Vehicle Dynamics and Control

RO47017 Homework 1

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I. INTRODUCTION

In this homework, the wheel slip control and mixed-slip-deceleration control were explored. MF-Tyre/MF-Swift 6.2.0.4 is implemented to MALTAB according to Installation Guide [1]. To use the MF-Tyre/MF-Swift model with MATLAB Simulink, the ”Toolbox” directory has to be on the MATLAB search path. Typically this directory has the name: **C:Delft-Tyre-Tyre MF-Swift 6.2.0.4**. After doing that, the model of tyre dynamics could be activated in Simulink. After that the two wheel controllers, wheel slip controller and mixed-slip-deceleration controller are designed and analysed. By comparing the braking performance of these two controllers, the conclusion of selection of controller in different scenarios and explain why the oscillations are desired in reference tracking of the wheel slip.

II. DESIGN OF WHEEL SLIP CONTROLLER

In this section, the focus is put on the design of a wheel slip controller and the analysis of simulation satisfying three different scenarios is made inluding the braking distance.

The definition of the longitudinal slip is firstly thrown out as the longitudinal response of a tire which is mainly generated by a relative motion between tire elements in contact with the road surface and tire body which causes slippage between the tire and the road [2]. In our case, the vehicle is of the braking state, thus the mathematical form of wheel slip could be written as:

$$\kappa = \frac{V_x - \omega_w r_e}{V_x} \tag{1}$$

where r_e represents effective rolling radius, V_x represents chassis velocity and $\omega_w r_r$ represents the wheel speed. The main concept of wheel slip controller is to regulate the calliper pressure such that the error of reference wheel slip κ_{ref} and wheel slip κ converges to a limit cycle in the optimal zone despite initial condition, disturbances, environmental conditions, etc. Based on that, the PID-based with gain scheduling controller could be determined as:

$$p_{br} = K_p(V_x)(\kappa_{ref} - \kappa) + K_i(V_x)\int(\kappa_{ref} - \kappa)dt + K_i(V_x)\frac{\mathrm{d}(\kappa_{ref} - \kappa)}{\mathrm{d}t} \tag{2}$$

By designing the PID-based with gain scheduling controller, the wheel slip error $\kappa_{ref} - \kappa$ could converge to zero, which means the wheel slip could track the reference wheel slip. At the same time the PID parameters K_p, K_i and K_d could be taken as the functions of V_x, which could be constructed as the piecewise affine functions. By doing so, the PID parameters could be chosen as the suitable values at different domains of V_x to have better control performance.

The following conditions are needed to be satisfied to design the wheel slip controller:

1) The type of surface wet asphalt $\mu = 0,6$.
2) The braking manoeuvre starting after 2 sec of free rolling.
3) Starting from 120 km/h until the speed of 10 km/h.

These conditions are already statisfied in the given files. First condition is satisfied by defining

```
par.friction = 0.6;       % Friction coefficient [-]
```

. The second condition is defined by define the reference pressure input as the ramp input followed by saturation, which has the start time $t = 2$s.

The third condition is to define the initial speed and minimal speed as:

```
par.V0 = 120/3.6;         % Initial speed, m/s
par.Vmin = 10 / 3.6;      % Minimal speed to stop simulation, m/s
```

A. The structure of wheel slip controller

The structure of the wheel slip controller is shown in Fig 1. The mathematical form of PID controller is shown as:

$$P_{br}(s) = (K_p + \frac{K_i}{s} + K_i \frac{N}{1+\frac{N}{s}})E(s) \tag{3}$$

where $P_{br}(s)$ and $E(s)$ are the Laplace transformation of $p_{br}(t)$ and $\kappa_{ref} - \kappa$ respectively.

In the wheel slip controller, the K_p parameter is defined as the the function of V_x, $K_p = f_1(V_x)$, which is built by the 1-D Lookup Table with table data

$$[110; 110 * 1.2; 110 * 1.2; 110 * 1.8; 110 * 2]$$

and breakpoints $[0; 5; 15; 25.3; 30]$ m/s. The breakpoints are designed according to the domain of speed, it was mentioned before the initial speed is $120/3.6 = 33.3333$ m/s and the minimal speed is $10/2.6 = 2.7778$ m/s. Thus the turning points of Piecewise affine functions of K_p to V_x should be in the domain $[33.3333, 2.7778]$. When V_x is large, which means the chassis velocity is closer to the initial speed, the $K_p = f_1(V_x)$ is turned to be large because at the start of the braking process, the error of wheel slip is relatively large, which needs larger K_p to accelerate adjustment and reduce slip error. When V_x is small, the velocity is closer to the minimal speed, which require smaller K_p to guarantee the stability of wheel slip control system.

Similarly, the K_i parameter of wheel slip controller is is defined as the the function of V_x, $K_i = f_2(V_x)$, which is built by the 1-D Lookup Table with table data

$$[70 * 0.9; 70 * 1.1; 70 * 1.1; 70 * 1.2; 70 * 1.5]$$

and breakpoints $[0; 5; 15; 25; 30]$ m/s. When V_x is larger, the $K_i = f_2(V_x)$ is larger as well, which leads to stronger integration effect to decrease the wheel slip error at the beginning of braking. When the V_x is smaller, the $K_i = f_2(V_x)$ is tuned to be smaller. This is to eliminate steady-state error and improve error tolerance in the system. Due to errors, Integral adjustment is carried out until there is no difference in which case the integral adjustment stops and outputs a constant value [3].

The K_d parameter is defined as the the function of V_x, $K_p = f_3(V_x)$, which is built by the 1-D Lookup Table with table data $[0.7; 0.8; 0.9; 0.9; 0.7]$ and breakpoints $[0; 10; 15; 25; 30]$ m/s. For the start and end of braking, the K_d is set to be small to increase the stability of system, in the middle of braking process, the K_d is turned on to reduce the overshoot and adjustment time can be reduced. The N parameter is set to be constant 100. The process to obtain the optimal PID controller is shown in Table I, it shows that different parameters of wheel slip controller and its corresponding stopping distance in the steps to get the optimal parameters.

B. The analysis of simulative results

According to the former design of wheel slip controller, the brake pressure is shown in Fig 2 sub-picture 1. In this figure, it is easily observed that the brake pressure started from 35 bar and converged to 70 bar, during this process the maximal brake pressure and minimal brake pressure reached 150 bar and -40 bar respectively and the amplitude of oscillation decreased from 95 bar to 10 bar, which indicates the good stability of wheel slip control system.

