Risk Management and Return Prediction

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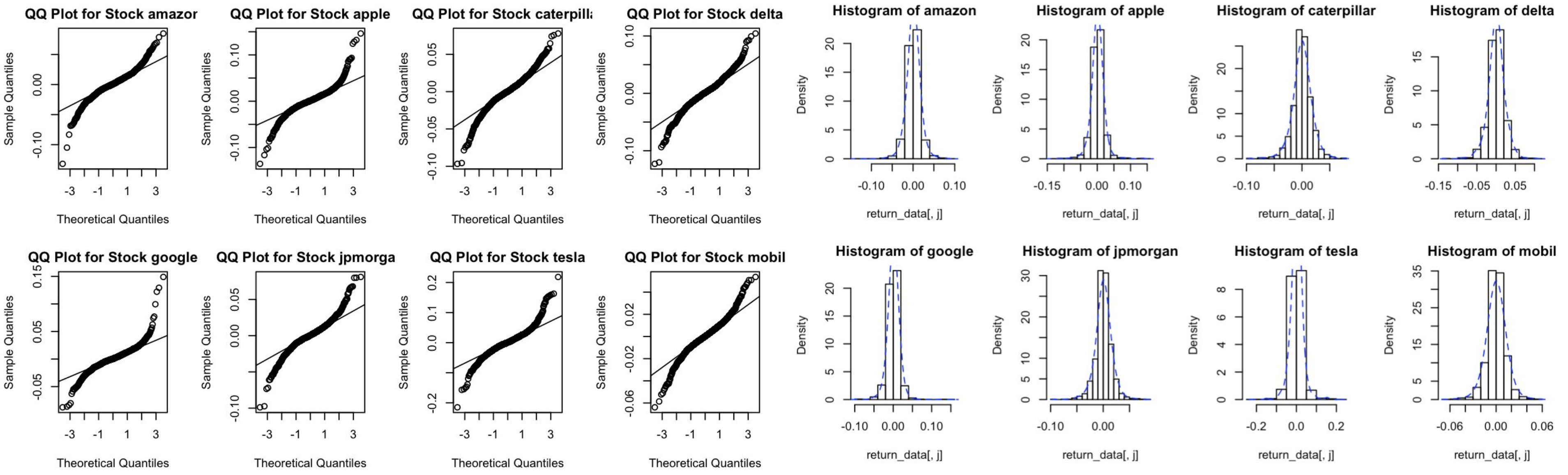
Outline

- Descriptive Statistics
- Data Overview
- Risk Management: Best Portfolios, VaR, ES
    - Universal Portfolio Theory
    - Copula (need to be revised)
    - Four Basic Models
        - Markowitz Portfolio Theory
        - Constant Correlation Model
        - Single Index Model
        - Multi Factor Model
- Return Prediction: Factor Analysis + Time series
    - Economic Factors
        - Index market return, Volume, Inflation Rate, Risk-free Rate, GDP, CPI, Unemployment Rate
    - ARMA + GARCH Model

