Vehicle Dynamics and Control

RO47017 Homework 2

MSc Systems & Control
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I. Introduction

In this homework, two vehicle stability controllers are implemented and analysed based on PD and LQR methods. The yaw rate is taken as the state variable of linear bicycle model and its reference generation is constructed from steer wheel angle ans longitudinal velocity. Then the PD and LQR based vehicle stability controllers are designed to achieve the best tracking performance which could helps maintain the stability of a vehicle by controlling its rotational motion around the vertical axis. By accurately tracking and controlling the yaw rate, the vehicle can remain balanced and prevent uncontrollable or undesired yawing movements.

II. Design vehicle dynamics controllers

In this section, the focus is put on the design of vehicle dynamics controllers. The essential dynamics and kinematics is yaw rate which is the angular rate about the vertical axis. Thus reference generator is implemented to generate desired yaw rate of the vehicle stability control system.

A. Reference generation

By employing the steer wheel angle and longitudinal velocity, the reference of yaw rate us given by $r = u/R$, where u is forward velocity and R is turn radius [2]. Then the steer wheel angle could be further represented as $\beta = \arctan(L/R) \approx L/R$ (as L/R is relatively small), by omitting friction estimation and ay-limiter, the reference yaw rate could be taken as:

$$r = \frac{u\beta}{L} \tag{1}$$

Note that the steering wheel angle is multiplied by the steer ratio to obtain the actual steering angle of the vehicle's wheels. Thus the steering wheel angle should be divided the steer ratio to obtain the actual steering angle of the vehicle's wheels. In Simulink, the reference generator is built and shown in Fig 1.

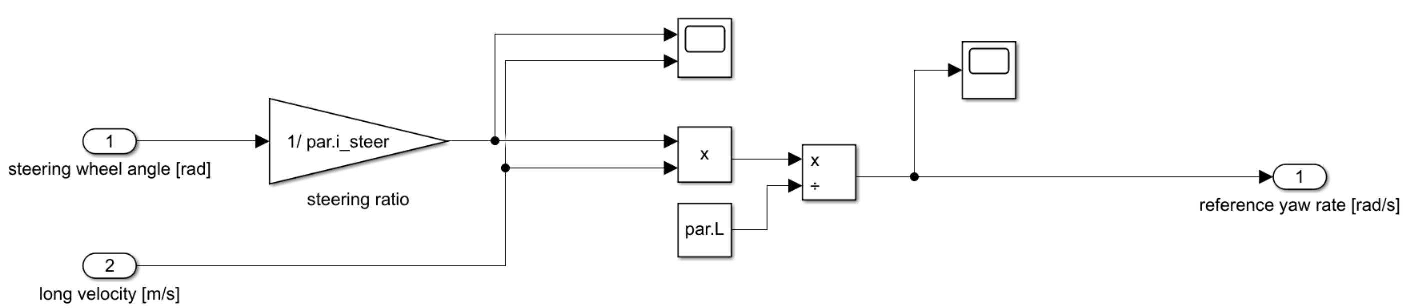


Fig. 1. The reference generator.

B. Feedback control based on PD controller

Then the feedback control based on PD controller is firstly designed to generate corrective control inputs based on the error between the reference yaw rate and the actual yaw rate of the vehicle. This controller takes the error of yaw rate as input and yaw moment as output, and combines the proportional part and derivative part to decrease the error of yaw rate. In Simulink, the model of PD controller is shown in Fig 2.

The PD-based with gain scheduling controller is employed to decrease the error of yaw rate to zero. At the same time the PD parameters K_P, K_D and K_N could be taken as the functions of error of yaw rate, which could be constructed as the piecewise affine functions in look-up tables. By doing so, the PD

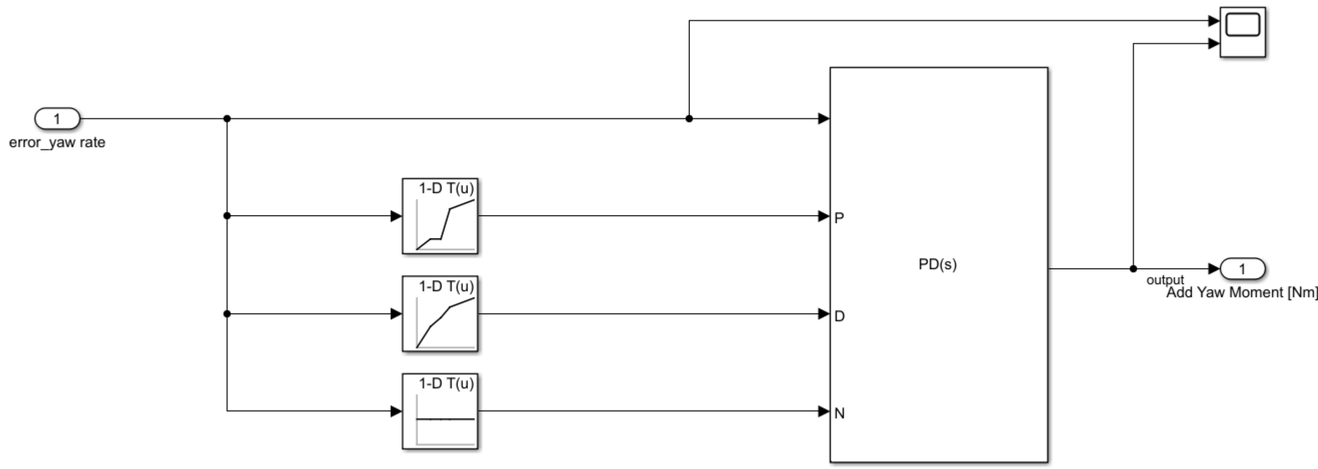


Fig. 2. The structure of PD controller.

parameters could be chosen as the suitable values at different intervals of error of yaw rate to have better control performance. K_P could help reduce steady-state errors and increase the speed of response, while K_D helps dampen the system's response. Based on that, the piecewise affine functions of K_P and K_D could be designed as increasing function of the error of yaw rate, which means when error of yaw rate is large, the values of K_P and K_D should also be large to increase the system's response and decease the steady-state errors. The Laplace mathematical form of PD controller is shown as:

$$P_{ym}(s) = (K_P + K_D \frac{K_N}{1 + \frac{K_N}{s}})E(s) \tag{2}$$

where $P_{ym}(s)$ and $E(s)$ are the Laplace transformation of yaw moment and the error of yaw rate respectively.