The wheel slip is shown in Fig 2 sub-picture 2. The wheel slip reference is 0.12, and the wheel slip converged to the wheel slip reference at time 7.269s, the amplitude of oscillation decreased from 1.1 to 0.1 bar, which indicates the good stability and convergence of wheel slip control system and verifies that there is no steady error of wheel slip.

Steps	K_p	K_i	K_d	Stopping distance [m]
1	0	1	0	210
2	0	10	0	208
3	0	100	0	230
4	0	200	0	235
5	0	70	0	220
6	1	70	0	220
7	10	70	0	220
8	100	70	0	208
9	110	70	0	205
10	110	70	0.5	200
11	Set PWA function	Set PWA function	Set PWA function	145

TABLE I

DIFFERENT PARAMETERS OF WHEEL SLIP CONTROLLER AND CORRESPONDING STOPPING DISTANCE IN DIFFERENT STEPS.

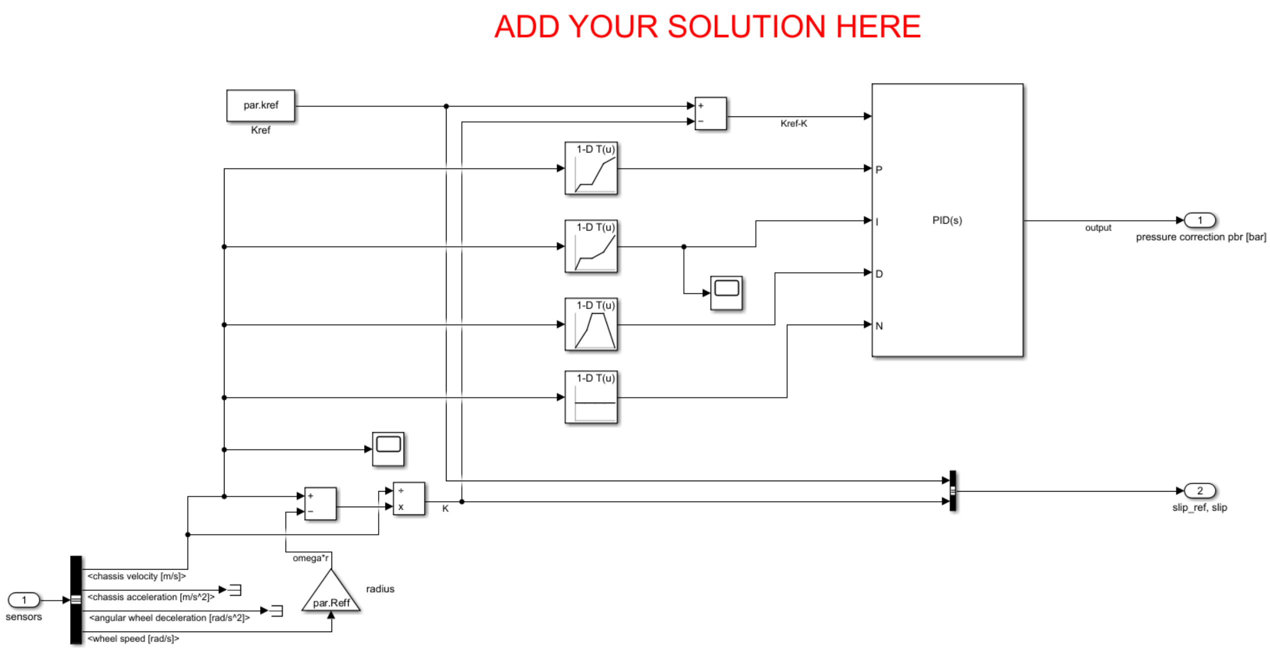

Fig. 1. The wheel slip controller

In Fig 3 sub-picture 1, the braking distance is growing from 0 to around 145 m. The chassis velocity gradually decreased from 33.3333 m/s to 2.778 m/s without the oscillation which shows the good flatness of the braking process. The wheel speed decreased from 110 m/s to the 10 rad/s with oscillation of which the amplitude of oscillation decreased to 0. The chassis acceleration keeps the value of 0 which verifies the flatness and smoothness of the braking process. The angular wheel deceleration converged to 0 at time 6.6s, meaning that the change of wheel speed is 0, which means after 6.6s, the wheel speed smoothly decreased to 10 rad/s.

III. DESIGN OF A MIXED-SLIP-DECELERATION CONTROLLER

In this section, the mixed-slip-deceleration controller is employed to the simplified vehicle model. The mixed-slip-deceleration controller is the combination of wheel slip controller and wheel deceleration. Wheel deceleration control uses the wheel deceleration signal $\dot{\omega}$ to predict if the wheel is about to lock

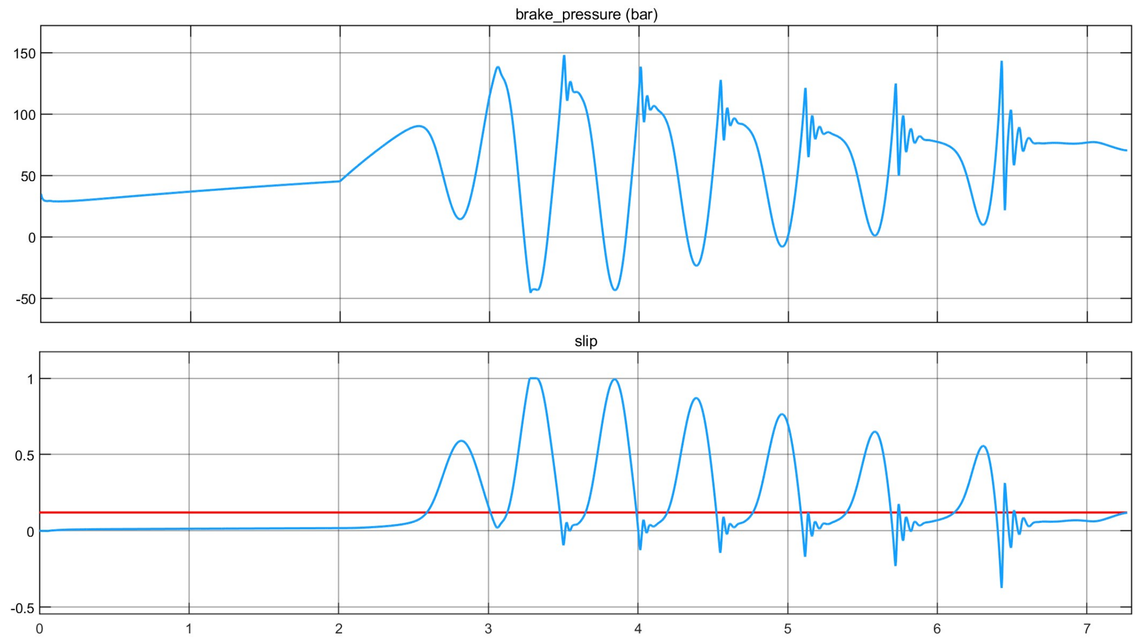

Fig. 2. Plots of brake pressure, wheel slip compared with wheel speed reference in the wheel slip control system.