Descriptive Statistics--Stock Price

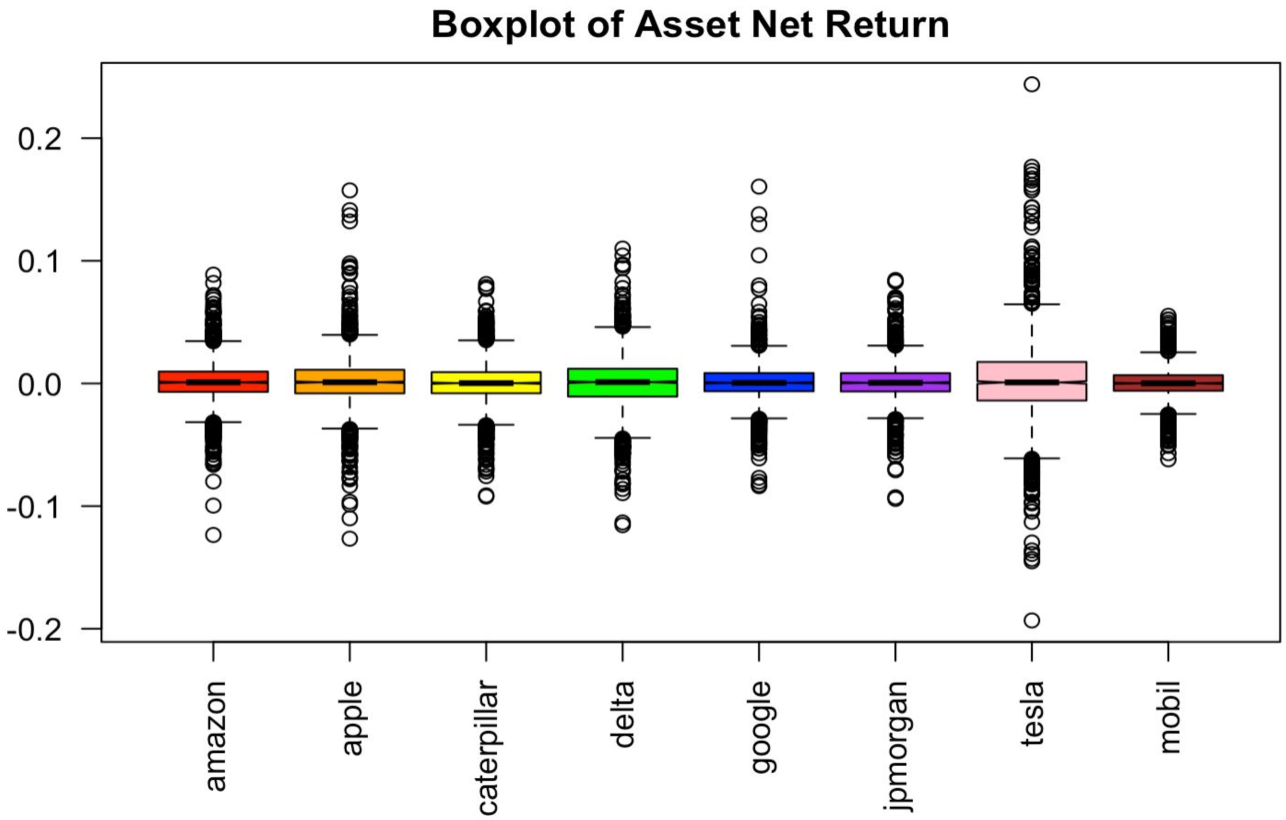
We have 8 stocks chosen from 6 different industries, Amazon(Consumer Cyclical), Apple(Technology), Caterpillar(Industrials), Delta(Industrials), Google(Communication Services), JP Morgan(Financial Services), Tesla(Consumer Cyclical), Mobil(Energy). Time period starts from Jan 1, 2011, to Dec 31, 2019, 2262 observations in total

