Problem a,b,c

Solution For this homework, I interpreted the "forward rate" in the problems as the instantaneous forward rate, which is defined in Curve Fitting Lecture Notes. I think the definition of "forward rate" if not clear enough in the problem. I have written two versions of definitions. If this one is not exact, I can resubmitted another version. But I think the basic principles and ideas are the same. Here are my results:
We need to use the instantaneous forward rate to represent the Discounted factors and LIBOR rates. Then we can calculate the present values of fixed leg and float leg. Then find the proper forward rate that makes these two legs equal. I wrote a Python class and use Newton methods to find the root of the difference of two legs:

Maturities	Swap Rates(%)	Forward Rates(%)
1	2.8438	2.8238
2	3.060	3.2565
3	3.126	3.2382
4	3.144	3.1762
5	3.150	3.1511
7	3.169	3.1965
10	3.210	3.2953
30	3.237	3.2324

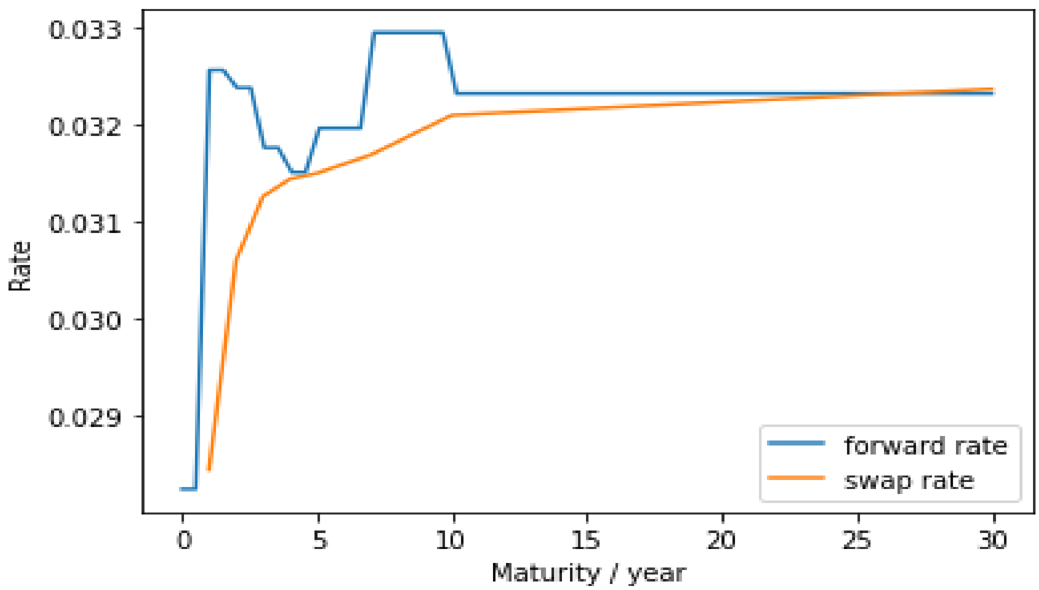

Figure 1: forward rates & swap rates

The forward rate generally is higher than the swap rate. That premium is because of the expectation of increasing interest rate in the future.

Problem d

Solution Just take the forward rate values into the equation of two legs, and set the maturity to 15 years, then we can get the result:
break even swap rate of a 15Y swap: 3.2237%

Problem e

Solution

By the formula:

$$D(S,T) = \exp\left(-\int_S^T f(s)ds\right)$$

We can get the Discount Factors. Then use the formula:

$$D(S,T) = \exp(-rt)$$

We can get the zero rates r.

Actually there are 60 points of values if we pay coupons semi-annually. Limited by space, I just show the values on the maturities, the other values can be checked in my Python file.

Maturities	Discount Factors(%)	Zero rates(%)
1	0.9722	0.0282
2	0.9410	0.0304
3	0.9110	0.0311
4	0.8825	0.0312
5	0.8552	0.0313
7	0.8022	0.0315
10	0.7267	0.0319
30	0.3807	0.0322

