Article

Supplementary Material for “Auxiliary Population Multitask Optimization based on Chinese Semantic Understanding”

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**Table S.1.** The comparison between APMTO and all the 12 algorithms on CEC2022

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| CEC2022 | | | | APMTO | MFEA | MFPSO | | MFDE | MFEA-II | | MFEA-AKT | | MFEA-DGD | |
| Benchmark1 | | T1 | | 6.28e+02 7.45e+00 - | **6.52e+02(+) 7.45e+00 0.0000** | 6.32e+02(+) 7.45e+00 0.0001 | | 6.06e+02(-) 7.45e+00 0.0000 | 6.16e+02(-) 7.45e+00 0.0000 | | 6.27e+02(=) 7.45e+00 0.9000 | | 6.17e+02(-) 7.45e+00 0.0000 | |
| T2 | | 6.26e+02 7.02e+00 - | **6.51e+02(+) 7.02e+00 0.0000** | 6.32e+02(+) 7.02e+00 0.0000 | | 6.06e+02(-) 7.02e+00 0.0000 | 6.17e+02(-) 7.02e+00 0.0000 | | 6.27e+02(=) 7.02e+00 0.0657 | | 6.21e+02(=) 7.02e+00 0.0877 | |
| Benchmark2 | | T1 | | 7.00e+02 9.10e−03 - | 7.01e+02(+) 9.10e-03 0.0000 | 7.13e+02(+) 9.10e-03 0.0000 | | **7.00e+02(=) 9.10e-03 0.5995** | 7.00e+02(+) 9.10e-03 0.0000 | | 7.01e+02(+) 9.10e-03 0.0000 | | 7.05e+02(+) 9.10e-03 0.0000 | |
| T2 | | **7.00e+02 9.28e−03 -** | 7.01e+02(+) 9.28e-03 0.0000 | 7.16e+02(+) 9.28e-03 0.0000 | | 7.00e+02(+) 9.28e-03 0.0061 | 7.00e+02(+) 9.28e-03 0.0000 | | 7.01e+02(+) 9.28e-03 0.0000 | | 7.14e+02(+) 9.28e-03 0.0000 | |
| Benchmark3 | | T1 | | **1.37e+05 6.81e+04 -** | 4.69e+06(+) 6.81e+04 0.0000 | 5.32e+06(+) 6.81e+04 0.0000 | | 5.75e+06(+) 6.81e+04 0.0000 | 2.63e+06(+) 6.81e+04 0.0000 | | 1.32e+06(+) 6.81e+04 0.0000 | | 3.13e+05(+) 6.81e+04 0.0169 | |
| T2 | | **1.58e+05 1.15e+05 -** | 4.41e+06(+) 1.15e+05 0.0000 | 5.51e+06(+) 1.15e+05 0.0000 | | 5.78e+06(+) 1.15e+05 0.0000 | 2.81e+06(+) 1.15e+05 0.0000 | | 1.65e+06(+) 1.15e+05 0.0000 | | 4.28e+06(+) 1.15e+05 0.0000 | |
| Benchmark4 | | T1 | | 1.30e+03 1.33e−01 - | **1.30e+03(-) 1.33e-01 0.0203** | 1.30e+03(+) 1.33e-01 0.0000 | | 1.30e+03(-) 1.33e-01 0.0028 | 1.30e+03(-) 1.33e-01 0.0001 | | 1.30e+03(-) 1.33e-01 0.0002 | | 1.30e+03(=) 1.33e-01 0.8883 | |
| T2 | | 1.30e+03 7.62e−02 - | **1.30e+03(-) 7.62e-02 0.0003** | 1.30e+03(+) 7.62e-02 0.0000 | | 1.30e+03(=) 7.62e-02 0.9234 | 1.30e+03(-) 7.62e-02 0.0004 | | 1.30e+03(-) 7.62e-02 0.0000 | | 1.30e+03(+) 7.62e-02 0.0000 | |
| Benchmark5 | | T1 | | 1.53e+03 1.00e+01 - | **1.57e+03(+) 1.00e+01 0.0000** | 9.60e+03(+) 1.00e+01 0.0000 | | 1.53e+03(+) 1.00e+01 0.0029 | 1.52e+03(-) 1.00e+01 0.0007 | | 1.56e+03(+) 1.00e+01 0.0000 | | 1.27e+04(+) 1.00e+01 0.0000 | |
| T2 | | 1.53e+03 1.02e+01 - | **1.57e+03(+) 1.02e+01 0.0000** | 1.42e+04(+) 1.02e+01 0.0000 | | 1.53e+03(+) 1.02e+01 0.0000 | 1.52e+03(-) 1.02e+01 0.0103 | | 1.56e+03(+) 1.02e+01 0.0000 | | 5.01e+04(+) 1.02e+01 0.0000 | |
| Benchmark6 | | T1 | | **1.05e+05 6.78e+04 -** | 2.54e+06(+) 6.78e+04 0.0000 | 1.38e+07(+) 6.78e+04 0.0000 | | 2.40e+06(+) 6.78e+04 0.0000 | 1.47e+06(+) 6.78e+04 0.0000 | | 2.11e+06(+) 6.78e+04 0.0000 | | 8.92e+06(+) 6.78e+04 0.0000 | |