Data Overview--Stock Return Distribution

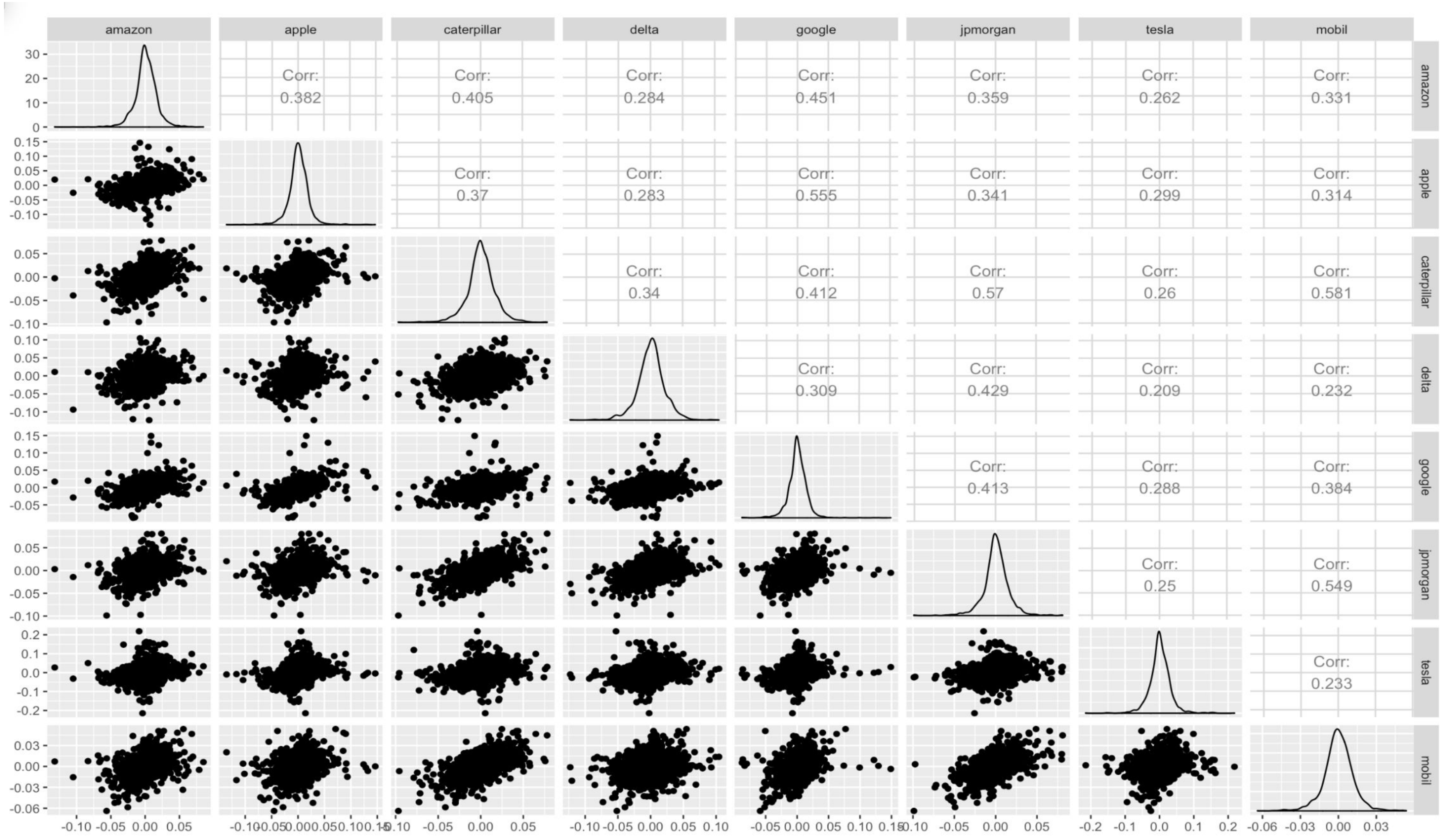


From QQ plot we can see that all of the eight stocks have heavy tail distribution, therefore we believe that each stock return follows a t-distribution. Moreover, histograms show the approximate density indicated by blue dash lines, and each stock return follows fat tail normal distribution, which can be approximate by t-distribution

