$$x(k+1) = A_i x(k) + B_i u(k) + f_i \quad \text{for} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \Omega_i$$
$$y(k) = C_i x(k) + D_i u(k) + g_i$$

x: energy stored

$$x_b(k+1) = x_b - s_b \cdot \eta_d \cdot u_b - (1-s_b) \cdot \eta_c \cdot u_b$$

during 1 sampling period: $T_s \cdot u_b$

$$x_b(k+1) = \begin{cases} x_b(k) - \eta_d \cdot T_s \cdot u_b(k) & u_b > 0 \quad (s_b(k)=0) \\ x_b(k) - \eta_c \cdot T_s \cdot u_b(k) & u_b \leq 0 \quad (s_b(k)=1) \end{cases}$$

2.2

MLD

$x(k)$: energy stored in battery x_b

$u(k)$ u_b

$\delta(k)$: s_b $[\delta(k) = 1] \Leftrightarrow u(x) \leq 0$

$z(k)$

$$x(k+1) = A x(k) + B_1 u(k) + B_2 \delta(k) + B_3 z(k) + B_4$$
$$E_1 x(k) + E_2 u(k) + E_3 \delta(k) + E_4 z(k) \leq g_5$$

$$\underline{u}_b \leq u(k) \leq \overline{u}_b$$
$$u(k) \leq \overline{u}_b(1 - s_b(k)) \qquad u(k) + \overline{u}_b s_b(k) \leq \overline{u}_b$$
$$u(k) \geq \varepsilon + (\underline{u}_b - \varepsilon) s_b(k) \qquad -u(k) + (\underline{u}_b - \varepsilon) s_b(k) \leq -\varepsilon$$

$$x(k+1) = x(k) - \eta_d \cdot T_s \cdot u(k) - (\eta_c - \eta_d) \cdot T_s \cdot \delta(k) \cdot u(k)$$
$$z(k) = \delta(k) u(k)$$

ε used to turn $>$ into $\geq$

$$x(k+1) = x(k) - \eta_d T_s\, u(k) + (\eta_d - \eta_c) T_s\, z(k)$$

$$z(k) \le \overline{u_b}\, s_b(k)$$
$$z(k) \ge \underline{u_b}\, s_b(k)$$
$$z(k) \le u(k) - \underline{u_b}(1 - s_b(k))$$
$$z(k) \ge u(k) - \overline{u_b}(1 - s_b(k))$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} z(k) + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} -\overline{u_b} \\ \underline{u_b} \\ -\underline{u_b} \\ \overline{u_b} \end{bmatrix} s_b(k) \le \begin{bmatrix} 0 \\ 0 \\ -\underline{u_b} \\ \overline{u_b} \end{bmatrix}$$

$$x(k) \le \overline{x_b} \qquad \begin{bmatrix} 1 \\ -1 \end{bmatrix} x(k) \le \begin{bmatrix} \overline{x_b} \\ 0 \end{bmatrix}$$
$$x(k) \ge 0$$

$$A = 1$$
$$B_1 = -\eta_d T_s$$
$$B_2 = 0$$
$$B_3 = (\eta_d - \eta_c) T_s$$

$$\text{s.t.} \quad \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}}_{E_1} x(k) + \underbrace{\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{E_2} u(k) + \underbrace{\begin{bmatrix} \underline{u_b} - \varepsilon \\ \overline{u_b} \\ -\overline{u_b} \\ \underline{u_b} \\ -\underline{u_b} \\ \overline{u_b} \\ 0 \\ 0 \end{bmatrix}}_{E_3} \delta(k) + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}}_{E_4} z(k) \le \underbrace{\begin{bmatrix} -\varepsilon \\ \overline{u_b} \\ 0 \\ 0 \\ -\underline{u_b} \\ \overline{u_b} \\ \overline{x_b} \\ 0 \end{bmatrix}}_{g_5}$$

