Exponential Growth and Percent Change

The graph shows **exponential growth**:

Numbers increasing faster . . .

. . . and faster . . .

. . . forever.

This function was based on a process where numbers kept increasing by 4%

In general, exponential functions come from a process of

percent change

Compound interest results in percent change, because interest is based on percent. Here are some other real-life contexts that involve percent change:

Population Growth

Epidemics

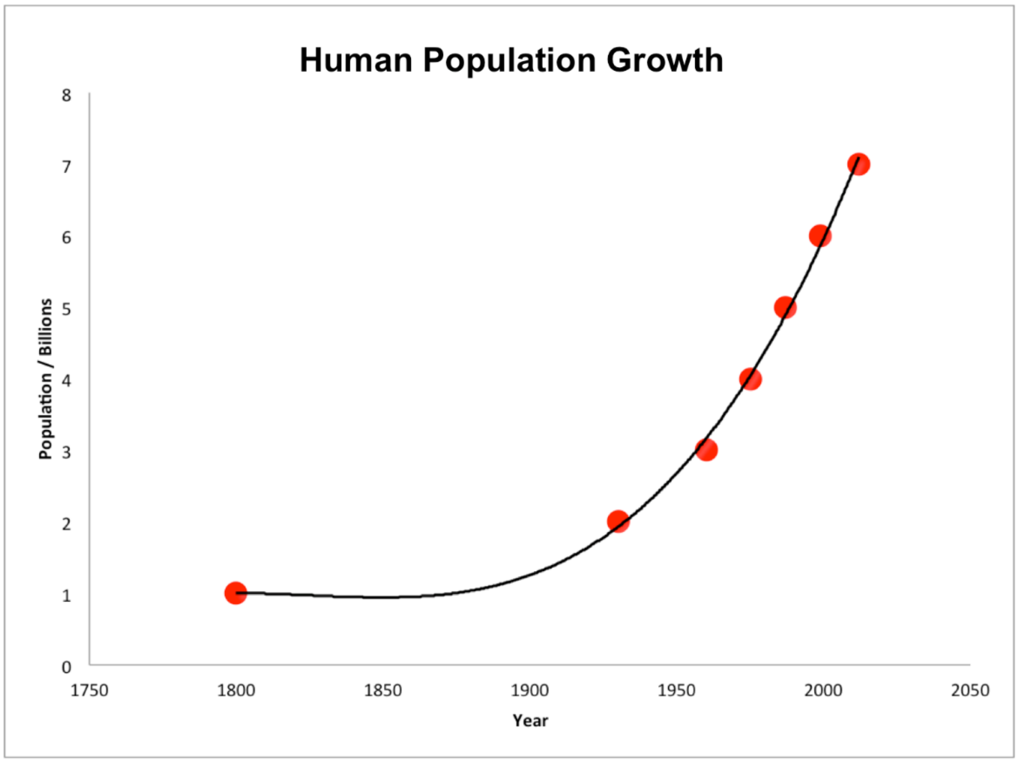
In both of these contexts, the new number depends on the old number.

For example, the number of new babies in a place depends on how many people are living there.

(why population grows exponentially)

The number of people who contract a disease depends on how many people can potentially spread it. (why epidemics grow exponentially)

Here's a graph of the world population since 1850:



The logic goes something like this:

The more people there are, the more babies are born . . .

. . . the more babies that are born, the more adults (eventually) who can have babies . . .

. . . etc.

This is different from linear processes, in which the amount goes up the same, for every unit of time.

In real life problems, the formula (or function) describing the process incorporates the percentage change:

$$A(t) = A_0(1 + r)^t$$

where

$$A_0 = \text{the initial amount}$$

$$r = \text{rate of change (percent as a decimal)}$$

$$t = \text{units of time elapsed}$$

Example:

The city of Dallas in the United States has been growing at the rate if 0.75% per year. If its population in 2018 was 1,341,200, what would its expected population be in the year 2028?

Since Dallas is growing according to a **percentage rate**, it is growing **exponentially**.

We will first show how to get the answer using a basic mathematical reasoning process:

Applying the principle of percentage change, we have that the Dallas population is increasing by 0.75% each year.

To increase a number by 0.75%, we multiply by 1.0075

this is percent as a decimal

1 + 0.0075

So each year that goes by, the population of Dallas is multiplied by 1.0075 to get the new population. After 10 years, the population would be

$$1{,}341{,}200 * \underbrace{1.0075 * 1.0075 * 1.0075 * \ldots * 1.0075}_{\text{10 times!}}$$

$$= 1{,}341{,}200 * 1.0075^{10}$$

$$= 1{,}445{,}254$$

Now, let's use the method of finding an exponential function using the general formula of percentage change.

First we need to define our function. All exponential functions use t as the unit of time, but it must be **elapsed time**, never an actual "year," like 2018. Here's how we go about that:

$$P(t) = Population\ of\ Dallas, t\ years\ after\ 2018$$

Here, the function is defined in such a way that $t = 0$ corresponds to the time we started growing the population, because "0 years after 2018" corresponds to 2018.

Now, to get the function, we can apply the general formula for percentage change:

$$A(t) = A_0(1 + r)^t$$

Where in this case,

$$A_0 = 1{,}341{,}200$$
$$r = 0.0075$$

So our exponential function is

$$A(t) = 1{,}341{,}200 * (1 + 0.0075)^t$$
$$A(t) = 1{,}341{,}200 * (1.0075)^t$$

And since 2028 happens in 10 years,

$$A(10) = 1{,}341{,}200 * 1.0075^{10} = 1{,}445{,}254$$