Functions

As noted earlier, this course has an applied side ("real-world"), and a theoretical side (abstract concepts).

We have been talking about two mathematical concepts in particular:

Equations *Formulas*

In our most recent problem, we came up with an equation that was also a formula:

$$V = -1500t + 20000$$

For an equation to be a formula…

one variable must be solved in terms of *the other*

That means that either V or t (for this example) must be ***by itself***.

In a sense, this means that the variable V is being "described."

Question: "What is *V*? Describe it for me!"

Answer: "*V* is what you get when you multiply -1500 and *t*, then add 20000"

Any mathematical statement of this form is a *Formula*.

Ready??

It’s also a function.

That’s right. **Functions are formulas**. That’s what they are.

Simple as that.

Well, not quite that simple. There are a couple of technicalities that we have to cover. But basically, functions are *formulas*.

This class is about the basic kinds of mathematical formulas.

Errr… I mean ***functions***.

Some of them are linear.

Some of them are quadratic.

Some of them are polynomial.

Some of them are rational.

Some of them are exponential.

Some of them are logarithmic.

Some of them are trigonometric.

the kinds of functions we will study

These are the specific types of functions we will be covering in this class.

First we will be talking about the basic aspects of functions as a concept.

This will be the more theoretical side of the course.

To begin theoretically, consider the following simple formula:

$$y = x^2 + 1$$

This is an equation, a *formula*, and a ***function***!!!

But to think about it, specifically, AS a function, we think of it in the following way:

Numbers go IN TO x

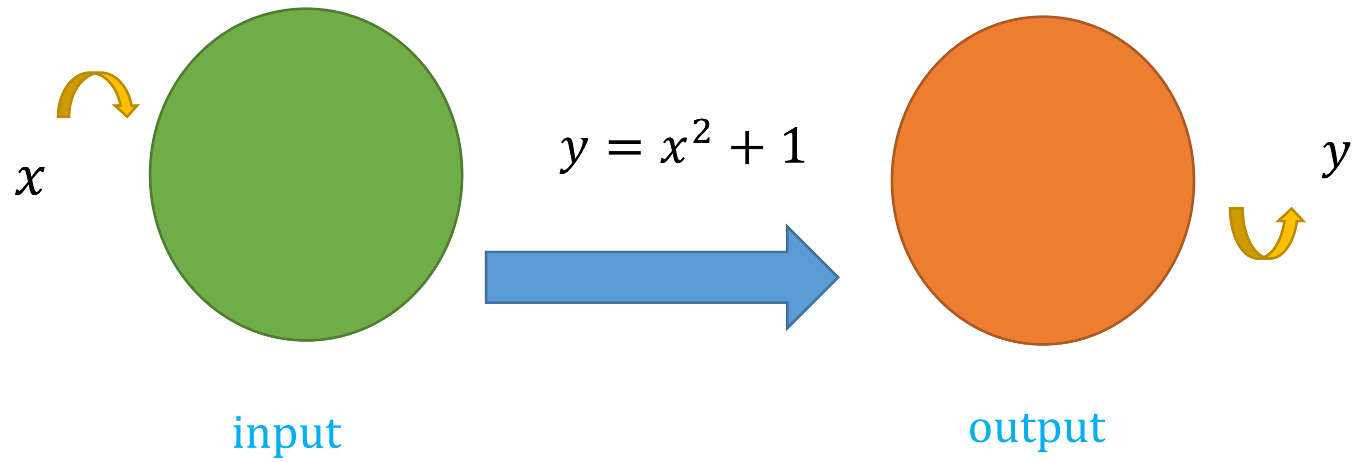
Numbers come OUT OF y

That's right, for an equation or a formula to be a function . . .

ONE VARIABLE STANDS FOR THE **input**

ONE VARIABLE STANDS FOR THE **output**.

