

Finding the quadratic function given the vertex and a point

The final type of problem from this section goes like this:

A quadratic function $p(x)$ has vertex $(2, -5)$ and passes through $(4, -1)$. Find the function.

The problem is asking us to find the formula for this function. We have seen that quadratic functions can be written two ways:

STANDARD FORM:

$$f(x) = a(x - h)^2 + k$$

GENERAL FORM:

$$f(x) = ax^2 + bx + c$$

The best way of solving this problem is aiming for **STANDARD FORM**:

This method is easy, because the problem gave us the vertex

$$(2, -5)$$

STANDARD FORM of a quadratic function has the vertex directly shown:

$$f(x) = a(x - h)^2 + k$$

So our function

$$p(x) = a(x - h)^2 + k$$

becomes

$$p(x) = a(x - 2)^2 - 5$$

x-coordinate of the vertex

y-coordinate of the vertex

and all we need now is the leading coefficient a .

To find a , let's plug the given point into our function!

NOTE: for the given point $(4, -1)$, we have that $x = 4$ and $y = p(4)$

Plugging this in, we have

$$p(4) = -1$$

$$a(4 - 2)^2 - 5 = -1$$

$$a(2)^2 - 5 = -1$$

$$4a - 5 = -1$$

$$4a = 4$$

$$a = 1$$

So,

$$p(x) = (x - 2)^2 - 5$$

Here's another one:

Find the quadratic function $q(x)$ with vertex $(-3, 4)$ passing through the point $(-5, -4)$.

Using the same method, we know that since the vertex is $(-3, 4)$,

$$q(x) = a(x + 3)^2 + 4$$

To find a , we use the given information that $q(-5) = -4$:

$$q(-5) = a(-5 + 3)^2 + 4 = -4$$

$$a(-2)^2 + 4 = -4$$

$$4a + 4 = -4$$

$$4a = -8$$

$$a = -2$$

So our function is

$$q(x) = -2(x + 3)^2 + 4$$

