

$$x(k+1) = A_i x(k) + B_i u(k) + f_i \quad \text{for } \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in \Omega_i$$

$$y(k) = C_i x(k) + D_i u(k) + g_i$$

x : energy stored

$$x_b(k+1) = x_b - s_b \cdot \eta_d \cdot u_b - (1-s_b) \cdot \eta_c \cdot u_b$$

during 1 sampling period: $T_s \cdot u_b$

$$x_b(k+1) = \begin{cases} x_b(k) - \eta_d \cdot T_s \cdot u_b(k) & u_b > 0 \\ x_b(k) - \eta_c \cdot T_s \cdot u_b(k) & u_b \leq 0 \end{cases} \quad \begin{array}{l} (s_b(k)=0) \\ (s_b(k)=1) \end{array}$$

2.2

MLD

$x(k)$: energy stored in battery x_b

$u(k)$ u_b

$\delta(k)$: s_b $[\delta(k)=1] \Leftrightarrow u(x) \leq 0$

$z(k)$

$$x(k+1) = A x(k) + B_1 u(k) + B_2 \delta(k) + B_3 z(k) + B_4$$

$$E_1 x(k) + E_2 u(k) + E_3 \delta(k) + E_4 z(k) \leq g_5$$

$$\underline{u}_b \leq u(k) \leq \bar{u}_b$$

$$u(k) \leq \bar{u}_b (1 - s_b(k)) \quad u(k) + \bar{u}_b s_b(k) \leq \bar{u}_b$$

$$u(k) \geq \varepsilon + (\underline{u}_b - \varepsilon) s_b(k) \quad -u(k) + (\underline{u}_b - \varepsilon) s_b(k) \leq -\varepsilon$$

$$x(k+1) = x(k) - \eta_d \cdot T_s \cdot u(k) - (\eta_c - \eta_d) \cdot T_s \cdot \delta(k) \cdot u(k)$$

$$z(k) = \delta(k) u(k)$$

ε used to turn $>$ into \geq

$$x(k+1) = x(k) - \eta_d T_s u(k) + (\eta_d - \eta_c) T_s z(k)$$

$$z(k) \leq \bar{u}_b s_b(k)$$

$$z(k) \geq \underline{u}_b s_b(k)$$

$$z(k) \leq u(k) - \underline{u}_b (1 - s_b(k))$$

$$z(k) \geq u(k) - \bar{u}_b (1 - s_b(k))$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} z(k) + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} -\bar{u}_b \\ \underline{u}_b \\ -\underline{u}_b \\ \bar{u}_b \end{bmatrix} s_b(k) \leq \begin{bmatrix} 0 \\ 0 \\ -\underline{u}_b \\ \bar{u}_b \end{bmatrix}$$

$$x(k) \leq \bar{x}_b$$

$$x(k) \geq 0$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} x(k) \leq \begin{bmatrix} \bar{x}_b \\ 0 \end{bmatrix}$$

$$A = 1$$

$$B_1 = -\eta_d T_s$$

$$B_2 = 0$$

$$B_3 = (\eta_d - \eta_c) T_s$$

$$\text{s.t. } \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} x(k) + \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} \underline{u}_b - \varepsilon \\ \bar{u}_b \\ -\bar{u}_b \\ \underline{u}_b \\ -\underline{u}_b \\ 0 \\ 0 \end{bmatrix} s(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} z(k) \leq \begin{bmatrix} -\varepsilon \\ \bar{u}_b \\ 0 \\ 0 \\ -\underline{u}_b \\ \bar{x}_b \\ 0 \end{bmatrix}$$

$E_1 \quad E_2 \quad E_3 \quad E_u \quad g_j$

2.6]

$$0 \leq u_d(h) \leq \bar{u}_d$$

$$\underline{x}_d \leq x_d(h) \leq \bar{x}_d$$

$$0 \leq u_d(h) < u_1$$

$$0 \leq u_d(h) < u_1(1 - \delta_1)$$

$$u_1 \leq u_d(h) < u_2$$

$$\varepsilon + (u_1 - \varepsilon)\delta \leq u_d(h) < u_2(1 - \delta)$$

$$0 \leq u_d(h) \leq \bar{u}_d + s_1(h)(u_1 - \bar{u}_d - \varepsilon)$$

$$0 \leq u(h) \leq \bar{u}_d + z_1(h) - \bar{u}_d \delta_1(h) - \in \delta_1(h)$$

$$\Rightarrow 0 \leq u(h) \leq z_1(h) - (\bar{u}_d + \varepsilon) \delta_1(h) + \bar{u}_d$$

$$u(h) \geq 0 \Rightarrow -u(h) \leq 0$$

$$u(h) - z_1(h) + (\bar{u}_d + \varepsilon) \delta_1(h) \leq \bar{u}_d$$

$$\Rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix} u(h) + \begin{bmatrix} 0 \\ -1 \end{bmatrix} z_1(h) + \begin{bmatrix} 0 \\ \bar{u}_d + \varepsilon \end{bmatrix} \delta_1(h) \leq \begin{bmatrix} 0 \\ \bar{u}_d \end{bmatrix}$$