C. Feedback control based on LQR controller

In this section, the linear bicycle model is firstly thrown out:

$$\dot{x}(t) = Ax(t) + Bu(t), x_0(t) = x_0 \tag{3}$$

By taking the error of yaw rate as the state variable $x(t) = r(t)$ and constant vehicle speed (the dynamics of vehicle speed could be removed), linear bicycle model could be further represented as:

$$\dot{r}(t) = -\frac{l_f^2 C_{\alpha f} + l_r^2 C_{\alpha r}}{I_z u} r(t) + \frac{1}{I_z}\delta(t) \tag{4}$$

where l_f represents distance from front axle to CoG [m], $C_{\alpha f}$ represents front axle cornering stiffness, l_r represents distance from rear axle to CoG [m], $C_{\alpha r}$ represents rear axle cornering stiffness, I_z represents body inertia around z-axis kg$\cdot m^2$, u represents forward speed [m/s] and control input δ represents yaw moment [Nm]. These parameters are given in this homework which is shown in Appendix B.

LQR controller is the method to construct quadratic optimization problem to obtain the control policy. This optimization trade off tracking desired states and minimizing control effort. By using weighting matrices Q and R which reflect the penalties associated with different states and inputs, the form of LQR is represented as:

$$\min \int_{t_0}^{\infty} (u(t)^T Ru(t) + x(t)^T Qx(t))dt \tag{5}$$

where R is a positive definite weighting matrix and Q is a positive semi-definite weighting matrix. Then the optimal control gains are obtained by

$$K = -(R + B^T P_n B)^{-1} B^T P_n A \tag{6}$$

where P_n represents solution to algebraic Riccati equation:

$$P_n = A^T P_n A - A^T P_n B(R + B^T P_n B)^{-1} B^T P_n A + Q$$

Then the optimal control input could be represented as:

$$u(t) = -Kx(t) \tag{7}$$

In MATLAB, the **lqr** function could be used to calculate the K under different values of forward speed u. Then K multiply by $x(t)$ to get the optimal control input according equation II-C. Based on that, the look-up table could be made by setting different values of forward speed u, the code is listed as followed:

```
B=1/par.Izz;
Q=530000000;
R=0.01;
u_in=1:28;
K_lp=zeros(size(u_in));
for i=1:length(u_in)
    A=-(par.l_f^2*par.Calpha_front+par.l_r^2*par.Calpha_rear)/par.Izz/i;
    [K,S,P] = lqr(A,B,Q,R);
    K_lp(i)=K;
end
```

The model of LQR is shown in Fig 3. The variables of forward velocity and its corresponding K are taken as breakpoints and table data separately in the look-up table.

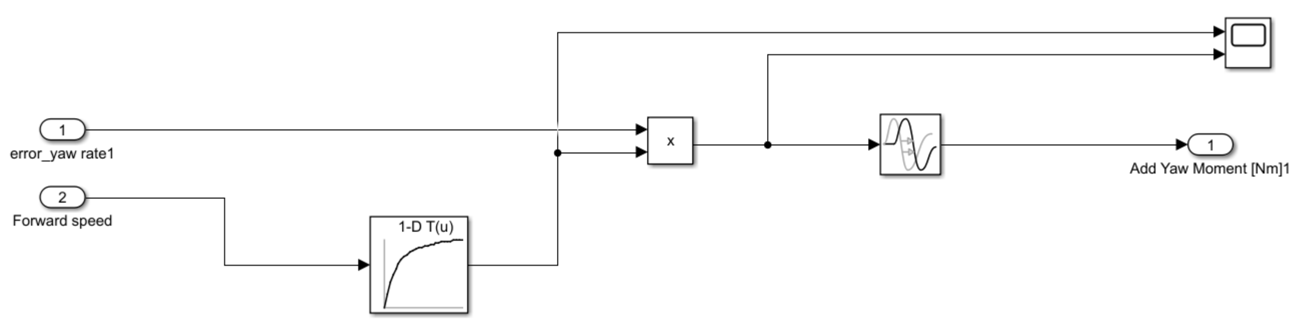


Fig. 3. The structure of LQR controller.

III. Simulation of Sine with Dwell manoeuvre

In this section, the simulation of the two dynamics controllers for Sine with Dwell manoeuvre at a high friction condition are analysed and compared.

A. Simulation of feedback control based on PD controller

1) Initial speed at 60 *km/h:* By using the gain scheduling, the parameters of PD controller are tuned to achieve the best performance [3]. K_p parameter is defined as the function of error of yaw rate, which is built by the 1-D Lookup Table with table data

$$[32000; 32000 * 1.2; 32000 * 1.2; 32000 * 1.3; 32000 * 1.3]$$

and breakpoints $[0; 0.02; 0.04; 0.06; 0.08]$. The breakpoints are designed according to the error of yaw rate.

Similarly, the K_D is built by the 1-D Lookup Table with table data

$$[32000; 32000 * 1.1; 32000 * 1.1; 32000 * 1.1; 32000 * 1.2]$$

and breakpoints$[0; 0.02; 0.04; 0.06; 0.08]$. The N parameter is set to be constant 100. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate are shown in Figs 4. It could be observed that the actual yaw rate could track the reference of yaw rate well except the extremum regions (around maximum and minimum values) of reference of yaw rate. Although continue to increase on the parameters of K_P and K_D could make the actual yaw rate expand its maximum and minimum values with small amount, the actual yaw rate still cannot fit the reference of yaw rate at the extremum values. By trading off the control performance against control effort (reflected by the PD gains), the values of K_P and K_D are chosen properly.

More specifically when the K_P is chosen as the look-up table

$$[64000; 64000 * 1.2; 64000 * 1.2; 64000 * 1.3; 64000 * 1.3]$$

and K_D is chosen as

$$[64000; 64000 * 1.1; 64000 * 1.1; 64000 * 1.1; 64000 * 1.2]$$

, the plots of reference yaw rate and actual yaw rate and the plot of error yaw rate using larger K_P and K_D are shown in Figs 5.

When tuning the PD parameters to the larger ones, the maximum and minimum values of actual rate increased from 0.71 to 0.74 and from -0.68 to -0.72 and the maximum magnitude of error of yaw rate is decreased from 0.75 to 0.44. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate for larger KP and KD are shown in Figs 5. While increasing the maximum and reducing the minimum values of the actual yaw rate and decreasing the amplitude of the yaw rate error can lead to some improvement in control performance, it is important to consider the values of the parameters in the PD controller. If the enhancement in control performance is only marginal, but it requires significantly larger values for the PD controller variables, it would be preferable to choose smaller values for the PD variables instead. In other words, if the gains of the PD controller need to be excessively increased to achieve a minor improvement in control performance, it is more advisable to opt for smaller values of the PD variables.

The process to obtain the optimal PD controller is shown in Table I, it shows that different parameters of PD controller and its corresponding maximum amplitude error of yaw rate in the steps to get the optimal parameters.

2) Initial speed at 100 *km/h:* By using the gain scheduling, the parameters of PD controller are tuned to achieve the best performance. K_p parameter is defined as the function of error of yaw rate, which is built by the 1-D Lookup Table with table data

$$[50000; 50000 * 1.1; 50000 * 1.1; 50000 * 1.2; 50000 * 1.2]$$

and breakpoints $[0; 0.3; 0.6; 0.9; 1.2]$. The breakpoints are designed according to the error of yaw rate.