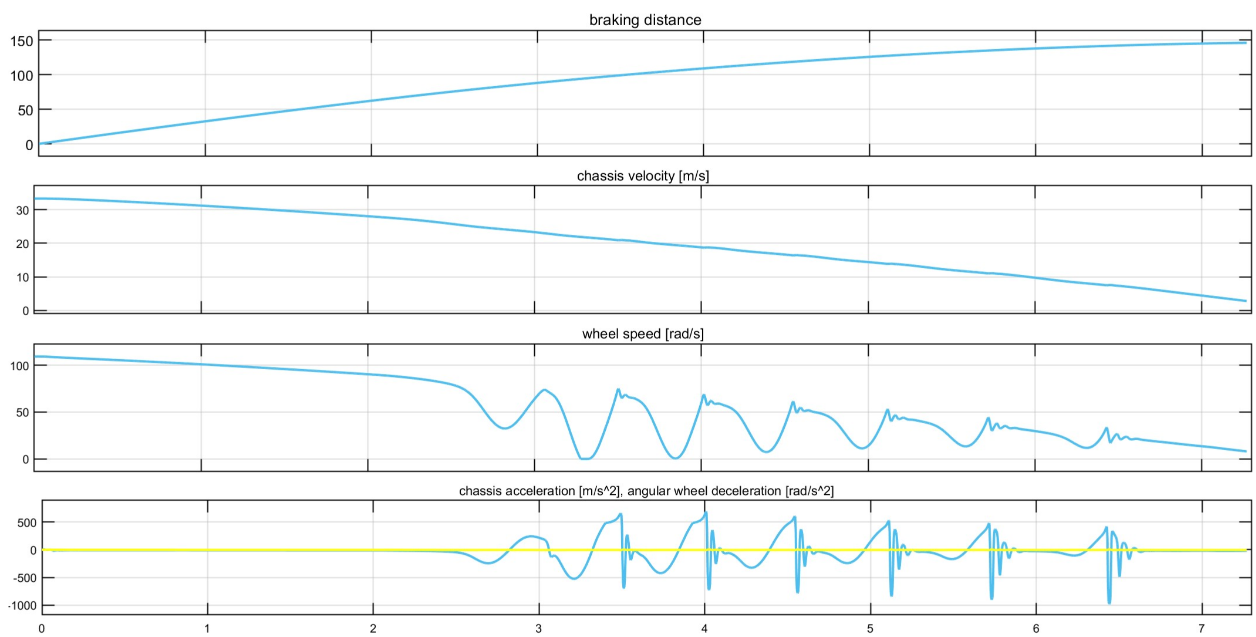

Fig. 3. Plots of brake distance [m], chassis velocity [m/s], wheel speed [rad/s], chassis acceleration [m/s^2] and angular wheel deceleration [rad/s^2] in the wheel slip control system.

and could make wheel acceleration converge to a limit cycle in the optimal zone by tuning the calliper pressure.

In this assignment, the mixed-slip-deceleration controller is chosen as PID type and could be written as:

$$p_{br} = K_p(V_x)(\kappa_{ref} - \kappa) + K_i(V_x) \int (\kappa_{ref} - \kappa)dt + K_d(V_x)\frac{r_w}{g}\dot{\omega} \tag{4}$$

A. *The structure of mixed-slip-deceleration controller*

The structure of mixed-slip-deceleration controller is shown in Fig 4. In this design, two PID blocks were used due to different input signals of mixed-slip-deceleration controller. The first PID block is used to represent $K_p(V_x)(\kappa_{ref} - \kappa) + K_i(V_x)\int(\kappa_{ref} - \kappa)dt$. This block is set to PI type and takes $\kappa_{ref} - \kappa$ as input signal. The second PID block is used to represent $K_d(V_x)\frac{r_w}{g}\dot{\omega}$. This block is set to PD type and takes ω as input signal, the value K_p is set to 0. Note that there is no pure derivative type of PID block, only PD could be used with P part being set to 0.

ADD YOUR SOLUTION HERE

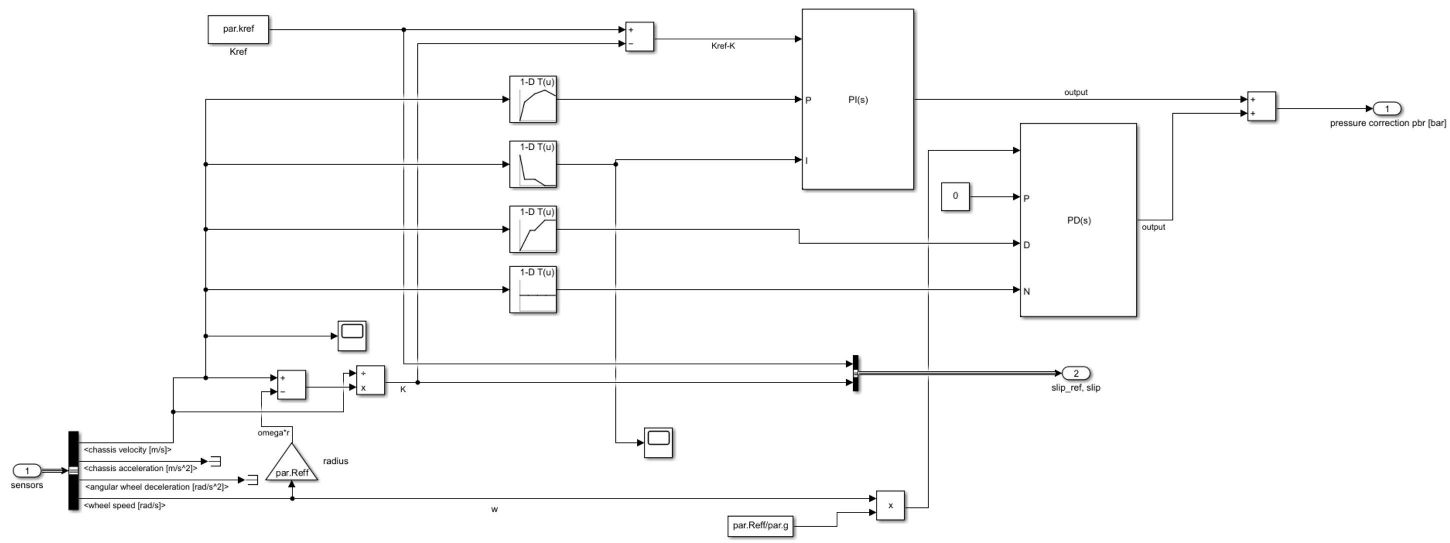

Fig. 4. The mixed-slip-deceleration controller

In the mixed-slip-deceleration controller, the K_p parameter is defined as the the function of V_x which is built by the 1-D Lookup Table with table data