| T2 | | **1.04e+05 5.77e+04 -** | 2.33e+06(+) 5.77e+04 0.0000 | 1.30e+07(+) 5.77e+04 0.0000 | | 2.08e+06(+) 5.77e+04 0.0000 | 1.55e+06(+) 5.77e+04 0.0000 | | 1.95e+06(+) 5.77e+04 0.0000 | | 4.09e+07(+) 5.77e+04 0.0000 | |
| Benchmark7 | | T1 | | 2.90e+03 3.06e+02 - | 3.40e+03(+) 3.06e+02 0.0000 | 3.43e+03(+) 3.06e+02 0.0000 | | 3.86e+03(+) 3.06e+02 0.0000 | 2.98e+03(=) 3.06e+02 0.2973 | | 3.35e+03(+) 3.06e+02 0.0000 | | 3.86e+03(+) 3.06e+02 0.0000 | |
| T2 | | 2.98e+03 3.28e+02 - | 3.38e+03(+) 3.28e+02 0.0008 | 3.62e+03(+) 3.28e+02 0.0000 | | 3.92e+03(+) 3.28e+02 0.0000 | 2.98e+03(=) 3.28e+02 0.9117 | | 3.38e+03(+) 3.28e+02 0.0000 | | 3.65e+03(+) 3.28e+02 0.0010 | |
| Benchmark8 | | T1 | | 5.21e+02 4.26e−02 - | **5.21e+02(-) 4.26e-02 0.0000** | 5.21e+02(=) 4.26e-02 0.4290 | | 5.21e+02(+) 4.26e-02 0.0242 | 5.21e+02(=) 4.26e-02 0.1297 | | 5.21e+02(-) 4.26e-02 0.0000 | | 5.21e+02(-) 4.26e-02 0.0000 | |
| T2 | | 5.21e+02 3.63e−02 - | **5.21e+02(-) 3.63e-02 0.0000** | 5.21e+02(-) 3.63e-02 0.0061 | | 5.21e+02(=) 3.63e-02 0.2340 | 5.21e+02(=) 3.63e-02 0.5793 | | 5.21e+02(-) 3.63e-02 0.0000 | | 5.21e+02(-) 3.63e-02 0.0000 | |
| Benchmark9 | | T1 | | 1.20e+04 3.96e+03 - | **8.39e+03(-) 3.96e+03 0.0061** | 1.28e+04(=) 3.96e+03 0.0933 | | 1.50e+04(+) 3.96e+03 0.0108 | 1.28e+04(=) 3.96e+03 0.6204 | | 8.41e+03(-) 3.96e+03 0.0040 | | 1.58e+04(+) 3.96e+03 0.0000 | |
| T2 | | 1.62e+03 4.34e−01 - | **1.62e+03(=) 4.34e-01 0.4918** | 1.62e+03(=) 4.34e-01 0.5793 | | 1.62e+03(+) 4.34e-01 0.0000 | 1.62e+03(+) 4.34e-01 0.0000 | | 1.62e+03(=) 4.34e-01 0.4035 | | 1.62e+03(-) 4.34e-01 0.0056 | |
| Benchmark10 | | T1 | | **8.56e+03 3.19e+03 -** | 4.80e+04(+) 3.19e+03 0.0000 | 5.90e+04(+) 3.19e+03 0.0000 | | 3.04e+04(+) 3.19e+03 0.0000 | 3.07e+04(+) 3.19e+03 0.0000 | | 4.16e+04(+) 3.19e+03 0.0000 | | 5.24e+04(+) 3.19e+03 0.0000 | |
| T2 | | **1.24e+05 6.51e+04 -** | 4.16e+06(+) 6.51e+04 0.0000 | 1.12e+07(+) 6.51e+04 0.0000 | | 2.54e+06(+) 6.51e+04 0.0000 | 2.67e+06(+) 6.51e+04 0.0000 | | 2.92e+06(+) 6.51e+04 0.0000 | | 6.71e+07(+) 6.51e+04 0.0000 | |
| Number of “+/=/−“ | | | | | 14/1/5 | 16/3/1 | | 14/3/3 | 9/5/6 | | 12/3/5 | | 14/2/4 | |
| CEC2022 | | | APMTO | | AEMTO | MTGA | MKTDE | | | OTMTO | | BLKT | | - |
| Benchmark1 | T1 | | 6.28e+02 7.45e+00 - | | 6.21e+02(-) 7.45e+00 0.0000 | 6.18e+02(-) 7.45e+00 0.0000 | 6.09e+02(-) 7.45e+00 0.0000 | | | 6.09e+02(-) 7.45e+00 0.0000 | | 6.17e+02(-) 7.45e+00 0.0000 | |  |
| T2 | | 6.26e+02 7.02e+00 - | | 6.21e+02(-) 7.02e+00 0.0000 | 6.17e+02(-) 7.02e+00 0.0000 | 6.08e+02(-) 7.02e+00 0.0000 | | | 6.09e+02(-) 7.02e+00 0.0000 | | 6.15e+02(-) 7.02e+00 0.0000 | |  |
| Benchmark2 | T1 | | 7.00e+02 9.10e−03 - | | 7.01e+02(+) 9.10e-03 0.0000 | 7.00e+02(+) 9.10e-03 0.0000 | 7.00e+02(+) 9.10e-03 0.0000 | | | 7.00e+02(=) 9.10e-03 0.4288 | | 7.00e+02(+) 9.10e-03 0.0000 | |  |
| T2 | | **7.00e+02 9.28e−03 -** | | 7.01e+02(+) 9.28e-03 0.0000 | 7.00e+02(+) 9.28e-03 0.0000 | 7.00e+02(+) 9.28e-03 0.0000 | | | 7.00e+02(+) 9.28e-03 0.0063 | | 7.00e+02(+) 9.28e-03 0.0000 | |  |
| Benchmark3 | T1 | | **1.37e+05 6.81e+04 -** | | 2.03e+07(+) 6.81e+04 0.0000 | 2.09e+06(+) 6.81e+04 0.0000 | 8.27e+06(+) 6.81e+04 0.0000 | | | 5.10e+07(+) 6.81e+04 0.0000 | | 3.31e+06(+) 6.81e+04 0.0000 | |  |
