Supplementary Material for "Orthogonal Transfer for Multitask Optimization"

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A. Construction of the Orthogonal Array with Two Levels for the D-Dimensional Problem [46]

Step 1: Determine the number of rows $M=2^{\lceil \log_2(D+1) \rceil}$, the number of columns D, and the number of basic columns $u=\log_2(M)$.

Step 2: The elements in the basic columns are set as:

$$L(a,b)=\left(\left\lfloor \frac{a-1}{2^{u-k}} \right\rfloor\right) \bmod 2 \tag{S1}$$

where a=1,2,…,M is the row index, $b=2^{k-1}$ is the basic column index, and k=1,2,…,u.

Step 3: The elements in other columns are set as:

$$L(a,b+s)=(L(a,s)+L(a,b)) \bmod 2 \tag{S2}$$

where a=1,2,…,M is the row index, $b=2^{k-1}$ is the basic column index, s=1,2,…,b-1, and k=2,…,u.

Step 4: For all the elements in the OA, transform the level value to 1 for the first level and the level value to 2 for the second level as:

$$L(a,b)=\begin{cases}1, & \text{if } L(a,b)=0\\ 2, & \text{if } L(a,b)=1\end{cases} \tag{S3}$$

where a=1,2,…,M is the row index, and b=1,2,…,N is the column index.

[46] Z. H. Zhan, J. Zhang, Y. Li, and Y. Shi, "Orthogonal learning particle swarm optimization," *IEEE Trans. Evol. Comput.*, vol. 15, no. 6, pp. 832-847, Dec. 2011.

B. Results of EMT Algorithms on MTOP-DD

TABLE S.I.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=20 AND D_2=20

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	3.83e-03	5.24e+01	2.12e-01(+)	7.20e+01(+)	9.09e-03(=)	1.80e+01(-)	3.29e-02(+)	2.38e+01(-)	1.68e-02(+)	1.14e+02(+)	2.32e-03(=)	1.16e+02(+)
2	9.69e-04	5.11e+01	2.67e+00(+)	6.63e+01(+)	8.27e-02(+)	7.50e+00(-)	4.81e-01(+)	2.10e+01(-)	2.19e-01(+)	1.12e+02(+)	5.67e-01(+)	1.14e+02(+)
3	9.61e-06	1.07e-10	1.68e+00(+)	2.66e+00(+)	1.09e-02(+)	6.42e-04(+)	1.70e-02(+)	1.41e-03(+)	2.19e-01(+)	1.68e-03(+)	5.42e-01(+)	4.53e-03(+)
4	3.88e+01	1.44e-07	1.00e+02(+)	4.46e+00(+)	6.50e+01(+)	6.63e-02(+)	2.14e+01(-)	7.22e-02(+)	9.92e+01(+)	1.24e-03(+)	1.20e+02(+)	4.01e-03(+)
5	1.19e-04	5.17e-03	2.71e+00(+)	2.61e-01(+)	9.07e-01(+)	2.51e-02(+)	3.19e-02(+)	6.57e-03(=)	2.17e-01(+)	3.05e-04(=)	4.98e-01(+)	9.26e-03(=)
6	2.38e-04	5.67e-01	6.67e+00(+)	6.24e+00(+)	3.13e+00(+)	3.66e+00(+)	6.78e-02(+)	9.58e-01(=)	2.06e-01(+)	3.04e-01(=)	4.59e-01(+)	7.37e-01(=)
7	5.01e+01	2.77e-07	7.16e+01(+)	3.85e+00(+)	2.76e+01(-)	1.71e-02(+)	2.18e+01(-)	6.55e-02(+)	9.57e+01(+)	1.52e-03(+)	1.19e+02(+)	4.06e-03(+)
8	7.99e-03	4.49e-01	2.05e-01(+)	2.65e+00(+)	1.44e-02(=)	1.26e+00(+)	7.63e-03(=)	9.02e-01(+)	1.27e-02(=)	3.75e-01(=)	4.13e-03(=)	8.63e-01(+)
9	6.40e-11	1.44e-04	2.83e+00(+)	2.27e+00(+)	1.42e-02(+)	1.65e-01(+)	1.90e-03(+)	1.66e-02(+)	1.13e-01(+)	3.09e-02(+)	4.37e-03(+)	3.08e-01(+)
Number of +/=/-			9/0/0	9/0/0	6/2/1	7/0/2	6/1/2	5/2/2	8/1/0	6/3/0	7/2/0	7/2/0

TABLE S.II.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=20 AND D_2=30

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	2.22e-03	3.37e+01	1.03e-01(+)	1.81e+02(+)	1.23e-02(+)	9.70e+01(+)	3.29e-02(+)	1.42e+02(+)	8.95e-04(=)	2.20e+02(+)	1.65e-05(=)	2.19e+02(+)
2	2.09e-05	3.06e+01	2.50e+00(+)	1.71e+02(+)	3.34e-01(+)	1.06e+02(+)	7.73e-01(+)	1.38e+02(+)	7.91e-03(+)	2.14e+02(+)	4.13e-02(+)	2.24e+02(+)
3	9.91e-07	2.30e-12	2.05e+00(+)	2.94e+01(+)	2.28e-02(+)	3.23e-01(+)	4.09e-01(+)	1.53e+01(+)	8.26e-03(+)	3.14e-01(+)	5.05e-02(+)	2.33e-01(+)
4	2.38e+01	3.14e-08	9.47e+01(+)	2.78e+01(+)	7.11e+01(+)	3.90e-01(+)	2.96e+01(=)	2.74e+01(+)	5.69e+01(+)	4.05e-01(+)	1.09e+02(+)	2.62e-01(+)
5	2.45e-06	1.85e-03	2.17e+00(+)	6.76e-01(+)	6.45e-01(+)	2.29e-02(+)	3.53e-01(+)	5.39e-01(+)	8.67e-03(+)	2.57e-02(+)	5.52e-02(+)	2.10e-02(+)
6	6.74e-06	8.19e-01	7.18e+00(+)	1.11e+01(+)	6.91e+00(+)	8.05e+00(+)	6.06e-01(+)	5.77e+00(+)	9.55e-03(+)	3.15e+00(+)	3.52e-02(+)	3.04e+00(+)
7	3.61e+01	9.99e-08	6.98e+01(+)	2.82e+01(+)	6.73e+01(+)	2.43e-01(+)	3.75e+01(=)	3.33e+01(+)	3.54e+01(=)	3.67e-01(+)	1.12e+02(+)	2.64e-01(+)
8	4.68e-03	1.17e+00	1.31e-01(+)	1.20e+01(+)	1.34e-02(+)	8.71e+00(+)	3.90e-02(+)	6.03e+00(+)	6.94e-05(=)	3.15e+00(+)	5.21e-04(=)	3.17e+00(+)
9	4.04e-14	1.40e-04	1.06e+00(+)	9.39e+00(+)	4.68e-03(+)	1.24e+01(+)	1.33e-01(+)	3.02e+00(+)	2.84e-04(+)	8.65e+00(+)	7.09e-05(+)	9.12e+00(+)
Number of +/=/-			9/0/0	9/0/0	9/0/0	9/0/0	7/2/0	9/0/0	6/3/0	9/0/0	7/2/0	9/0/0