$$[0; 30 * 1.1; 40 * 1.2; 50 * 1.1; 50 * 0.9]$$

and breakpoints $[3; 5; 15; 25; 30]$ m/s. When V_x is large, which means the chassis velocity is closer to the initial speed, the $K_p = f_1(V_x)$ is chosen to be larger to accelerate adjustment and reduce slip error. While when velocity is close to the minimal speed (in this case chassis velocity ≤ 3 m/s), in order the enhance the instability of wheel slip, the K_p is set to be 0.

The K_i parameter of wheel slip controller is is defined as the the function of V_x which is built by the 1-D Lookup Table with table data

$$[3000 * 3; 3000 * 1.8; 3000 * 1.8; 3000 * 1.5; 3000 * 1.5]$$

and breakpoints $[0; 5; 15; 25; 30]$ m/s. K_i is tuned to be quite large when the chassis speed approaches the minimal speed to eliminate steady-state error, and enhance the instability.

The K_d parameter is defined as the the function of wheel speed V_x which is built by the 1-D Lookup Table with table data $[40; 50; 50; 55; 55]$ and breakpoints $[0; 10; 15; 25; 30]$ m/s of chassis speed. For the end of braking, the K_d is set to be small to be less and slowly responsive to changes in the signal ω to decrease the overshoot and enhance the instability. The N parameter is set to be constant 100.

The process to obtain the optimal PID controller is shown in Table II, it shows that different parameters of mixed-slip-deceleration controller and its corresponding stopping distance in the steps to get the optimal parameters.

Steps	K_p	K_i	K_d	Stopping distance [m]
1	0	1	0	210
2	0	100	0	230
3	0	1000	0	215
4	0	3000	0	240
5	10	3000	0	245
6	50	3000	0	250
7	50	3000	50	253
8	Set PWA function	Set PWA function	Set PWA function	250

TABLE II
DIFFERENT PARAMETERS OF MIXED-SLIP-DECELERATION CONTROLLER AND CORRESPONDING STOPPING DISTANCE IN DIFFERENT STEPS.

B. The analysis of simulative results

By simulating the mixed-slip-deceleration control system, the brake pressure is shown in Fig 5 sub-picture 1. In this figure, it is easily observed that the brake pressure started from 18000 bar and converged to 0 bar, during this process the amplitude of oscillation remains around 2000 bar. But there is no obvious trend of the amplitude of oscillation which indicates the poor convergence of brake pressure.

The wheel slip is shown in Fig 5 sub-picture 2. The wheel slip reference is 0.12, and the wheel slip oscillated around the wheel slip reference. While the wheel slip is not stable because the amplitude of the wheel slip is even increasing. The possible reason for this scenario is that due to the interaction between the tire and the road surface the wheel slip is unstable as it generates a force that is proportional to the wheel slip, which can cause the tire to oscillate back and forth between static and dynamic friction. This can lead to a loss of traction and control of the vehicle. Another reason might be that the a vehicle with a limited-slip differential, instability in wheel slip can occur when the rotational speeds of the wheels oscillate out of phase, the differential switch from locks and unlocks leading to which makes oscillations in the wheel slip and instability in the vehicle's handling.

In Fig 6 sub-picture 1, by calculating the integration to chassis velocity, the braking distance is growing from 0 to around 250 m. The chassis velocity gradually decreased from 33.3333 m/s to 2.778 m/s with slight oscillation. The wheel speed decreased from 110 m/s with large oscillation of which the amplitude does not have obvious tendency to become small. The chassis acceleration keeps the value around 0 which verifies the flatness and smoothness of the braking process. The angular wheel deceleration oscillated from 800 to -2000 rad/s^2 and its amplitude is not decreasing. It could be showed that the instability of wheel slip deceleration in the braking process.

Overall, because of the instability of wheel slip and wheel deceleration in the braking process, the mixed-slip-deceleration control system is not stable one but provide larger angular wheel deceleration which help the vehicle to stop quickly.

IV. CONTROLLERS PERFORMANCE WITH NOISE

A. The noises added to the mixed-slip-deceleration control system

In this part, the noise for both wheel slip κ of 0.025 and wheel deceleration $\dot{\omega}$ of 0.5 rad/s^2 is applied to wheel slip control system and mixed-slip-deceleration control system. The noises could be taken as the

