Linear and Quadratic Programming Assignment 2022

SC42056 Optimization for Systems and Control

E_1, E_2, and E_3 are parameters ranging from 0 to 18 for each group according to a specific sum of the last three numbers of the student or employee IDs:

$$E_1 = D_{a,1} + D_{b,1}, \quad E_2 = D_{a,2} + D_{b,2}, \quad E_3 = D_{a,3} + D_{b,3}$$

with $D_{a,3}$ the right-most digit of the ID of the first group member, $D_{b,3}$ the right-most digit of the ID of the other group member, $D_{a,2}$ the one but last digit of the ID of the first group member, etc.

Important: Please note that all questions regarding this assignment should be asked via the Brightspace Discussion forum.

Cryptocurrencies are omnipresent in today's world, so much so that the university is contemplating to allow students to pay their tuition fee in DelftCoin, a new cryptocurrency that will go to the moon.
In this assignment we will explore some options for acquiring a setup of multiple mining units and how we can optimally control one such unit to provide enough DelftCoin for your tuition and perhaps some extra coffee now and then.

1. Configuring the mining setup
To mine for DelftCoin, the university offers two models of USB-powered mining devices: GreenMine which can mine 3.6 coins per day on average and a BlueMine model which mine 2.2 coins per day. They are priced at €250 and €120 respectively. You fear that only having one USB port hinders you from getting multiple devices, but these mining devices can be connected in series into a larger mining cluster. Sadly, your computer is USB bandwidth limited, so you can only install up to 10 mining devices. Additionally, your budget is limited to €2000 + $20E_1$.

1. If we want to get fastest mining rate of DelftCoin, expressed in coins per day, which miners should we buy? Formulate the optimization problem and transform it into the *standard* form for linear programming problems considering the number of GreenMiners (G) and number of BlueMiners (B) as continuous variables.

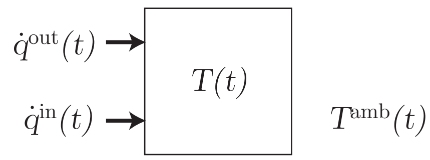
2. Find the optimal solution of the LP problem using MATLAB (use the continuous formulation and `linprog`; do not use `intlinprog`). What is the mining rate of the resulting mining setup, and the optimal number of G and B to be installed? Note: take into account G and B should be integers, but still do not use `intlinprog`. Which of the constraints is limiting the setup?

3. While you could leave the setup running continuously to gather DelftCoin, you would rather balance the system so that you mine just enough to pay for a year's worth of tuition fee in 365 days, which is 8000 + $20E_2$ DelftCoin. What is the cheapest setup to mine enough for your tuition fee in one year (365 days)? Formulate the new LP and solve it using `linprog`.

Note that exercises 2-4 below only consider *one* mining device, you don't need to use the results or information from the first exercise.

2. Obtaining a discrete-time model for a single mining device
After you bought the initial setup you plug it in and start your mining journey. You hear the internal fans spin up, which at first sounds like money to your ears. A few hours later you get worried about the cost of computation and cooling; you contemplate whether you could hack and adjust the miners to re-balance their cooling system or maybe even under- or overclock the mining device to be more energy efficient.

You realize that the temperature dynamics of one of these mining devices can be modeled by one state $T(t)$ representing the internal temperature at time t, an input for the ambient temperature $T^{\text{amb}}(t)$, and two inputs $\dot{q}^{\text{in}}(t)$ representing the power used by the mining device for computation and $\dot{q}^{\text{out}}(t)$ representing the power used to cool the device.



The dynamic model for the temperature inside the mining device, $T(t)$, is represented by the following differential equation:

$$\frac{\mathrm{d}T(t)}{\mathrm{d}t} = a_1[T^{\text{amb}}(t) - T(t)] + a_2[\dot{q}^{\text{in}}(t) - \dot{q}^{\text{out}}(t)] \tag{1}$$

where a_1 and a_2 are model parameters and $\dot{q}^{\text{in}}(t)$, $\dot{q}^{\text{out}}(t) \geq 0$.