Similarly, the K_D is built by the 1-D Lookup Table with table data

$$[50000; 50000 * 1.1; 50000 * 1.2; 50000 * 1.2; 50000 * 1.2]$$

and breakpoints $[0; 0.3; 0.6; 0.9; 1.2]$. The N parameter is set to be constant 100. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate are shown in Figs 6.

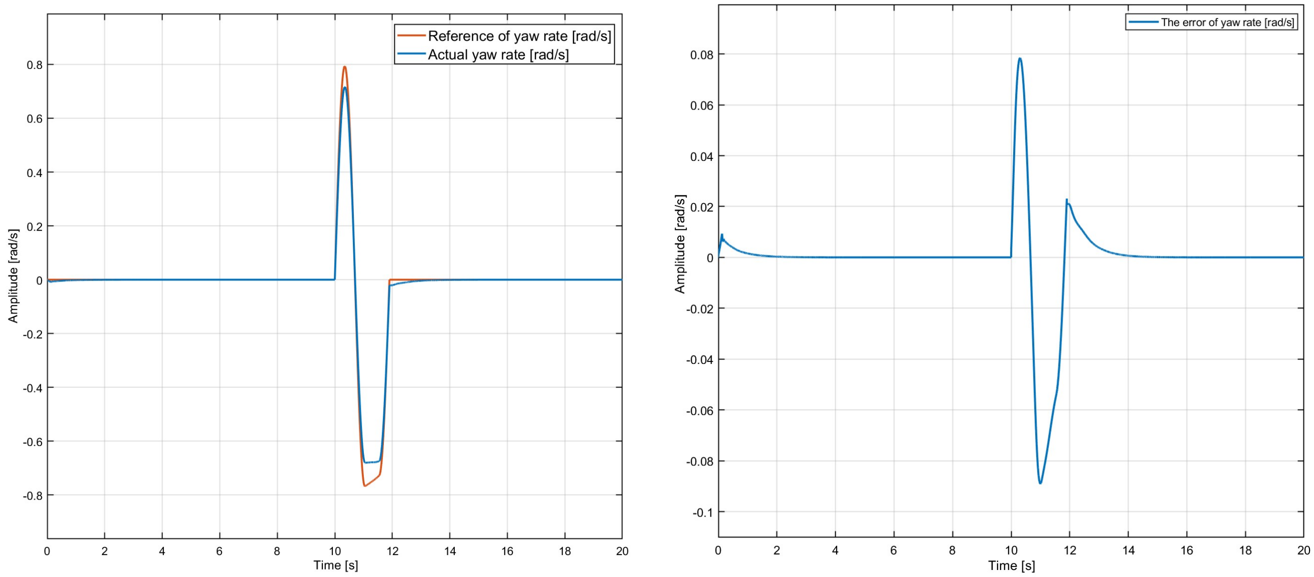


Fig. 4. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate for chosen K_P and K_D.

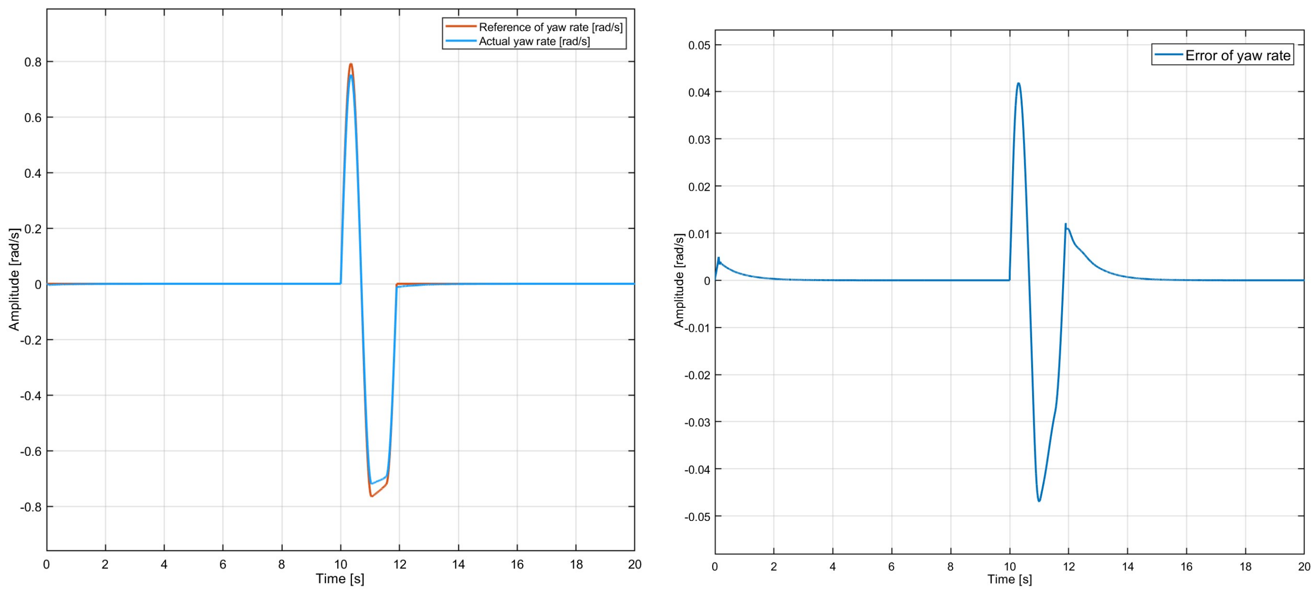


Fig. 5. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate for larger K_P and K_D.

The maximum error in the yaw rate is measured at 0.065 rad/s. Overall, the actual yaw rate tracks the reference signal effectively apart from the spikes in occurrence if the maximum and minimum regions of the reference of yaw rate. When the yaw rate error recovers from the largest negative values and approaches zero, It was noticed that the convergence is relatively slow, resulting in a prolonged tail where the value remains close to zero but not exactly reaching it.

The process to obtain the optimal PD controller is shown in Table II, it shows that different parameters of PD controller and its corresponding maximum amplitude error of yaw rate in the steps to get the optimal parameters.

Steps	K_P	K_D	K_N	Maximum error of yaw rate [rad/s]
1	1	1	100	0.6271
2	100	100	100	0.5987
3	1000	100	100	0.5943
4	1000	1000	100	0.4384
5	3000	1000	100	0.4284
6	3000	3000	100	0.3495
7	10000	10000	100	0.1957
8	30000	30000	100	0.0836
9	32000	32000	100	0.079
10	Set PWA function	Set PWA function	100	0.0754

TABLE I
DIFFERENT PARAMETERS OF PD CONTROLLER AND CORRESPONDING MAXIMUM AMPLITUDE OF ERROR OF YAW RATE OF INITIAL SPEED 60 KM/H.