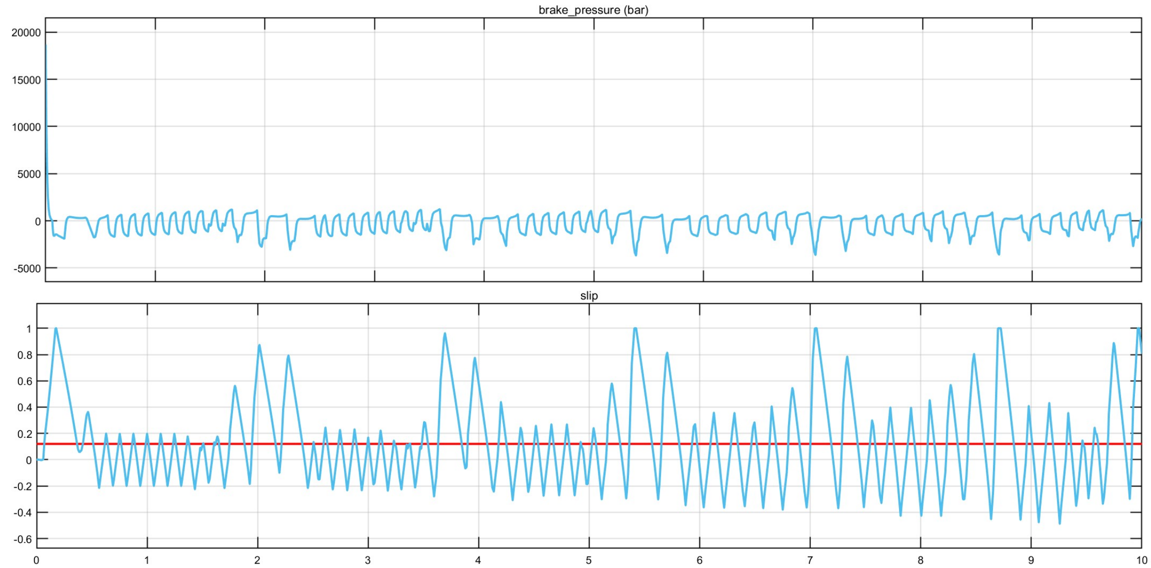

Fig. 5. Plots of brake pressure, wheel slip compared with wheel speed reference in the mixed-slip-deceleration control system.

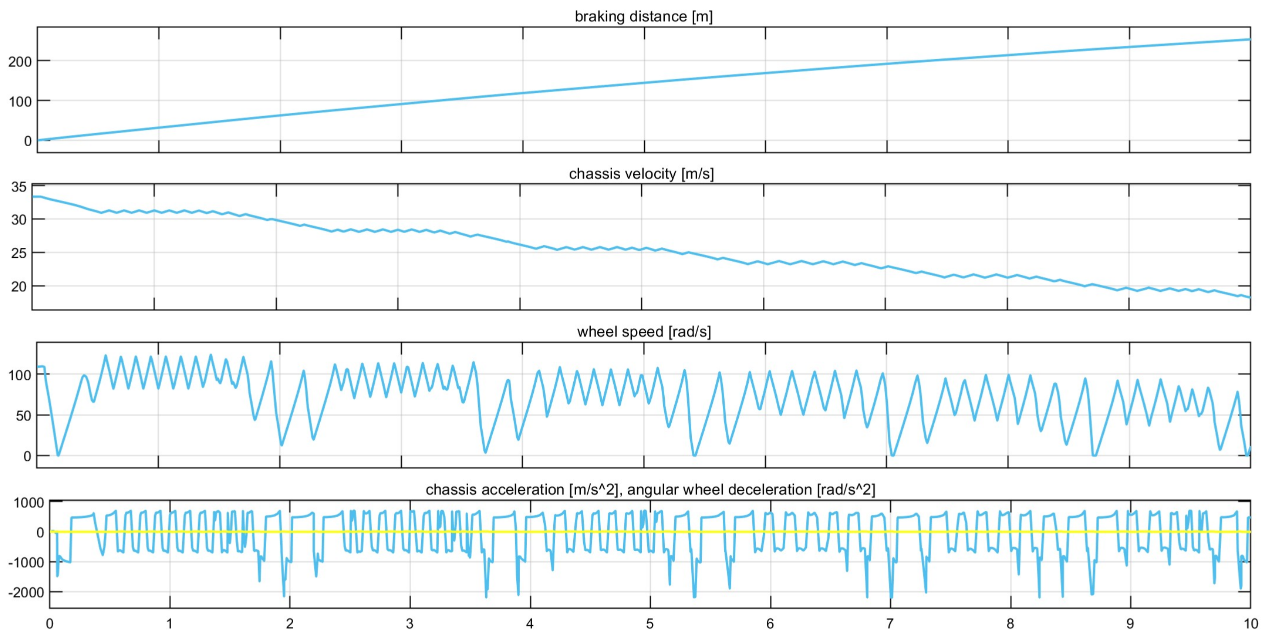

Fig. 6. Plots of brake distance [m], chassis velocity [m/s], wheel speed [rad/s], chassis acceleration [m/s^2] and angular wheel deceleration [rad/s^2] in the mixed-slip-deceleration control system.

uniform random noises and implemented to the system by adding the 'Uniform Random Number' blocks in Simulink.

The added noises are shown in Figs 7. The noise with the maximum and minimum values of 0.025 and -0.025 for wheel slip κ is added to the κ in the denoted red rectangular in 'Your WSC controller' block and the noise with the maximum and minimum values of 0.5 and -0.5 rad/s^2 for wheel deceleration $\dot{\omega}$ is added in the denoted red rectangular in 'Single Corner Dynamics' block.

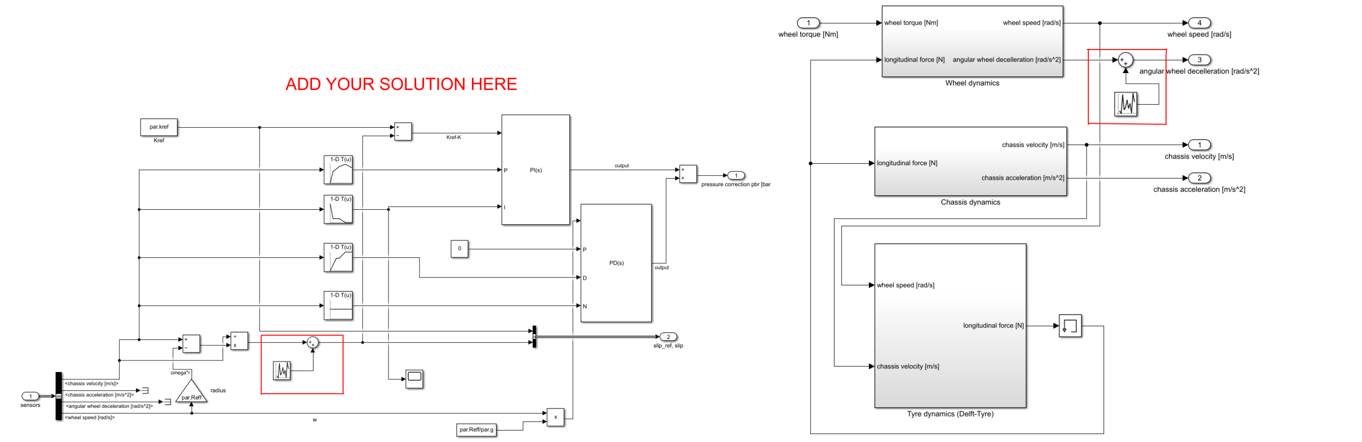

Fig. 7. The noise for wheel slip κ of 0.025 is added to the κ and the noise for wheel deceleration $\dot{\omega}$ of 0.5 rad/s^2 is added is added to $\dot{\omega}$.