2.6

$$0 \le ud(k) \le \bar{u}d$$
$$\underline{x}d \le xd(k) \le \bar{x}d$$

$$0 \le ud(k) < u_1$$
$$0 \le ud(k) < u_1(1-\delta_1)$$

$$u_1 \le ud(k) < u_2$$
$$\varepsilon + (u_1 - \varepsilon)\delta \le ud(k) < u_2(1-\delta)$$

$$0 \le ud(k) \le \bar{u}d + \delta_1(k)(u_1 - \bar{u}d - \varepsilon)$$

$$0 \le u(k) \le \bar{u}d + z_1(k) - \bar{u}d\,\delta_1(k) - \epsilon\,\delta_1(k)$$
$$\Rightarrow 0 \le u(k) \le z_1(k) - (\bar{u}d + \epsilon)\delta_1(k) + \bar{u}d$$
$$u(k) \ge 0 \Rightarrow -u(k) \le 0$$
$$u(k) - z_1(k) + (\bar{u}d + \epsilon)\delta_1(k) \le \bar{u}d$$
$$\Rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ -1 \end{bmatrix} z_1(k) + \begin{bmatrix} 0 \\ \bar{u}d + \epsilon \end{bmatrix} \delta_1(k) \le \begin{bmatrix} 0 \\ \bar{u}d \end{bmatrix}$$

$$z_i(k) \le u_i\,\delta_i(k)$$
$$z_i(k) \ge u_{i-1}\,\delta_i(k)$$
$$z_i(k) \le u(k) - \underline{u}d\,(1 - \delta_i(k))$$
$$z_i(k) \ge u(k) - \bar{u}d\,(1 - \delta_i(k))$$
where $\underline{u}d = 0$

2.8 $T_1 = \begin{bmatrix} B_1 & & & 0 \\ AB_1 & \ddots & & \\ \vdots & & \ddots & \\ A^{Np-1}B_1 & A^{Np-2}B_2 & \cdots & B_1 \end{bmatrix}$ $\hat{u}$

$B_1 =$

$B_2 \in N$

$\underset{1 \times x}{A} \; \underset{1 \times 4}{B_2}$

$[\,\cdot\,][\,\cdot\quad\cdot\,] = [\quad]$

1×4

$N_x \times N_{delta}$

$$T_1 = \underbrace{\begin{bmatrix} \underbrace{B_1}_{N_x \times N_u} & & & \\ \underbrace{AB_1}_{N_x \times N_u} & \underbrace{B_1}_{N_x \times N_u} & \ddots & \\ \vdots & & \ddots & \\ \underbrace{A^{Np-1}B_1}_{N_x \times N_u} & & & \underbrace{B_1}_{N_x \times N_u} \end{bmatrix}}_{N_p \times N_u} \Bigg\} N_p \times N_x$$

$T_1(1 : N_x ; 1 : N_u) = B_1$

$T_1(N_x + 1 : 2 \cdot N_x ; N_u + 1 : 2 \cdot N_u) = B_1$

$(i-1) \cdot N_x + 1 : i \cdot N_x$

Cost function:

$$\sum_{j=0}^{N_p-1} P_{imp}(k+j)\, C_e(k+j) = \sum_{j=0}^{N_p-1} \left(P_{load}(k+j) - u_d(k+j) - \sum_{i=1}^{N_b} u_{b_i,j}(k+j) \right) \cdot C_e(k+j)$$

$$(\text{for } N_b = 2) \quad = \sum_{j=0}^{N_p-1} \left(P_{load}(k+j) + \begin{bmatrix} -1 & -1 & -1 \end{bmatrix} u(k+j) \right) \cdot C_e(k+j)$$

$$P_{load}(k)\, C_e(k) + P_{load}(k+1)\, C_e(k) + \dots + P_{load}(k+N_p-1)\, C_e(k+N_p-1)$$

$$\tilde{P}_{load}(k) = \begin{bmatrix} P_{load}(k) \\ P_{load}(k+1) \\ \vdots \\ P_{load}(k+N_p-1) \end{bmatrix} \qquad \tilde{C}_e(k) = \begin{bmatrix} C_e(k) \\ \vdots \\ C_e(k+N_p-1) \end{bmatrix}$$

so $\tilde{P}_{load}^T(k)\, \tilde{C}_e(k)$

or $\underline{\tilde{C}_e^T(k)\, \tilde{P}_{load}(k)}$

so we get
$$\tilde{C}_e^T(k)\, \tilde{P}_{load}(k) + \tilde{C}_e^T(k) \underbrace{\begin{bmatrix} -1 & -1 & -1 & & & & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & & & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & & 0 & -1 & -1 & -1 \end{bmatrix}}_{C_u} \hat{u}(k)$$

$$\hat{u}(k) = K_u\, \tilde{u}(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ \vdots \\ u(k+N_p-1) \end{bmatrix} \quad \text{where} \quad u(k) = \begin{bmatrix} u_{b1}(k) \\ u_{b2}(k) \\ u_{dies}(k) \end{bmatrix}$$