Steps	K_P	K_D	K_N	Maximum error of yaw rate [rad/s]
1	1	1	100	0.9625
2	100	100	100	0.9488
3	1000	100	100	0.9185
4	1000	1000	100	0.8339
5	5000	5000	100	0.4878
6	10000	10000	100	0.3082
7	50000	50000	100	0.0765
8	55000	55000	100	0.0699
9	Set PWA function	Set PWA function	100	0.065

TABLE II
DIFFERENT PARAMETERS OF PD CONTROLLER AND CORRESPONDING MAXIMUM AMPLITUDE OF ERROR OF YAW RATE OF INITIAL SPEED 100 KM/H.

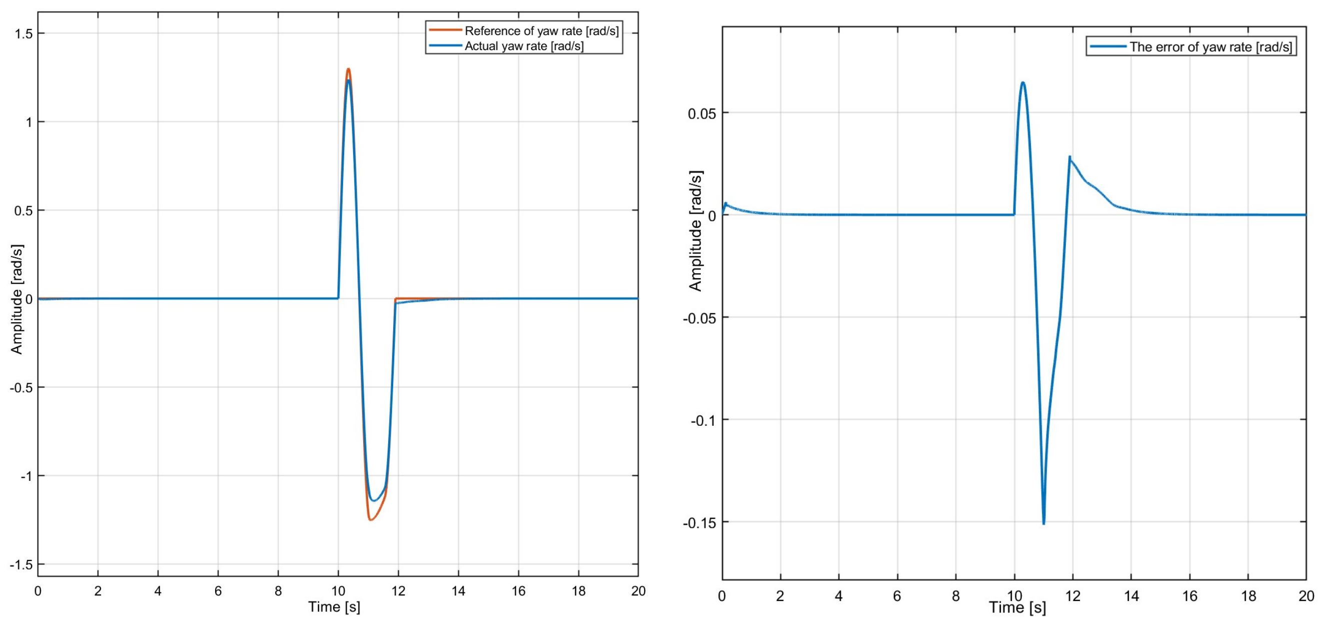


Fig. 6. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate for larger K_P and K_D of initial speed 100 km/h.

B. *Simulation of feedback control based on LQR controller*

1) Initial speed at 60 *km/h:* In this case weighting matrices R and Q are chosen as $R = 0.01$ and $Q = 530000000$, the plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate are shown in Figs 7. The maximum error in the yaw rate is measured at 0.0724 rad/s. Overall, the actual yaw rate tracks the reference signal effectively, even in the presence of spikes in the maximum and minimum regions. The only exception is at $t = 0$s, where the yaw rate exhibits minimal variation around zero.

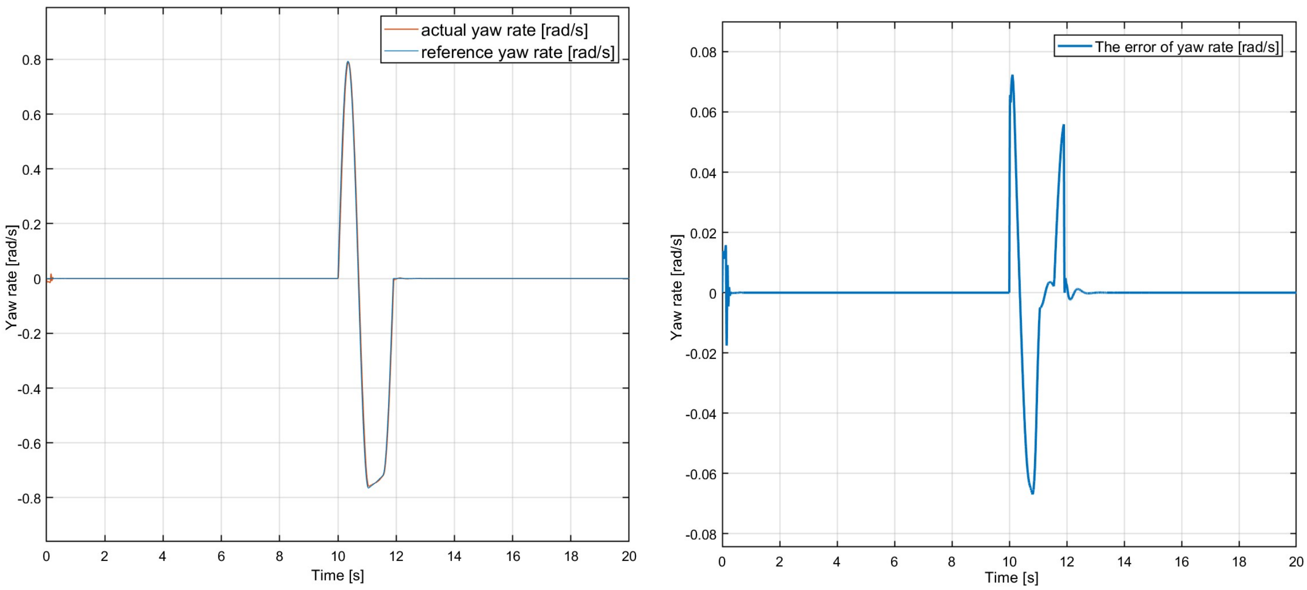


Fig. 7. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate with initial speed 100 km/h.

The process to obtain the optimal LQR controller is shown in Table III, it shows that different values of weighting matrices R and Q of LQR controller and corresponding maximum amplitude of error of yaw rate in the steps to get the optimal parameters.