B. The comparison of simulative results

First of all the simulation of noised wheel slip is shown in Figs . Compared with Figs 2, it could be observed that by adding the noises for wheel slip and angular wheel deceleration, the wheel slip controlled system is no longer stable any more, after 2.3 s, the plots of brake pressure and wheel slip diverge with increasing amplitudes of oscillation. From this, it could be concluded that wheel slip control system is sensitive to external noise and not robust to variations. In Figs 11, the braking distance was increased to 200 m which shows that the brake performance is violated. Also, the chassis velocity was not decreased with the same slope, which means that the flatness of braking process was harmed as well. For the wheel speed, the amplitudes of oscillation increased after 2.5s, which indicates that the wheel slip control system is not stable. For angular wheel deceleration, the amplitudes of oscillation stay unchanged (does not have tendency to become smaller). Overall, it could be inferred that the noised wheel slip control system was sensitive to external noises and less robust to external variation, even the noise-free wheel slip control system show good braking performance (small braking distance, high flatness and smoothness of the decrease of chassis velocity and stability of wheel slip control system).

Compared with noised wheel slip control system, the noised mixed-slip-deceleration control system shows strong robustness and are less sensitive to the external variations and noises. In Figs 10 and 11, the plots of brake pressure [bar], wheel slip, brake distance [m], chassis velocity [m/s], wheel speed [rad/s], chassis acceleration [m/s2] and angular wheel deceleration [rad/s2] in noised mixed-slip-deceleration control system are almost the same as the plots in the noised-free mixed-slip-deceleration control system, apart from negligible oscillation in some points.

Ideally, braking distance can be a useful indicator of the performance of the wheel controller. A shorter braking distance generally reflects better performance, while a longer braking distance may indicate issues with the system. The decrease in chassis velocity should be flat and consistent without any sudden spikes or drops. This is important because sudden changes in velocity can cause instability in the vehicle and lead to loss of control or skidding. The instability caused by wheel slip can have several negative influences on a vehicle's handling and safety, including reduced traction which can cause the vehicle to slide or skid,longer braking distances, uneven tire wear and reduced fuel efficiency.

Overall, if the roads are not slippery, the wheel slip controller shows better braking performance than mixed-slip-deceleration system for having smaller braking distance, smoothness and flatness and stability of wheel slip but when noise is added to it, the wheel slip control system may show the worse braking performance including longer braking distance, low smoothness and flatness of decrease of chassis velocity when braking and violated stability of wheel slip. While noised mixed-slip-deceleration controller shows better robustness to noise and maintain its braking properties compared with noised wheel slip control system. However when the roads are slippery, such as during rain, snow, or ice, it should prevent chassis velocity from smooth and stable deceleration.

Based on the former analysis, the wheel slip controller should be chosen in the ideally accurate system, of which the variables could be measured correctly, on not slippery roads, or not making the urgent stops. It will show optimal braking performance compared to the mix-slip-deceleration controller. While, when noise exists and cannot be measured properly, the road is slippery or make the urgent brake, the mix-slip-deceleration controller could absorb noise, keep the braking performance and provide large traction to avoid skid or slide.

What is more, some amount of oscillation or tracking error can be desirable in the reference tracking of the wheel slip because it indicates that the control system is actively adjusting to maintain traction with the road surface. In another word, it can ensure that the wheel control system is responding appropriately to changes in vehicle parameters and road conditions. However, excessive oscillation can lead to instability and reduced vehicle control which may lead to slide or skid, so it is important to trade off the level of oscillation against the requirement for stability and control.

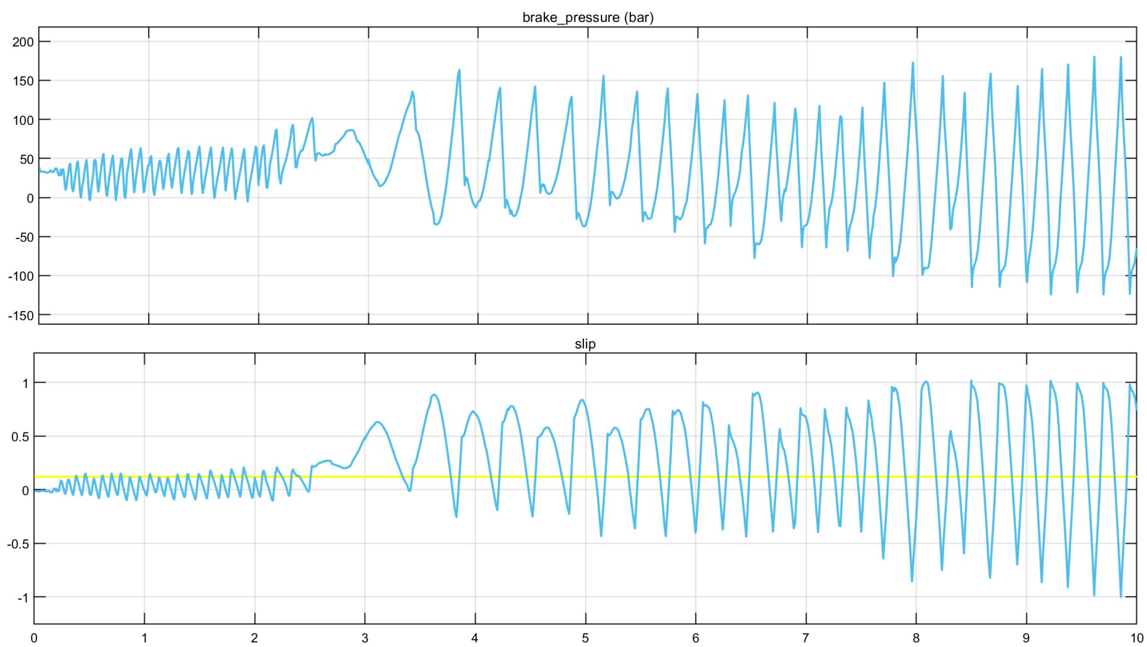

Fig. 8. Plots of brake pressure, wheel slip compared with wheel speed reference in the noised wheel slip control system.

V. Self reflection

Firstly, the most difficult part is about how to interpret the wheel slip(the definition and meaning of wheel slip) and how the vehicle model functions together. This takes me long time to have the basic idea for it. After doing it, the difficulty is to design the PID controllers and tune the parameters, by trying thousands times, it could be found that the constant values of PID controller do not have the good performance, thus the piecewise affine function should be used to enhance the performance of PID controller. But it also takes me so much time the choose the optimal pair of turning points of PID parameters. Even this task is not easy for me, still do I understood the wheel slip control and mixed-slip-deceleration control.