This can be verified by box-plot…

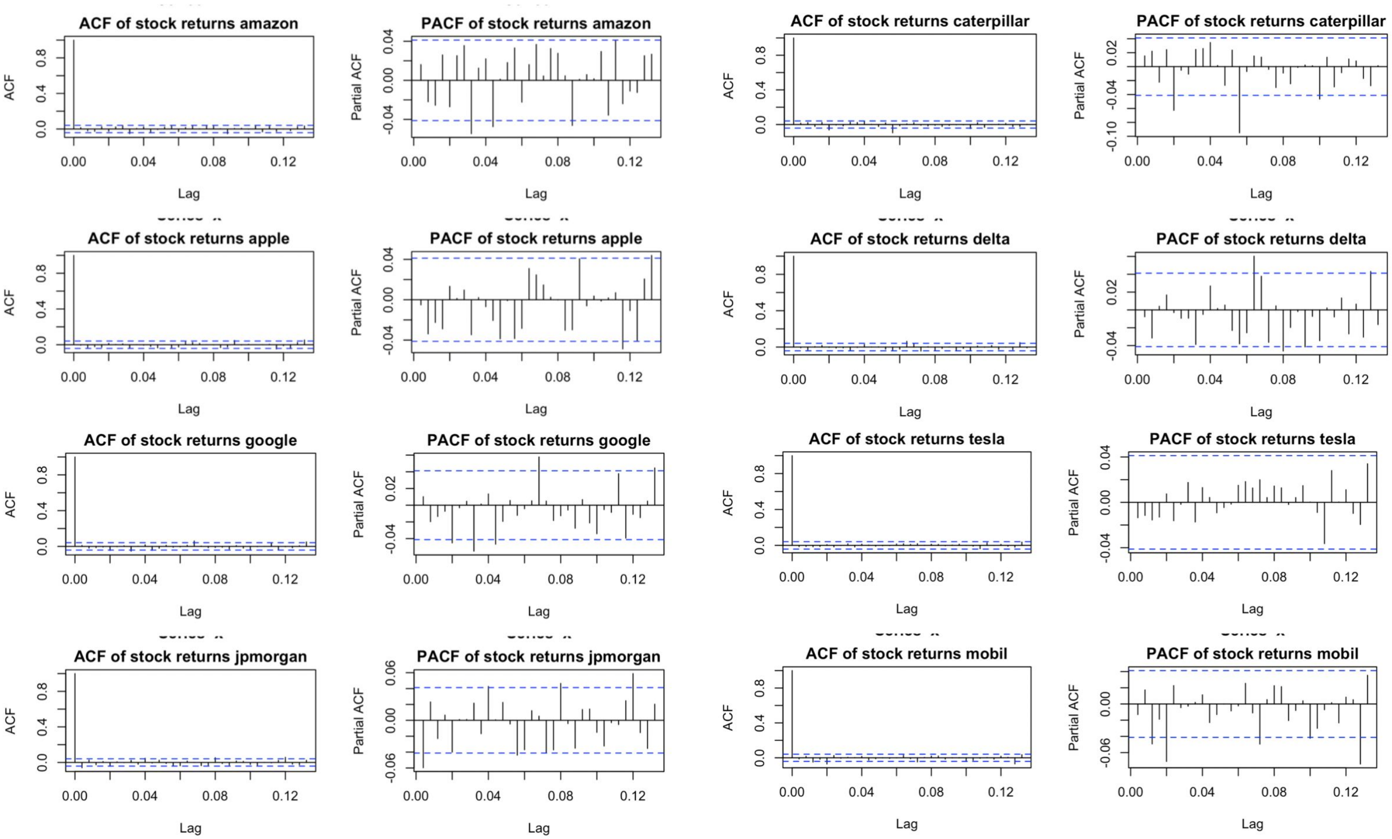


Data Overview--Stock Return Correlation



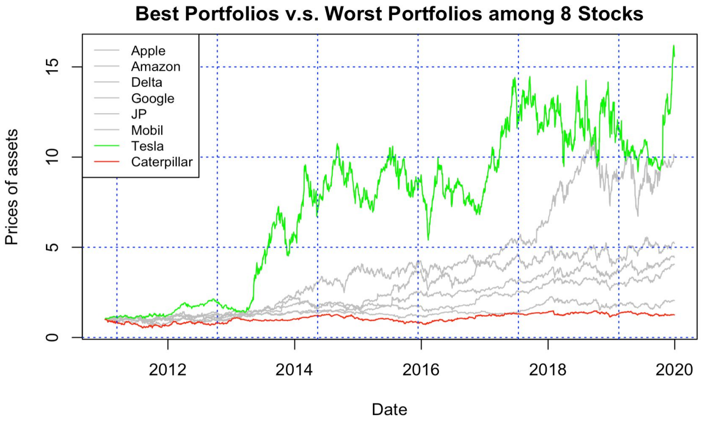
Each pair of stock has around 0.3 correlation

Data Overview--(Partial) Autocorrelation



Each stock return has irregular partial autocorrelation, but little autocorrelation

