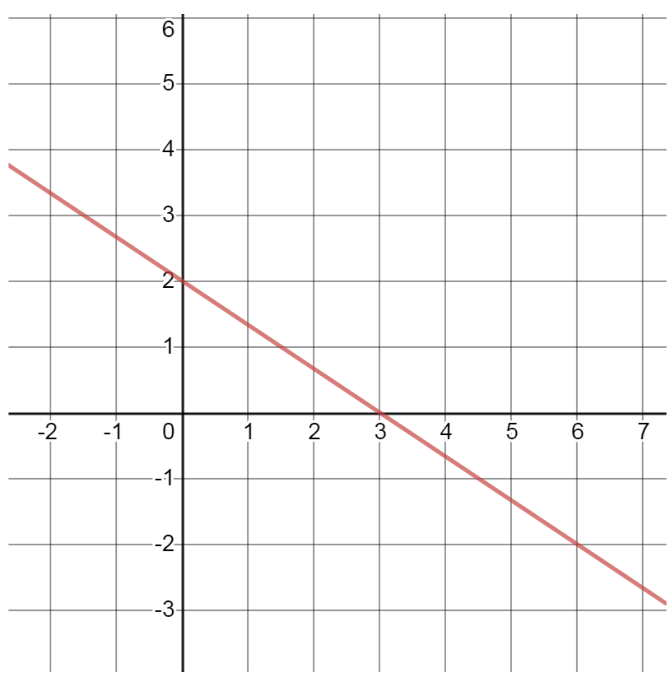
Linear Equations

In your previous math classes, you should have learned about a particular kind of equation in two variables. This is ***the linear equation in two variables***. Here are two examples:

$$2x + 3y = 6$$

$$y = -\frac{2}{3}x + 2$$

Actually, these equations are really the same. The way this could be shown is by drawing the graph of each equation. **Both of the above equations have the same graph**:



There are two main ways to read this graph. Consider the first version of the equation:

$$2x + 3y = 6$$

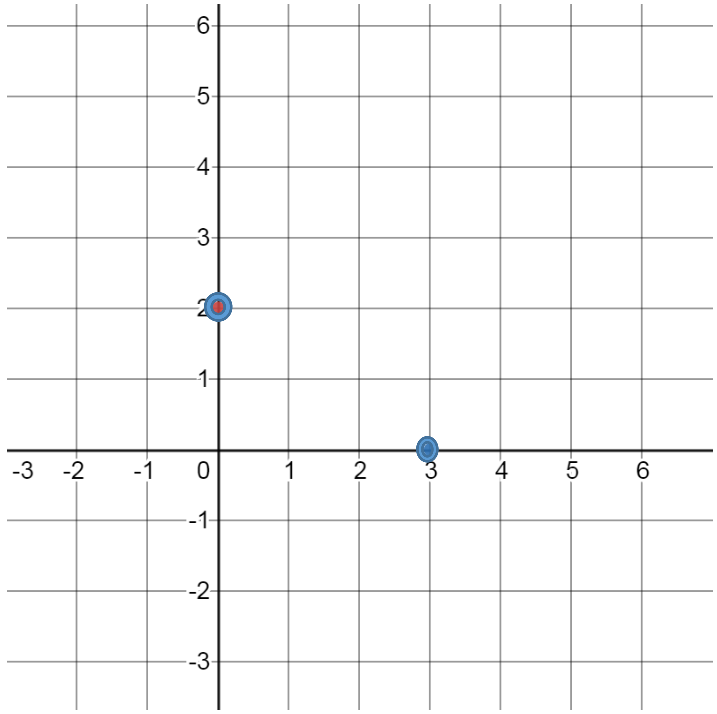
(standard form of a linear equation)

By plugging in zero for both variables, x and y, we could have very easily found two points on the graph:

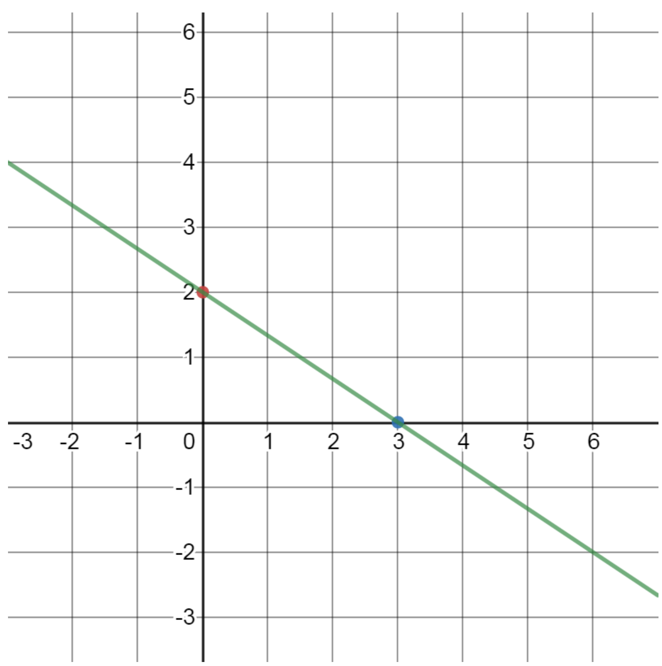
$$\begin{array}{rr} let\ x = 0 & let\ y = 0 \\ 2(0) + 3y = 6 & 2x + 3(0) = 6 \\ 3y = 6 & 2x = 6 \\ y = 2 & x = 3 \end{array}$$

$\boldsymbol{x}$	$\boldsymbol{y}$
0	2
3	0

Then plotted those points:



And connected them with a straight line:



This method of graphing a straight line is called the **intercept method**.

However, you may have learned a different method of graphing linear equations.

If we take our original equation in standard form:

$$2x + 3y = 6$$

And solve that equation for y by first moving the $2x$ to the other side:

$$3y = -2x + 6$$

And then dividing both sides of the equation by 3:

$$y = -\frac{2}{3}x + 2$$

We have a different version of the equation called slope-intercept form.

You have seen this before! Its form is:

$$y = mx + b$$

Where

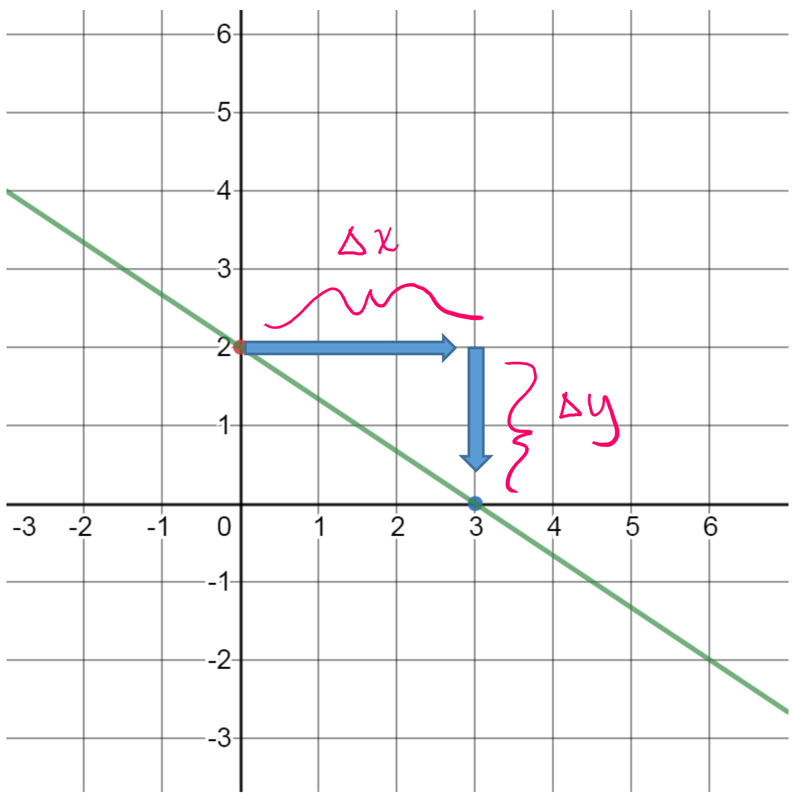
m stands for slope

and

b stands for the y-intercept.