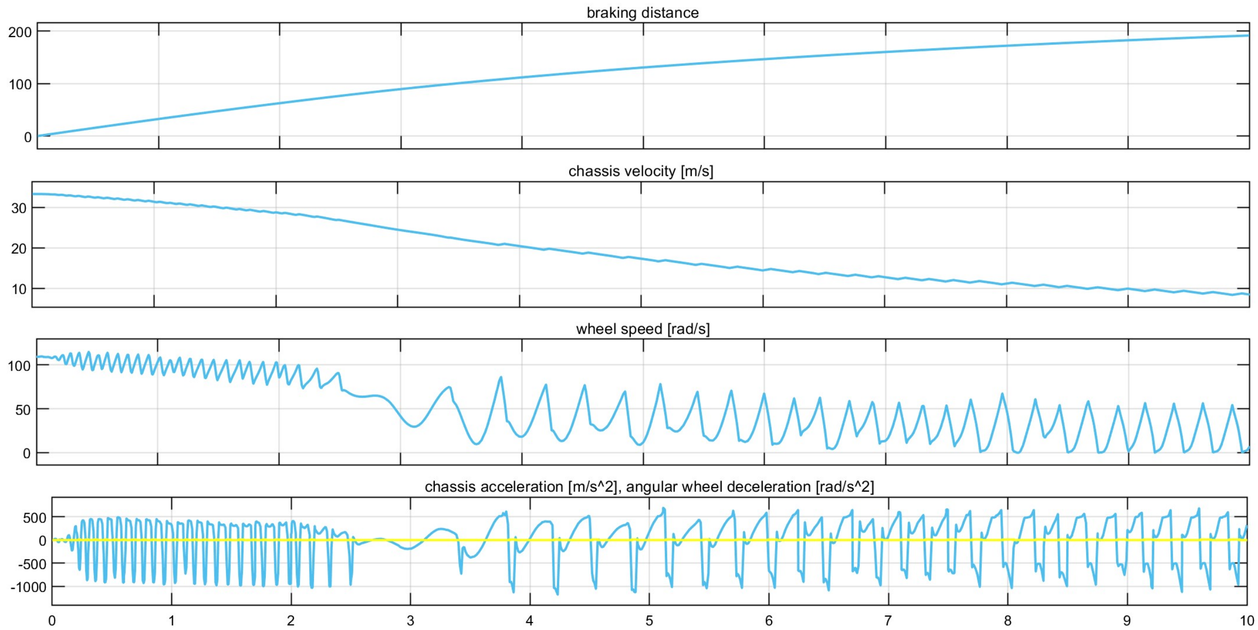

Fig. 9. Plots of brake distance [m], chassis velocity [m/s], wheel speed [rad/s], chassis acceleration [m/s^2] and angular wheel deceleration [rad/s^2] in noised the wheel slip control system.

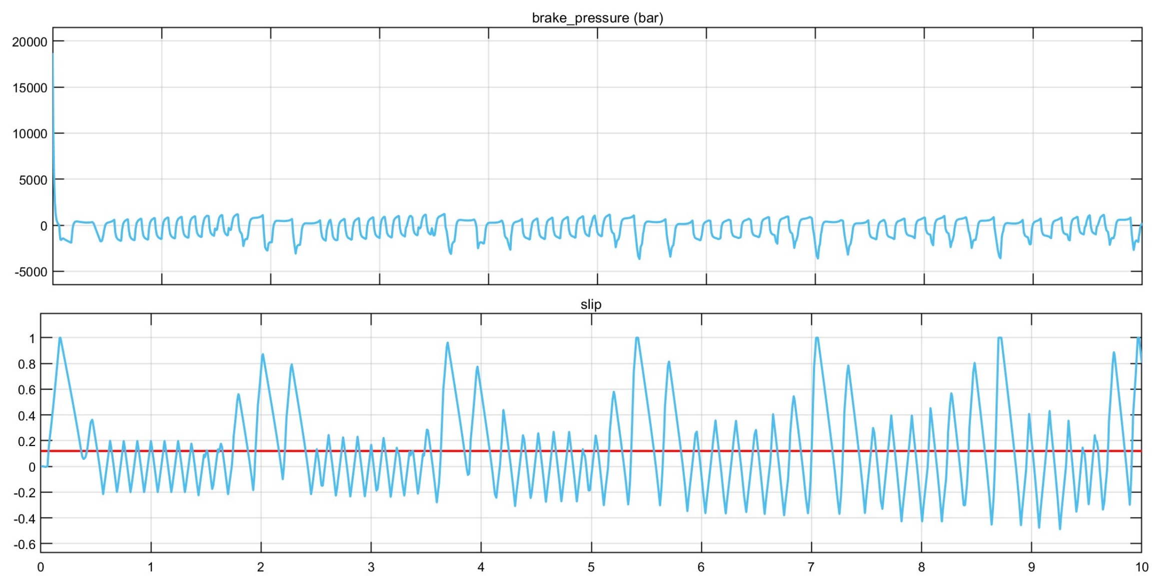

Fig. 10. Plots of brake pressure, wheel slip compared with wheel speed reference in the noised mixed-slip-deceleration control system.

I am glad that effort leads to great harvest and I will keep on exploring the course of vehicle dynamics and control.

VI. Conclusions

In this homework, two wheel controllers, wheel slip controller and mixed-slip-deceleration controller are designed and analysed. The wheel slip controller should be chosen in the ideally accurate system, of which the variables could be measured correctly, on not slippery roads, or in the case of not making the