Steps	Q	R	Maximum error of yaw rate [rad/s]
1	1	1	0.6274
2	100	1	0.6274
3	10000	1	0.6274
4	1000000	1	0.6273
5	100000000	1	0.6218
6	500000000	1	0.5765
7	500000000	0.1	0.2392
8	500000000	0.01	0.0746
9	530000000	0.01	0.0724

TABLE III

DIFFERENT VALUES OF WEIGHTING MATRICES R AND Q OF LQR CONTROLLER AND CORRESPONDING MAXIMUM AMPLITUDE OF ERROR OF YAW RATE UNDER INITIAL SPEED 60KM/H.

2) Initial speed at 100 *km/h:* In this case weighting matrices R and Q are chosen as $R = 0.05$ and $Q = 10000000000$, the plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate are shown in Figs 8. The maximum error of yaw rate is 0.0821 rad/s and the actual yaw rate track the reference well including spikes in the maximum and minimum regions. At $t = 0$s and $t = 12$s, the yaw rate exhibits minimal variation around zero. In general, the LQR controller demonstrates excellent tracking performance, effectively capturing and accommodating spikes in the data. However, unavoidable variations persist, which may have a slight impact on the overall tracking performance.

The process to obtain the optimal LQR controller is shown in Table IV, it shows that different values of weighting matrices R and Q of LQR controller and corresponding maximum amplitude of error of yaw rate in the steps to get the optimal parameters.

Steps	Q	R	Maximum error of yaw rate [rad/s]
1	1	1	0.9627
2	100	1	0.9627
3	10000	1	0.9627
4	1000000	1	0.9619
5	100000000	1	0.8941
6	1000000000	1	0.7264
7	10000000000	1	0.2421
8	10000000000	0.1	0.092
9	10000000000	0.05	0.0821

TABLE IV

DIFFERENT VALUES OF WEIGHTING MATRICES R AND Q OF LQR CONTROLLER AND CORRESPONDING MAXIMUM AMPLITUDE OF ERROR OF YAW RATE UNDER INITIAL SPEED 100 KM/H.

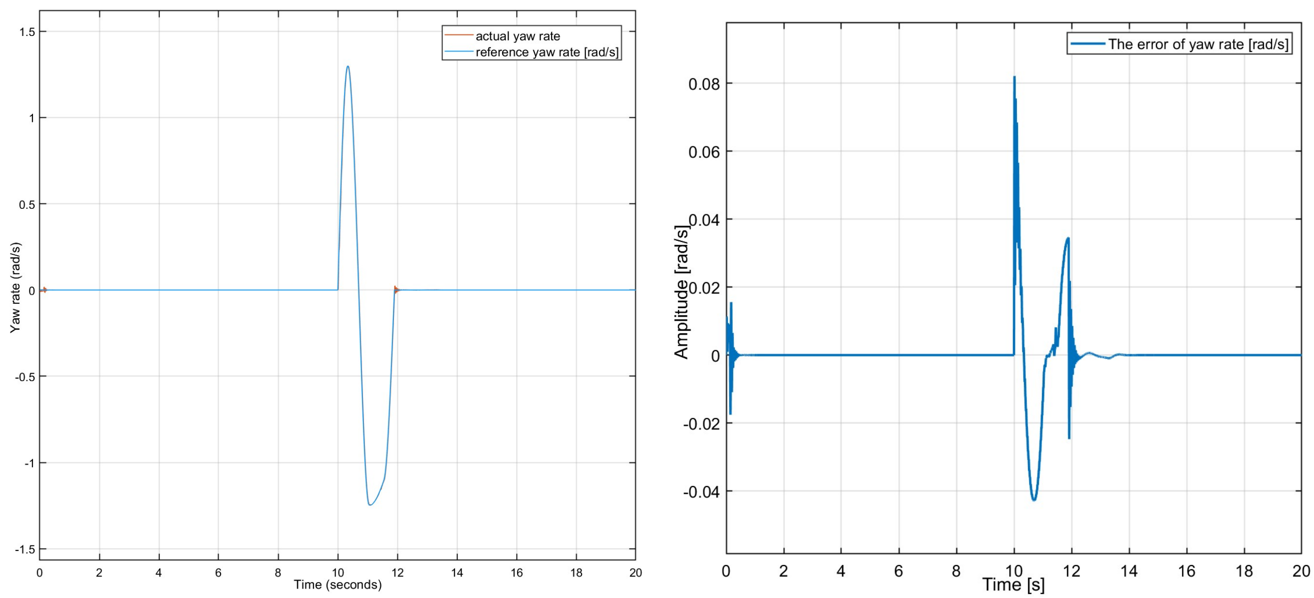


Fig. 8. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate.

IV. CONTROLLER PERFORMANCE WITH SENSOR NOISE

In this part, the noise for yaw rate of 1 deg/sec is applied to PD based and LQR based vehicle stability control system. The noises could be taken as the uniform random noises and implemented to the system by adding the 'Uniform Random Number' blocks in Simulink. It is worth mentioning that the unit of the noise should be converged from deg/s to rad/s. (1 deg/s 0.01745 rad/s)

A. *Simulation of feedback control based on PD controller with sensor noise*

1) Initial speed at 60 *km/h:* Tn the noised system, K_p parameter is defined as the function of error of yaw rate, which is built by the 1-D Lookup Table with table data

$$[55000; 55000 * 1.1; 55000 * 1.1; 55000 * 1.2; 55000 * 1.2]$$

and breakpoints $[0; 0.03; 0.06; 0.09; 0.12]$. The breakpoints are designed according to the error of yaw rate.

Similarly, the K_D is built by the 1-D Lookup Table with table data

$$[55000; 55000 * 1.1; 55000 * 1.2; 55000 * 1.2; 55000 * 1.2]$$

and breakpoints$[0; 0.03; 0.06; 0.09; 0.12]$. The N parameter is set to be constant 100. In Figs 9, the plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate are shown. It is easily observed that the noise in the actual yaw rate is conspicuous in the regions with the yaw rate around zero and the spike region in occurrence of the maximum and minimum value regions. Apart from these two parts, the actual yaw rate could track the reference of yaw rate efficiently and the noise could be eliminated to large extent. The maximum amplitude of the error of yaw rate 0.066 rad/s. Compared with the no noise case, the K_P and K_D parameters are chosen much larger than those used in non noise case.

The process to obtain the optimal PD controller is shown in Table V, it shows that different parameters of PD controller and its corresponding maximum amplitude error of yaw rate in the steps to get the optimal parameters. Then the optimal parameters are chosen and are shown in Figs 9.