In our original graph, we can see that the line crosses the y-axis at **y = 2**.

We can also see that the line has a direction. Every 2 units it moves to the right, it moves 3 down:



This is because the slope is a ratio between two numbers, the change in y and the change in x:

$$m = \frac{-2}{3} = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x}$$

The Greek symbol “delta” is the triangle Δ. In math it represents change. This symbol becomes very important in Calculus.

This should all be review! But it’s very important to this class and to Calculus!

Let’s look at a real-life example of a linear equation.

A college purchased exercise equipment for $20,000. Suppose that the equipment will only be worth $5000 in ten years. Assuming linear depreciation, predict the re-sale value of the equipment for any year in the near future.

This is a real-life problem. That means that our equation must be connected to the real-life quantities. We need to **define our variables**!

This is very important for real-world problems! The variables we use must have the words connected to them so we know what they stand for!

$$Let\ V = resale\ value\ of\ the\ equipment$$

$$Let\ t = the\ number\ of\ years\ of\ use$$

These are our variables because they are the numbers that can change. $20000 and $5000 are not variables. They represent given information for the problem.

How to begin? The problem gave us a big clue. It said linear depreciation.

So we know that it must have a slope. What would the slope be?

We noted before the following definition of slope:

$$slope = m = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x}$$

BUT WAIT!

We don't have "x" and "y" in this problem! We have "V" and "t"!

This is actually very important.

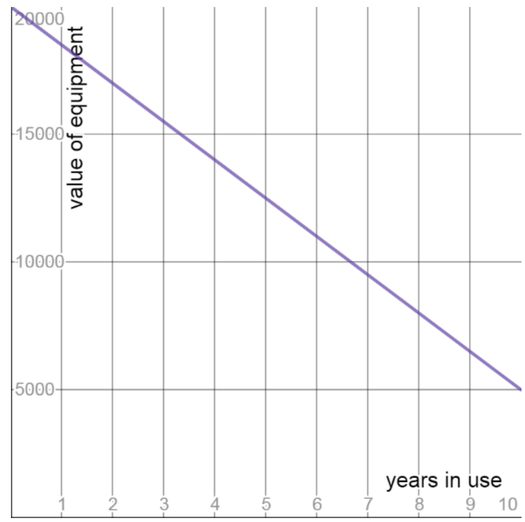
The reason it's important is this:

The "y" variable goes on the **vertical axis** of the graph (up and down).

The "x" variable goes on the **horizontal axis** of the graph (left to right).

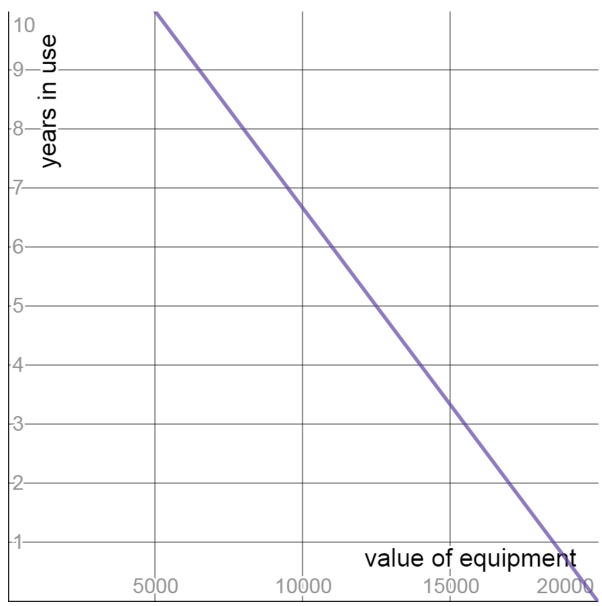
We read graphs in a certain way. For one thing, anytime ***time*** is involved,

We read ***time*** from left-to-right:



"time" is on the horizontal axis (good)

Instead of reading time from down-to-up:



"time" is on the vertical axis (confusing)

Compare the two graphs above. They are from the SAME information!!

WHICH ONE MAKES MORE SENSE?

I think the first one, don't you?