| T2 | | **1.58e+05 1.15e+05 -** | | 2.13e+07(+) 1.15e+05 0.0000 | 1.86e+06(+) 1.15e+05 0.0000 | 8.44e+06(+) 1.15e+05 0.0000 | | | 4.89e+07(+) 1.15e+05 0.0000 | | 4.16e+06(+) 1.15e+05 0.0000 | |  |
| Benchmark4 | T1 | | 1.30e+03 1.33e−01 - | | 1.30e+03(=) 1.33e-01 0.0724 | 1.30e+03(-) 1.33e-01 0.0000 | 1.30e+03(=) 1.33e-01 0.9705 | | | 1.30e+03(-) 1.33e-01 0.0004 | | 1.30e+03(=) 1.33e-01 0.6952 | |  |
| T2 | | 1.30e+03 7.62e−02 - | | 1.30e+03(+) 7.62e-02 0.0000 | 1.30e+03(-) 7.62e-02 0.0000 | 1.30e+03(+) 7.62e-02 0.0226 | | | 1.30e+03(=) 7.62e-02 0.5201 | | 1.30e+03(+) 7.62e-02 0.0141 | |  |
| Benchmark5 | T1 | | 1.53e+03 1.00e+01 - | | 1.54e+03(+) 1.00e+01 0.0000 | 1.51e+03(-) 1.00e+01 0.0000 | 1.54e+03(+) 1.00e+01 0.0004 | | | 1.53e+03(+) 1.00e+01 0.0019 | | 1.54e+03(+) 1.00e+01 0.0002 | |  |
| T2 | | 1.53e+03 1.02e+01 - | | 1.54e+03(+) 1.02e+01 0.0000 | 1.51e+03(-) 1.02e+01 0.0000 | 1.54e+03(+) 1.02e+01 0.0000 | | | 1.53e+03(+) 1.02e+01 0.0000 | | 1.54e+03(+) 1.02e+01 0.0000 | |  |
| Benchmark6 | T1 | | **1.05e+05 6.78e+04 -** | | 8.75e+06(+) 6.78e+04 0.0000 | 1.20e+06(+) 6.78e+04 0.0000 | 2.13e+07(+) 6.78e+04 0.0000 | | | 2.27e+07(+) 6.78e+04 0.0000 | | 1.96e+06(+) 6.78e+04 0.0000 | |  |
| T2 | | **1.04e+05 5.77e+04 -** | | 9.23e+06(+) 5.77e+04 0.0000 | 9.40e+05(+) 5.77e+04 0.0000 | 2.11e+07(+) 5.77e+04 0.0000 | | | 2.09e+07(+) 5.77e+04 0.0000 | | 1.79e+06(+) 5.77e+04 0.0000 | |  |
| Benchmark7 | T1 | | 2.90e+03 3.06e+02 - | | 4.09e+03(+) 3.06e+02 0.0000 | **2.87e+03(=) 3.06e+02 0.6843** | 4.25e+03(+) 3.06e+02 0.0000 | | | 4.33e+03(+) 3.06e+02 0.0000 | | 2.95e+03(=) 3.06e+02 0.4825 | |  |
| T2 | | 2.98e+03 3.28e+02 - | | 4.13e+03(+) 3.28e+02 0.0000 | **2.91e+03(=) 3.28e+02 0.3112** | 4.24e+03(+) 3.28e+02 0.0000 | | | 4.28e+03(+) 3.28e+02 0.0000 | | 3.09e+03(=) 3.28e+02 0.1958 | |  |
| Benchmark8 | T1 | | 5.21e+02 4.26e−02 - | | 5.21e+02(=) 4.26e-02 0.1761 | 5.21e+02(+) 4.26e-02 0.0069 | 5.21e+02(=) 4.26e-02 0.0877 | | | 5.21e+02(+) 4.26e-02 0.0163 | | 5.21e+02(=) 4.26e-02 0.2340 | |  |
| T2 | | 5.21e+02 3.63e−02 - | | 5.21e+02(=) 3.63e-02 0.6627 | 5.21e+02(=) 3.63e-02 0.2973 | 5.21e+02(=) 3.63e-02 0.8073 | | | 5.21e+02(=) 3.63e-02 0.3555 | | 5.21e+02(=) 3.63e-02 0.9470 | |  |
| Benchmark9 | T1 | | 1.20e+04 3.96e+03 - | | 1.51e+04(+) 3.96e+03 0.0056 | 7.41e+03(-) 3.96e+03 0.0002 | 1.49e+04(+) 3.96e+03 0.0141 | | | 1.50e+04(+) 3.96e+03 0.0121 | | 8.98e+03(-) 3.96e+03 0.0126 | |  |
| T2 | | 1.62e+03 4.34e−01 - | | 1.62e+03(+) 4.34e-01 0.0000 | 1.62e+03(-) 4.34e-01 0.0000 | 1.62e+03(+) 4.34e-01 0.0000 | | | 1.62e+03(+) 4.34e-01 0.0000 | | 1.62e+03(+) 4.34e-01 0.0000 | |  |
| Benchmark10 | T1 | | **8.56e+03 3.19e+03 -** | | 5.57e+04(+) 3.19e+03 0.0000 | 1.51e+04(+) 3.19e+03 0.0000 | 6.65e+04(+) 3.19e+03 0.0000 | | | 7.76e+04(+) 3.19e+03 0.0000 | | 3.77e+04(+) 3.19e+03 0.0000 | |  |
| T2 | | **1.24e+05 6.51e+04 -** | | 1.03e+07(+) 6.51e+04 0.0000 | 1.85e+06(+) 6.51e+04 0.0000 | 2.30e+07(+) 6.51e+04 0.0000 | | | 2.12e+07(+) 6.51e+04 0.0000 | | 2.71e+06(+) 6.51e+04 0.0000 | |  |
