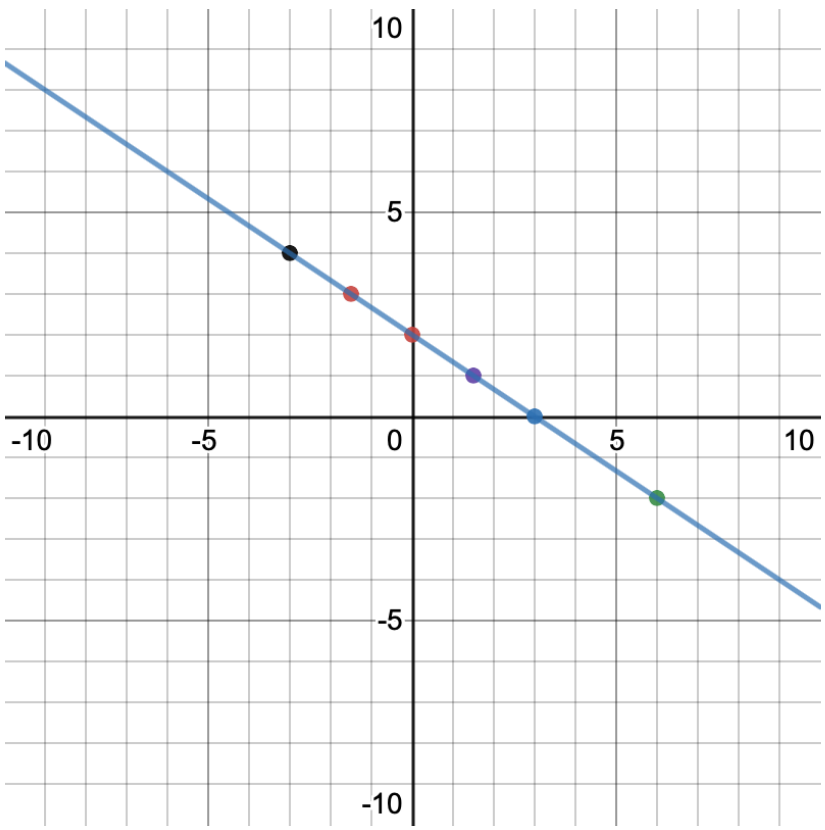
means change

$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{3}$$

$$y = mx + b$$

The ***y*-intercept** shows where the line intersects the $y$-axis:

$$y = -\frac{2}{3}x + 2$$



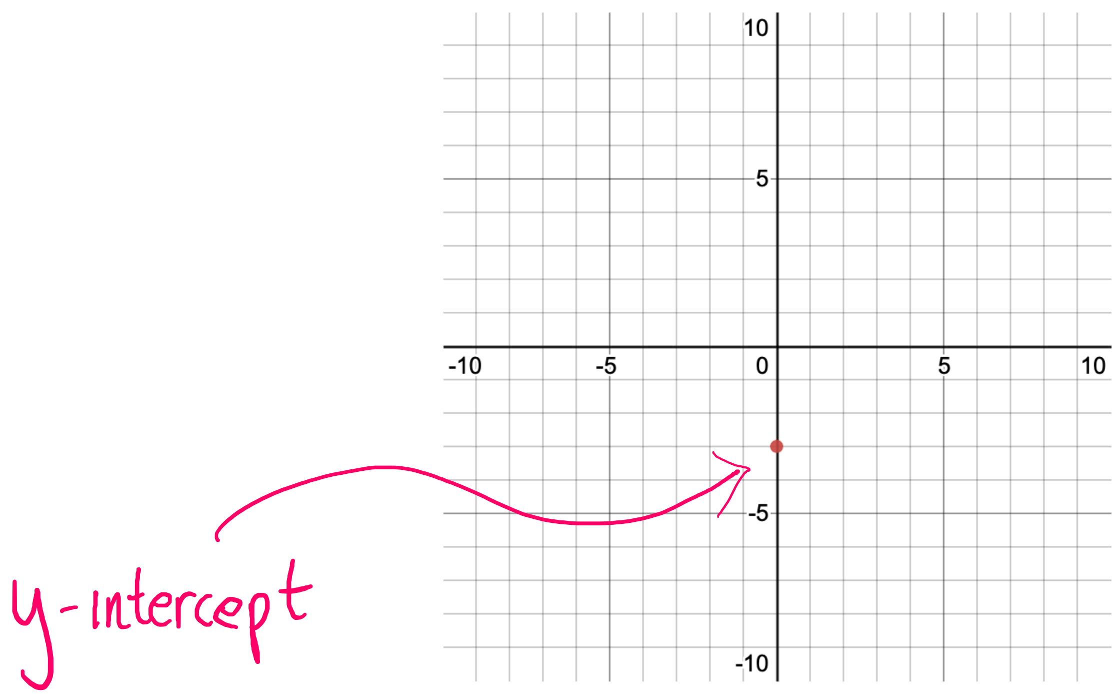

$$b = 2$$

You can even graph a linear equation in slope-intercept form:

Graph the linear equation:

$$y = \frac{2}{5}x - 3$$

Let’s start the graph at the $y$-intercept, which is the point $(0, -3)$:

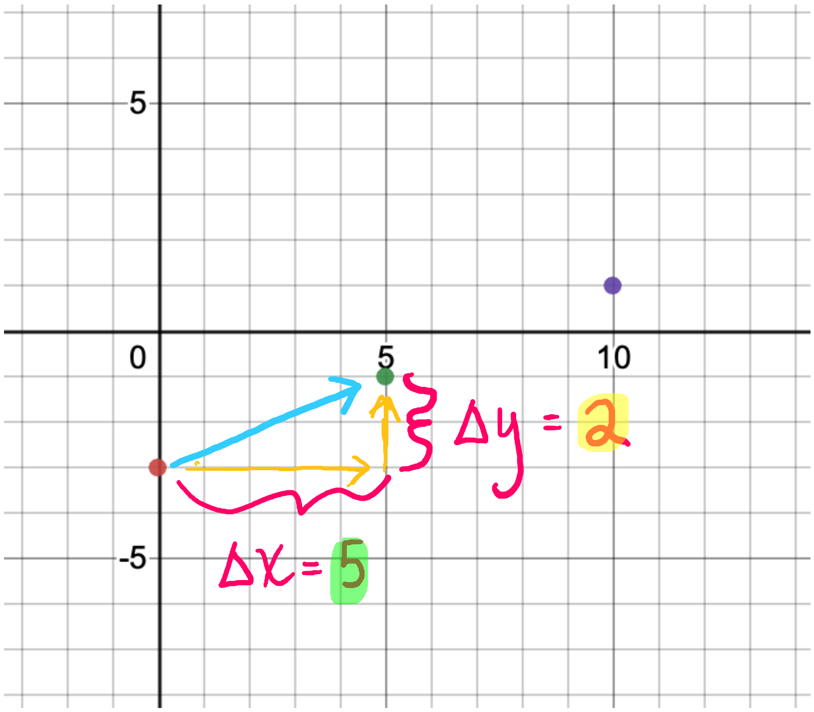



Next, we will draw the direction of line, by noting the slope:

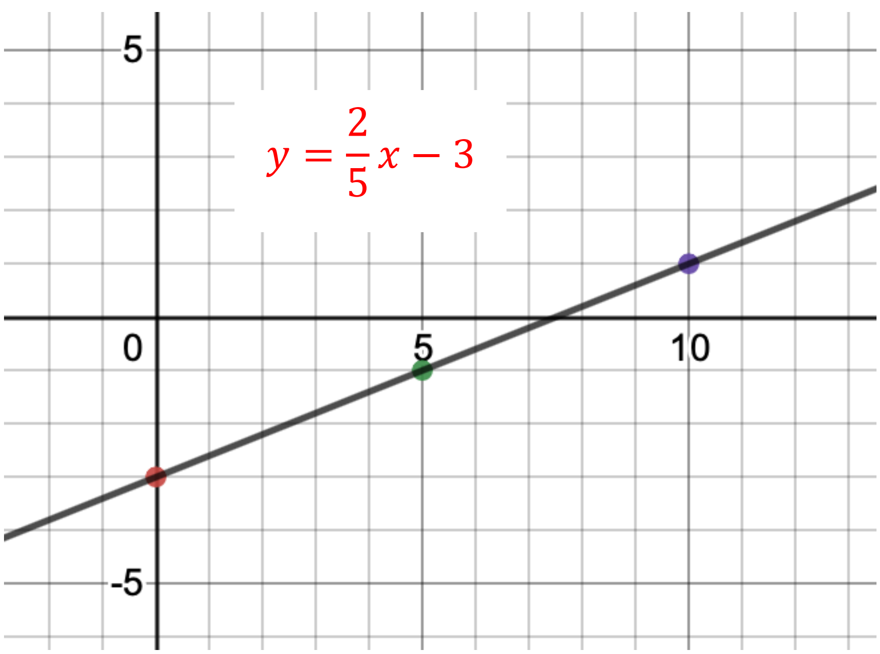
$$m = \frac{\Delta y}{\Delta x} = \frac{2}{5}$$

We can get other points on the line by *rising* by 2 . . .

. . . and *running* by 5 . . .




$$m = \frac{\Delta y}{\Delta x} = \frac{2}{5}$$



We can also *find the equation of a line* . . .

. . . by knowing the ***slope*** . . .

. . . and ***y-intercept***

Find the equation of the line with slope $-\frac{1}{2}$ and y-intercept 4.

We simply plug these values into the slope-intercept equation of the line:

$$y = mx + b$$

$$y = -\frac{1}{2}x + 4$$

What if you don't know the y-intercept but you know a point on the line?

Find the equation of the line with slope $\frac{2}{3}$ passing through (6,2).

We can plug in the slope:

$$y = \frac{2}{3}x + b$$

And to find $b$, let's plug in the point and solve:

$$2 = \frac{2}{3}(6) + b$$

$$2 = \ 4 + b$$

$$-2 = b$$

So the equation is

$$y = \frac{2}{3}x - 2$$

Here's a variation:

Find the equation of the line passing through $(-4, 2)$ and $(2, 5)$

We need to first find the slope.

We know that

$$m = \frac{\Delta y}{\Delta x}$$

Those Δ symbols “delta” represent change.

The change between two numbers is the difference.

The difference between two numbers is found by subtraction.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_2}$$

We call this the ***slope formula.***

Find the equation of the line passing through $(-4, 2)$ and $(2, 5)$

We will plug in these numbers:

$$\frac{y_2 - y_1}{x_2 - x_2}$$

$$= \frac{5 \ - \ 2}{2 - (-4)}$$

$$= \frac{3}{6} = \frac{1}{2} = m$$

So our equation has the form:

$$y = \frac{1}{2}x + b$$

Find the equation of the line passing through $(-4, 2)$ and $(2, 5)$

To find $b$, we can plug in either point! Let's plug in $(-4, 2)$:

$$(2) = \frac{1}{2}(-4) + b$$

$$2 = -2 + b$$

$$4 = b$$

So the equation is

$$y = \frac{1}{2}x + 4$$