TABLE S.III.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=20 AND D_2=40

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	2.59e-03	4.53e+01	7.98e-02(+)	2.56e+02(+)	1.92e-02(+)	1.39e+02(+)	3.07e-02(+)	2.42e+02(+)	3.70e-04(=)	3.32e+02(+)	5.33e-07(=)	3.39e+02(+)
2	7.78e-06	4.64e+01	1.94e+00(+)	2.50e+02(+)	9.17e-01(+)	1.54e+02(+)	2.74e-01(+)	2.56e+02(+)	4.75e-04(+)	3.40e+02(+)	3.33e-03(+)	3.40e+02(+)
3	5.15e-07	1.78e-12	2.23e+00(+)	6.41e+01(+)	2.29e-02(+)	9.20e-01(+)	3.77e-01(+)	1.06e+02(+)	4.21e-04(+)	3.86e+00(+)	5.30e-03(+)	3.20e+00(+)
4	2.95e+01	4.17e-07	1.06e+02(+)	5.79e+01(+)	6.28e+01(+)	8.41e-01(+)	4.67e+01(+)	1.16e+02(+)	2.27e+01(=)	3.53e+00(+)	1.05e+02(+)	3.01e+00(+)
5	7.62e-07	1.97e-03	2.30e+00(+)	9.13e-01(+)	1.24e+00(+)	4.51e-02(+)	4.58e-01(+)	9.54e-01(+)	4.44e-04(+)	1.56e-01(+)	1.09e-02(+)	1.17e-01(+)
6	3.44e-06	2.81e+00	7.79e+00(+)	1.88e+01(+)	6.63e+00(+)	1.30e+01(+)	5.07e-01(+)	1.23e+01(+)	4.60e-04(+)	9.19e+00(+)	3.32e-03(+)	8.27e+00(+)
7	3.23e+01	1.39e-07	8.62e+01(+)	5.79e+01(+)	6.02e+01(+)	6.24e-01(+)	4.03e+01(=)	1.16e+02(+)	1.62e+01(-)	3.59e+00(+)	1.04e+02(+)	2.66e+00(+)
8	4.06e-03	2.26e+00	7.45e-02(+)	1.93e+01(+)	2.64e-02(+)	1.35e+01(+)	3.63e-02(+)	1.19e+01(+)	4.93e-04(=)	8.80e+00(+)	9.86e-04(=)	9.39e+00(+)
9	8.02e-15	3.92e-02	6.35e-01(+)	1.92e+01(+)	9.72e-03(+)	2.00e+01(+)	9.44e-02(+)	4.36e+00(+)	7.83e-07(+)	1.88e+01(+)	1.17e-06(+)	1.79e+01(+)
Number of +/=/-			9/0/0	9/0/0	9/0/0	9/0/0	8/1/0	9/0/0	5/3/1	9/0/0	7/2/0	9/0/0

TABLE S.IV.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=20 AND D_2=50

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	1.85e-03	5.97e+01	5.16e-02(+)	4.12e+02(+)	2.00e-02(+)	2.45e+02(+)	1.67e-02(+)	3.89e+02(+)	7.40e-04(=)	4.58e+02(+)	4.97e-09(=)	4.71e+02(+)
2	2.74e-07	5.93e+01	2.42e+00(+)	3.93e+02(+)	7.37e-01(+)	2.38e+02(+)	1.82e-01(+)	3.93e+02(+)	2.86e-05(+)	4.75e+02(+)	7.61e-04(+)	4.71e+02(+)
3	2.82e-08	2.35e-14	2.14e+00(+)	1.06e+02(+)	5.35e-02(+)	1.89e+00(+)	2.39e-01(+)	2.08e+02(+)	2.43e-05(+)	1.96e+01(+)	3.01e-04(+)	1.59e+01(+)
4	1.95e+01	9.44e-09	1.14e+02(+)	1.07e+02(+)	7.35e+01(+)	2.19e+00(+)	3.81e+01(+)	2.27e+02(+)	1.69e+01(=)	2.13e+01(+)	1.05e+02(+)	1.64e+01(+)
5	2.55e-08	7.40e-04	2.39e+00(+)	9.74e-01(+)	1.11e+00(+)	6.76e-02(+)	3.61e-01(+)	1.03e+00(+)	2.55e-05(+)	4.72e-01(+)	2.19e-03(+)	4.25e-01(+)
6	1.15e-07	4.71e+00	1.11e+01(+)	2.75e+01(+)	5.00e+00(+)	1.91e+01(+)	2.83e-01(+)	1.64e+01(+)	2.77e-05(+)	1.71e+01(+)	7.31e-04(+)	1.64e+01(+)
7	2.18e+01	1.98e-08	9.32e+01(+)	1.02e+02(+)	6.03e+01(+)	1.36e+00(+)	4.01e+01(+)	2.06e+02(+)	1.43e+01(=)	1.97e+01(+)	1.06e+02(+)	1.62e+01(+)
8	1.97e-03	4.72e+00	5.52e-02(+)	2.88e+01(+)	1.66e-02(+)	2.31e+01(+)	2.31e-02(+)	1.80e+01(+)	1.36e-03(=)	1.82e+01(+)	1.25e-08(=)	1.67e+01(+)
9	3.29e-18	2.73e-01	4.31e-01(+)	1.93e+01(+)	7.70e-03(+)	2.00e+01(+)	4.23e-02(+)	5.38e+00(+)	2.09e-09(+)	2.07e+01(+)	1.57e-08(+)	2.06e+01(+)
Number of +/=/-			9/0/0	9/0/0	9/0/0	9/0/0	9/0/0	9/0/0	5/4/0	9/0/0	7/2/0	9/0/0