$$-W_{fuel}(x_d(k+N_p) - x_d(k)) - W_e \sum_{i=1}^{N_b} (x_{b,i}(k+N_p) - x_{b,i}(k))$$

$$= \begin{bmatrix} W_e & W_e & W_{fuel} \end{bmatrix} \left(\underbrace{\begin{bmatrix} x_{b1}(k) \\ x_{b2}(k) \\ x_d(k) \end{bmatrix}}_{x(k)} + \begin{bmatrix} 0 & \cdots & 0 & -1 & 0 & 0 \\ 0 & \cdots & 0 & 0 & -1 & 0 \\ 0 & \cdots & 0 & 0 & 0 & -1 \end{bmatrix} \hat{x}(k) \right)$$

$$= \begin{bmatrix} W_e & W_e & W_{fuel} \end{bmatrix} x(k) + \underbrace{\begin{bmatrix} 0 & \cdots & 0 & -W_e & -W_e & -W_{fuel} \end{bmatrix}}_{C_x} \cdot \hat{x}(k) \qquad \hat{x}(k) = \begin{bmatrix} x(k+1) \\ \vdots \\ x(k+N_p) \end{bmatrix}$$

$$\sum_{j=0}^{N_p-1} \left(\sum_{i=1}^{N_b} w_{b,i} |\Delta s_{b,i}(k+j)| + w_d |\Delta s_d(k+j)| \right)$$

$$= \sum_{j=0}^{N_p-1} (w_{b1} |\Delta s_{b1}(k+j)| + w_{b2} |\Delta s_{b2}(k+j)| + w_{b3} |\Delta s_{b3}(k+j)| + w_d |\Delta s_d(k+j)|)$$

$$S_d(k) = \delta_{d1}(k) + \delta_{d2}(k) + \delta_{d3}(k) + \delta_{d4}(k)$$

$$\Delta s(k) = \begin{bmatrix} \Delta s_1(k) \\ \Delta s_2(k) \\ \Delta s_3(k) \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & -1 \end{bmatrix} \delta(k-1) + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \delta(k)$$

W_b & $W_d \geq 0$

$$\sum_{j=0}^{N_p-1} \left\| \begin{bmatrix} -W_{b1} & 0 & 0 & 0 & 0 & 0 \\ 0 & -W_{b2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -W_d & -W_d & -W_d & -W_d \end{bmatrix} \delta(k+j-1) + \begin{bmatrix} W_{b1} & 0 & 0 & 0 & 0 & 0 \\ 0 & W_{b2} & 0 & 0 & 0 & 0 \\ 0 & 0 & W_d & W_d & W_d & W_d \end{bmatrix} \delta(k+j) \right\|_1$$

$$\hat{\delta}(k) = \begin{bmatrix} \delta(k) \\ \vdots \\ \delta(k+N_p-1) \end{bmatrix} \qquad \delta(k) = \begin{bmatrix} \delta_{b1} \\ \delta_{b2} \\ \delta_{d1} \\ \delta_{d2} \\ \delta_{d3} \\ \delta_{d4} \end{bmatrix}$$

$$= \left\| \underbrace{\begin{bmatrix} W_\delta & 0 & \cdots & 0 \\ -W_\delta & W_\delta & & 0 \\ \vdots & -W_\delta & W_\delta & \\ \vdots & & \ddots & \ddots \\ 0 & & -W_\delta & W_\delta \end{bmatrix}}_{C_{\delta 1}} \hat{\delta}(k) + \underbrace{\begin{bmatrix} -W_\delta \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}}_{C_{\delta 2}} \delta(k+1) \right\|_1$$

use $\displaystyle \min_{p,\theta \in \mathbb{R}^n} \; \underset{[1 \cdots 1]\, p(k)}{p_1 + p_2 + p_3 \cdots + p_n}$ s.t. $-p \leq \theta \leq p$ and $A\theta \leq b$

$$-p(k) \leq C_{\delta 1} \hat{\delta}(k) + C_{\delta 2} \delta(k-1) \leq p(k)$$

Then

$$J(k) = [1\ 1 \cdots 1]\, p(k) + C_x \hat{x}(k) + C_u \hat{u}(k) + [W_e \ W_e \ W_{fuel}]\, x(k) + \tilde{C}_e^T(k) P_{load}(k)$$

replace $\hat{x}(k)$ & $\hat{u}(k)$ & omit constants.