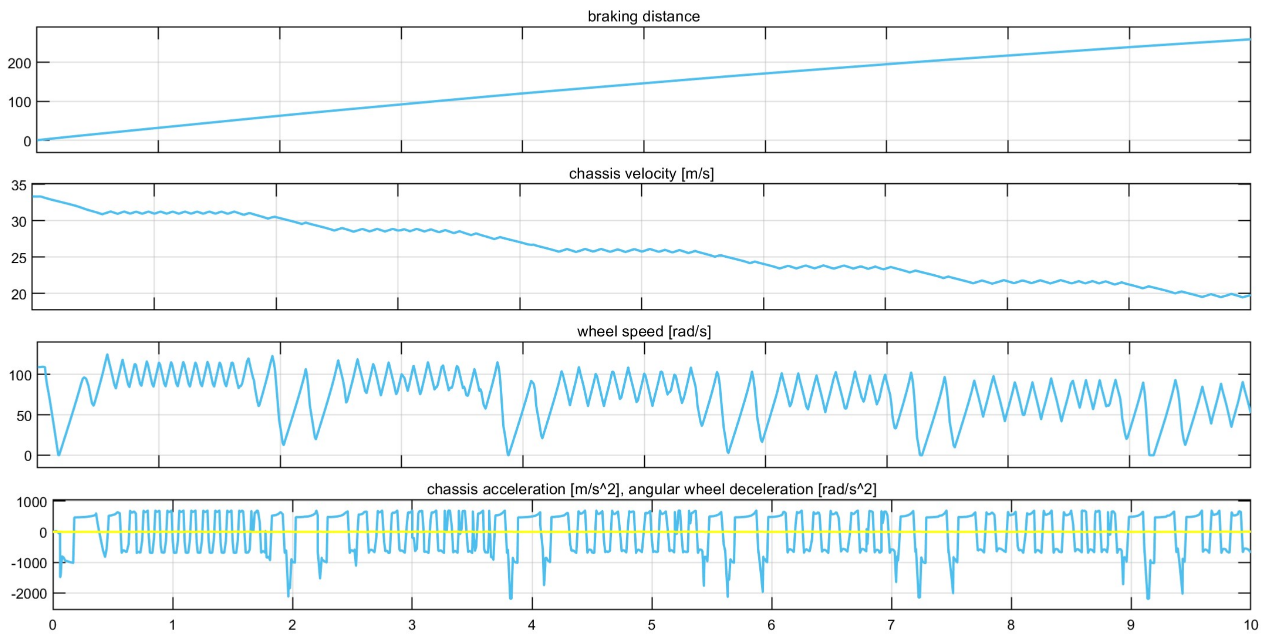

Fig. 11. Plots of brake distance [m], chassis velocity [m/s], wheel speed [rad/s], chassis acceleration [m/s^2] and angular wheel deceleration [rad/s^2] in noised the mixed-slip-deceleration control system.

urgent stops. It would show optimal braking performance compared to the mix-slip-deceleration controller. While, when noise exists and cannot be measured properly, under the condition that the road is slippery or make the urgent stop, the mix-slip-deceleration controller could absorb noise, keep the braking performance and provide large traction to avoid skid or slide.

References

[1] MF-Tyre/MF-Swift 6.2.0.4 Installation Guide. (2017, March 1). http://www.delft-tyre.nl.

[2] Vehicle Dynamics and Control (RO47017). (2023, May). Retrieved from https://brightspace.tudelft.nl/d2l/le/content/500965/viewContent/2936161/View

[3] PID controller - MATLAB Simulink. (2023, May 15). Retrieved from https://nl.mathworks.com/help/simulink/slref/pidcontroller.html

Appendix

A. *Simulink block diagrams.*

All the Simulink block diagrams have the same block diagram of simplified vehicle model in Fig 12. The only different parts are different designs of 'Your WSC controller' which are already shown before.

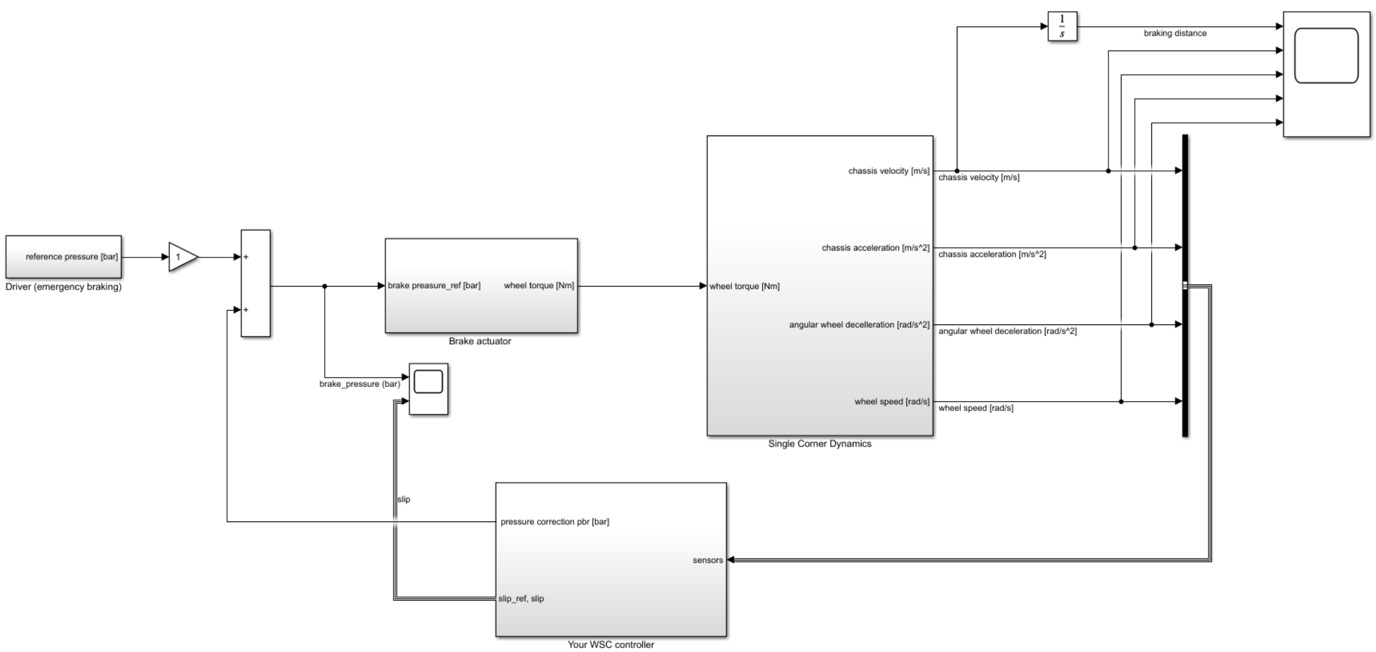

Fig. 12. The block diagram of simplified vehicle model with controller.

B. *Vehicle parameters.*

Vehicle parameters	Values
Quarter car mass	450 kg
Inertia of the wheel	1.2 kg*m^2
Wheel effective radius	0.305 m
Conversion from brake pressure to brake torque	11.25
Maximal brake pressure	160 bar
Initial speed	120 km/h
Minimal speed to stop simulation	10km/h
Friction coefficient	0.6

TABLE III
VEHICLE PARAMETERS.