TABLE S.V.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=30 AND D_2=20

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	8.63e-04	2.37e+01	6.72e-01(+)	7.81e+01(+)	1.82e-02(+)	3.77e+01(=)	7.56e-01(+)	3.09e+01(=)	4.29e-02(+)	1.05e+02(+)	1.69e-02(+)	1.09e+02(+)
2	5.12e-04	3.38e+01	7.27e+00(+)	8.62e+01(+)	1.72e+01(+)	5.15e+01(+)	3.66e+00(+)	3.37e+01(=)	1.17e+00(+)	1.11e+02(+)	7.50e+00(+)	1.11e+02(+)
3	4.67e-02	3.79e-14	7.18e+00(+)	1.11e+00(+)	8.70e+00(+)	4.28e-03(+)	3.27e+00(+)	1.71e-01(+)	9.46e-01(+)	1.13e-04(+)	8.19e+00(+)	7.72e-05(+)
4	4.10e+01	1.26e-10	1.77e+02(+)	1.50e+00(+)	1.21e+02(+)	9.22e-03(+)	1.41e+02(+)	1.26e-01(+)	1.86e+02(+)	7.56e-05(+)	2.20e+02(+)	7.19e-05(+)
5	5.79e-02	3.82e-03	8.08e+00(+)	1.25e-01(+)	9.94e+00(+)	2.21e-02(+)	3.44e+00(+)	4.14e-02(+)	1.08e+00(+)	4.37e-04(=)	7.47e+00(+)	3.54e-05(=)
6	1.23e-04	8.71e-01	1.76e+01(+)	7.46e+00(+)	1.45e+01(+)	4.70e+00(+)	3.60e+00(+)	2.19e+00(+)	1.16e+00(+)	2.28e-01(-)	9.82e+00(+)	3.22e-01(=)
7	3.49e+01	1.56e-10	1.94e+02(+)	9.75e-01(+)	1.28e+02(+)	3.00e-03(+)	1.50e+02(+)	1.98e-01(+)	1.89e+02(+)	8.37e-05(+)	2.20e+02(+)	6.93e-05(+)
8	1.85e-03	4.84e-01	6.49e-01(+)	2.74e+00(+)	3.21e-02(+)	2.00e+00(+)	6.98e-01(+)	1.68e+00(+)	4.91e-02(+)	1.81e-01(-)	2.37e-02(+)	4.98e-01(=)
9	2.02e-12	3.11e-06	2.53e+01(+)	2.05e+00(+)	3.37e-01(+)	2.73e-01(+)	2.18e+01(+)	3.41e-01(+)	7.57e-01(+)	6.04e-03(+)	2.60e-01(+)	3.95e-02(+)
Number of +/=/-			9/0/0	9/0/0	9/0/0	8/1/0	9/0/0	7/2/0	9/0/0	6/1/2	9/0/0	6/3/0

TABLE S.VI.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=30 AND D_2=30

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	2.22e-03	3.00e+01	2.94e-01(+)	1.08e+02(+)	5.80e-03(=)	4.84e+01(+)	1.24e-02(+)	3.51e+01(+)	8.14e-03(+)	2.06e+02(+)	1.71e-03(=)	2.08e+02(+)
2	1.84e-05	2.92e+01	2.93e+00(+)	1.11e+02(+)	3.25e-01(+)	2.66e+01(=)	1.89e-01(+)	3.45e+01(=)	1.53e-01(+)	2.10e+02(+)	3.98e+00(+)	2.11e+02(+)
3	6.12e-08	9.14e-15	1.76e+00(+)	4.37e+00(+)	7.88e-03(+)	4.89e-04(+)	1.53e-01(+)	2.78e-03(+)	1.67e-01(+)	6.73e-03(+)	6.49e+00(+)	2.27e-02(+)
4	3.07e+01	1.17e-11	1.61e+02(+)	1.01e+01(+)	1.10e+02(+)	1.19e-01(+)	3.35e+01(=)	1.21e-02(+)	1.63e+02(+)	5.43e-03(+)	2.12e+02(+)	2.01e-02(+)
5	3.42e-05	2.10e-03	2.92e+00(+)	3.61e-01(+)	1.69e+00(+)	1.69e-02(+)	7.88e-02(+)	2.14e-03(=)	1.47e-01(+)	3.70e-04(=)	4.64e+00(+)	1.94e-03(=)
6	8.43e-03	1.09e+00	1.05e+01(+)	1.10e+01(+)	4.09e+00(+)	6.35e+00(+)	4.68e-02(+)	1.49e+00(=)	1.56e-01(+)	6.55e-01(-)	4.56e+00(+)	1.66e+00(+)
7	2.73e+01	2.63e-11	1.12e+02(+)	7.50e+00(+)	3.61e+01(+)	2.14e-02(+)	4.01e+01(+)	3.31e-02(+)	1.70e+02(+)	6.86e-03(+)	2.12e+02(+)	1.94e-02(+)
8	2.95e-03	1.29e+00	2.72e-01(+)	3.83e+00(+)	7.93e-03(+)	1.63e+00(=)	2.86e-03(=)	9.59e-01(=)	6.22e-03(+)	8.13e-01(=)	1.86e-03(=)	1.67e+00(=)
9	8.31e-15	4.43e-03	5.38e+00(+)	2.28e+00(+)	1.46e-02(+)	1.11e-01(+)	4.04e-03(+)	4.26e-02(+)	1.09e-01(+)	4.78e-02(+)	2.06e-02(+)	3.78e+00(+)
Number of +/=/-			9/0/0	9/0/0	8/1/0	7/2/0	7/2/0	5/4/0	9/0/0	6/2/1	7/2/0	7/2/0

TABLE S.VII.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=30 AND D_2=50

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	2.34e-03	6.23e+01	9.23e-02(+)	3.88e+02(+)	1.11e-02(+)	2.40e+02(+)	9.80e-03(+)	2.53e+02(+)	2.13e-06(=)	4.54e+02(+)	1.01e-05(=)	4.49e+02(+)
2	3.07e-07	6.16e+01	2.68e+00(+)	3.76e+02(+)	1.18e+00(+)	2.23e+02(+)	3.31e-01(+)	2.47e+02(+)	2.42e-03(+)	4.41e+02(+)	2.52e+00(+)	4.48e+02(+)
3	4.66e-02	7.24e-17	2.50e+00(+)	6.37e+01(+)	2.40e-02(-)	5.94e-01(+)	4.61e-01(+)	6.06e+01(+)	2.32e-03(-)	6.21e+00(+)	9.91e-01(+)	4.52e+00(+)
4	3.00e+01	2.00e-11	1.48e+02(+)	5.71e+01(+)	1.06e+02(+)	8.36e-01(+)	6.51e+01(+)	8.26e+01(+)	4.08e+01(=)	5.70e+00(+)	2.03e+02(+)	4.58e+00(+)
5	4.22e-07	4.53e-03	2.35e+00(+)	8.47e-01(+)	1.37e+00(+)	5.24e-02(+)	7.13e-01(+)	7.51e-01(+)	2.25e-03(+)	1.88e-01(+)	1.45e+00(+)	1.60e-01(+)
6	4.25e-07	4.84e+00	1.56e+01(+)	2.83e+01(+)	1.71e+01(+)	1.94e+01(+)	5.17e-01(+)	1.20e+01(+)	2.46e-03(+)	1.25e+01(+)	7.27e-01(+)	1.20e+01(+)
7	2.64e+01	8.89e-13	1.43e+02(+)	6.30e+01(+)	9.93e+01(+)	8.56e-01(+)	6.28e+01(+)	8.13e+01(+)	4.36e+01(+)	5.69e+00(+)	1.98e+02(+)	4.70e+00(+)
8	1.85e-03	4.84e+00	1.07e-01(+)	2.81e+01(+)	1.46e-02(+)	2.44e+01(+)	2.25e-02(+)	1.26e+01(+)	1.62e-06(=)	1.34e+01(+)	3.88e-04(=)	1.25e+01(+)
9	9.88e-18	2.73e-01	2.02e+00(+)	1.85e+01(+)	2.76e-02(+)	2.00e+01(+)	1.65e-01(+)	3.63e+00(+)	2.07e-05(+)	2.04e+01(+)	1.33e-04(+)	2.06e+01(+)
Number of +/=/-			9/0/0	9/0/0	8/0/1	9/0/0	9/0/0	9/0/0	5/3/1	9/0/0	7/2/0	9/0/0