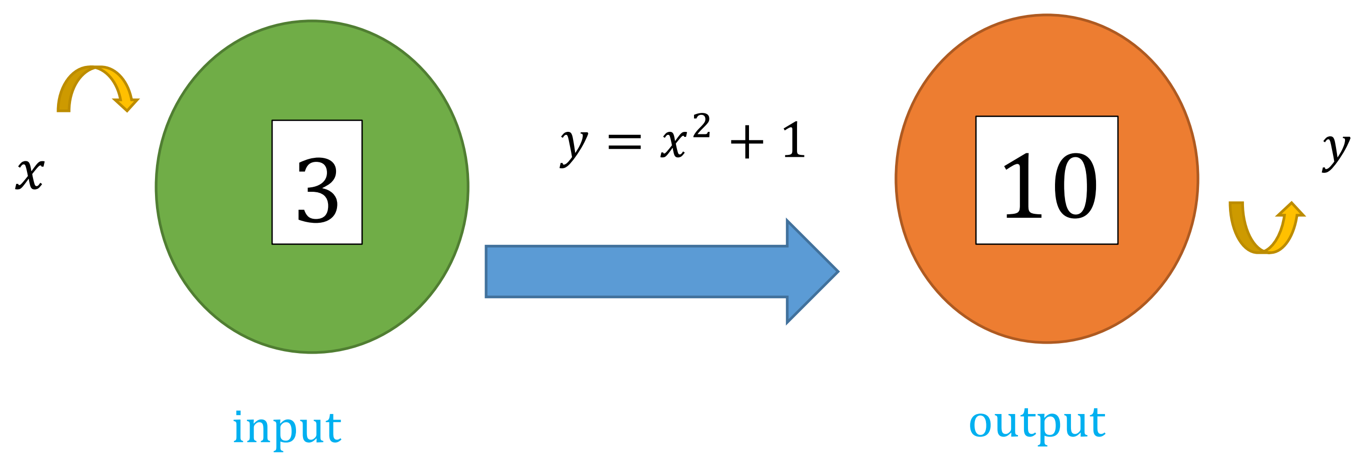
We can visualize it as follows:



Let's say we decide to let $x = 3$.

Then putting in 3 for the input ….

….. produces $3^2 + 1 = 10$ for the output!



Pretty simple, right?

There's just one catch.

It's a very important catch. You might even call it a rule.

For an equation to be a function . . .

There can only be one output

for any input

To understand this rule for a function, remember where this whole thing started: equations.

We can **turn** equations **into** functions.

In fact, we have already done this . . . when we converted a linear equation in standard form:

$$2x + 3y = 6$$

into a formula:

$$y = -\frac{2}{3}x + 2$$

BY SOLVING FOR $\boldsymbol{y}$

(the output variable)

But what would happen if we tried to do the same thing to the equation,

$$x^2 + y^2 = 9?$$

Well, let's see.

First, we would isolate the y^2:

$$y^2 = 9 - x^2$$

Then, we would find y by taking the square root of both sides:

$$\sqrt{y^2} = \pm\sqrt{9 - x^2}$$

$$y = \pm\sqrt{9 - x^2}$$

Yes, you need the $\pm$ sign before the radical, because all positive numbers have two square roots, a positive one and a negative one.

Also, if you want to turn the right side of the equation into

$$3 - x$$

You would not be the first person to do so! However . . .

$$\sqrt{9 - x^2} \neq 3 - x.$$

this is similar to the way that $(x+y)^2 \neq x^2+y^2$

It's an algebra thing.

So our final equation, which is to say our *formula*, is

$$y = \pm\sqrt{9 - x^2}$$

But can this be a **function**?

NO!

