Adding and Subtracting Rational Expressions

Rational expressions are fractions.

If you remember how to add and subtract fractions . . .

. . . you remember that you need a **common denominator**.

Ideally the

lowest common denominator

$$\frac{1}{6}+\frac{1}{8}$$

$$=\frac{4}{\mathbf{24}}+\frac{3}{\mathbf{24}}$$

$$=\frac{7}{24}$$

In this case, 24 is the LCD . . .

. . . because it's the smallest number 6 and 8 divide into.

Here's another way you could think about it:

$$\frac{1}{6}+\frac{1}{8}$$

$$=\frac{1}{2\cdot 3}+\frac{1}{2\cdot 4}$$

Both fractions have a 2 in the denominator . . .

. . . the left-hand fraction needs a 4 . . .

. . . the right-hand fraction needs a 3:

$$\frac{1}{2\cdot 3}+\frac{1}{2\cdot 4}$$

$$=\frac{(4)\,1}{(4)\,2\cdot 3}+\frac{(3)\,1}{(3)\,2\cdot 4}$$

But what would be the LCD here?

$$\frac{1}{x^2-9}+\frac{3x}{x^2+2x-15}$$

The only way to know is to factor!

$$\frac{1}{x^2-9}+\frac{3x}{x^2+2x-15}$$

$$=\frac{1}{(x+3)(x-3)}+\frac{3x}{(x+5)(x-3)}$$

Now, **both** denominators have a factor of $(x-3)$.

So to make the denominators the same . . .

. . . we need a factor of $(x+5)$ on the left . . .

. . . and a factor of $(x+3)$ on the right:

$$\frac{1}{(x+3)(x-3)}+\frac{3x}{(x+5)(x-3)}$$

$$=\frac{(x+5)}{(x+5)}\frac{1}{(x+3)(x-3)}+\frac{(x+3)}{(x+3)}\frac{3x}{(x+5)(x-3)}$$

This is the key step in combining rational expressions!

Next, we simplify the numerators and then combine them:

$$= \frac{(x+5)1}{(x+5)(x+3)(x-3)} + \frac{(x+3)3x}{(x+3)(x+5)(x-3)}$$

$$= \frac{x+5+3x^2+3x}{(x+5)(x+3)(x-3)}$$

$$= \frac{3x^2+4x+5}{(x+5)(x+3)(x-3)}$$

There's an additional challenge to **subtracting** rational expressions:

$$\frac{2x}{3x^2-6x} - \frac{x+1}{2x^2-8}$$

This challenge happens near the end of the problem!

First we factor, so that we can find the LCD:

$$\frac{2x}{3x(x-2)} - \frac{x+1}{2(x^2-4)}$$

$$\frac{2x}{3x(x-2)} - \frac{x+1}{2(x+2)(x-2)}$$

Both denominators have a factor of $(x-2)$ . . .

. . . the left side needs a factor of $2(x+2)$ . . .

. . . the right side needs a factor of $3x$:

$$\frac{2x}{3x(x-2)} - \frac{x+1}{2(x+2)(x-2)}$$

$$= \frac{2(x+2)}{2(x+2)}\frac{2x}{3x(x-2)} - \frac{x+1}{2(x+2)(x-2)}\frac{(3x)}{(3x)}$$

Now, first we should simplify each numerator:

$$\frac{2(x+2)2x}{2(x+2)3x(x-2)} - \frac{3x(x+1)}{(3x)2(x+2)(x-2)}$$

$$\frac{4x^2+8x}{6x(x+2)(x-2)} - \frac{3x^2+3x}{6x(x+2)(x-2)}$$

Now, here's the crucial part!

We need to make sure the ***minus sign distributes***!

$$\frac{4x^2+8x}{6x(x+2)(x-2)} + \frac{-3x^2+3x}{6x(x+2)(x-2)}$$

$$= \frac{4x^2+8x}{6x(x+2)(x-2)} + \frac{-3x^2-3x}{6x(x+2)(x-2)}$$

The most common mistake is not carrying the – sign through all the way!

$$\frac{4x^2+8x}{6x(x+2)(x-2)} + \frac{-3x^2-3x}{6x(x+2)(x-2)}$$

$$= \frac{x^2+5x}{6x(x+2)(x-2)}$$

And then we have one last step . . . we must simplify!

$$\frac{x^2+5x}{6x(x+2)(x-2)}$$

$$= \frac{x(x+5)}{6x(x+2)(x-2)}$$

$$= \frac{x+5}{6(x+2)(x-2)}$$