TABLE S.VIII.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=40 AND D_2=20

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	8.63e-04	2.27e+01	9.02e-01(+)	8.31e+01(+)	3.59e-02(+)	3.03e+01(=)	9.85e-01(+)	4.02e+01(+)	9.24e-02(+)	1.08e+02(+)	1.09e-01(+)	1.00e+02(+)
2	7.35e-04	3.02e+01	1.92e+01(+)	1.06e+02(+)	2.00e+01(+)	5.94e+01(+)	4.68e+00(+)	3.96e+01(=)	1.96e+00(+)	1.05e+02(+)	1.78e+01(+)	1.03e+02(+)
3	3.84e-03	3.28e-15	1.92e+01(+)	6.21e-01(+)	1.99e+01(+)	1.05e-02(+)	4.50e+00(+)	6.97e-02(+)	1.88e+00(+)	2.22e-06(+)	1.65e+01(+)	1.04e-06(+)
4	4.63e+01	4.47e-12	3.10e+02(+)	4.82e-01(+)	1.91e+02(+)	2.11e-03(+)	2.74e+02(+)	3.26e-02(+)	2.79e+02(+)	1.89e-06(+)	3.44e+02(+)	9.24e-07(+)
5	9.46e-02	9.86e-04	2.01e+01(+)	8.17e-02(+)	1.91e+01(+)	1.51e-02(+)	4.52e+00(+)	2.98e-02(+)	1.87e+00(+)	3.71e-04(-)	1.97e+01(+)	4.91e-07(-)
6	7.67e-02	7.53e-01	2.00e+01(+)	8.24e+00(+)	2.00e+01(+)	4.79e+00(+)	4.98e+00(+)	2.20e+00(+)	2.03e+00(+)	9.05e-02(-)	1.72e+01(+)	2.35e-01(-)
7	5.10e+01	4.45e-11	3.00e+02(+)	5.44e-01(+)	1.84e+02(+)	7.50e-03(+)	2.55e+02(+)	3.56e-02(+)	2.92e+02(+)	1.57e-06(+)	3.44e+02(+)	1.02e-06(+)
8	3.20e-03	9.22e-01	8.82e-01(+)	2.97e+00(+)	3.87e-02(+)	2.51e+00(+)	1.00e+00(+)	2.14e+00(+)	1.02e-01(+)	6.89e-02(-)	1.32e-01(+)	1.64e-01(-)
9	5.73e-12	3.93e-02	5.47e+01(+)	1.89e+00(+)	7.17e-01(+)	7.21e-01(+)	1.05e+02(+)	3.55e-01(+)	2.30e+00(+)	1.04e-03(-)	2.71e+00(+)	5.41e-03(-)
Number of +/=/-			9/0/0	9/0/0	9/0/0	8/1/0	9/0/0	8/1/0	9/0/0	5/0/4	9/0/0	5/0/4

TABLE S.IX.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=40 AND D_2=30

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	2.34e-03	2.79e+01	6.64e-01(+)	1.39e+02(+)	2.82e-02(+)	8.76e+01(+)	6.24e-01(+)	5.93e+01(+)	2.70e-02(+)	2.07e+02(+)	2.37e-02(+)	2.01e+02(+)
2	3.29e-05	3.02e+01	9.82e+00(+)	1.40e+02(+)	8.44e+00(+)	9.40e+01(+)	3.24e+00(+)	5.70e+01(+)	9.19e-01(+)	2.11e+02(+)	1.36e+01(+)	2.06e+02(+)
3	5.83e-02	1.11e-15	8.99e+00(+)	2.87e+00(+)	9.96e+00(+)	4.35e-02(+)	2.56e+00(+)	1.35e-01(+)	8.60e-01(+)	2.65e-03(+)	1.73e+01(+)	1.68e-03(+)
4	4.81e+01	2.68e-13	2.92e+02(+)	4.03e+00(+)	1.74e+02(+)	3.01e-02(+)	1.55e+02(+)	5.43e-02(+)	2.64e+02(+)	1.69e-03(+)	3.23e+02(+)	1.71e-03(+)
5	7.39e-02	3.08e-03	6.43e+00(+)	1.76e-01(+)	1.29e+01(+)	1.02e-02(+)	2.56e+00(+)	1.35e-02(+)	1.14e+00(+)	1.94e-04(=)	1.51e+01(+)	1.36e-04(=)
6	1.25e-05	1.43e+00	1.92e+01(+)	1.24e+01(+)	1.82e+01(+)	7.06e+00(+)	2.75e+00(+)	2.71e+00(+)	8.61e-01(+)	8.21e-01(-)	1.69e+01(+)	8.44e-01(-)
7	5.05e+01	1.34e-11	2.82e+02(+)	2.91e+00(+)	1.84e+02(+)	1.52e-02(+)	1.57e+02(+)	6.43e-02(+)	2.62e+02(+)	1.85e-03(+)	3.28e+02(+)	1.51e-03(+)
8	2.09e-03	1.32e+00	6.34e-01(+)	4.15e+00(+)	2.29e-02(+)	2.59e+00(=)	4.94e-01(+)	3.10e+00(+)	2.61e-02(+)	6.00e-01(-)	2.53e-02(+)	7.78e-01(=)
9	3.22e-15	2.73e-06	3.03e+01(+)	2.39e+00(+)	2.20e-01(+)	6.56e-01(+)	1.55e+01(+)	3.07e-01(+)	5.91e-01(+)	2.13e-02(+)	5.15e-01(+)	1.95e+00(+)
Number of +/=/-			9/0/0	9/0/0	9/0/0	8/1/0	9/0/0	9/0/0	9/0/0	6/1/2	9/0/0	6/2/1

TABLE S.X.

THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=40 AND D_2=40

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	1.11e-03	4.29e+01	3.63e-01(+)	1.73e+02(+)	7.57e-03(+)	6.51e+01(+)	5.79e-03(+)	4.85e+01(=)	6.78e-03(+)	3.08e+02(+)	4.76e-03(+)	3.20e+02(+)
2	1.04e-05	3.88e+01	3.13e+00(+)	1.57e+02(+)	8.50e-01(+)	7.04e+01(+)	1.64e-01(+)	4.71e+01(+)	1.55e-01(+)	3.11e+02(+)	1.14e+01(+)	3.19e+02(+)
3	8.16e-02	1.93e-14	1.92e+00(+)	7.35e+00(+)	9.14e-03(-)	8.19e-04(+)	6.13e-02(-)	3.08e-03(+)	1.54e-01(+)	3.06e-02(+)	1.26e+01(+)	9.43e-02(+)
4	4.59e+01	8.45e-13	2.76e+02(+)	1.40e+01(+)	1.58e+02(+)	2.66e-01(+)	4.77e+01(=)	6.99e-03(+)	2.20e+02(+)	2.29e-02(+)	3.18e+02(+)	9.64e-02(+)
5	8.31e-02	1.72e-03	3.30e+00(+)	4.29e-01(+)	1.98e+00(+)	1.98e-02(+)	4.39e-02(-)	2.72e-03(+)	1.47e-01(+)	1.05e-03(=)	1.30e+01(+)	4.10e-03(+)
6	9.92e-02	2.76e+00	1.43e+01(+)	1.57e+01(+)	1.19e+01(+)	1.23e+01(+)	1.42e-01(+)	3.30e+00(=)	1.80e-01(+)	1.56e+00(-)	1.53e+01(+)	3.71e+00(+)
7	4.84e+01	4.56e-13	1.63e+02(+)	1.37e+01(+)	7.11e+01(=)	6.17e-02(+)	5.23e+01(=)	2.45e-02(+)	2.09e+02(+)	3.39e-02(+)	3.20e+02(+)	1.03e-01(+)
8	1.48e-03	2.17e+00	3.32e-01(+)	5.17e+00(+)	2.49e-03(+)	1.07e+00(-)	1.55e-03(=)	3.20e+00(=)	6.45e-03(+)	1.84e+00(=)	5.10e-03(+)	3.87e+00(+)
9	7.30e-15	1.61e-02	7.99e+00(+)	2.29e+00(+)	1.57e-02(+)	4.40e-02(+)	2.64e-03(+)	1.15e-01(+)	1.38e-01(+)	1.20e-01(+)	8.68e-02(+)	1.69e+01(+)
Number of +/=/-			9/0/0	9/0/0	7/1/1	8/0/1	4/3/2	6/3/0	9/0/0	6/2/1	9/0/0	9/0/0

TABLE S.XI.

THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=40 AND D_2=50

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	9.86e-04	6.17e+01	2.03e-01(+)	3.68e+02(+)	6.56e-03(+)	2.22e+02(+)	1.19e-02(+)	1.60e+02(+)	2.58e-04(-)	4.42e+02(+)	9.10e-04(-)	4.41e+02(+)
2	2.21e-05	6.96e+01	3.09e+00(+)	3.36e+02(+)	1.68e+00(+)	2.24e+02(+)	4.45e-01(+)	1.73e+02(+)	3.31e-02(+)	4.34e+02(+)	1.11e+01(+)	4.37e+02(+)
3	1.64e-01	1.23e-16	2.68e+00(+)	3.40e+01(+)	3.24e-02(=)	1.93e-01(+)	3.02e-01(+)	7.59e+00(+)	3.21e-02(=)	1.72e+00(+)	1.19e+01(+)	1.41e+00(+)
4	3.93e+01	2.00e-14	2.57e+02(+)	3.69e+01(+)	1.81e+02(+)	5.88e-01(+)	7.90e+01(+)	2.40e+01(+)	9.86e+01(+)	1.84e+00(+)	3.10e+02(+)	1.31e+00(+)
5	1.56e-02	1.11e-03	3.39e+00(+)	6.79e-01(+)	1.88e+00(+)	3.06e-02(+)	3.90e-01(+)	2.08e-01(+)	3.33e-02(+)	6.24e-02(+)	1.06e+01(+)	5.32e-02(+)
6	6.42e-07	4.83e+00	1.58e+01(+)	2.82e+01(+)	1.28e+01(+)	2.06e+01(+)	8.08e-01(+)	9.66e+00(+)	3.55e-02(+)	9.64e+00(+)	7.51e+00(+)	9.44e+00(+)
7	4.49e+01	1.42e-12	1.98e+02(+)	3.49e+01(+)	1.56e+02(+)	3.56e-01(+)	6.83e+01(+)	1.88e+01(+)	1.13e+02(+)	2.07e+00(+)	3.08e+02(+)	1.22e+00(+)
8	1.85e-03	3.89e+00	2.34e-01(+)	2.06e+01(+)	1.87e-02(+)	1.94e+01(+)	1.95e-02(+)	9.27e+00(+)	2.98e-04(=)	1.00e+01(+)	8.23e-04(=)	8.94e+00(+)
9	7.60e-18	3.29e-01	4.80e+00(+)	4.68e+00(+)	4.33e-02(+)	8.26e+00(+)	1.30e-01(+)	1.95e+00(+)	7.31e-03(+)	2.09e+01(+)	1.70e-02(+)	2.12e+01(+)
Number of +/=/-			9/0/0	9/0/0	8/1/0	9/0/0	9/0/0	9/0/0	6/2/1	9/0/0	7/1/1	9/0/0

TABLE S.XII.

THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=50 AND D_2=20

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	1.23e-03	1.76e+01	9.87e-01(+)	8.11e+01(+)	4.62e-02(+)	2.99e+01(+)	1.05e+00(+)	3.71e+01(+)	1.63e-01(+)	1.00e+02(+)	3.92e-01(+)	9.93e+01(+)
2	1.76e-04	2.20e+01	2.01e+01(+)	8.98e+01(+)	2.00e+01(+)	6.45e+01(+)	5.35e+00(+)	3.94e+01(+)	2.84e+00(+)	1.00e+02(+)	2.09e+01(+)	9.79e+01(+)
3	2.22e-01	3.87e-18	2.01e+01(+)	4.95e-01(+)	2.00e+01(+)	1.93e-03(+)	5.43e+00(+)	4.91e-02(+)	2.65e+00(+)	4.72e-08(+)	2.04e+01(+)	1.92e-08(+)
4	5.33e+01	4.13e-15	3.86e+02(+)	3.40e-01(+)	2.21e+02(+)	1.42e-03(+)	3.90e+02(+)	1.91e-02(+)	3.78e+02(+)	4.42e-08(+)	4.60e+02(+)	1.95e-08(+)
5	1.76e-01	4.68e-03	2.01e+01(+)	7.12e-02(+)	2.00e+01(+)	2.86e-02(+)	5.49e+00(+)	2.68e-02(+)	2.63e+00(+)	1.47e-08(=)	2.03e+01(+)	9.56e-09(=)
6	1.21e-01	8.16e-01	2.01e+01(+)	8.21e+00(+)	2.00e+01(+)	5.45e+00(+)	5.65e+00(+)	3.54e+00(+)	2.63e+00(+)	3.54e-02(-)	1.91e+01(+)	2.44e-02(-)
7	6.45e+01	2.42e-15	4.01e+02(+)	3.52e-01(+)	2.17e+02(+)	3.10e-03(+)	3.81e+02(+)	2.23e-02(+)	3.98e+02(+)	4.72e-08(+)	4.55e+02(+)	1.86e-08(+)
8	2.71e-03	5.88e-01	9.59e-01(+)	2.90e+00(+)	6.02e-02(+)	1.11e+00(=)	1.05e+00(+)	2.05e+00(+)	1.35e-01(+)	3.15e-02(-)	4.14e-01(+)	9.40e-02(-)
9	1.15e-14	3.63e-08	1.15e+02(+)	2.11e+00(+)	1.65e+00(+)	6.30e-01(+)	2.25e+02(+)	4.19e-01(+)	5.26e+00(+)	2.24e-04(+)	1.62e+01(+)	5.38e-04(+)
Number of +/=/-			9/0/0	9/0/0	9/0/0	8/1/0	9/0/0	9/0/0	9/0/0	6/1/2	9/0/0	6/1/2