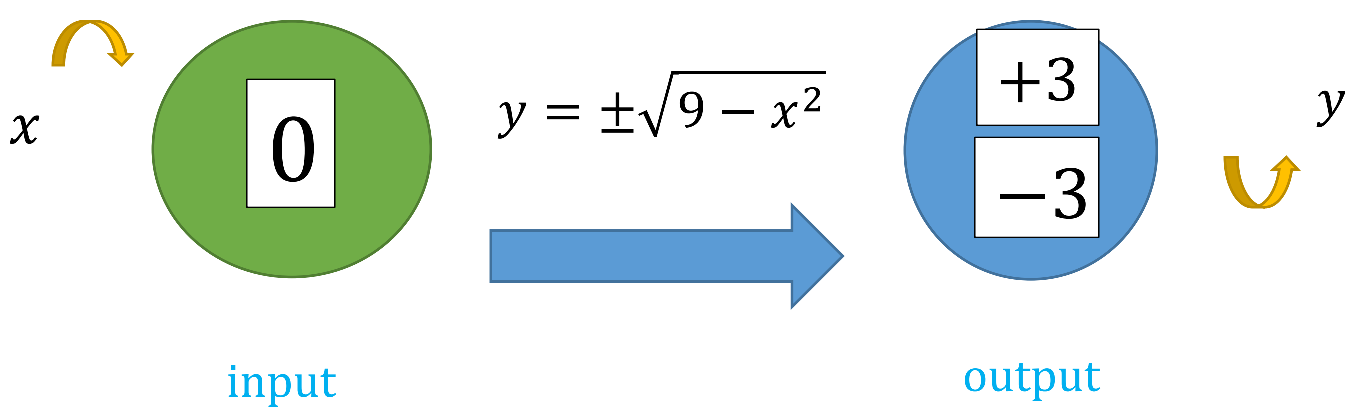
Why not?

Because of the ***rule about functions***:

There can only be one output

for any input

Suppose we tried putting $x = 3$ in for the input. What would happen?



In the words of the Jack Nicholson character in a very old movie called Prizzi's Honor, which one of 'dese?

This formula is NOT a function, because one input produces **two** outputs.

We only want there to be **one** output. It's kind of like wanting there to be a clear answer.

By the way, "input" and "output" are good simple words to describe the **one-way process** of a **function**.

But it turns out there are some additional words.

The "input" is the number that goes in for the **INDEPENDENT VARIABLE.**

The "output" is the number that comes out of the **DEPENDENT VARIABLE**.

Get why?

The OUTPUT ***depends*** on the INPUT.

You start with something independent…..

And finish with something that depended….

on what you started with.

Let's recall a real life example to understand the meaning.

Remember the story of Jeremy?

> Jeremy has a job offer for a sales position selling furniture. He will be paid \$400 per week base salary. In addition, he will be paid 5% of the price of the furniture he has sold (this is called a commission). What is the total weekly amount Jeremy will be paid?

We derived the following equation I mean ***formula*** I mean **function**:

$$S = 400 + 0.03x$$

Where

$$S = weekly\ salary$$

And

$$x = total\ price\ value\ of\ his\ sales$$

Which is the **DEPENDENT VARIABLE?**

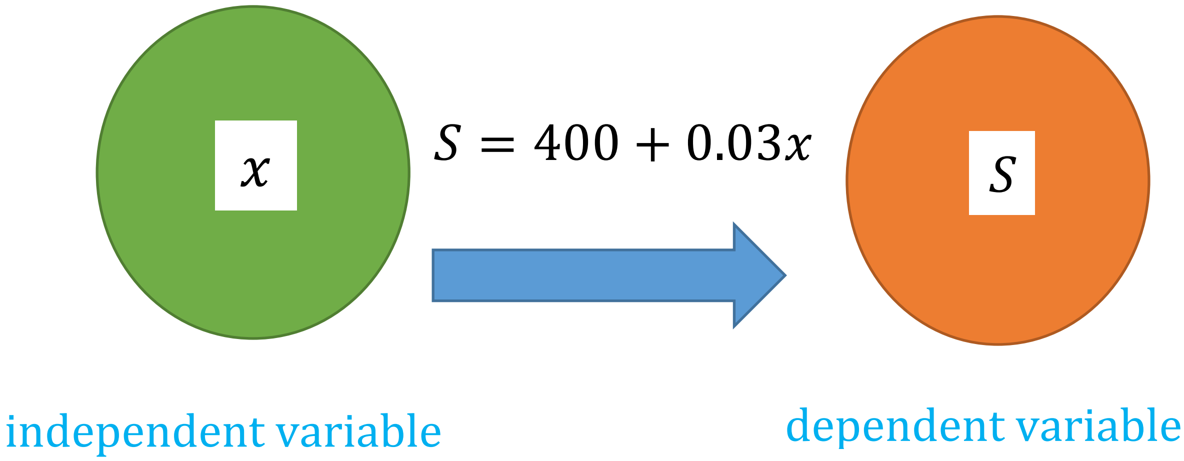
And which is the **INDEPENDENT VARIABLE?**