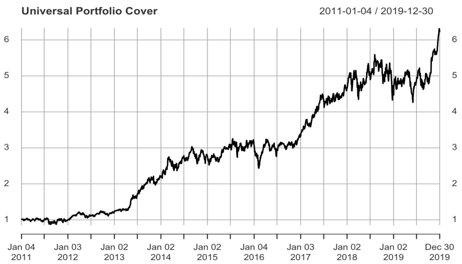
Risk Management--Universal Portfolio

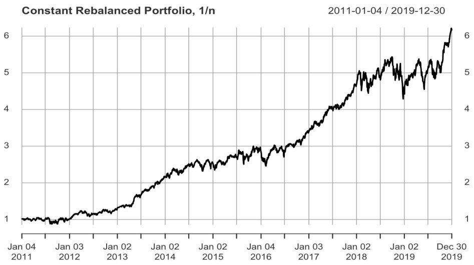


Initial wealth is $1. We invest it on 1/3/2011. The best asset Tesla can bring us $15.57 on 12/31/2019, while the worst asset Caterpillar can only bring us $1.25.

	Weight <dbl>	Wealth <dbl>
apple	0.3035004	3.042731
tesla	0.6964996	10.850428

If we only invest in any two stocks of them, the best combination would be investing 0.3 in Apple and 0.7 in Tesla; at the end of year 2019, we will have 3.04 in Apple and 10.85 in Tesla





1. Calculate current portfolio weights as a weighted average of the best CRP until now
2. Universal portfolio selection algorithm from Cover
3. Take the weights for the best CRP in the past as the weights for the next period
4. Constant rebalanced portfolio with weight 1/8

Risk Management (4 Basic Models)

Markowitz Model:

- Use historical return and risk as reference
- Return: Historical Return
- Covariance Formula: ${\sigma_{ij}}^2$

Single Index Model:

- Use market index return (S&P 500) for return and risks
- Return: $R_i = \alpha_i + \beta_i * R_M$
- Covariance Formula: ${\sigma_i}^2 = {\beta_i}^2 * {\sigma_{RM}}^2$ ${\sigma_{ij}}^2 = \beta_i * \beta_j * {\sigma_{RM}}^2$

Constant Correlation Model:

- Use past average correlation structure (constant) as future correlation
- Return: Historical Return
- Covariance Formula:

$$\rho = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{ij}}{\frac{N(N-1)}{2}} \qquad \sigma_{ij}^2 = \sigma_i * \sigma_j * \rho$$

Multi Factor Model:

- Use fama-french factors to calculate returns and risks
- Return:

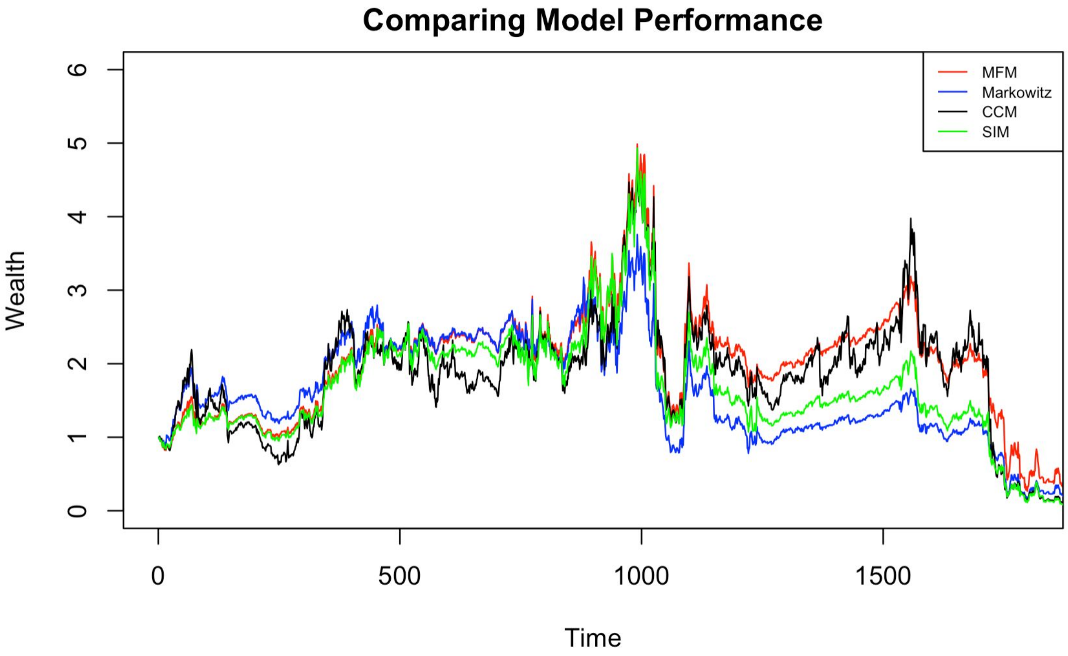
$$R_{it} - R_{ft} = \alpha_{it} + \beta_{i1} * (R_{Mt} - R_{ft}) + \beta_{i2} * SMB + \beta_{i3} * HML$$