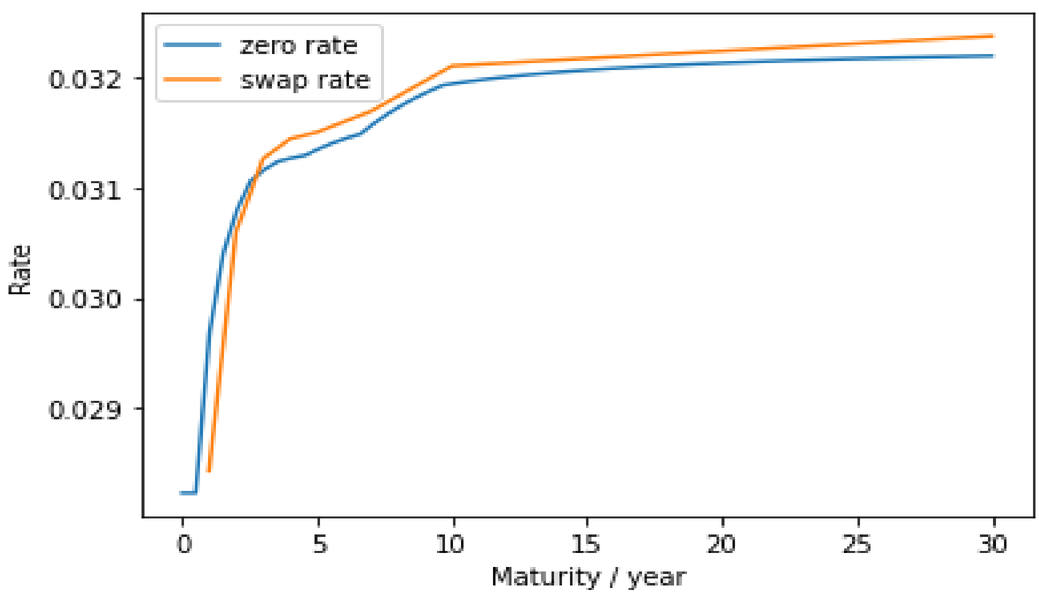

Figure 2: zero rates & swap rates

The trends of these two curves are the same and the values are very close. The zero rates are generally lower than the swap rates. This is just like the CDS-bond basis in lecture 2, the difference is small but non-zero, which mainly because of the coupon payments in this problem.

Problem f

Solution We just need to shift the forward rates up 100 bps and use the same method as problem (d) to get the break even swap rates for all these maturities:

Maturities	Old Swap(%)	New Swap(%)	Difference(bps)
1	2.8438	3.8606	101.6759
2	3.060	4.0767	101.6739
3	3.126	4.1427	101.6746
4	3.144	4.1609	101.6849
5	3.150	4.1671	101.7052
7	3.169	4.1860	101.6968
10	3.210	4.2260	101.601
30	3.237	4.2528	101.5779

The difference is a bit larger than shift the swap rates directly. Generally it shift up 1.6 bps more than the direct shift.

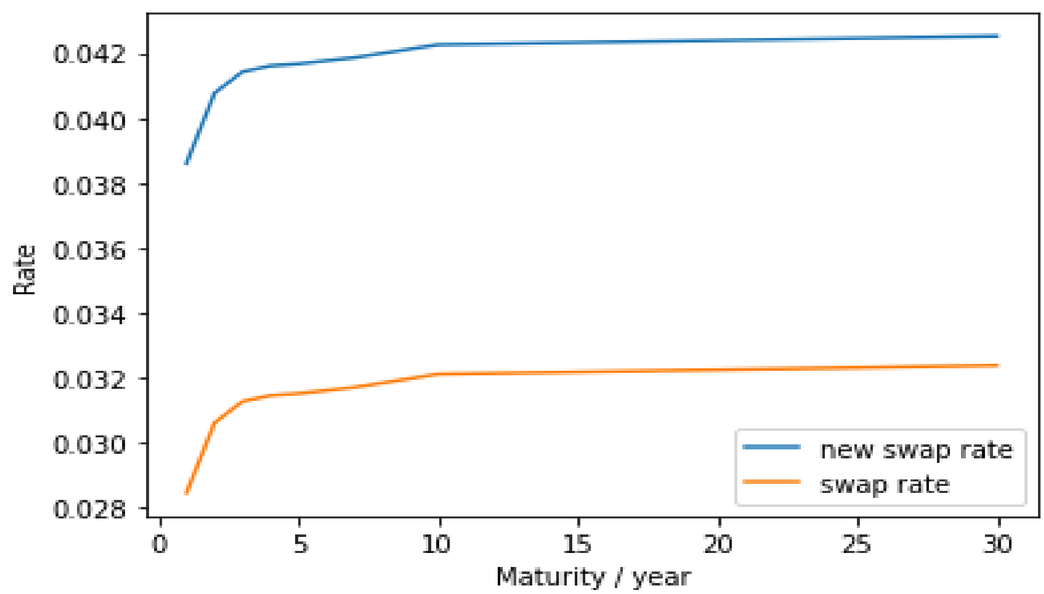

Figure 3: shift of swap rates

Problem g,h

Solution All the procedure is the same as problem(a,b,c):

Maturities	Bear Swap Rates(%)	Bear Forward Rates(%)
1	2.8438	2.8238
2	3.060	3.2565
3	3.126	3.2382
4	3.194	3.3831
5	3.250	3.4635
7	3.319	3.4830
10	3.460	3.8171
30	3.737	3.9393

