Domain and Range

We've got one final very important (and somewhat theoretical) component to the concept of a **function**.

Remember the function for the value of our depreciating equipment:

$$V = -1500t + 20000$$

Where

$$V = resale\ value\ of\ the\ equipment$$

$$t = the\ number\ of\ years\ of\ use$$

And which I will now re-express using ***Newtonian functional notation***:

$$V(t) = resale\ value\ of\ equipment\ after\ t\ years\ of\ use$$

$$V(t) = -1500t + 20000$$

What would happen if I was to plug into the function the value $t = -5$?

$$V(-5) = -1500(-5) + 200000$$

$$= 7500 + 20000$$

$$= 27500$$

Here's what this would mean for the real-life situation:

"If we were to go back in time 5 years with a time machine, the equipment would be worth $27500, which is more than it was worth brand new."

This makes no sense!

Some numbers don't make sense to put into a function.

In this case, that would be negative numbers. Also, the real-life situation was only designed to go ten years into the future, because we don't know what the equipment will be worth after that.

Therefore the *domain* of the function is:

$$0 \leq t \leq 10$$

That's right, the "*domain*" means the numbers that can ***go into*** a function.

And in this class, we will often use *interval notation* to state the domain.

That means that instead of

$$0 \leq t \leq 10$$

We will write

$$[0, 10]$$

Where the "closed brackets" [ ] mean that the ***boundary is included***.

In this last example, the *domain* is determined by ***real-life meaning***.

In this next example, the *domain* is determined by ***math meaning***.

Consider the following function:

$$f(x) = \sqrt{x}$$

This is called the “radical function” or “square-root function”

And it works very well if you plug in positive numbers:

$$f(4) = \sqrt{4} = 2$$

$$f(81) = \sqrt{81} = 9$$

$$f(8) = \sqrt{8} = 2\sqrt{2}$$

But if you try to plug in a negative number,

$$f(-4) = \sqrt{-4} = not\ a\ real\ number$$

And in this course, we are only dealing with real numbers . . . partly because only real-numbers can be placed on graphs.

Therefore, we say that negative numbers are not in the domain of $f(x)$.

So the domain of $f(x)$ is

$$[0, \infty)$$

Note that $f(0) = 0$ so $x = 0$ **is** in the domain of $f(x)$.

That's why we use the closed bracket next to the ***lower bound***.

All numbers greater than zero have a real number radical, so the ***upper bound*** of the domain is infinity.

We cannot include infinity in any set (because it's not a number) so we must use the **open parenthesis** next to the upper bound, meaning "not included."

$$[0, \infty)$$

Here's another example….

Consider the function:

$$f(x) = \frac{x}{x^2 - 1}$$

What is its domain?

Answer:

This function "works" as long as you don't give it two particular numbers!

Can you see what those numbers are?

$f(x)$ is in the form of a fraction, and it's denominator can be zero . . .

Which makes it . . . **UNDEFINED**

For example, suppose we plug in $x = 1$:

$$f(1) = \frac{1}{(1)^2 - 1}$$
$$= \frac{1}{0}$$

Since this **input** does not give us a real-number **output** . . .

. . . it's ***not in the domain!***

So what IS the domain for this function?

To find out what IS the domain, find out what's NOT in the domain!

$f(x)$ is undefined where its denominator is zero:

$$x^2 - 1 = 0$$
$$x^2 = 1$$
$$x = \pm\sqrt{1}$$
$$x = \pm 1$$

So we can say that the domain includes all real numbers **except** ± 1.

We need to state this in math language. There are two ways.

One is interval notation: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

"union" symbol connects the included sets (intervals)

The other is the language of sets: $\{x \mid x \neq \pm 1\}$

means x such that x doesn't equal ± 1

The domain is half of the story here . . .

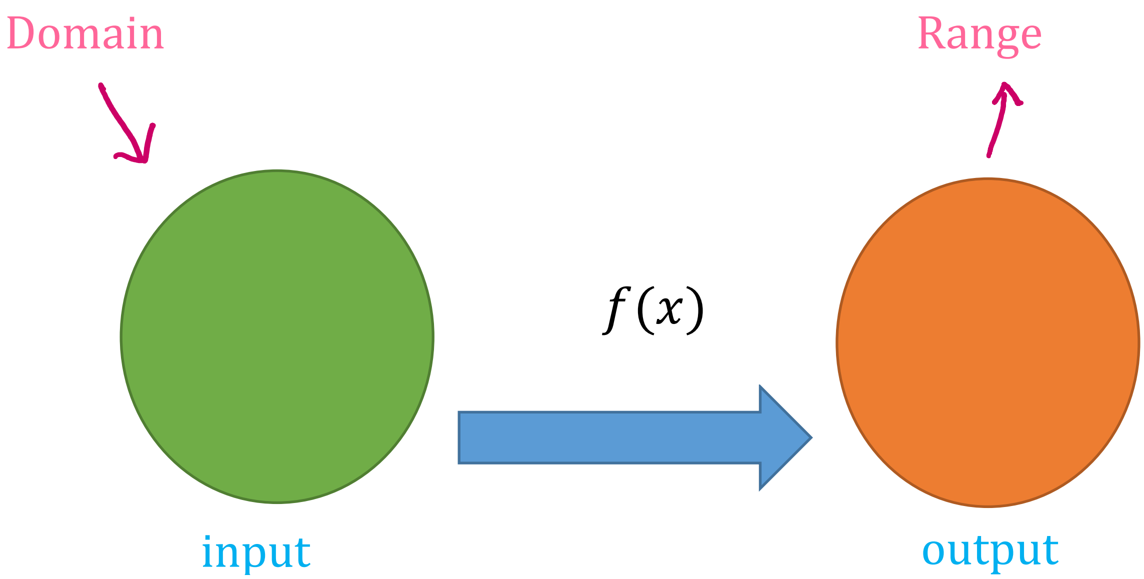
Just as we have only certain numbers ***allowed to go in*** to the function…