As a first step towards performing the optimization, you want to identify the model parameters that govern the mining device. To do so, transform the continuous-time model into a discrete-time version using the following approximation:

$$\frac{\mathrm{d}T_k}{\mathrm{d}t} \approx \frac{T_{k+1} - T_k}{\Delta t} \tag{2}$$

where T_k represents the internal temperature at time step k. The resulting model has to be of the form:

$$T_{k+1} = AT_k + B[\dot{q}_k^{\text{in}}, \dot{q}_k^{\text{out}}, T_k^{\text{amb}}]^\top \tag{3}$$

Provide values of A and B as a function of a_1 and a_2.

3. Identifying the model parameters of a single mining device
You let the mining device run for a while and record the power consumption for computation and fans respectively, as well as the internal and ambient temperature. These are sampled at intervals of 1 minute (i.e., $\Delta t = 60s$) and can be obtained by downloading the file `measurements22.csv` from Brightspace. Use the Matlab function `readtable` to read in the available data. Then, formulate and solve the following optimization problem:

$$\min_{a_1,a_2} \sum_{k=1}^{N} (T_{k+1} - (AT_k + B[\dot{q}_k^{\text{in}}, \dot{q}_k^{\text{out}}, T_k^{\text{amb}}]^\top))^2 \tag{4}$$

What are the estimated values for a_1 and a_2?

4. Optimizing the cost for mining of a single mining device

You must now maximize the mining rate while minimizing the cost for operating the miner over some horizon of N steps, starting at $k = 1$ for some standardized input signals, while making sure that the dynamic model is followed throughout the time horizon.

Running the system has taught you some additional things. The rate of mining is directly proportional to $\dot{q}^{\text{in}}$, which is capped at $\dot{q}^{\text{in}}_{\text{max}} = 125$ W.
To relate the yield of DelftCoin to the energy costs, you avoid speculation and set the price at €0.38 + 0.01$E3$ per coin. You further find that it takes 8 hours at full power to mine one coin, which results in a yield of $\Psi = 0.38 + 0.01E3$ €/kWh for power spent on $\dot{q}^{\text{in}}$.
The cost of energy Φ_k (€/kWh) is provided in the `measurements22.csv` file. You can also use values for $\dot{q}^{\text{out}}_k$ and T^{amb}_k from this file.
Additionally you should constrain the internal temperature T as to satisfy the operational range of the mining device, notably $T_{\text{max}} = 95$ °C.

Combining this information, you can define the optimization problem as:

$$
\begin{aligned}
\min_{\substack{T_2,\dots,T_{N+1}\\ \dot{q}^{\text{in}}_1,\dots,\dot{q}^{\text{in}}_N}} \quad & \sum_{k=1}^{N} \Phi_k(\dot{q}^{\text{in}}_k + \dot{q}^{\text{out}}_k)\Delta t - \Psi \dot{q}^{\text{in}}_k \Delta t \\
\text{s.t.} \quad & T_{k+1} = AT_k + B[\dot{q}^{\text{in}}_k, \dot{q}^{\text{out}}_k, T^{\text{amb}}_k]^\top, && k = 1, \dots, N, \\
& 0 \leq \dot{q}^{\text{in}}_k \leq \dot{q}^{\text{in}}_{\text{max}}, && k = 1, \dots, N, \\
& T_k \leq T^{\max}, && k = 2, \dots, N+1.
\end{aligned}
\tag{5}
$$

Consider a horizon of 1 day cycle, i.e. N = 1440, assume that $T_1 = 23.3$ °C, and use the model parameters found in the previous task. Make sure you transform all quantities to the right units. Is this a quadratic optimization problem? Motivate your answer. What is the optimized cost of mining with this specific mining device?

Hint: If needed, you may get some inspiration from the goal attainment method for multi-objective optimization.

The solutions of the assignment should be uploaded to Brightspace before Monday, October 3, 2022 at 17:00 as two separate files:

1. A written report on the practical exercise as a single .pdf file (no other formats allowed).
2. A single .m file with the Matlab code you used; please make sure that the code is error free.

After uploading, please verify the uploaded files so as make sure that you have uploaded the correct files and that they are not broken.

Please also note that you will lose 0.5 point from your grade for this assignment for each (started) day of delay in case you exceed the deadline.