Put another way, does the salary depend on how much he sells, or does how much he sells depends on his salary?

If you think about it, it's the first one.

his salary depends on sales

(not the other way around)



His salary depends on how much he sells.

Let's consider one final **(and very important to this class!)** real life example:

> You want to start a business selling personally designed t-shirts. Before you put your t-shirts on sale, you must decide on the best price. You do some market research. You sell the t-shirts at two different prices: \$8 and \$12. Unsurprisingly, you sell more of them at \$8 than at \$12.
>
> At \$8, you sell 100 per day.
>
> At \$12, you sell 50 per day.
>
> You want to consider other prices, too, but there isn't enough time to do more market research. **You must decide your price**. You want to sell them for the price that brings in the most money, that is, revenue. To figure that out, you assume that the relationship between price and quantity sold is **linear**.
>
> **This will give you a way to predict how many you will sell at any given price!**
>
> Find a linear equation between the **price** and the **quantity sold.**
>
> **And turn the equation into a function.**

First (**as always with real-life problems!**) we must define our variables:

$$Let\ p = price\ (in\ dollars)$$

$$Let\ q = quantity\ sold$$

Now, since we know that this **function** will be linear, we know that it will be in the following form:

$$y = mx + b$$

The first thing we need to decide is . . .

Which of our variables will be the y, and which one will be the x.

In other words, are we looking for

$$p = mq + b$$

Or

$$q = mp + b$$

This question cuts to the very heart of what a function is, and how it is used, in real-life problems.

We know that a function that represents a real-life situation is a ***formula.***

that gives the OUTPUT in terms of the INPUT.

The question here is . . . does the price depend on the quantity sold?

Or does the quantity sold depend on the price?

Think about this. It’s important.

the higher the price, the less you will sell

I think that the quantity sold depends on the price.

Thus, q is the **DEPENDENT VARIABLE**

And p is the **INDEPENDENT VARIABLE**.

The price is independent, because we can make it be whatever we want.

But how much we sell? We can't really make that what we want.

We can only try to choose a price that will result in selling a certain amount.

Therefore we are looking for an equation (I mean **function**) of the form:

output (sales)

$$q = mp + b$$

input (price)

This function is set up for us to plug in some number for p (price),

And depending on that price, predict how many t-shirts we will sell.

First let's find m.

$$slope = m = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x} = \frac{\Delta q}{\Delta p}$$

And given that we have two points on the line,

$$(p_1, q_1) = (8, 100)$$

And

$$(p_2, q_2) = (12, 50)$$

$$\frac{\Delta q}{\Delta p} = \frac{50 - 100}{12 - 8} = -\frac{50}{4} = -12.5$$

As I noted earlier, all slopes corresponding to real-life linear equations have some real-life meaning. What is the real life meaning of the slope here?