| Number of “+/=/−“ | | | | | 15/3/2 | 9/3/8 | 15/3/2 | | | 14/3/3 | | 12/5/3 | |  |

**Table S.2.** The detailed comparison among APMTO with different *top*.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *top* | | APMTO | 10 | 40 | 50 | 70 | 100 |
| Benchmark1 | T1 | 6.28e+02 7.45e+00 - | 6.30e+02(=) 7.45e+00 0.46 | **6.27e+02(=) 7.45e+00 0.95** | 6.28e+02(=) 7.45e+00 0.52 | 6.28e+02(=) 7.45e+00 0.71 | 6.31e+02(=) 7.45e+00 0.33 |
| T2 | **6.26e+02 7.02e+00 -** | 6.30e+02(+) 7.02e+00 0.01 | 6.28e+02(=) 7.02e+00 0.33 | 6.27e+02(=) 7.02e+00 0.84 | 6.32e+02(+) 7.02e+00 0.01 | 6.28e+02(=) 7.02e+00 0.11 |
| Benchmark2 | T1 | 7.00e+02 9.10e−03 - | 7.00e+02(=) 9.10e−03 0.86 | **7.00e+02(=) 9.10e−03 0.41** | 7.00e+02(=) 9.10e−03 0.15 | 7.00e+02(+) 9.10e−03 0.02 | 7.04e+02(+) 9.10e−03 0.00 |
| T2 | **7.00e+02 9.28e−03 -** | 7.00e+02(+) 9.28e−03 0.01 | 7.00e+02(+) 9.28e−03 0.01 | 7.00e+02(+) 9.28e−03 0.00 | 7.00e+02(+) 9.28e−03 0.00 | 7.04e+02(+) 9.28e−03 0.00 |
| Benchmark3 | T1 | **1.37e+05 6.81e+04 -** | 1.39e+05(=) 6.81e+04 0.83 | 1.68e+05(=) 6.81e+04 0.63 | 1.90e+05(=) 6.81e+04 0.08 | 2.20e+05(+) 6.81e+04 0.02 | 3.06e+05(+) 6.81e+04 0.00 |
| T2 | 1.58e+05 1.15e+05 - | **1.09e+05(=) 1.15e+05 0.07** | 1.76e+05(=) 1.15e+05 0.43 | 1.34e+05(=) 1.15e+05 0.49 | 2.14e+05(+) 1.15e+05 0.00 | 2.48e+05(+) 1.15e+05 0.01 |
| Benchmark4 | T1 | 1.30e+03 1.33e−01 - | **1.30e+03(=) 1.33e−01 0.46** | 1.30e+03(=) 1.33e−01 0.80 | 1.30e+03(=) 1.33e−01 0.96 | 1.30e+03(=) 1.33e−01 0.56 | 1.30e+03(=) 1.33e−01 0.63 |
| T2 | 1.30e+03 7.62e−02 - | 1.30e+03(=) 7.62e−02 0.46 | 1.30e+03(=) 7.62e−02 0.80 | **1.30e+03(=) 7.62e−02 0.20** | 1.30e+03(=) 7.62e−02 0.43 | 1.30e+03(=) 7.62e−02 0.76 |
| Benchmark5 | T1 | **1.53e+03 1.00e+01 -** | 1.53e+03(=) 1.00e+01 0.44 | 1.53e+03(=) 1.00e+01 0.60 | 1.53e+03(=) 1.00e+01 0.46 | 1.53e+03(=) 1.00e+01 0.25 | 1.54e+03(=) 1.00e+01 0.21 |
| T2 | 1.53e+03 1.02e+01 - | 1.53e+03(=) 1.02e+01 0.85 | 1.53e+03(=) 1.02e+01 0.06 | 1.53e+03(=) 1.02e+01 0.85 | 1.53e+03(=) 1.02e+01 0.40 | 1.53e+03(=) 1.02e+01 0.26 |
| Benchmark6 | T1 | 1.05e+05 6.78e+04 - | **9.74e+04(=) 6.78e+04 0.78** | 1.22e+05(=) 6.78e+04 0.35 | 1.25e+05(=) 6.78e+04 0.22 | 1.55e+05(=) 6.78e+04 0.12 | 2.10e+05(+) 6.78e+04 0.00 |
| T2 | 1.04e+05 5.77e+04 - | 9.37e+04(=) 5.77e+04 0.30 | **8.53e+04(=) 5.77e+04 0.18** | 1.09e+05(=) 5.77e+04 0.76 | 1.29e+05(=) 5.77e+04 0.15 | 1.24e+05(=) 5.77e+04 0.19 |
| Benchmark7 | T1 | 2.90e+03 3.06e+02 - | **2.88e+03(=) 3.06e+02 0.70** | 3.03e+03(=) 3.06e+02 0.14 | 2.91e+03(=) 3.06e+02 0.90 | 2.92e+03(=) 3.06e+02 0.70 | 2.98e+03(=) 3.06e+02 0.56 |
| T2 | 2.98e+03 3.28e+02 - | 2.99e+03(=) 3.28e+02 0.91 | 3.05e+03(=) 3.28e+02 0.54 | 2.96e+03(=) 3.28e+02 0.74 | 3.06e+03(=) 3.28e+02 0.55 | **2.87e+03(=) 3.28e+02 0.11** |
| Benchmark8 | T1 | **5.21e+02 4.26e−02 -** | 5.21e+02(=) 4.26e−02 0.09 | 5.21e+02(+) 4.26e−02 0.01 | 5.21e+02(=) 4.26e−02 0.30 | 5.21e+02(+) 4.26e−02 0.00 | 5.21e+02(+) 4.26e−02 0.00 |