$$z_i(h) \leq u_i \delta_i(h)$$

$$z_i(h) \geq u_{i-1} \delta_i(h)$$

$$z_i(h) \leq u(h) - \underline{u}_d (1 - \delta_i(h))$$

$$z_i(h) \geq u(h) - \bar{u}_d (1 - \delta_i(h))$$

where $\underline{u}_d = 0$

$$2.8] T_1 = \begin{bmatrix} B_1 & & & \\ AB_1 & / & & \\ \vdots & & \ddots & \\ A^{NP-1}B_1 & A^{NP-2}B_2 & \cdots & B_1 \end{bmatrix} \overset{\text{U}}{\sim}$$

$$B_1 =$$

$$B_2 \in N$$

$$\begin{array}{c} A \quad B_2 \\ 1 \times x \quad 1 \times 4 \end{array}$$

$$\left[\cdot \right] \left[\cdot \quad \right] = \left[\quad \right]_{1 \times 4}$$

$N_x \times N_{\text{delta}}$

$$T_1 = \begin{bmatrix} B_1 & & & \\ \underbrace{AB_1}_{N_x \times N_u} & \underbrace{B_1}_{N_x \times N_u} & & \\ \vdots & & \ddots & \\ \underbrace{A^{NP-1}B_1}_{N_x \times N_u} & & & \underbrace{B_1}_{N_x \times N_u} \end{bmatrix} \quad NP < N_x$$

$NP \times N_u$

$$T_1 \quad (1: Nx^0, 1: Nu) = B_1$$

$$T_1(Nx+1 : 2 \cdot Nx; Nu+1 : 2 \cdot Nu) = B_1$$

$$(i-1) \cdot Nx + 1 \leq i \leq Nx$$

Cost function

$$\sum_{j=0}^{N_p-1} Dimp(k+j) \cdot ce(k+j) = \sum_{j=0}^{N_p-1} \left(p_{load}(k+j) - ud(k+j) - \sum_{i=1}^{Nb} u_{b,i}(k+j) \right)$$

$$P_{load}(k) C_e(k) + P_{load}(k+1) C_e(k) + \dots + P_{load}(k+N_p-1) C_e(k+N_p-1)$$

$$\tilde{P}_{load}(k) = \begin{bmatrix} P_{load}(k) \\ P_{load}(k+1) \\ \vdots \\ P_{load}(k+N_p-1) \end{bmatrix} \quad \tilde{C}_e(k) = \begin{bmatrix} C_e(k) \\ \vdots \\ C_e(k+N_p-1) \end{bmatrix}$$

$$\left[\begin{array}{c} \text{so } \tilde{P}_{\text{load}}^T(h) \tilde{C}_e(h) \\ \text{or } \underline{\tilde{C}_e^T(h) \tilde{P}_{\text{load}}(h)} \end{array} \right]$$

so we get $\tilde{C}e^T(k) \tilde{P}_{load}(k) + \tilde{C}e^T(k) \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \hat{u}(k)$

$$\tilde{C}e^T(h) \begin{bmatrix} -1 & -1 & -1 & & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & - & - & 0 & -1 & -1 & -1 \end{bmatrix}$$

$$\hat{u}(h) = Ku \hat{u}(h) = \begin{bmatrix} u(h) \\ u(h+1) \\ \vdots \\ u(h+N_p-1) \end{bmatrix}$$

where $u(h) = \begin{bmatrix} u_{b_1}(h) \\ u_{b_2}(h) \\ u_{dies}(h) \end{bmatrix}$

$$Ku = \begin{bmatrix} I & & & \\ & I & & \\ & & I & \\ & & & I \\ & & & & I \end{bmatrix}$$

$$-W_fuel(x_d(h+N_p) - x_d(h)) - W_e \sum_{i=1}^{N_b} (x_{b,i}(h+N_p) - x_{b,i}(h))$$

$$= [W_e \quad W_e \quad W_fuel] \begin{bmatrix} x_{b_1}(h) \\ x_{b_2}(h) \\ x_d(h) \end{bmatrix} + \begin{bmatrix} 0 & \dots & 0 & -1 & 0 & 0 \\ 0 & \dots & 0 & 0 & -1 & 0 \\ 0 & \dots & 0 & 0 & 0 & -1 \end{bmatrix} \hat{x}(h)$$

$$= [W_e \quad W_e \quad W_fuel] x_c(h) + \underbrace{\begin{bmatrix} 0 & \dots & 0 & -W_e & -W_e & -W_{fuel} \end{bmatrix}}_{C_x} \hat{x}(h) = \begin{bmatrix} x(h+1) \\ \vdots \\ x(h+N_p) \end{bmatrix}$$

$$\sum_{j=0}^{N_p-1} \left(\sum_{i=1}^{N_b} |w_{b,i}| \Delta s_{b,i}(h+j) | + w_d | \Delta s_d(h+j) | \right)$$

