Simplifying Rational Expressions

One of the first things you learn about fractions . . .

. . . is that they can be written in different ways:

$$\frac{24}{72} = \frac{12}{36} = \frac{6}{18} = \frac{3}{9} = \frac{1}{3}$$

In math, we have agreed that one form is the best: **simplified form**

$$\mathbf{\frac{1}{3}}$$

Notice that in the example above . . .

. . . we can get to the *more* simplified fraction . . .

. . . by dividing top and bottom by 2:

$$\frac{24 \div 2}{72 \div 2} = \frac{12}{36}$$

Here's the reason that works:

$$\frac{24}{72} = \frac{12 \cdot 2}{36 \cdot 2} = \frac{12}{36} \cdot 1 = \frac{12}{36}$$

In other words, we are effectively **factoring** the top and bottom . . .

. . . and ***canceling*** like factors!

We will use this strategy with algebra . . .

. . . to simplify **rational expressions**!

Simplify:

$$\frac{x^2 - 3x + 2}{x^2 - 4}$$

First, I need to tell you what **not** to do!

You **can't** do this:

$$\frac{\cancel{x^2} - 3x + 2}{\cancel{x^2} - 4}$$

More generally, there's a rule:

You can only cancel factors

That means that you cannot cancel any term connected by a + or -- . . .

. . . only terms connected by multiplication.

Which means, that to **simplify rational expressions** . . .

. . . the first thing to do is factor:

$$\frac{x^2 - 3x + 2}{x^2 - 4} = \frac{(x-1)(x-2)}{(x+2)(x-2)}$$

Then, **cancel like factors**:

$$\frac{(x-1)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \frac{x-1}{x+2}$$

And we're done!

Simplify:

$$\frac{4x^2 - 12x}{6x^2 - 6x - 36}$$

Here, we must factor first . . .

. . . and we must remember that the **first** step in factoring is to

factor out the greatest common factor.

$$\frac{4x^2 - 12x}{6x^2 - 6x - 36} = \frac{4x(x - 3)}{6(x^2 - x - 6)}$$

Next, we want to factor the trinomial in the denominator:

$$\frac{4x(x - 3)}{6(x^2 - x - 6)} = \frac{4x(x - 3)}{6(x - 3)(x + 2)}$$

Finally, we can cancel like factors, while reducing the coefficent part:

$$\frac{\cancel{4x}(\cancel{x - 3})}{\cancel{6}(\cancel{x - 3})(x + 2)} = \frac{2x}{3(x + 2)}$$

Simplify:

$$\frac{12x^2 + 33x - 9}{3 - 48x^2}$$

We begin by factoring out the GCF:

$$\frac{12x^2+33x-9}{3-48x^2}=\frac{3(4x^2+11x-3)}{3(1-16x^2)}$$

Next we factor the polynomials in the top and bottom:

$$\frac{\cancel{3}(4x^2+11x-3)}{\cancel{3}(1-16x^2)}=\frac{(4x-1)(x+3)}{(1+4x)(1-4x)}$$

At this point it looks like nothing will cancel!

But wait . . . the factor of $(4x-1)$ on top . . .

. . . is the ***negative*** of $(1-4x)$ on the bottom!

We can make them the same by factoring out a -1 from the bottom factor:

$$\frac{(4x-1)(x+3)}{(1+4x)(1-4x)}=\frac{(4x-1)(x+3)}{(1+4x)(-1)(4x-1)}$$

And now we can cancel:

$$\frac{\cancel{(4x-1)}(x+3)}{(1+4x)(-1)\cancel{(4x-1)}}=-\frac{x+3}{1+4x}$$