| T2 | 5.21e+02 3.63e−02 - | **5.21e+02(=) 3.63e−02 0.20** | 5.21e+02(=) 3.63e−02 0.83 | 5.21e+02(=) 3.63e−02 0.28 | 5.21e+02(=) 3.63e−02 0.33 | 5.21e+02(=) 3.63e−02 0.31 |
| Benchmark9 | T1 | 1.20e+04 3.96e+03 - | **9.89e+03(−) 3.96e+03 0.01** | 1.27e+04(=) 3.96e+03 0.85 | 1.40e+04(=) 3.96e+03 0.29 | 1.45e+04(=) 3.96e+03 0.16 | 1.50e+04(+) 3.96e+03 0.00 |
| T2 | 1.62e+03 4.34e−01 - | 1.62e+03(=) 4.34e−01 0.51 | 1.62e+03(=) 4.34e−01 0.71 | 1.62e+03(+) 4.34e−01 0.01 | 1.62e+03(+) 4.34e−01 0.00 | 1.62e+03(+) 4.34e−01 0.00 |
| Benchmark10 | T1 | 8.56e+03 3.19e+03 - | 8.01e+03(=) 3.19e+03 0.48 | 1.01e+04(=) 3.19e+03 0.22 | 9.53e+03(=) 3.19e+03 0.43 | 1.08e+04(=) 3.19e+03 0.11 | 1.33e+04(+) 3.19e+03 0.00 |
| T2 | 1.24e+05 6.51e+04 - | **1.02e+05(=) 6.51e+04 0.07** | 1.22e+05(=) 6.51e+04 0.76 | 1.79e+05(+) 6.51e+04 0.01 | 1.59e+05(=) 6.51e+04 0.12 | 1.64e+05(=) 6.51e+04 0.19 |
| Number of “+/=/−“ | | | 2/17/1 | 2/18/0 | 3/17/0 | 7/13/0 | 9/11/0 |

**Table S.3** The detailed comparison among APMTO with different *APMaxFEs*.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *APMaxFEs* | | APMTO | 0 | 50 | 500 |
| Benchmark1 | T1 | 6.28e+02 7.45e+00 - | **6.20e+02(−) 7.45e+00 0.0000** | 6.22e+02(−) 7.45e+00 0.0000 | 6.23e+02(−) 7.45e+00 0.0000 |
| T2 | 6.26e+02 7.02e+00 - | **6.23e+02(−) 7.02e+00 0.0014** | 6.23e+02(−) 7.02e+00 0.0043 | 6.25e+02(−) 7.02e+00 0.0014 |
| Benchmark2 | T1 | 7.00e+02 9.10e−03 - | 7.00e+02(=) 9.10e−03 0.1333 | **7.00e+02(=) 9.10e−03 0.1152** | 7.00e+02(+) 9.10e−03 0.0175 |
| T2 | **7.00e+02 9.28e−03 -** | 7.00e+02(+) 9.28e−03 0.0030 | 7.00e+02(+) 9.28e−03 0.0006 | 7.00e+02(+) 9.28e−03 0.0029 |
| Benchmark3 | T1 | **1.37e+05 6.81e+04 -** | 5.50e+05(+) 6.81e+04 0.0000 | 4.17e+05(+) 6.81e+04 0.0000 | 3.55e+05(+) 6.81e+04 0.0000 |
| T2 | **1.58e+05 1.15e+05 -** | 4.48e+05(+) 1.15e+05 0.0000 | 3.82e+05(+) 1.15e+05 0.0000 | 4.32e+05(+) 1.15e+05 0.0000 |
| Benchmark4 | T1 | **1.30e+03 1.33e−01 -** | 1.30e+03(=) 1.33e−01 0.7618 | 1.30e+03(=) 1.33e−01 0.7731 | 1.30e+03(=) 1.33e−01 0.6735 |
| T2 | **1.30e+03 7.62e−02 -** | 1.30e+03(=) 7.62e−02 0.6100 | 1.30e+03(=) 7.62e−02 0.4553 | 1.30e+03(=) 7.62e−02 0.5298 |
| Benchmark5 | T1 | **1.53e+03 1.00e+01 -** | 1.54e+03(+) 1.00e+01 0.0001 | 1.54e+03(+) 1.00e+01 0.0001 | 1.54e+03(+) 1.00e+01 0.0007 |
| T2 | **1.53e+03 1.02e+01 -** | 1.54e+03(+) 1.02e+01 0.0000 | 1.54e+03(+) 1.02e+01 0.0000 | 1.54e+03(+) 1.02e+01 0.0000 |
| Benchmark6 | T1 | **1.05e+05 6.78e+04 -** | 3.38e+05(+) 6.78e+04 0.0000 | 3.37e+05(+) 6.78e+04 0.0000 | 3.58e+05(+) 6.78e+04 0.0000 |
| T2 | **1.04e+05 5.77e+04 -** | 2.66e+05(+) 5.77e+04 0.0000 | 2.75e+05(+) 5.77e+04 0.0000 | 3.15e+05(+) 5.77e+04 0.0000 |
| Benchmark7 | T1 | 2.90e+03 3.06e+02 - | **2.71e+03(−) 3.06e+02 0.0075** | 2.75e+03(=) 3.06e+02 0.0615 | 2.67e+03(−) 3.06e+02 0.0029 |
| T2 | 2.98e+03 3.28e+02 - | **2.63e+03(−) 3.28e+02 0.0000** | 2.74e+03(−) 3.28e+02 0.0017 | 2.82e+03(−) 3.28e+02 0.0157 |
| Benchmark8 | T1 | **5.21e+02 4.26e−02 -** | 5.21e+02(+) 4.26e−02 0.0152 | 5.21e+02(+) 4.26e−02 0.0126 | 5.21e+02(+) 4.26e−02 0.0141 |