Steps	K_P	K_D	K_N	Maximum error of yaw rate [rad/s]
1	1	1	100	0.6423
2	100	100	100	0.6141
3	1000	1000	100	0.4545
4	10000	10000	100	0.2173
5	50000	50000	100	0.0806
6	55000	55000	100	0.0757
7	Set PWA function	Set PWA function	100	0.066

TABLE V

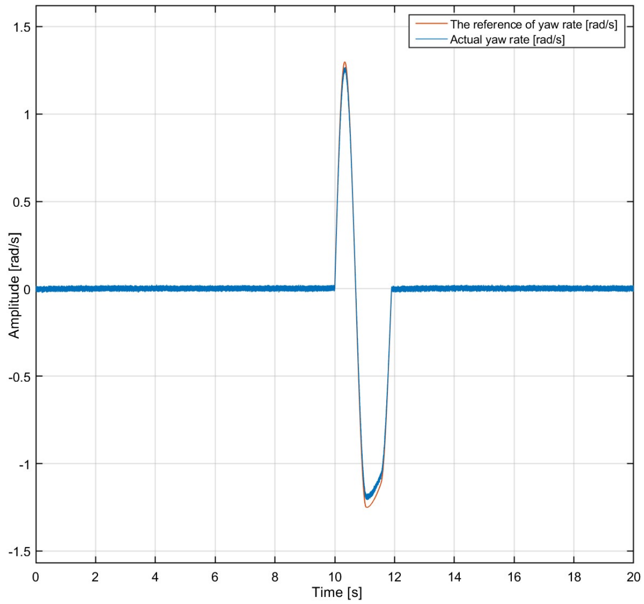
DIFFERENT PARAMETERS OF PD CONTROLLER AND CORRESPONDING MAXIMUM AMPLITUDE OF ERROR OF YAW RATE OF INITIAL SPEED 60 KM/H WITH SENSOR NOISE.

2) Initial speed at 100 *km/h:* The process to obtain the optimal PD controller is shown in Table VI, it shows that different parameters of PD controller and its corresponding maximum amplitude error of yaw rate in the steps to get the optimal parameters.

Steps	K_P	K_D	K_N	Maximum error of yaw rate [rad/s]
1	1	1	100	0.9788
2	100	100	100	0.9657
3	1000	1000	100	0.8521
4	10000	10000	100	0.3296
5	50000	50000	100	0.1041
6	80000	80000	100	0.0676
7	Set PWA function	Set PWA function	100	0.0604

TABLE VI

DIFFERENT PARAMETERS OF PD CONTROLLER AND CORRESPONDING MAXIMUM AMPLITUDE OF ERROR OF YAW RATE OF INITIAL SPEED 100 KM/H WITH SENSOR NOISE.



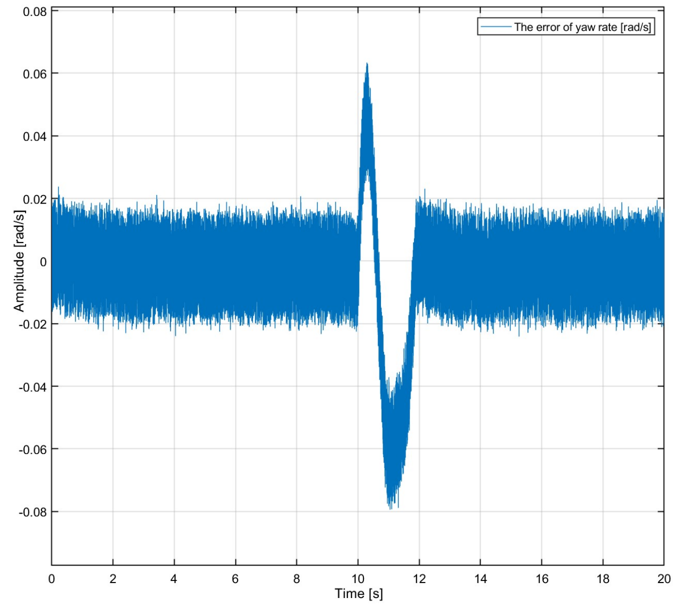


Fig. 9. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate at initial speed 60km/h with sensor noise.

K_p parameter is defined as the function of error of yaw rate, which is built by the 1-D Lookup Table with table data

$$[80000; 80000 * 1.2; 80000 * 1.2; 80000 * 1.3; 80000 * 1.3]]$$

and breakpoints $[0; 0.02; 0.04; 0.06; 0.08]$. The breakpoints are designed according to the error of yaw rate.

Similarly, the K_D is built by the 1-D Lookup Table with table data

$$[80000; 80000 * 1.1; 80000 * 1.1; 80000 * 1.1; 80000 * 1.2]$$

and breakpoints$[0; 0.02; 0.04; 0.06; 0.08]$. The N parameter is set to be constant 100.

In Figs 10, it is readily apparent that the noise in the actual yaw rate becomes particularly prominent and not eliminated largely in regions where the yaw rate hovers around zero and in the occurrence of spike regions associated with maximum and minimum values. However, excluding these two segments, the actual yaw rate demonstrates efficient tracking of the reference yaw rate, and the noise can be significantly reduced. The maximum error amplitude of the yaw rate amounts to 0.0604 rad/s. In comparison to the scenario without noise, significantly larger K_P and K_D parameters are selected for this case.

B. *Simulation of feedback control based on LQR controller with sensor noise*

1) Initial speed at 60 *km/h:* In this case weighting matrices R and Q are chosen as $R = 0.01$ and $Q = 2000000000$, the plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate are shown in Figs 11. The maximum error in the yaw rate is measured at 0.0681 rad/s. Overall, the actual yaw rate tracks the reference signal effectively. The regions characterized by the yaw rate hovering around zero and the occurrence of spike regions associated with maximum and minimum values exhibit existing noise with a relatively small amplitude.

The process to obtain the optimal LQR controller is shown in Table VII, it shows that different values of weighting matrices R and Q of LQR controller and corresponding maximum amplitude of error of yaw rate in the steps to get the optimal parameters.

