Rational Functions

Consider the following function:

$$g(x) = \frac{x+1}{x-1}$$

This function has a different form than any we have seen in this course!

What makes it different? It's a fraction with **variables in the denominator**.

This is an important kind of function . . . we call it a **rational function**.

$$\text{"rational"} \longleftrightarrow \text{"ratio"}$$

Our main goal in this section is to learn how to graph these functions.

Let's try with $g(x)$.

We start by plugging in points to $g(x)$. Synthetic substitution will not work on this type of function. We must do it the old fashioned way. Here's what we come up with when we try some positive and negative values (and zero of course):

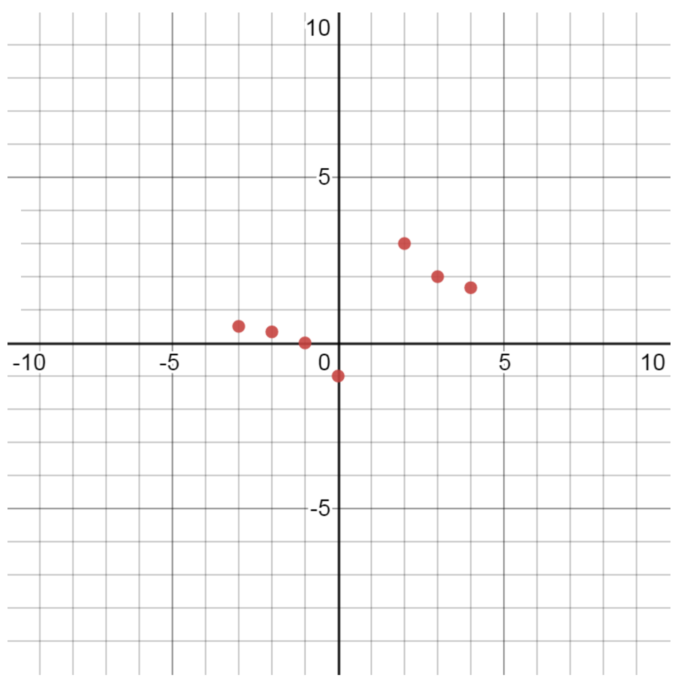
x	$g(x)$
$\mathbf{0}$	-1
$\mathbf{1}$	$undefined$
$\mathbf{2}$	3
$\mathbf{3}$	2
$\mathbf{4}$	1.67
$\mathbf{-1}$	0
$\mathbf{-2}$	0.33
$\mathbf{-3}$	0.5

finding these y-values the old-fashioned way

Very early on, something unusual happens: when we plug in $x = 1$, we get an output that has zero in the denominator. This result is called "undefined" and cannot be plotted. In fact, $x = 1$ is not technically in the domain of $g(x)$.

Now let's plot our values on a graph:

$$g(x) = \frac{x+1}{x-1}$$

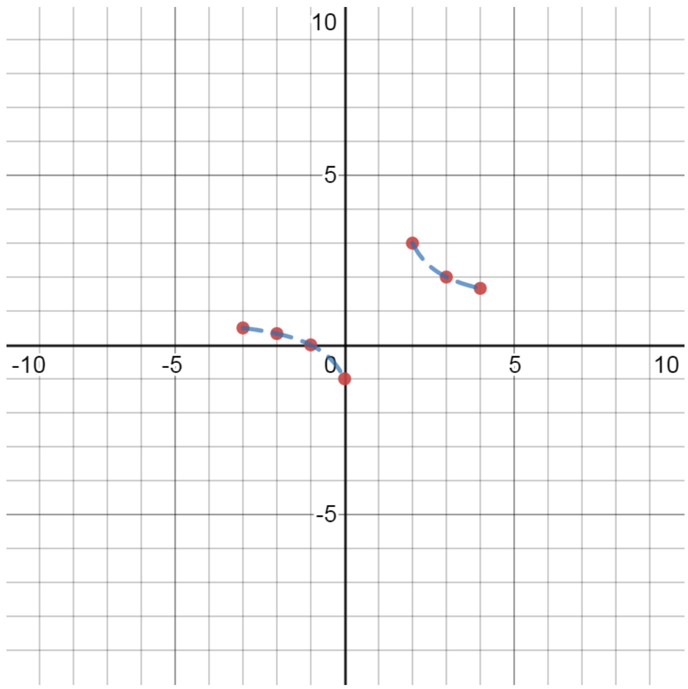


The results are a bit confusing!!