TABLE S.XIII.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=50 AND D_2=30

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	1.97e-03	3.06e+01	8.24e-01(+)	1.35e+02(+)	3.14e-02(+)	7.86e+01(+)	8.75e-01(+)	5.43e+01(+)	5.66e-02(+)	1.97e+02(+)	1.36e-01(+)	1.98e+02(+)
2	4.72e-06	2.58e+01	1.84e+01(+)	1.44e+02(+)	2.00e+01(+)	9.95e+01(+)	4.03e+00(+)	6.04e+01(+)	1.93e+00(+)	2.03e+02(+)	2.08e+01(+)	1.99e+02(+)
3	1.00e-01	1.97e-18	2.00e+01(+)	2.17e+00(+)	1.90e+01(+)	2.05e-02(+)	3.67e+00(+)	1.23e-01(+)	1.75e+00(+)	2.25e-04(+)	2.01e+01(+)	1.33e-04(+)
4	5.80e+01	3.14e-15	3.39e+02(+)	1.69e+00(+)	2.15e+02(+)	2.64e-02(+)	2.57e+02(+)	4.50e-02(+)	3.37e+02(+)	2.06e-04(+)	4.47e+02(+)	1.50e-04(+)
5	3.83e-01	2.21e-03	1.99e+01(+)	1.00e-01(+)	1.64e+01(+)	9.17e-03(+)	3.78e+00(+)	1.64e-02(+)	1.68e+00(+)	1.85e-05(=)	1.97e+01(+)	1.33e-05(=)
6	6.77e-03	1.18e+00	2.00e+01(+)	1.27e+01(+)	2.00e+01(+)	7.62e+00(+)	4.00e+00(+)	3.30e+00(+)	1.58e+00(+)	4.64e-01(-)	1.98e+01(+)	4.01e-01(-)
7	5.22e+01	1.03e-14	3.38e+02(+)	1.45e+00(+)	2.27e+02(+)	1.11e-02(+)	2.57e+02(+)	4.31e-02(+)	3.54e+02(+)	2.21e-04(+)	4.46e+02(+)	1.15e-04(+)
8	3.81e-03	9.43e-01	8.19e-01(+)	4.31e+00(+)	4.05e-02(+)	3.53e+00(+)	8.98e-01(+)	2.94e+00(+)	5.11e-02(+)	3.86e-01(-)	1.50e-01(+)	4.29e-01(-)
9	8.63e-15	5.10e-03	6.00e+01(+)	2.43e+00(+)	7.74e-01(+)	5.24e-01(+)	5.23e+01(+)	3.14e-01(+)	1.74e+00(+)	8.70e-03(+)	4.78e+00(+)	1.30e+00(+)
Number of +/=/-			9/0/0	9/0/0	9/0/0	9/0/0	9/0/0	9/0/0	9/0/0	6/1/2	9/0/0	6/1/2

TABLE S.XIV.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=50 AND D_2=40

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	3.70e-04	4.44e+01	6.53e-01(+)	1.84e+02(+)	2.36e-02(+)	1.13e+02(+)	4.59e-01(+)	7.61e+01(+)	2.22e-02(+)	3.09e+02(+)	4.28e-02(+)	3.06e+02(+)
2	4.29e-06	3.54e+01	4.62e+00(+)	1.89e+02(+)	6.43e+00(+)	1.26e+02(+)	2.98e+00(+)	7.85e+01(+)	8.32e-01(+)	3.02e+02(+)	1.94e+01(+)	3.07e+02(+)
3	3.23e-01	1.46e-16	5.80e+00(+)	5.43e+00(+)	8.06e+00(+)	2.90e-02(+)	2.06e+00(+)	1.35e-01(+)	9.44e-01(+)	2.64e-02(+)	2.04e+01(+)	1.60e-02(+)
4	5.89e+01	2.26e-15	3.29e+02(+)	6.81e+00(+)	2.11e+02(+)	1.22e-01(+)	1.73e+02(+)	9.75e-02(+)	2.92e+02(+)	1.69e-02(+)	4.39e+02(+)	1.60e-02(+)
5	1.99e-01	1.23e-03	5.88e+00(+)	2.29e-01(+)	5.58e+00(+)	8.09e-03(+)	2.29e+00(+)	9.95e-03(+)	8.90e-01(+)	1.17e-03(=)	1.81e+01(+)	8.18e-04(=)
6	5.14e-02	2.77e+00	2.00e+01(+)	1.83e+01(+)	1.63e+01(+)	1.34e+01(+)	2.50e+00(+)	6.10e+00(+)	7.91e-01(+)	2.51e+00(=)	1.97e+01(+)	2.45e+00(=)
7	6.50e+01	8.82e-14	3.20e+02(+)	5.77e+00(+)	2.01e+02(+)	2.86e-02(+)	1.69e+02(+)	9.63e-02(+)	3.15e+02(+)	1.85e-02(+)	4.46e+02(+)	1.63e-02(+)
8	1.48e-03	2.41e+00	6.52e-01(+)	5.40e+00(+)	2.33e-02(+)	3.37e+00(=)	3.76e-01(+)	4.45e+00(+)	2.20e-02(+)	1.56e+00(-)	4.53e-02(+)	2.46e+00(=)
9	4.24e-16	1.31e-01	3.33e+01(+)	2.75e+00(+)	2.28e-01(+)	6.22e-01(+)	1.04e+01(+)	4.52e-01(+)	6.58e-01(+)	7.26e-02(=)	1.38e+00(+)	1.25e+01(+)
Number of +/=/-			9/0/0	9/0/0	9/0/0	8/1/0	9/0/0	9/0/0	9/0/0	5/3/1	9/0/0	6/3/0