| T2 | 5.21e+02 3.63e−02 - | 5.21e+02(=) 3.63e−02 0.6520 | 5.21e+02(=) 3.63e−02 0.5493 | **5.21e+02(=) 3.63e−02 0.0824** |
| Benchmark9 | T1 | **1.20e+04 3.96e+03 -** | 1.50e+04(+) 3.96e+03 0.0099 | 1.50e+04(+) 3.96e+03 0.0157 | 1.48e+04(=) 3.96e+03 0.1297 |
| T2 | **1.62e+03 4.34e−01 -** | 1.62e+03(+) 4.34e−01 0.0000 | 1.62e+03(+) 4.34e−01 0.0004 | 1.62e+03(+) 4.34e−01 0.0008 |
| Benchmark10 | T1 | **8.56e+03 3.19e+03 -** | 1.62e+04(+) 3.19e+03 0.0000 | 1.50e+04(+) 3.19e+03 0.0000 | 1.48e+04(+) 3.19e+03 0.0001 |
| T2 | **1.24e+05 6.51e+04 -** | 3.87e+05(+) 6.51e+04 0.0000 | 3.97e+05(+) 6.51e+04 0.0000 | 4.08e+05(+) 6.51e+04 0.0000 |
| Number of “+/=/−“ | | | 12/4/4 | 12/5/3 | 12/4/4 |

**Table S.4** The detailed comparison among APMTO with different *N*.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *N* | | APMTO | 25 | 50 | 75 |
| Benchmark1 | T1 | **6.28e+02 7.45e+00 -** | 6.43e+02(+) 7.45e+00 0.0000 | 6.34e+02(+) 7.45e+00 0.0000 | 6.30e+02(=) 7.45e+00 0.0798 |
| T2 | **6.26e+02 7.02e+00 -** | 6.45e+02(+) 7.02e+00 0.0000 | 6.35e+02(+) 7.02e+00 0.0000 | 6.32e+02(+) 7.02e+00 0.0001 |
| Benchmark2 | T1 | 7.00e+02 9.10e−03 - | 7.00e+02(+) 9.10e−03 0.0002 | **7.00e+02(=) 9.10e−03 0.2007** | 7.00e+02(=) 9.10e−03 0.5581 |
| T2 | **7.00e+02 9.28e−03 -** | 7.00e+02(+) 9.28e−03 0.0000 | 7.00e+02(+) 9.28e−03 0.0003 | 7.00e+02(+) 9.28e−03 0.0034 |
| Benchmark3 | T1 | 1.37e+05 6.81e+04 - | 4.94e+05(+) 6.81e+04 0.0000 | 1.42e+05(=) 6.81e+04 0.7283 | **1.28e+05(=) 6.81e+04 0.5895** |
| T2 | 1.58e+05 1.15e+05 - | 4.98e+05(+) 1.15e+05 0.0000 | 1.41e+05(=) 1.15e+05 0.5793 | **1.26e+05(=) 1.15e+05 0.2772** |
| Benchmark4 | T1 | 1.30e+03 1.33e−01 - | **1.30e+03(=) 1.33e−01 0.1580** | 1.30e+03(=) 1.33e−01 0.7845 | 1.30e+03(=) 1.33e−01 0.1373 |
| T2 | 1.30e+03 7.62e−02 - | **1.30e+03(−) 7.62e−02 0.0242** | 1.30e+03(=) 7.62e−02 0.0501 | 1.30e+03(=) 7.62e−02 0.4825 |
| Benchmark5 | T1 | **1.53e+03 1.00e+01 -** | 2.73e+03(+) 1.00e+01 0.0000 | 1.59e+03(+) 1.00e+01 0.0000 | 1.55e+03(+) 1.00e+01 0.0000 |
| T2 | **1.53e+03 1.02e+01 -** | 2.14e+03(+) 1.02e+01 0.0000 | 1.60e+03(+) 1.02e+01 0.0000 | 1.54e+03(+) 1.02e+01 0.0001 |
| Benchmark6 | T1 | 1.05e+05 6.78e+04 - | 3.12e+05(+) 6.78e+04 0.0000 | 9.75e+04(=) 6.78e+04 0.7506 | **7.96e+04(=) 6.78e+04 0.0933** |
| T2 | 1.04e+05 5.77e+04 - | **2.11e+05(+) 5.77e+04 0.0000** | 7.04e+04(−) 5.77e+04 0.0189 | 8.09e+04(=) 5.77e+04 0.1154 |
| Benchmark7 | T1 | **2.90e+03 3.06e+02 -** | 3.29e+03(+) 3.06e+02 0.0001 | 3.04e+03(=) 3.06e+02 0.2116 | 3.03e+03(=) 3.06e+02 0.1624 |
| T2 | **2.98e+03 3.28e+02 -** | 3.37e+03(+) 3.28e+02 0.0005 | 3.10e+03(=) 3.28e+02 0.2707 | 3.02e+03(=) 3.28e+02 0.9234 |
| Benchmark8 | T1 | **5.21e+02 4.26e−02 -** | 5.21e+02(+) 4.26e−02 0.0069 | 5.21e+02(+) 4.26e−02 0.0028 | 5.21e+02(=) 4.26e−02 0.1335 |
| T2 | 5.21e+02 3.63e−02 - | 5.21e+02(=) 3.63e−02 0.4204 | 5.21e+02(=) 3.63e−02 0.8303 | **5.21e+02(=) 3.63e−02 0.9000** |
| Benchmark9 | T1 | 1.20e+04 3.96e+03 - | **8.12e+03(−) 3.96e+03 0.0025** | 9.08e+03(−) 3.96e+03 0.0064 | 9.29e+03(−) 3.96e+03 0.0069 |