It doesn't seem to be linear . . . but it's not clear how to draw the graph!

On the left side, the points seem to be in a curve, and on the right side too:

$$g(x) = \frac{x+1}{x-1}$$



But what about at the ends? And between the points in the middle?

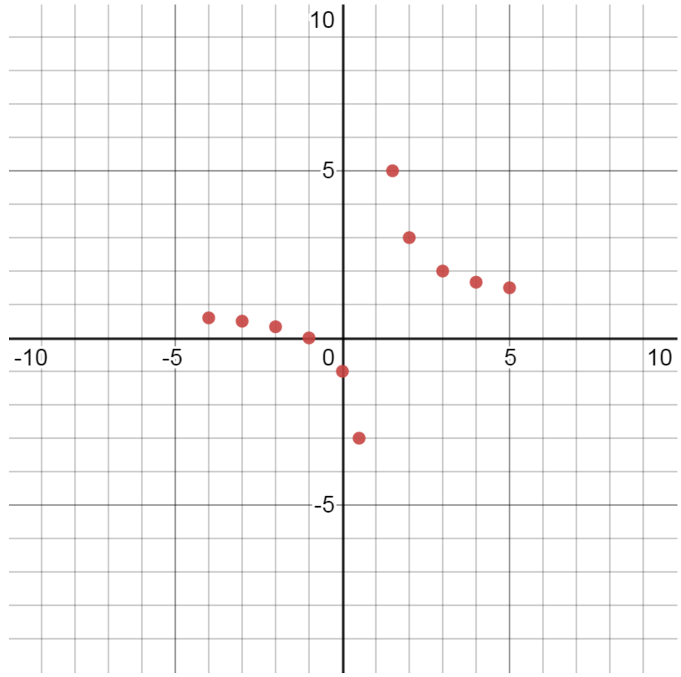
We may have no choice but to **plug in more x-values** to see!

Let's try a couple of points in the middle, and a couple more at the end:

$\boldsymbol{x}$	$\boldsymbol{g(x)}$
$\mathbf{0.5}$	-3
$\mathbf{1.5}$	5
$\mathbf{5}$	1.5
$\mathbf{-4}$	0.6

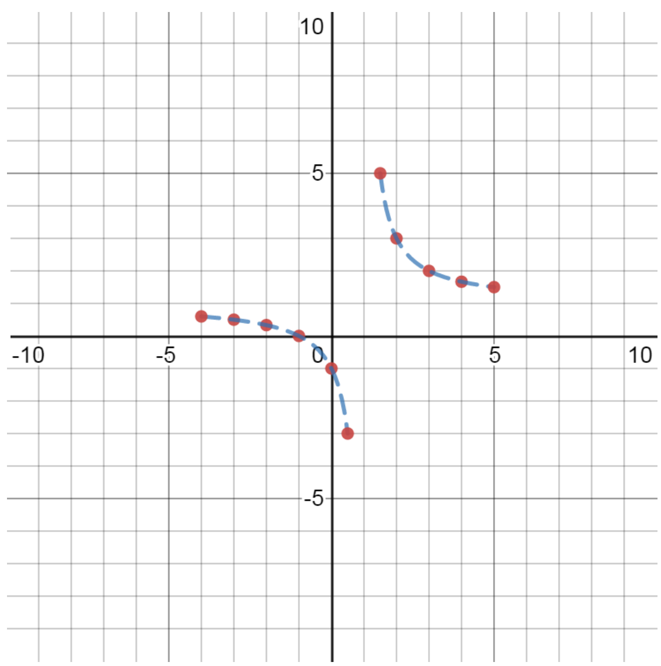
So plotting these points, we now have this:

$$g(x) = \frac{x+1}{x-1}$$



Now let's see what happens when we connect the points on the left and right with a smooth curve:

$$g(x) = \frac{x+1}{x-1}$$



It seems clearer, now that the curves on either side . . . will never meet!!!