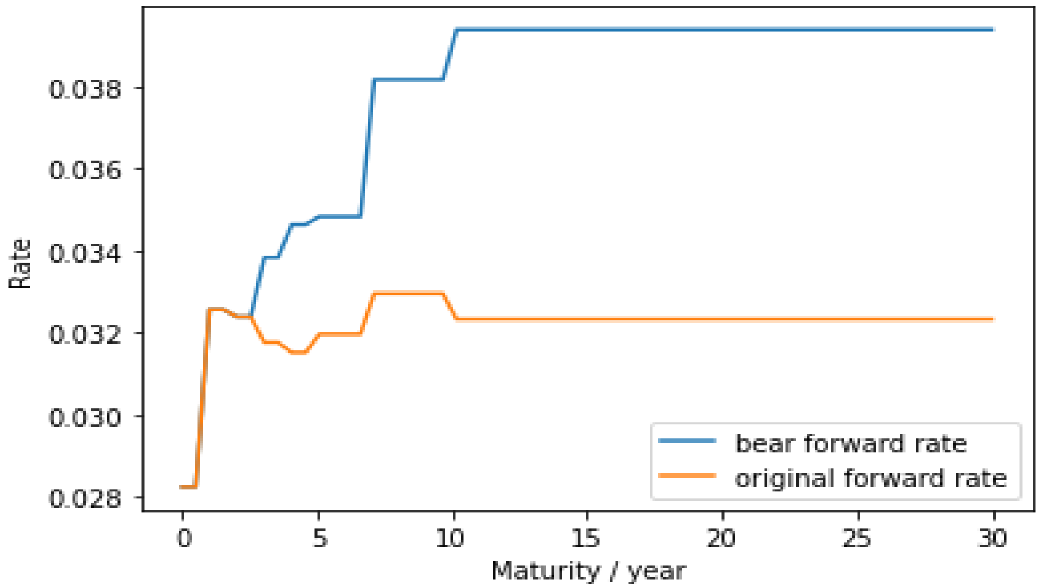

Figure 4: shift of bear forward rates

They are the same at the beginning, but when swap rates become steep since year 4, the forward rates also increase and also get steep.

Problem i,j

Solution All the procedure is the same as problem(a,b,c):

Maturities	Bull Swap Rates(%)	Bull Forward Rates(%)
1	2.3438	2.3302
2	2.81	3.2638
3	2.976	3.2970
4	3.044	3.2354
5	3.1	3.3155
7	3.169	3.3348
10	3.21	3.2957
30	3.237	3.2324

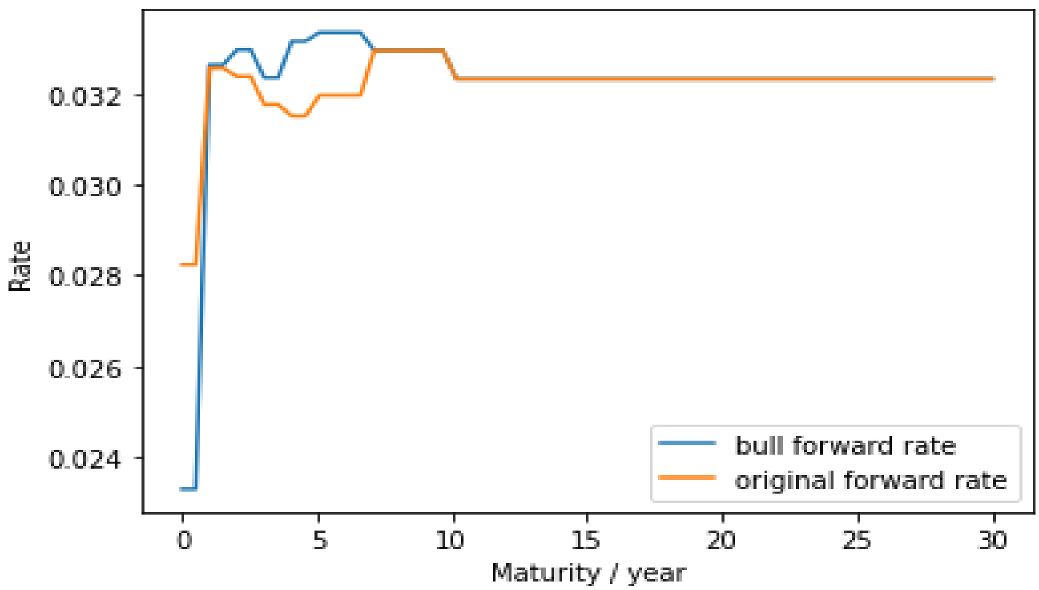

Figure 5: shift of bull forward rates

When swap rates become steep at the beginning, the forward rates also increase and also get steep. When the swap rates go back to the same level as origin, the bull forward rate converges back to the original one.