| T2 | 1.62e+03 4.34e−01 - | 1.62e+03(=) 4.34e−01 0.0933 | **1.62e+03(=) 4.34e−01 0.0993** | 1.62e+03(=) 4.34e−01 0.7731 |
| Benchmark10 | T1 | 8.56e+03 3.19e+03 - | **8.80e+03(=) 3.19e+03 0.7731** | 6.46e+03(−) 3.19e+03 0.0023 | 6.36e+03(−) 3.19e+03 0.0016 |
| T2 | 1.24e+05 6.51e+04 - | **4.15e+05(+) 6.51e+04 0.0000** | 8.45e+04(−) 6.51e+04 0.0088 | 8.26e+04(−) 6.51e+04 0.0028 |
| Number of "+/=/**−**" | | | 14/4/2 | 6/10/4 | 4/13/3 |

**Table S.5** The detailed comparison among APMTO with different *F*.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *F* | | APMTO(0.5) | 0.6 | 0.7 | 0.9 |
| Benchmark1 | T1 | 6.28e+02 7.45e+00 - | **6.21e+02(−) 7.45e+00 0.0000** | 6.17e+02(−) 7.45e+00 0.0000 | 6.26e+02(=) 7.45e+00 0.3790 |
| T2 | 6.26e+02 7.02e+00 - | **6.22e+02(−) 7.02e+00 0.0001** | 6.16e+02(−) 7.02e+00 0.0000 | 6.27e+02(=) 7.02e+00 0.1537 |
| Benchmark2 | T1 | 7.00e+02 9.10e−03 - | **7.00e+02(=) 9.10e−03 0.1713** | 7.02e+02(+) 9.10e−03 0.0025 | 7.27e+02(+) 9.10e−03 0.0000 |
| T2 | **7.00e+02 9.28e−03 -** | 7.00e+02(+) 9.28e−03 0.0017 | 7.02e+02(+) 9.28e−03 0.0003 | 7.26e+02(+) 9.28e−03 0.0000 |
| Benchmark3 | T1 | **1.37e+05 6.81e+04 -** | 4.19e+05(+) 6.81e+04 0.0000 | 3.74e+06(+) 6.81e+04 0.0000 | 1.28e+07(+) 6.81e+04 0.0000 |
| T2 | **1.58e+05 1.15e+05 -** | 4.11e+05(+) 1.15e+05 0.0000 | 3.25e+06(+) 1.15e+05 0.0000 | 1.43e+07(+) 1.15e+05 0.0000 |
| Benchmark4 | T1 | 1.30e+03 1.33e−01 - | **1.30e+03(=) 1.33e−01 0.8303** | 1.30e+03(+) 1.33e−01 0.0081 | 1.30e+03(+) 1.33e−01 0.0000 |
| T2 | **1.30e+03 7.62e−02 -** | 1.30e+03(+) 7.62e−02 0.0099 | 1.30e+03(+) 7.62e−02 0.0001 | 1.30e+03(+) 7.62e−02 0.0000 |
| Benchmark5 | T1 | **1.53e+03 1.00e+01 -** | 1.54e+03(+) 1.00e+01 0.0008 | 1.54e+03(+) 1.00e+01 0.0001 | 1.56e+03(+) 1.00e+01 0.0000 |
| T2 | **1.53e+03 1.02e+01 -** | 1.54e+03(+) 1.02e+01 0.0000 | 1.54e+03(+) 1.02e+01 0.0000 | 1.56e+03(+) 1.02e+01 0.0000 |
| Benchmark6 | T1 | **1.05e+05 6.78e+04 -** | 3.21e+05(+) 6.78e+04 0.0000 | 2.08e+06(+) 6.78e+04 0.0000 | 9.51e+06(+) 6.78e+04 0.0000 |
| T2 | **1.04e+05 5.77e+04 -** | 3.01e+05(+) 5.77e+04 0.0000 | 1.78e+06(+) 5.77e+04 0.0000 | 9.90e+06(+) 5.77e+04 0.0000 |
| Benchmark7 | T1 | 2.90e+03 3.06e+02 - | **2.68e+03(−) 3.06e+02 0.0045** | 3.54e+03(+) 3.06e+02 0.0000 | 4.12e+03(+) 3.06e+02 0.0000 |
| T2 | 2.98e+03 3.28e+02 - | **2.75e+03(−) 3.28e+02 0.0024** | 3.52e+03(+) 3.28e+02 0.0000 | 4.12e+03(+) 3.28e+02 0.0000 |
| Benchmark8 | T1 | **5.21e+02 4.26e−02 -** | 5.21e+02(+) 4.26e−02 0.0141 | 5.21e+02(=) 4.26e−02 0.0701 | 5.21e+02(=) 4.26e−02 0.1297 |
| T2 | 5.21e+02 3.63e−02 - | 5.21e+02(=) 3.63e−02 0.2772 | **5.21e+02(=) 3.63e−02 0.2170** | 5.21e+02(=) 3.63e−02 0.5895 |
| Benchmark9 | T1 | **1.20e+04 3.96e+03 -** | 1.48e+04(=) 3.96e+03 0.1260 | 1.49e+04(=) 3.96e+03 0.0748 | 1.51e+04(+) 3.96e+03 0.0028 |
| T2 | **1.62e+03 4.34e−01 -** | 1.62e+03(+) 4.34e−01 0.0018 | 1.62e+03(+) 4.34e−01 0.0000 | 1.62e+03(+) 4.34e−01 0.0000 |
| Benchmark10 | T1 | **8.56e+03 3.19e+03 -** | 1.88e+04(+) 3.19e+03 0.0000 | 2.88e+04(+) 3.19e+03 0.0000 | 6.46e+04(+) 3.19e+03 0.0000 |