This is because . . .

. . . as we choose x-values that get close to $x = 1$ on either side . . .

. . . the **denominators** of the fractions get very **small** . . .

. . . which makes the **value get very big** . . .

. . . in both a positive and negative way.

$\boldsymbol{x}$	$\boldsymbol{g(x)}$
$\mathbf{0.9}$	-19
$\mathbf{0.99}$	-199
$\mathbf{0.999}$	-1999
$\mathbf{0.9999}$	-19999

$$1 \qquad -\infty$$

$\boldsymbol{x}$	$\boldsymbol{g(x)}$
$\mathbf{1.1}$	21
$\mathbf{1.01}$	201
$\mathbf{1.001}$	2001
$\mathbf{1.0001}$	20001

$$1 \qquad \infty$$

The fancy math way of saying this . . .

. . . which you will need to know in Calculus . . .

means "approaches"

Goes like this:

$$as\ x \to 1^-, \qquad g(x) \to +\infty$$

$$as\ x \to 1^+, \qquad g(x) \to -\infty$$

Here, the notation

$$x \to 1^{-}$$

Means that x is "approaching" 1 from **below** . . .

. . . or on the **left** side of $x = 1$ on a number line.

Similarly, the notation

$$x \to 1^{+}$$

Means that x is "approaching" 1 from **above** . . .

. . . or on the **right** side of $x = 1$ on a number line.

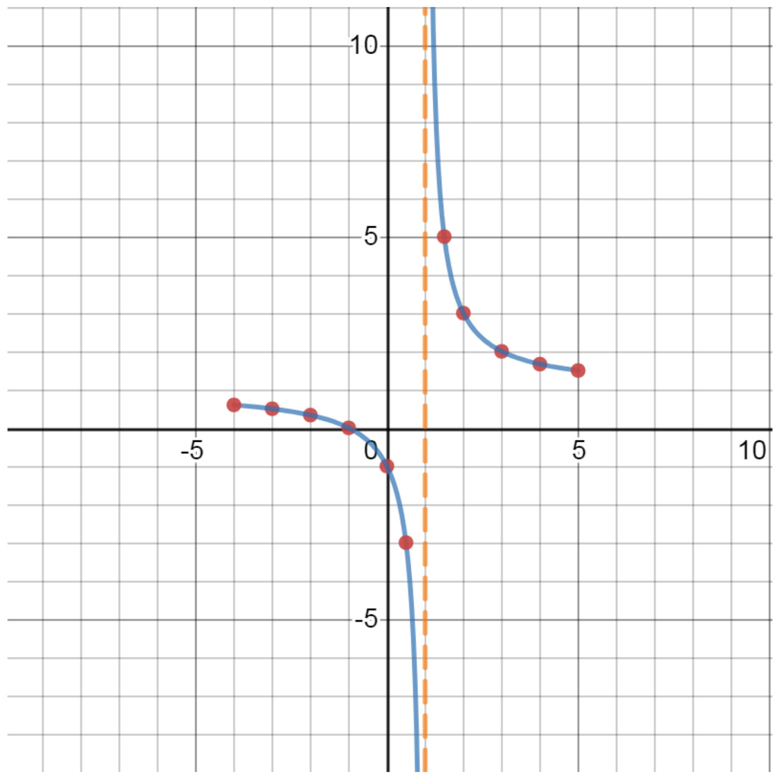
So on the graph, we draw the curve so that it **goes to** $\pm\infty$ . . .

. . . but without ever touching the vertical line $x = 1$ . . .

. . . as there can be no points with $x = 1$ . . .