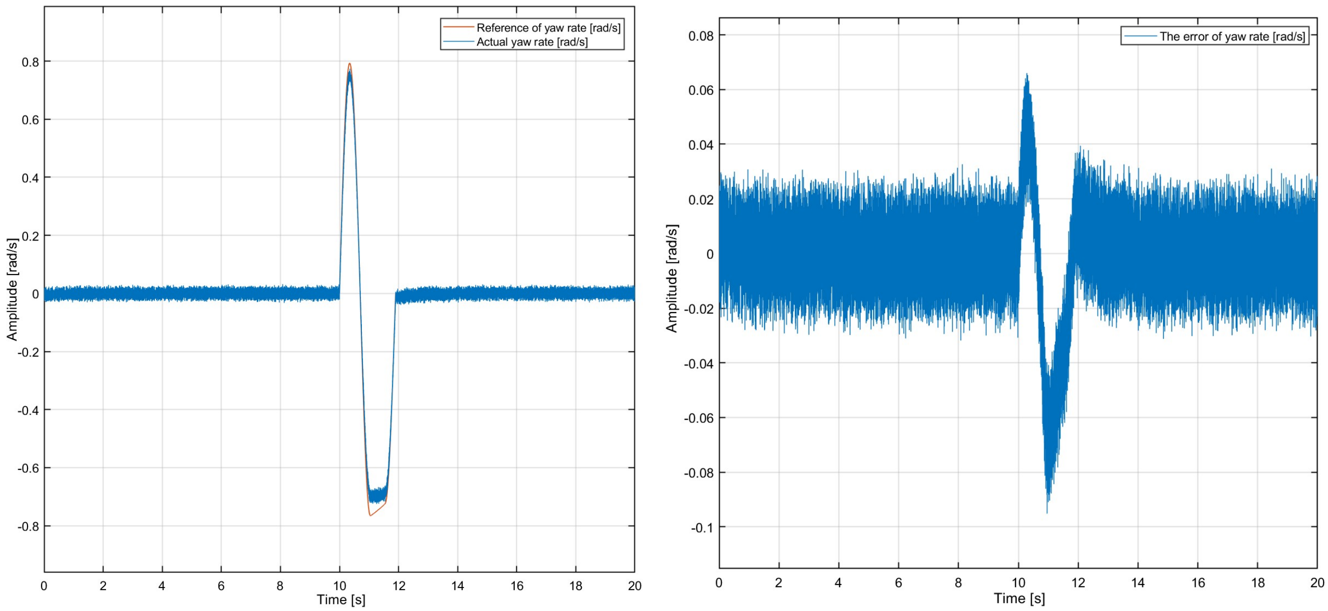


Fig. 10. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate at initial speed 60km/h with sensor noise.

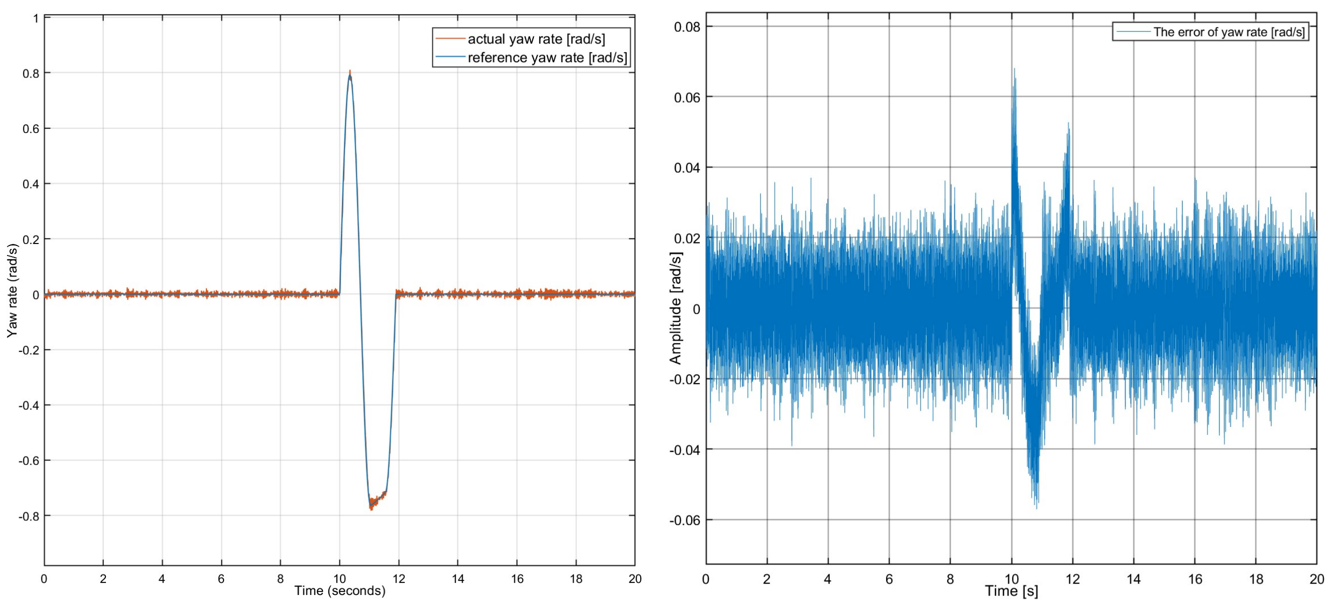


Fig. 11. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate at initial speed 60km/h with sensor noise.

2) Initial speed at 100 *km/h:* In this case weighting matrices R and Q are chosen as $R = 0.05$ and $Q = 10000000000$, the plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate are shown in Figs 12. The maximum error of yaw rate is 0.0918 rad/s and the actual yaw rate track the reference well except noise left in yaw rate hovering around zero spikes and yaw rate with maximum and minimum value regions. In general, the LQR controller demonstrates excellent tracking performance, effectively capturing and accommodating spikes in the data. However, unavoidable noise persist in some regions, which may have a slight impact on the overall tracking performance.

Steps	Q	R	Maximum error of yaw rate [rad/s]
1	1	1	0.6426
2	100	1	0.6426
3	10000	1	0.6426
4	1000000	1	0.6426
5	100000000	1	0.6372
6	1000000000	1	0.5208
7	1000000000	0.1	0.1830
8	1000000000	0.01	0.0705
9	2000000000	0.01	0.0681

TABLE VII

DIFFERENT VALUES OF WEIGHTING MATRICES R AND Q OF LQR CONTROLLER AND CORRESPONDING MAXIMUM AMPLITUDE OF ERROR OF YAW RATE UNDER INITIAL SPEED 60KM/H WITH SENSOR NOISE.

The process to obtain the optimal LQR controller is shown in Table VIII, it shows that different values of weighting matrices R and Q of LQR controller and corresponding maximum amplitude of error of yaw rate in the steps to get the optimal parameters.

The maximum error in the yaw rate is measured at 0.0681 rad/s. Overall, the actual yaw rate tracks the reference signal effectively. The regions characterized by the yaw rate hovering around zero and the occurrence of spike regions associated with maximum and minimum values exhibit existing noise with a relatively small amplitude.

Steps	Q	R	Maximum error of yaw rate [rad/s]
1	1	1	0.9789
2	100	1	0.9789
3	10000	1	0.9789
4	1000000	1	0.9781
5	100000000	1	0.9070
6	1000000000	1	0.7415
7	10000000000	1	0.2569
8	10000000000	0.1	0.1011
9	10000000000	0.05	0.0918

TABLE VIII

DIFFERENT VALUES OF WEIGHTING MATRICES R AND Q OF LQR CONTROLLER AND CORRESPONDING MAXIMUM AMPLITUDE OF ERROR OF YAW RATE UNDER INITIAL SPEED 100 KM/H WITH SENSOR NOISE.

V. SELF REFLECTION

Firstly, the most difficult part is about how to interpret the bicycle model and relation between vehicle parameters and how the vehicle model functions together. This takes me long time to have the basic idea for it. After doing it, the difficulty is to design the PD controllers and tune the parameters, by trying thousands times, it could be found that the constant values of PD controller do not have the good performance, thus the piecewise affine function should be used to enhance the performance of PD controller. But it also takes me so much time the choose the optimal pair of turning points of PD parameters. Then learning the LQR strategy is also a big challenge. Even this task is not easy for me, still do I understood the vehicle stability control. I am glad that effort leads to great harvest and I will keep on exploring the course of vehicle dynamics and control.

