Factoring Part II

(factoring simple trinomials)

Check out this (quadratic) equation:

$$x^2 + 8x + 15 = 0$$

Can you remember solving equations like this?

You can't do it by isolating the x!

You need to use the zero product rule:

The Zero Product Rule
If $A \cdot B = 0$
Then $A = 0 \quad or \quad B = 0$

That means we need to factor!

How to factor $x^2 + 8x + 15$?

We will assume it's the product of two binomials:

$$(\quad\quad)(\quad\quad) = x^2 + 8x + 15$$

And try to ***reverse the FOIL method***!

We know that the first terms must multiply to be x^2:

$$(x\quad\quad)(x\quad\quad) = x^2 + 8x + 15$$

first

And we figure that the last terms must multiply to be 15:

$$(x + 3)(x + 5) = x^2 + 8x + 15$$

last

Checking, we see that it works out to get the $8x$ as well:

$$(x + 3)(x + 5)$$

$$= x^2 + \underbrace{5x + 3x}_{} + 15$$

$$x^2 + 8x + 15$$

That one was kind of easy!!!

Factor the expression:

$$x^2 + 12x + 20$$

Here there are three ways we could factor the last term:

$$1 \cdot 20 = 20$$
$$2 \cdot 10 = 20$$
$$4 \cdot 5 = 20$$

That means three possible ways to factor:

$$(x+1)(x+20)$$

or

$$(x+2)(x+10)$$

or

$$(x+4)(x+5)$$

All of these factorizations get the first and last terms of the trinomial right.

But only **one** gets the **middle term** right:

$(x+1)(x+20)$	$(x+2)(x+10)$	$(x+4)(x+5)$
$= x^2 + 20x + x + 21$	$= x^2 + 10x + 2x + 20$	$= x^2 + 4x + 5x + 20$
$= x^2 + 21x + 20$	$= x^2 + 12x + 20$	$= x^2 + 9x + 20$

Let's take a closer look, so we can see how we could have known:

$$(x + 2)(x + 10)$$

$$= x^2 + 10x + 2x + 20$$

$$= x^2 + 12x + 20$$

We see that the constant terms . . .

. . . 10 and 2 . . .

. . . add up to be 12 . . .

which is the middle coefficient.

This gives us our method for factoring simple trinomials.

$$x^2 + bx + c$$

$$= (x + \quad)(x + \quad)$$

multiply to c
add to b

Okay, now that we have a method, try this one:

Factor the expression:

$$x^2 - 2x - 8$$

Our method says:

$$x^2 - 2x - 8$$

$$= (x + \quad)(x + \quad)$$

multiply to be -8

and

add to be -2.

Can you think of them?

They are -4 and +2

We get

$$x^2 - 2x - 8$$

$$= (x - 4)(x + 2)$$

Check:

$$(x-4)(x+2)$$

$$= x^2 + 2x - 4x - 8$$

$$= x^2 - 2x - 8$$

What about this one:

Factor the expression:

$$4x^2 - 28x + 48$$

Wait . . . this is not a simple trinomial!

There is a leading coefficient 4 . . . we haven't factored this kind!

But remember the ***first rule of factoring***:

The **first** step in factoring is . . .

. . . factor out the greatest common factor.

$$4x^2 - 28x + 48$$

$$= 4(x^2 - 7x + 12)$$

Then we try to factor the resulting simple trinomial:

$$= 4(x^2 - 7x + 12)$$

$$= 4(x + \quad)(x + \quad)$$

multiply to 12
add to −7

Those numbers are -3 and -4.

$$= 4(x^2 - 7x + 12)$$

$$= 4(x + \quad)(x + \quad)$$

$$= 4(x - 3)(x - 4)$$

Let's reiterate this very important lesson:

The **first** step in factoring is . . .

. . . factor out the greatest common factor!