| T2 | **1.24e+05 6.51e+04 -** | 4.34e+05(+) 6.51e+04 0.0000 | 2.31e+06(+) 6.51e+04 0.0000 | 1.14e+07(+) 6.51e+04 0.0000 |
| Number of “+/=/−“ | | | 12/4/4 | 15/3/2 | 16/4/0 |

**Table S.6** The detailed comparison among APMTO with different *CR*.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| CR | | APMTO(0.7) | 0.6 | 0.8 | 0.9 |
| Benchmark1 | T1 | 6.28e+02 7.45e+00 - | **6.17e+02(−) 7.45e+00 0.0000** | 6.28e+02(=) 7.45e+00 0.3790 | 6.32e+02(+) 7.45e+00 0.0000 |
| T2 | 6.26e+02 7.02e+00 - | **6.21e+02(−) 7.02e+00 0.0001** | 6.27e+02(=) 7.02e+00 0.1907 | 6.33e+02(+) 7.02e+00 0.0000 |
| Benchmark2 | T1 | 7.00e+02 9.10e−03 - | **7.00e+02(=) 9.10e−03 0.2086** | 7.00e+02(=) 9.10e−03 0.9822 | 7.00e+02(=) 9.10e−03 0.1974 |
| T2 | **7.00e+02 9.28e−03 -** | 7.00e+02(+) 9.28e−03 0.0030 | 7.00e+02(=) 9.28e−03 0.1011 | 7.00e+02(+) 9.28e−03 0.0174 |
| Benchmark3 | T1 | 1.37e+05 6.81e+04 - | **4.62e+05(+) 6.81e+04 0.0000** | 1.44e+05(=) 6.81e+04 0.8534 | 4.65e+04(−) 6.81e+04 0.0000 |
| T2 | 1.58e+05 1.15e+05 - | **4.41e+05(+) 1.15e+05 0.0000** | 1.29e+05(=) 1.15e+05 0.2519 | 4.47e+04(−) 1.15e+05 0.0000 |
| Benchmark4 | T1 | 1.30e+03 1.33e−01 - | 1.30e+03(=) 1.33e−01 0.9587 | 1.30e+03(=) 1.33e−01 0.3183 | **1.30e+03(=) 1.33e−01 0.2707** |
| T2 | 1.30e+03 7.62e−02 - | 1.30e+03(=) 7.62e−02 0.5395 | 1.30e+03(=) 7.62e−02 0.0555 | **1.30e+03(=) 7.62e−02 0.0615** |
| Benchmark5 | T1 | 1.53e+03 1.00e+01 - | **1.52e+03(=) 1.00e+01 0.0905** | 1.53e+03(=) 1.00e+01 0.2340 | 1.56e+03(+) 1.00e+01 0.0000 |
| T2 | 1.53e+03 1.02e+01 - | **1.52e+03(−) 1.02e+01 0.0054** | 1.52e+03(=) 1.02e+01 0.5895 | 1.55e+03(+) 1.02e+01 0.0000 |
| Benchmark6 | T1 | 1.05e+05 6.78e+04 - | **3.23e+05(+) 6.78e+04 0.0000** | 1.03e+05(=) 6.78e+04 1.0000 | 5.50e+04(−) 6.78e+04 0.0001 |
| T2 | 1.04e+05 5.77e+04 - | **2.35e+05(+) 5.77e+04 0.0000** | 8.93e+04(=) 5.77e+04 0.5592 | 5.01e+04(−) 5.77e+04 0.0000 |
| Benchmark7 | T1 | 2.90e+03 3.06e+02 - | **2.87e+03(=) 3.06e+02 0.5298** | 2.95e+03(=) 3.06e+02 0.4553 | 3.05e+03(=) 3.06e+02 0.0575 |
| T2 | 2.98e+03 3.28e+02 - | **2.84e+03(−) 3.28e+02 0.0189** | 3.03e+03(=) 3.28e+02 0.9587 | 3.03e+03(=) 3.28e+02 0.7394 |
| Benchmark8 | T1 | **5.21e+02 4.26e−02 -** | 5.21e+02(+) 4.26e−02 0.0038 | 5.21e+02(=) 4.26e−02 0.0615 | 5.21e+02(=) 4.26e−02 0.2226 |
| T2 | 5.21e+02 3.63e−02 - | 5.21e+02(=) 3.63e−02 0.6309 | **5.21e+02(=) 3.63e−02 0.7394** | 5.21e+02(=) 3.63e−02 0.5692 |
| Benchmark9 | T1 | 1.20e+04 3.96e+03 - | **1.41e+04(=) 3.96e+03 0.3871** | 1.01e+04(−) 3.96e+03 0.0126 | 8.10e+03(−) 3.96e+03 0.0005 |
| T2 | 1.62e+03 4.34e−01 - | 1.62e+03(+) 4.34e−01 0.0112 | 1.62e+03(=) 4.34e−01 0.7506 | **1.62e+03(=) 4.34e−01 0.2226** |
| Benchmark10 | T1 | 8.56e+03 3.19e+03 - | **1.48e+04(+) 3.19e+03 0.0001** | 8.29e+03(=) 3.19e+03 0.3403 | 3.18e+03(−) 3.19e+03 0.0000 |
| T2 | 1.24e+05 6.51e+04 - | **4.11e+05(+) 6.51e+04 0.0000** | 1.29e+05(=) 6.51e+04 0.7172 | 5.26e+04(−) 6.51e+04 0.0000 |
| Number of "+/=/**−**" | | | 9/7/4 | 0/19/1 | 5/8/7 |