TABLE S.XV.
THE EXPERIMENTAL RESULTS ON THE PROPOSED BENCHMARK PROBLEMS WITH D_1=50 AND D_2=50

Algorithm	OTMTO		MFEA		MFEA2		MTGA		EMTEA		SODE	
Problem	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	0.00e+00	6.87e+01	3.73e-01(+)	2.27e+02(+)	7.07e-03(+)	7.63e+01(=)	6.08e-03(+)	7.07e+01(=)	6.38e-03(+)	4.22e+02(+)	1.16e-02(+)	4.25e+02(+)
2	5.14e-02	6.32e+01	3.10e+00(+)	2.37e+02(+)	9.76e-01(+)	8.45e+01(=)	4.13e-01(+)	7.31e+01(=)	1.82e-01(+)	4.24e+02(+)	1.91e+01(+)	4.19e+02(+)
3	2.61e-01	7.62e-18	2.00e+00(+)	9.80e+00(+)	8.72e-03(=)	9.65e-04(+)	2.38e-01(=)	4.43e-03(+)	1.86e-01(=)	2.09e-01(+)	1.97e+01(+)	3.88e-01(+)
4	5.76e+01	7.79e-16	2.86e+02(+)	1.94e+01(+)	1.84e+02(+)	3.64e-01(+)	6.85e+01(=)	6.62e-03(+)	2.39e+02(+)	1.67e-01(+)	4.28e+02(+)	3.84e-01(+)
5	3.13e-01	2.95e-03	3.09e+00(+)	4.75e-01(+)	1.73e+00(+)	1.65e-02(+)	1.54e-01(=)	1.43e-03(=)	1.85e-01(=)	4.41e-03(=)	1.62e+01(+)	1.74e-02(+)
6	5.53e-02	5.00e+00	1.91e+01(+)	2.84e+01(+)	1.46e+01(+)	2.09e+01(+)	2.30e-01(+)	5.52e+00(=)	1.94e-01(+)	3.44e+00(-)	1.82e+01(+)	6.86e+00(+)
7	5.83e+01	7.04e-15	2.00e+02(+)	1.72e+01(+)	1.11e+02(+)	1.17e-01(+)	7.16e+01(+)	8.89e-03(+)	1.93e+02(+)	2.08e-01(+)	4.18e+02(+)	3.75e-01(+)
8	2.70e-03	4.56e+00	3.17e-01(+)	6.44e+00(+)	7.01e-03(+)	1.84e+00(-)	4.75e-03(+)	4.99e+00(=)	7.25e-03(+)	3.68e+00(=)	1.24e-02(+)	6.65e+00(+)
9	3.40e-18	1.69e-01	1.12e+01(+)	2.45e+00(+)	1.86e-02(+)	4.34e-02(=)	4.83e-03(+)	8.46e-02(=)	1.80e-01(+)	2.81e-01(=)	3.95e-01(+)	1.80e+01(+)
Number of +/=/-			9/0/0	9/0/0	8/1/0	5/3/1	6/3/0	3/6/0	7/2/0	5/3/1	9/0/0	9/0/0

TABLE S.XVI.
THE NUMBERS OF OBTAINED '+', '=', AND '-' WHEN COMPARING THE OTMTO ALGORITHM WITH OTHER ALGORITHMS ON THE PROPOSED MTOP-DD BENCHMARK PROBLEMS

Algorithm	Task	D_1=20,D_2=20	D_1=20,D_2=30	D_1=20,D_2=40	D_1=20,D_2=50
MFEA	1	9/0/0	9/0/0	9/0/0	9/0/0
	2	9/0/0	9/0/0	9/0/0	9/0/0
MFEA2	1	6/2/1	9/0/0	9/0/0	9/0/0
	2	7/0/2	9/0/0	9/0/0	9/0/0
MTGA	1	6/1/2	7/2/0	8/1/0	9/0/0
	2	5/2/2	9/0/0	9/0/0	9/0/0
EMTEA	1	8/1/0	6/3/0	5/3/1	5/4/0
	2	6/3/0	9/0/0	9/0/0	9/0/0
SODE	1	7/2/0	7/2/0	7/2/0	7/2/0
	2	7/2/0	9/0/0	9/0/0	9/0/0
Algorithm	Task	D_1=30,D_2=20	D_1=30,D_2=30	D_1=30,D_2=40	D_1=30,D_2=50
MFEA	1	9/0/0	9/0/0	9/0/0	9/0/0
	2	9/0/0	9/0/0	9/0/0	9/0/0
MFEA2	1	9/0/0	8/1/0	9/0/0	8/0/1
	2	8/1/0	7/2/0	9/0/0	9/0/0
MTGA	1	9/0/0	7/2/0	9/0/0	9/0/0
	2	7/2/0	5/4/0	9/0/0	9/0/0
EMTEA	1	9/0/0	9/0/0	6/0/3	5/3/1
	2	6/1/2	6/2/1	9/0/0	9/0/0
SODE	1	9/0/0	7/2/0	8/0/1	7/2/0
	2	6/3/0	7/2/0	9/0/0	9/0/0
Algorithm	Task	D_1=40,D_2=20	D_1=40,D_2=30	D_1=40,D_2=40	D_1=40,D_2=50
MFEA	1	9/0/0	9/0/0	9/0/0	9/0/0
	2	9/0/0	9/0/0	9/0/0	9/0/0
MFEA2	1	9/0/0	9/0/0	7/1/1	8/1/0
	2	8/1/0	8/1/0	8/0/1	9/0/0
MTGA	1	9/0/0	9/0/0	4/3/2	9/0/0
	2	8/1/0	9/0/0	6/3/0	9/0/0
EMTEA	1	9/0/0	9/0/0	9/0/0	6/2/1
	2	5/0/4	6/1/2	6/2/1	9/0/0
SODE	1	9/0/0	9/0/0	9/0/0	7/1/1
	2	5/0/4	6/2/1	9/0/0	9/0/0
Algorithm	Task	D_1=50,D_2=20	D_1=50,D_2=30	D_1=50,D_2=40	D_1=50,D_2=50
MFEA	1	9/0/0	9/0/0	9/0/0	9/0/0
	2	9/0/0	9/0/0	9/0/0	9/0/0
MFEA2	1	9/0/0	9/0/0	9/0/0	8/1/0
	2	8/1/0	9/0/0	8/1/0	5/3/1
MTGA	1	9/0/0	9/0/0	9/0/0	6/3/0
	2	9/0/0	9/0/0	9/0/0	3/6/0
EMTEA	1	9/0/0	9/0/0	9/0/0	7/2/0
	2	6/1/2	6/1/2	5/3/1	5/3/1
SODE	1	9/0/0	9/0/0	9/0/0	9/0/0
	2	6/1/2	6/1/2	6/3/0	9/0/0

C. Quantitative Analysis on the Performance of the EMT Algorithms

According to the CEC17 benchmark, we use the performance metric in [52] to quantitatively evaluate the performance of the EMT algorithms. Say we have N stochastic algorithms, A_1, A_2;…, A_N for a specific test problem with K minimization tasks T_1,T_2,…,T_K, and each algorithm is run for L times.