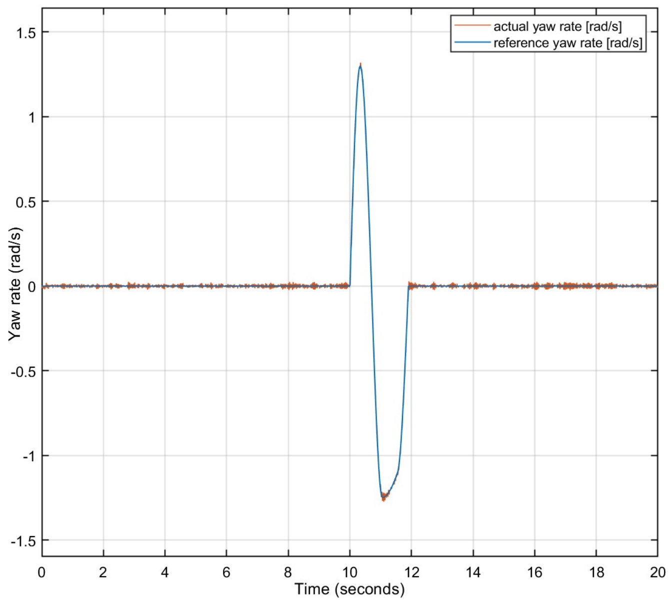


Fig. 12. The plots of reference yaw rate and actual yaw rate, and the plot of error of yaw rate of initial speed 100 km/h with sensor noise.

VI. Conclusions

In this homework, two vehicle stability controllers are designed and applied to control the actual yaw rate track the reference of yaw rate.

The tracking performance of the yaw rate can have a significant impact on the stability of a vehicle. If the tracking of the yaw rate is poor, it can lead to instability and compromised handling. In this case, it may be difficult to control the vehicle to maintain the desired direction of the vehicle during cornering or sudden maneuvers.

By analysis, LQR shows an excellent tracking performance compared with PD controller. The actual yaw rate of PD controller could not track the reference yaw rate fully in the regions in the occurrence of spike regions associated with maximum and minimum values, while LQR demonstrates the ability to effectively track the reference yaw rate in these challenging regions.

Furthermore, when the vehicle starts at a higher initial speed, the PD controller's K_P and K_D parameters, as well as the LQR controller's parameters Q and R, are intentionally selected to be larger compared to the scenario with a lower initial speed. Additionally, even under the same initial speed, when there is noise present in the yaw rate, the PD and LQR controllers necessitate larger parameter values to effectively handle the dynamic challenges and track the yaw rate.

In noised case, in PD and LQR controller the noise in the actual yaw rate is not eliminated fully in regions where the yaw rate hovers around zero and in the occurrence of spike regions associated with maximum and minimum values. However, excluding these two segments, the actual yaw rate demonstrates efficient tracking of the reference yaw rate, and the noise can be significantly reduced. What is more, LQR shows stronger robustness to noise because in the regions where the yaw rate hovers around zero and in the occurrence of spike regions associated with maximum and minimum values, the actual yaw rate has existing noise with smaller amplitude.

In conclude, LQR shows better tracking performance and better robustness to noise compared with PD controller which could make timely adjustments and maintain the desired trajectory to ensure vehicle stability.

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APPENDIX

A. Simulink block diagrams.

All the Simulink block diagrams have the same block diagram of simplified vehicle model in Fig 13. The only different parts are different designs of 'Your WSC controller' which are already shown before.

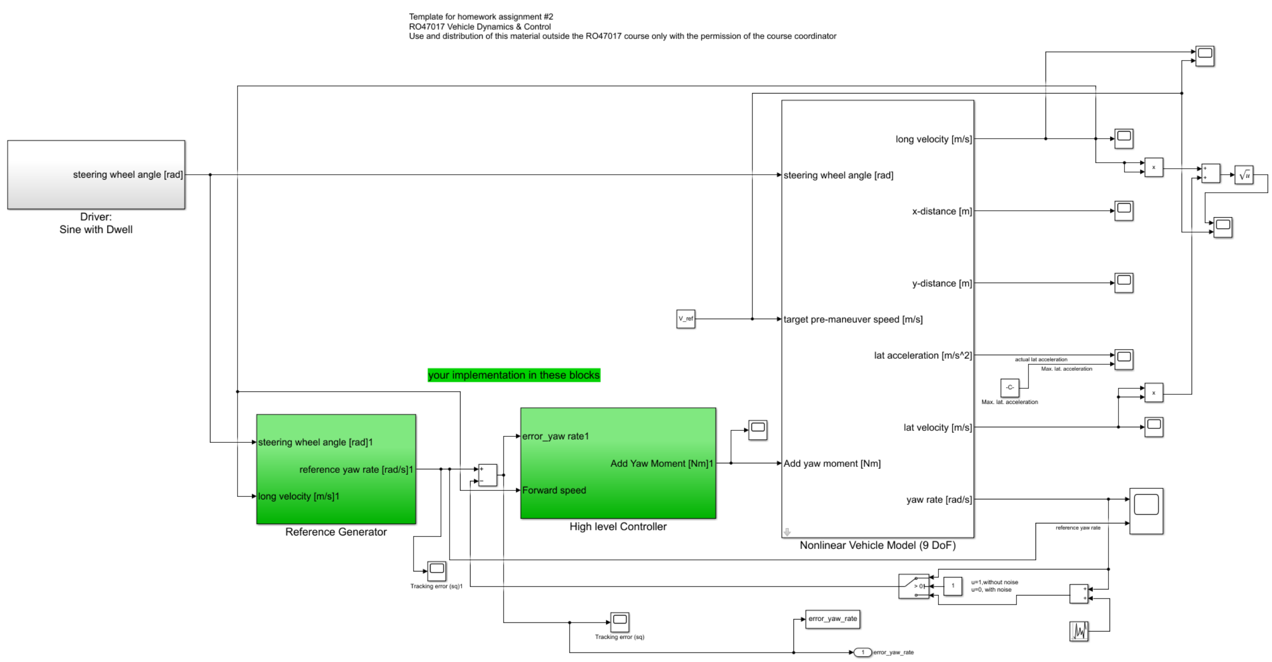

Fig. 13. The block diagram of simplified vehicle model with controller.

B. Vehicle parameters.

Vehicle parameters	Values
Initialization velocity	50 km/h
Vehicle mass	1380 kg
Body inertia around z-axis	2634.5 kgm^2
Wheelbase	2.79 m
Distance from front axle to CoG	1.384 m
Steering ratio	15.4
Front axle cornering stiffness	120000
Rear axle cornering stiffness	190000
Friction coefficient	1

TABLE IX
VEHICLE PARAMETERS.