This is because we are trained to read sentences from left-to-right.

Graphs are like math "stories."

The “story” of our equipment is a story that happens in ***time***. And time going forward makes more sense from left-to-right, just like stories that we read are told from left-to-right.

Thus, going back to our problem, we will make

“V” be the y-variable

and

“t” be the x-variable

So, the slope of our equation will be:

$$slope = m = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x} = \frac{\Delta V}{\Delta t}$$

What is ΔV?

It’s the change in the value of the equipment.

Change between two numbers is the difference.

The difference is obtained by subtracting them . . . *starting with the last number*!

So . . . $\Delta V = 5000 - 20000 = -15000.$

Does it make sense that ΔV is negative??

Because it goes down.

What about Δt?

It’s the change in time.

$$\Delta t = 10 - 0 = 10$$

The change in time is always positive . . . because we don’t have time machines . . .

Yet.

Therefore we get that the slope is equal to:

$$\frac{\Delta V}{\Delta t} = -\frac{15000}{10} = -1500$$

Since this is a real-life problem, slope has a meaning. It means . . .

rate of change

It turns out that the rate of change is the whole basis for Calculus I

So it's important.

What is the meaning of the rate of change here???

Think about it: the value of the equipment is changing over time.

If it goes down by \$15000 in ten years, that's \$1500 per year.

Or rather: ***negative*** 1500

Because it's going down.

Now . . . *why did we want the slope???*

This problem was about finding the equation.

Equations of line are easy to get because they ***all*** have the form:

$$y = mx + b$$

Which in this problem would be

$$V = mt + b$$

again, t is the "x" variable because we read time left to right

Given that V is the y-variable and t is the x-variable.

So now we know that our equation looks like this:

$$V = -1500t + b$$

We just have to find b!

We can do that by plugging in the information we have for V and t, either the fact that

V = 20000 when t = 0

OR that

V = 5000 when t = 10

Either of these would work because (0, 20000) and (10, 5000) are both points on the line.

Let's plug in the first point:

$$V = -1500t + b$$

$$20000 = -1500(0) + b$$

$$20000 = b$$

This should not be surprising, because b represents the y-intercept, which stands for the **STARTING POINT** in any linear equation describing a real-life situation.

This is worth emphasizing! In linear equations, the numbers

m

And

b

Have a purely graphical meaning. Those are "slope" and "y-intercept".

But in real-life situations (called real-life "models") these numbers have real-life meanings. Those are, again:

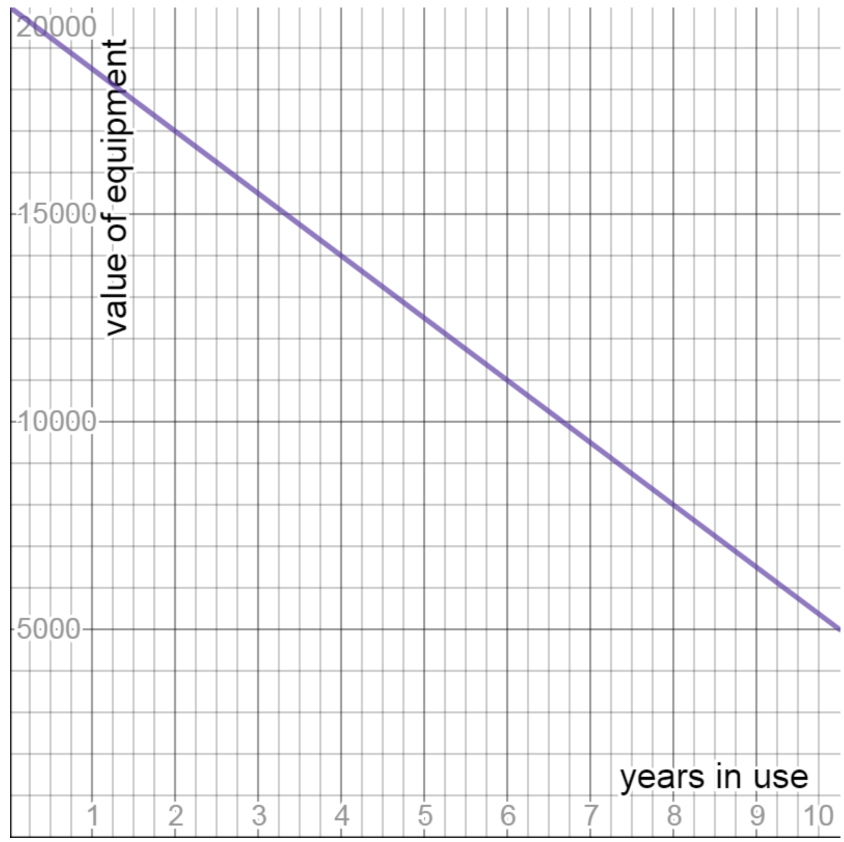
$m =$ rate of change

$b =$ **STARTING POINT**

real-life meaning

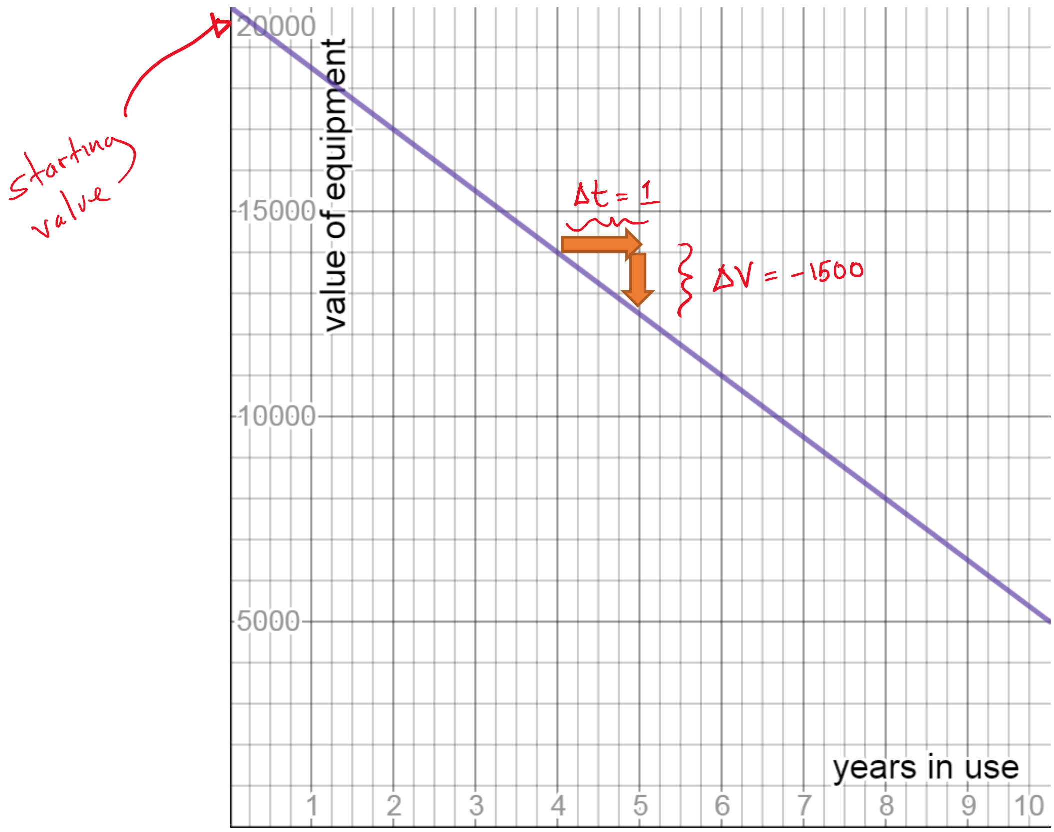
To understand this, and finish the problem, let's look at the graph of our finished equation,

$$V = -1500t + 20000$$



It should be easy to see that $20000 is the starting point.

To see that -1500 is the rate of change, look how far the line goes down for any unit of time:



Here you can see that from t = 4 years to 5 years, the value of the equipment goes from 14000 to 12500. That's a decrease of 1500 dollars per year! That's the rate of change . . . and the slope!!!