- Covariance Formula:

$${\sigma_i}^2 = {\beta_{i1}}^2 * {\sigma_{(RM-Rf)}}^2 + {\beta_{i2}}^2 * {\sigma_{(SMB)}}^2 + {\beta_{i3}}^2 * {\sigma_{(HML)}}^2$$

$${\sigma_{ij}}^2 = \beta_{i1} * \beta_{j1} * {\sigma_{(RM-Rf)}}^2 + \beta_{i2} * \beta_{j2} * {\sigma_{(SMB)}}^2 + \beta_{i3} * \beta_{j3} * {\sigma_{(HML)}}^2$$

Risk Management (4 Basic Models) cont'



Model	Wealth
Markowitz Model	0.1877
Constant Correlation Model	0.1086
Single Index Model	0.0948
Multi Factor Model	0.4099

From both the plot and the chart we can see, if the initial wealth is $1, after 9 years everyday reweighted, tangent portfolio based on all 4 models are going to lose money. Multi-factor model has best performance which end up with $0.4099

Risk Management--...(Time Series)

Value-at-risk (VaR) is a bound such that the loss over the horizon is less than this bound with probability equal to the confidence coefficient $1 - \alpha$

VaR uses two parameters: The time horizon T and the confidence level $1 - \alpha$

Expected shortfall (ES) is average of VaR(u) over all u that are less than or equal to α

Time series: use ARMA+GARCH(1,1) model to predict VaR and ES for t+1.
At 12/31/2019:

	Markowitz	**CCM**	**SIM**	**MFM**
VaR	0.040	0.054	0.036(min)	0.037
ES	0.058	0.060	0.053(min)	0.053

Risk Management--VaR & ES(Parametric)

Parametric: returns have a joint parametric distribution (e.g. multivariate t-distribution)

$\widehat{\mathrm{VaR}}^{\mathrm{par}}(\alpha) = -S \times F^{-1}(\alpha\|\widehat{\boldsymbol{\theta}})$	where S is the size of current position
$\widehat{\mathrm{ES}}^{\mathrm{par}}(\alpha) = -\frac{S}{\alpha} \times \int_{-\infty}^{F^{-1}(\alpha\|\widehat{\boldsymbol{\theta}})} x f(x\|\widehat{\boldsymbol{\theta}})\, dx$	where "par" denotes parametric
$\widehat{\mathrm{ES}}^{\mathrm{t}}(\alpha) = S \times \left\{-\mu + \lambda\left(\frac{f_\nu\{F_\nu^{-1}(\alpha)\}}{\alpha}\left[\frac{\nu + \{F_\nu^{-1}(\alpha)\}^2}{\nu - 1}\right]\right)\right\}$	where "t" denotes t-distribution, with mean = μ, scale parameter = λ, df = ***v***

	Markowitz	CCM	SIM	MFM
VaR	0.065(min)	0.089	0.072	0.065(min)
ES	0.003	0.026	0.003	0.001(min)