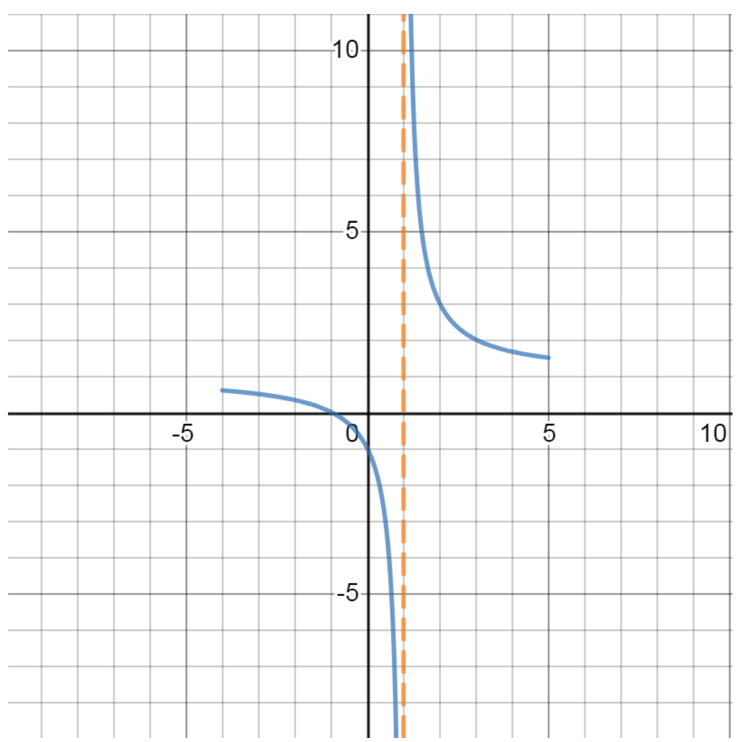
. . . since $g(x)$ is undefined!!!!

$$g(x) = \frac{x+1}{x-1}$$



or

$$g(x) = \frac{x+1}{x-1}$$



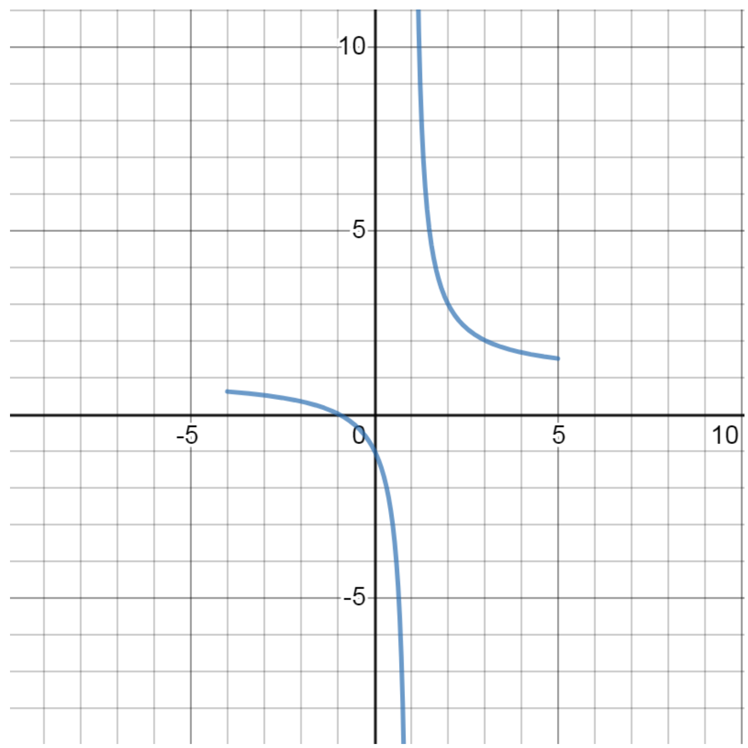
The dotted line is not part of the graph itself!

It is to **help draw the graph**!!!

The actual graph would simply look like this:

$$g(x) = \frac{x+1}{x-1}$$

$$-4 \le x \le 5$$



But since you will have to draw your graphs by hand . . .

. . . the dotted line is useful.

It has a name:

vertical asymptote