$$= \sum_{j=0}^{N_p-1} (w_{b_1} |\Delta s_{b_1}(h+j)| + w_{b_2} |\Delta s_{b_2}(h+j)| + w_{b_3} |\Delta s_{b_3}(h+j)| + w_d |\Delta s_d(h+j)|)$$

$$s_d(h) = \delta_{d_1}(h) + \delta_{d_2}(h) + \delta_{d_3}(h) + \delta_{du}(h)$$

$$\Delta s(h) = \begin{bmatrix} \Delta s_1(h) \\ \Delta s_2(h) \\ \Delta s_3(h) \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix} s(h-1) + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \delta(h)$$

$$w_b \& w_d \geq 0$$

$$\left\| \begin{bmatrix} -w_{b_1} & 0 & 0 & 0 & 0 \\ 0 & -w_{b_2} & 0 & 0 & 0 \\ 0 & 0 & -w_d & -w_d & -w_d \end{bmatrix} \delta(h+j-1) + \begin{bmatrix} w_{b_1} & 0 & 0 & 0 & 0 \\ 0 & w_{b_2} & 0 & 0 & 0 \\ 0 & 0 & w_d & w_d & w_d \end{bmatrix} \delta(h+j) \right\|_1$$

$$\hat{\delta}(h) = \begin{bmatrix} \delta(h) \\ \vdots \\ \delta(h+N_p-1) \end{bmatrix}$$

$$s(h) = \begin{bmatrix} \delta_{b_1} \\ \delta_{b_2} \\ \delta_{d_1} \\ \delta_{d_2} \\ \delta_{d_3} \\ \delta_{du} \end{bmatrix}$$

$$= \left\| \begin{bmatrix} w_8 & 0 & \dots & 0 \\ -w_8 & w_8 & & 0 \\ \vdots & -w_8 & w_8 & \\ 0 & \ddots & \ddots & -w_8 & w_8 \end{bmatrix} \hat{\delta}(k) + \begin{bmatrix} -w_8 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \delta(k+1) \right\|,$$

C_{δ_1} C_{δ_2}

use $\min_{p, \theta \in \mathbb{R}^n} p_1 + p_2 + p_3 \dots + p_n$ s.t. $-p \leq \theta \leq p$ and $A\theta \leq b$

$$-p(k) \leq C_{\delta_1} \hat{\delta}(k) + C_{\delta_2} \delta(k-1) \leq p(k)$$

$$p(k) = \begin{bmatrix} p(1) \\ \vdots \\ p(n) \end{bmatrix}$$

Then

$$J(k) = [1 1 \dots 1] p(k) + Cx \hat{x}(k) + Cu \hat{u}(k) + [w_e w_e w_{fuel}] x(k) + \tilde{e}^T(k) P_{load}(k)$$

replace $\hat{x}(k)$ & $\hat{u}(k)$ & omit constants.

$$\text{use } \hat{x}(k) = M_1 \tilde{v}(k) + M_2 x(k) + M_3$$

& $\hat{u}(k) = k_u \tilde{u}(k)$

$$\begin{aligned} J(k) &= [1 1 \dots 1] p(k) + Cx M_1 \tilde{v}(k) + Cx M_2 x(k) + Cx M_3 \\ &+ [Cu k_u \ 0 \ 0] \tilde{v}(k) + [w_e w_e w_{fuel}] x(k) + \tilde{e}^T(k) P_{load}(k) \\ &= [1 1 \dots 1] p(k) + (Cx M_1 + [Cu k_u \ 0 \ 0]) \tilde{v}(k) \\ &+ (\cancel{Cx M_2 + [w_e w_e w_{fuel}]}) x(k) + \tilde{e}^T(k) P_{load}(k) \end{aligned}$$

doesn't depend on $\tilde{v}(k)$ so doesn't influence optimal value.

$$\text{Then } J(k) = \underbrace{[1 1 \dots 1] ([Cu k_u \ 0 \ 0] + Cx M_1)}_{S_p} \begin{bmatrix} \tilde{p}(k) \\ \tilde{v}(k) \\ \tilde{u}(k) \end{bmatrix}$$

now we only need to group the constraints together.

$$F_1 \tilde{V}(k) \leq F_2 + F_3 x(k)$$

$$-p(k) - C\delta \hat{\delta}(k) = -p(k) - C\delta K_S \tilde{\delta}(k) \leq F_4 \delta(k-1)$$

$$-p(k) + C\delta \hat{\delta}(k) = -p(k) + C\delta K_S \tilde{\delta}(k) \leq -F_4 \delta(k-1)$$

$$\left[\begin{array}{c|cc} 0 & F_1 \\ -I & [0 \ -C\delta K_S \ 0] \\ -I & [0 \ C\delta K_S \ 0] \end{array} \right] \tilde{V}_p(k) \leq \left[\begin{array}{c} F_2 + F_3 x(k) \\ F_4 \delta(k-1) \\ -F_4 \delta(k-1) \end{array} \right]$$

$F_{p,1}$ $F_{p,2}$

$$\text{so min. } S_p \tilde{V}_p(k)$$

$$\text{s.t. } F_{p,1} \tilde{V}_p(k) \leq \bar{F}_{p,2}$$