

# Solving Quadratic Equations by factoring

Solve the equation:

$$x^2 - 5x - 6 = 0$$

This is a classic type of problem from algebra . . . maybe the most classic!

If we tried solving this equation by **isolating the variable** . . .

. . . we wouldn't get very far:

$$5x = x^2 - 6$$

$$x = \frac{x^2 - 6}{5}$$

**We will solve it by factoring.**

This method is based on a very important rule for math:

## The Zero Product Rule

$$\text{If } A \cdot B = 0$$

$$\text{Then } A = 0 \text{ or } B = 0$$

First, note that this method *only works for expressions that equal zero*.

So if we can write

$$x^2 - 5x - 6$$

as two factors multiplied, we can set each of them equal to zero and solve:

$$x^2 - 5x - 6 = 0$$

$$(x - 6)(x + 1) = 0$$

$$x - 6 = 0 \qquad x + 1 = 0$$

$$x = 6 \qquad x = -1$$

The solution set to this equation is  $\{6, -1\}$

Note: we could directly factor the quadratic expression here because it was **already equal to zero**:

$$x^2 - 5x - 6 = \mathbf{0}$$

When the quadratic expression is equal to zero, we say that it is in

## Standard Form

In general, a quadratic equation is in standard form if it is written:

$$ax^2 + bx + c = 0$$

*standard form of a  
quadratic equation*

Consider this next equation:

$$2(x^2 + 3x) = 36$$

Because this equation contains an  $x^2$  term . . .

. . . we see it's a **quadratic equation**.

But we are ***not ready to factor!***


The equation must first be put into *standard form*.

$$2(x^2 + 3x) = 36$$

First we bring the constant term over, so we have the **expression = 0**:

$$2(x^2 + 3x) - 36 = 0$$

Next we need to multiply out the parenthesis:


$$2(x^2 + 3x) - 36 = 0$$

$$2x^2 + 6x - 36 = 0$$

And only now do we factor!

To begin, we must remember the first rule of factoring:

The **first** step in factoring is . . .

*. . . factor out the greatest common factor.*

$$2x^2 + 6x - 36 = 0$$

$$2(x^2 + 3x - 18) = 0$$

Now we look for two numbers that multiply to be -18 . . . and add to be 6:

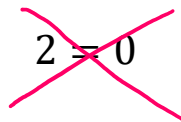
$$2(x + 6)(x - 3) = 0$$

From here we can use the **zero product rule** . . . for **three** factors:

$$2(x + 6)(x - 3) = 0$$

$$2 = 0 \quad x + 6 = 0 \quad x - 3 = 0$$

The first equation is never true! That doesn't give us any solution!


$$~~2 = 0~~$$

The next two equations do produce solutions:

$$x + 6 = 0$$

$$x - 3 = 0$$

$$x = -6$$

$$x = 3$$

So our solution set to our original equation is

$$\{-6, 3\}$$