Factoring Part III

special factoring

Consider the expression

$$x^2 - 9$$

This looks a little different than the expressions we've been factoring.

It's *not* a trinomial . . .

. . . and there is *no* greatest common factor.

But something else is different about this expression . . .

. . . both terms are **perfect squares**.

A *perfect square* is . . .

. . . the **square** of a whole number . . .

. . . or algebraic term.

Here are some examples of *perfect squares*:

4 100

25

x^2 t^4 $16x^2$

If the expression with two perfect squares involves subtraction . . .

. . . we call that a *difference of two squares*.

It is always factorable in the following way:

The Difference of Two Squares

$$a^2 - b^2$$

$$(a + b)(a - b)$$

To see that it works, just FOIL the binomial. You'll get $a^2 - b^2$.

So

$$x^2 - 9$$

becomes

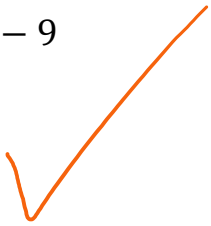
$$(x + 3)(x - 3)$$

Check:

$$(x + 3)(x - 3)$$

$$= x^2 - 3x + 3x - 9$$

$$= x^2 - 9$$



Factor:

$$36x^2 - 1$$

Once again, we will use the formula for the difference of two squares:

$$a^2 - b^2 = (a + b)(a - b)$$

Here, let a equal $6x$ and b equal 1 . . .

So we get

$$36x^2 - 1$$
$$= (6x + 1)(6x - 1)$$

What about this one?

Factor:

$$2x^3 - 50x$$

This is NOT the difference of two squares!

How would we do this one??

Remember our first rule of factoring:

The **first** step in factoring is . . .

. . . factor out the greatest common factor.

So

$$2x^3 - 50x$$

Becomes

$$2x(x^2 - 25)$$

Now we have the difference of two squares! We can factor the binomial:

$$2x(x + 5)(x - 5)$$

What about this one?

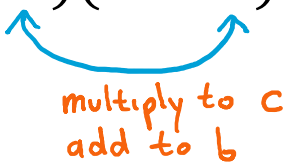
$$2x^2 + 7x + 3$$

Here we have a trinomial with leading coefficient $\neq 1$.

So we can't use the rule that we learned for simple trinomials:

$$x^2 + bx + c$$

$$= (x + \quad)(x + \quad)$$



This won't work for this trinomial because of the leading coefficient:

$$2x^2 + 7x + 3$$

One way to do this is to try to factor the $2x^2$:

$$(2x \quad + \quad)(x \quad + \quad)$$

Now try to come up with numbers that multiply to equal 3. There are two possibilities:

$$2x^2 + 7x + 3$$

$$= (2x + 1)(x + 3)$$

or

$$= (2x + 3)(x + 1)$$

Which one of these results in the middle term of $7x$?

$$(2x + 1)(x + 3)$$

$$= (2x^2 + 6x + x + 3)$$

$$= 2x^2 + 7x + 3$$

$$(2x + 3)(x + 1)$$

$$= (2x^2 + 2x + 3x + 3)$$

$$= 2x^2 + 5x + 3$$

This strategy of factoring might be called the **trial-and-error method**.

There is another method for factoring harder trinomials called . . .

. . . the a/c Method . . . .

. . . or the Master Product Method.

I will elaborate on this other method in the video version of this lesson!