Suppose $I(i, j)_l$ denotes the best obtained result on the lth run by Algorithm A_i on the task T_j. Next, let μ_j and σ_j be the mean and the standard deviation with respect to task T_j over all the runs of all algorithms. Thereafter, consider the normalized performance $I'(:, j)_l = (I(:, j)_l - \mu_j) / \sigma_j$. This normalization procedure is repeated for all tasks on all the runs. Based on the above, for each algorithm A_i, its final performance score is given as $score_i = \sum_{j=1}^{K} \sum_{l=1}^{L} I'(i, j)_l$. A smaller score suggests that the corresponding algorithm has a superior overall performance over all tasks in the multitasking environment.

The results of the scores are reported in Table S.I and Table S.II. The result of the best algorithm of a problem is marked in **boldface**. It can be seen that OTMTO ranks first on most of the problems in CEC17 benchmark while ranks first on all problems in the proposed MTOP-DD benchmark. This indicates that the proposed methods are not only useful on traditional MTOPs but also offer great advantages on MTOP-DDs.

TABLE S.XVII.
THE PERFORMANCE SCORES OF EMT ALGORITHMS ON THE CEC17 BENCHMARK PROBLEMS

Problem	OTMTO	MFEA	MFEA2	MTGA	EMTEA	SODE
1	**-31.041**	53.346	-16.670	-20.926	-30.523	45.814
2	**-30.375**	21.233	-17.648	-19.562	-30.257	76.609
3	-4.340	-39.466	**-46.929**	8.671	40.335	41.728
4	**-34.531**	71.739	-3.039	-33.432	-2.695	1.957
5	-26.575	28.750	-19.488	-15.786	**-26.898**	59.997
6	**-40.480**	45.080	46.554	-10.933	-40.461	0.240
7	0.290	43.380	-13.312	-29.816	**-39.872**	39.330
8	**-33.553**	58.901	7.052	-27.720	-30.672	25.993
9	**-27.467**	21.160	-24.139	-21.812	23.239	29.019

TABLE S.XVIII.
THE PERFORMANCE SCORES OF EMT ALGORITHMS ON THE PROPOSED BENCHMARK PROBLEM WITH D_1=30 AND D_2=40

Problem	OTMTO	MFEA	MFEA2	MTGA	EMTEA	SODE
1	**-41.670**	46.326	-18.046	-11.518	12.177	12.731
2	**-44.838**	26.767	-8.044	-12.184	10.581	27.718
3	**-19.497**	49.243	-18.856	0.016	-18.010	7.104
4	**-35.550**	52.925	-15.678	5.185	-23.561	16.680
5	**-24.397**	44.801	-11.960	8.578	-22.288	5.265
6	**-31.733**	55.870	30.735	-15.998	-22.906	-15.967
7	**-35.957**	36.610	-14.481	8.978	-17.164	22.014
8	**-30.621**	74.794	6.062	-11.551	-18.861	-19.822
9	**-31.183**	34.383	5.379	-23.829	6.678	8.572

[52] B. Da *et al.*, "Evolutionary multitasking for single-objective continuous optimization: Benchmark problems, performance metric, and baseline results," Nanyang Technol. Univ., Singapore, Rep., 2016. [Online]. Available: http://www.cil.ntu.edu.sg/mfo/download.html

D. Results of EMT Algorithms on Three-Task Optimization Problem

TABLE S.XIX.
PROBLEM SETTING AND EXPERIMENTAL RESULTS OF THE OTMTO AND COMPETING ALGORITHMS ON THE THREE-TASK OPTIMIZATION PROBLEMS

Problem	Task	Function	Dimensionality	Optimum Similarity	OTMTO	MFEA	SODE
1	1	Griewank	50	High	2.96e-03	1.03e+00(+)	1.31e-02(+)
	2	Ackley	50		8.33e-01	4.80e+00(+)	1.99e+01(+)
	3	Rastrigin	50		8.78e-01	4.77e+00(+)	2.11e+01(+)
2	1	Griewank	30	High	2.95e-03	6.56e-01(+)	1.07e-05(=)
	2	Ackley	40		4.99e-01	1.06e+01(+)	1.49e+01(+)
	3	Rastrigin	50		8.70e-01	1.32e+01(+)	2.01e+01(+)
3	1	Griewank	50	Medium	0.00e+00	1.07e+00(+)	1.28e-02(+)
	2	Ackley	50		8.78e-01	5.72e+00(+)	1.76e+01(+)
	3	Rastrigin	50		8.81e-01	6.23e+00(+)	1.92e+01(+)
4	1	Griewank	30	Medium	1.72e-03	7.63e-01(+)	1.12e-05(=)
	2	Ackley	40		4.30e-01	9.16e+00(+)	1.51e+01(+)
	3	Rastrigin	50		6.59e-01	1.57e+01(+)	2.06e+01(+)
5	1	Griewank	50	Low	1.32e-03	1.07e+00(+)	1.19e-02(+)
	2	Ackley	50		1.16e+00	6.30e+00(+)	2.01e+01(+)
	3	Rastrigin	50		1.05e+00	6.98e+00(+)	1.79e+01(+)
6	1	Griewank	30	Low	2.71e-03	7.45e-01(+)	1.05e-05(=)
	2	Ackley	40		8.05e-01	9.95e+00(+)	1.18e+01(+)
	3	Rastrigin	50		8.91e-01	1.84e+01(+)	2.06e+01(+)
+/=/-						18/0/0	15/3/0

E. Running Time of the EMT Algorithms on Double Pole Balancing Problem

TABLE S.XX.
RUNNING TIME (SECONDS) OF THE OTMTO AND COMPETING ALGORITHMS NEEDED TO OBTAIN THE OPTIMAL POLICY ON THE DOUBLE POLE BALANCING PROBLEMS

Algorithm	DPB1 (T1, T2)	DPB2 (T1, T3)	DPB3 (T2, T3)	DPB4 (T2, T4)	DPB5 (T3, T4)	DPB6 (T3, T5)
OTMTO	1275±298	1185±350	1333±398	1371±432	1339±387	1426±591
SODE	1317±523	1283±364	1389±586	1415±655	1491±610	1486±652
EMTEA	1834±784	1644±646	2069±1512	1864±715	1648±599	1783±977
MTGA	1505±418	1498±369	1607±501	1685±704	1583±447	1574±377