We only have certain numbers that . . . . ***come out***.

Domain: ***all possible inputs***

Range: ***all resulting outputs***

Complete Concept Model for a Function



Finding the range of a function can be a bit trickier than finding the domain.

Let's take another look at our real life example:

$$V = -1500t + 20000$$

Where

$$V = resale\ value\ of\ the\ equipment$$

$$t = the\ number\ of\ years\ of\ use$$

What is the range for this function?

Here, the numbers that come out of the function are values for V.

There is only a certain range of numbers that make sense for V.

The **greatest** value for V that makes sense is 20000 . . .

. . . because that is the value of the equipment brand-new.

The **least** value for V that makes sense is 5000 . . .

. . . because that's the value after 10 years (the end of the domain).

So the range of the function is $[5000, 20000]$.

Note that the range has to begin with 5000, because it is the ***lower bound***.

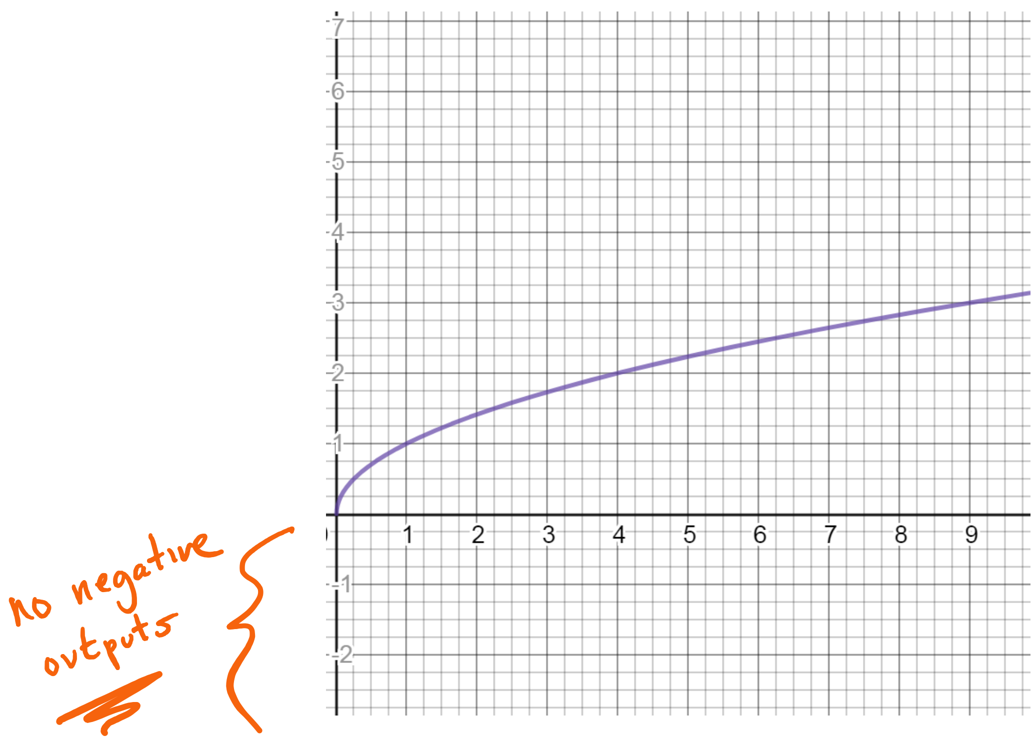
$$[5000, 20000]$$

Now, let's find the range for

$$f(x) = \sqrt{x}.$$

What set of numbers ***come out of*** $f(x)$?

To see this, it's helpful to look at the graph:



What story does the graph tell about the numbers that come out of $f(x)$?

One thing that I note is that they seem to "start" at . . . $y = 0.$

And from there? They go up. And up and up . . . forever.

The range of this function is . . . . $[0, \infty)$.