Problem a

Solution For this homework, I constructed a python class to do the bootstrap and some calculation and it replicates all the functions and results of the R package "credule". Here are my results:
We just need to use these formulas iteratively:

$$S \times RPV01 = (1 - R) \sum_{n=1}^{N} DF(t_n)(P(t_{n-1}) - P(t_n))$$

$$P(t) = \exp\left(-\int_0^t h(s) ds\right)$$

$$h(t_{m-1}) = \frac{\log P(t_{m-1}) - \log P(t_m)}{t_m - t_{m-1}}$$

h is the only unknown variable in this situation, we just need to take the values in and calculate the hazard rate and survival probabilities.

If the payment period is one year:

Maturity	hazard rate	survival probability	spread(bps)
1	0.01666705	0.9834711	100
2	0.02006583	0.9639336	110
3	0.02354460	0.9415032	120
4	0.02889339	0.9146600	132.527
5	0.02889339	0.8885822	140

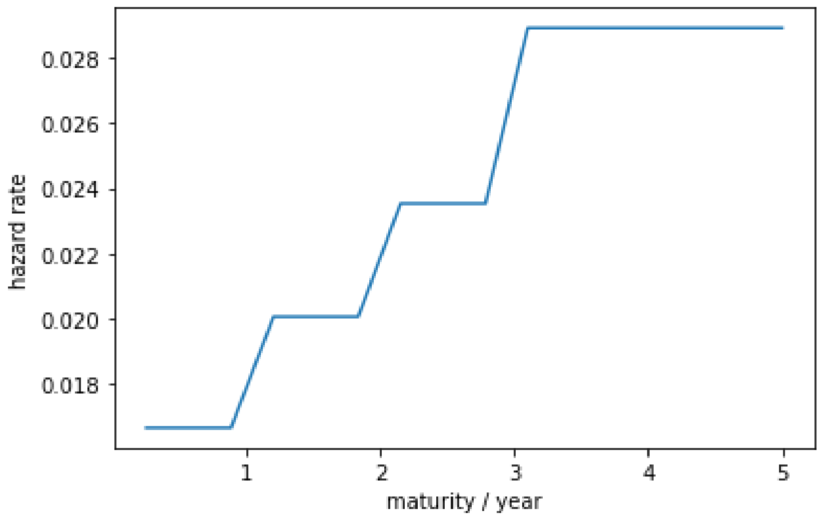
Table 1: Bootstrap with one year payment period

If the payment period is one quarter:

Maturity	hazard rate	survival probability	spread(bps)
1	0.01666669	0.9834714	100
2	0.02006521	0.9639345	110
3	0.02354360	0.9415051	120
4	0.02892354	0.9146634	132.527
5	0.02892354	0.8885871	140

Table 2: Bootstrap with one quarter payment period

The CDS curve(hazard rate vs maturity) with 4 payment one year bootstrap:



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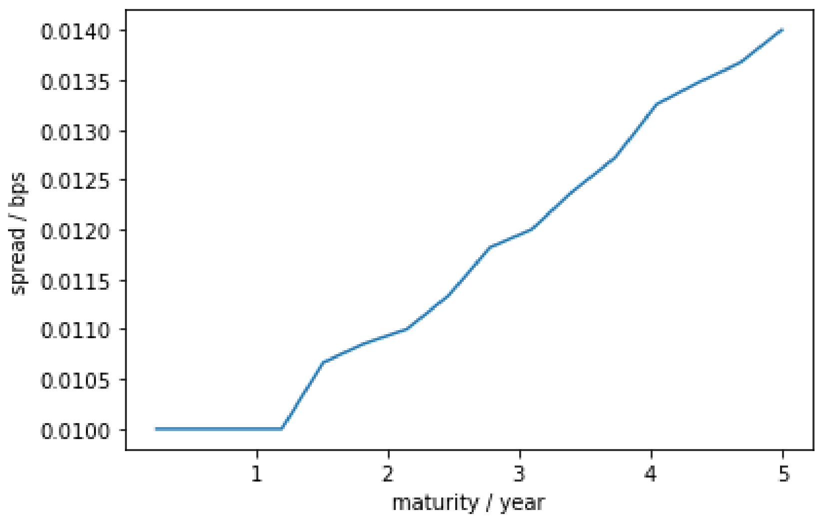

In the following problems, my answers are all based on the quarterly bootstrapped results via my python program.

Problem b

Solution This problem has been solved in (a), by bootstrapping, we can get the hazard rate, $h_4 = h_5$, then we can get P_4 by integral. Then, just take into the formula and get the spread is around 132.527 bps.

Problem c

Solution By the formula:

$$RPV01(t) = \frac{1}{2}\sum_{t_n>t} \Delta\left(t_{n-1}, t_n\right) DF\left(t_n\right)\left(P\left(t_{n-1}\right) + P\left(t_n\right)\right)$$

We can get:

$$RPV01(1) = \frac{1}{2}\sum_{t_n>1} \Delta\left(t_{n-1}, t_n\right) DF\left(t_n\right)\left(P\left(t_{n-1}\right) + P\left(t_n\right)\right) \approx 3.69$$

Then by the formula:

$$S(t) \times RPV01(t) = (1 - R) \sum_{t_n > t} DF(t_n)(P(t_{n-1}) - P(t_n))$$

We can get:

$$S(1) \approx 132.527 \ bps$$

Then the price to buy it off:

$$(S(1) - S(0)) \times RPV01(1) = (132.527 - 80) \times 3.69 \approx 193.78 \ bps$$

The content on slides here is somewhat ambiguous. If the we re-count the time from point 0, then the results above will be correct, which in fact uses P(0), P(1), P(2), P(3), P(4). However, if we just continue to count from 1, we should use P(1), P(2), P(3), P(4),P(5). Then the same formula will give us the results as 254.38 bps.

Problem d

Solution Actually, the DV01 of a CDS depends on the contract spread. The formula should be :

$$V_{CDS} = \text{ Notional } \times (C - S) \times RPV01$$

By calculating RPV01 like in (c), we can get:

$$RPV01 \approx 3.6892$$

New spread for 4-year CDS:

$$S_{new} = 133.527 bps$$

DV01 at 132.527 bps is actually the RPV01:

$$CREDIT \quad DV01 = 3.6892 \ bps$$

Problem e

Solution When the interest rate goes up 1bp, we can recalculate the RPV01 and spread for 4y CDS:

$$RPV01_{new} \approx 3.68843 bps$$

$$S_{new} \approx 132.524 bps$$

$$IR \quad DV01 = \Delta S * RPV01_{new} = -0.011 \ bps$$

Generally, the sensitivity to interest rate is not big.

Problem f

Solution When the recovery rate goes up 1bp, we can recalculate the RPV01 and spread for 4y CDS:

$$RPV01_{new} \approx 3.689193 bps$$

$$S_{new} \approx 132.504 bps$$

$$REC01 = \Delta S * RPV01_{new} = -0.0815 \ bps$$

Similarly, if the definition here is to take the negative value:

$$REC01 = -0.0815 \ bps$$

In the end, I have to mention that I spent a lot of time on this homework, especially replicating the R package in Python. In fact, what I have finished can be capsuled to be a new package in Python. However, I cannot find clear requirements or definitions of some problems, especially the last three problems. If all the conditions and definitions are clear and unambiguous, I can get any correct result with my code. Thank you for your guidance and help.