What we have is

$$\frac{\Delta q}{\Delta p} = \frac{-12.5}{1}$$

Meaning that every time that the price is ***is increased by one dollar***,

The number of t-shirts sold ***goes down by 12.5***.

you lose about 12 customers for every dollar you increase the price

Our equation (I mean **function**) is now:

$$q = -12.5p + b$$

As before, to find b, we can plug in either data point into the equation:

$$100 = -12.5(8) + b$$

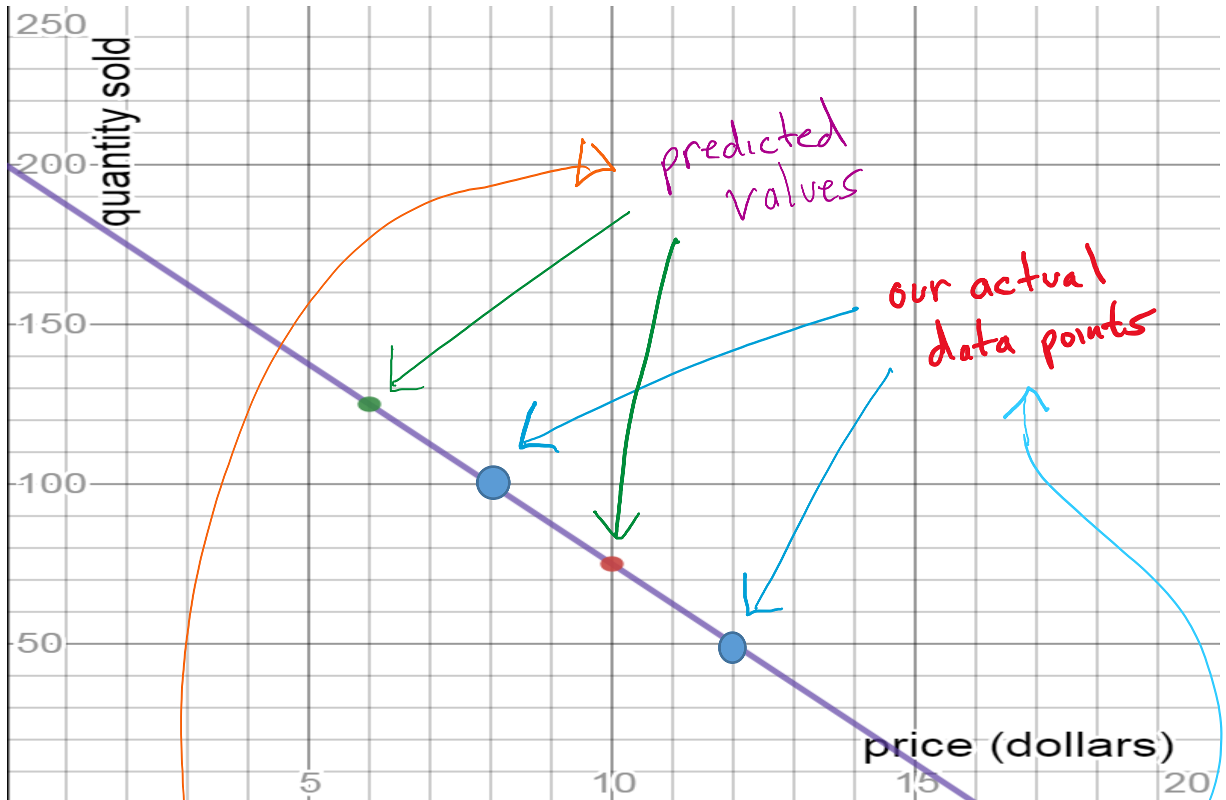
$$100 = -100 + b$$

$$200 = b$$

So

$$q = -12.5p + 200$$

This function allows us to predict what would happen at different prices. For example, look at the graph of this equation:



Here, we used two data points we knew to be true, the fact that:

$$At\ \$8, 100\ are\ sold$$

$$At\ \$12, 50\ are\ sold$$

To predict at least two other data points:

$$At\ \$6, 125\ are\ sold$$

$$At\ \$10, \$75\ are\ sold$$

Note: these are just predictions based on the two original data points. But many other predictions are possible, for example,

At \$9.35, we would predict that $q = -12.5(9.35) + 200 = 83.125$

Or approximately 83 t-shirts would be sold.

On a later problem, we will use this function to find the **best price**!

….the price that will bring in the most money.