

## Factoring Part I

*(factoring out the greatest common factor)*

Soon we will see how important factoring is to

... solving equations ...

... and simplifying expressions.

For example, consider the following problem:

$$4x^3 - 20x^2 = 0$$

To solve this equation, we must factor the expression on the left:

$$4x^3 - 20x^2 = 0$$

$$4x^2(x - 5) = 0$$

Now we have two factors that multiply to be **zero**!

This can only happen if one of the factors is zero!

$$\underbrace{4x^2}_{\uparrow}(\underbrace{x - 5}_{\uparrow}) = 0$$

one of these must be zero!

This will give us two solutions to the equation:

$$\begin{array}{ll} 4x^2 = 0 & x - 5 = 0 \\ \downarrow & \downarrow \\ x^2 = 0 & \\ \downarrow & \\ x = 0 & x = 5 \end{array}$$

The principle I used to solve this equation will be very useful!

It is called the *Zero Product Rule*.

### The Zero Product Rule

$$\text{If } A \cdot B = 0$$

$$\text{Then } A = 0 \text{ or } B = 0$$

Writing an expression as the product of two smaller expressions is called

**factoring**

For example, we can **factor** the number 35 . . .

. . . as (7)(5):

$$35 \\ = 7 \cdot 5$$

With algebra, we can factor

$$4a + 4b$$

as

$$4(a + b)$$

check:

$$4(a + b) \\ = 4a + 4b$$

Do do this we ...

... **reversed the distribution** ...

... of the factor 4.

This will always be our **first** step in factoring!

It's called ...

... factoring out the greatest common factor.

always step # 1

Factor out the greatest common factor:

$$12x - 6y + 15z$$

First we decide . . . what **is** the greatest common factor?

Which is to say . . . what can be factored out?

The answer is that all three of those terms are divisible by 3.

That means that . . .

*. . . 3 is the greatest common factor.*


So we get

$$\begin{aligned} 12x - 6y + 15z \\ = 3(4x - 2y + 5z) \end{aligned}$$

We factored out 3 . . .

*. . . reversing the distribution.*

*check:*

$$\begin{aligned} & 3(4x - 2y + 5z) \\ = & 12x - 6y + 15z \end{aligned}$$


Factor out the greatest common factor:

$$8x^2 - 24x$$

First we must decide . . .

. . . what goes into both  $8x^2$  and  $24x$ ?

The answer is that there are a number of **common factors**:

$$2$$

$$4$$

$$8$$

$$8x$$


All of these terms are factors of both  $8x^2$  and  $24x$

The **greatest** of them is  $8x$ .

So we factor it out:

$$\begin{aligned} 8x^2 - 24x \\ = 8x(x - 3) \end{aligned}$$

To see that we factored correctly, we can check:



$$8x(x - 3)$$

$$= 8x^2 - 24x$$

Factor out the greatest common factor:

$$12x^4 - 18x^3 + 24x^2$$

First find the GCF of the coefficients . . .

. . . which is **6**.

Then find the GCF of the variable part . . .

. . . which is  **$x^2$** .

Putting them together, the greatest common factor of the expression is

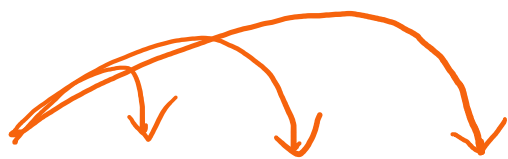
$$6x^2$$

Factoring it out, we get

$$12x^4 - 18x^3 + 24x^2$$

$$= 6x^2(2x^2 - 3x + 4)$$

check :


$$6x^2(2x^2 - 3x + 4)$$

$$= 12x^4 - 18x^3 + 24x$$