Risk Management--...(Nonparametric)

Nonparametric: the loss distribution is not assumed to be in a parametric family such as normal or t-distribution.

$$\widehat{\text{VaR}}^{\text{np}}(\alpha) = -S \times \widehat{q}(\alpha)$$

where S is the size of current position
q(α) is α-quantile of a sample of historic returns

$$\widehat{\text{ES}}^{\text{np}}(\alpha) = \frac{\sum_{i=1}^{n} \mathcal{L}_i \, I\{\mathcal{L}_i > \widehat{\text{VaR}}(\alpha)\}}{\sum_{i=1}^{n} I\{\mathcal{L}_i > \widehat{\text{VaR}}(\alpha)\}} = -S \times \frac{\sum_{i=1}^{n} R_i \, I\{R_i < \widehat{q}(\alpha)\}}{\sum_{i=1}^{n} I\{R_i < \widehat{q}(\alpha)\}}$$

where R is historic returns
Define L = -S * R

	Markowitz	**CCM**	**SIM**	**MFM**
VaR	0.072(min)	0.099	0.081	0.073
ES	0.140(min)	0.167	0.156	0.139

Risk Management--...(Bootstrap CI)

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> Bootstrap.CI
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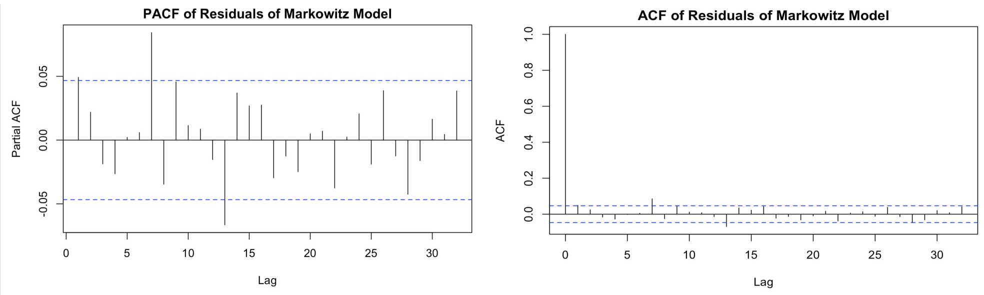
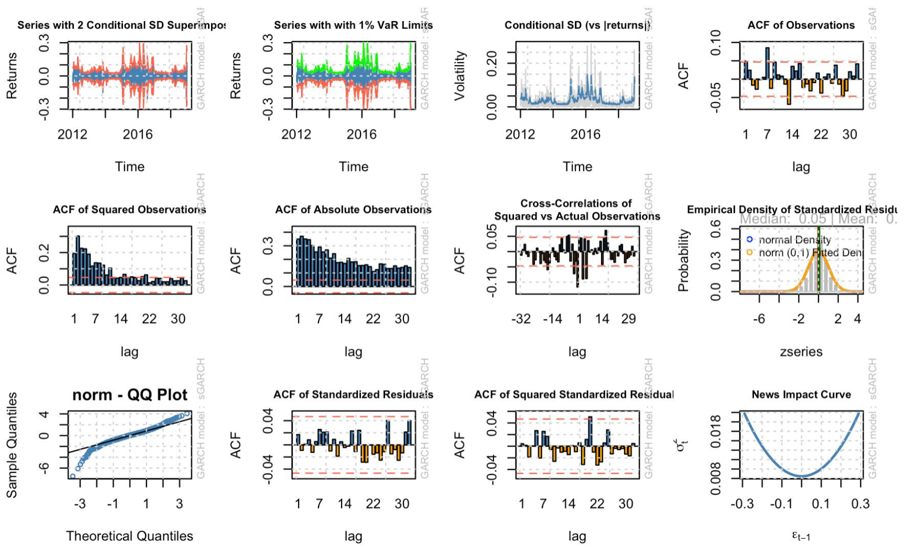
	VaR_LowerBond	VaR	VaR_UpperBond	ES_LowerBond	ES	ES_UpperBond
Markowitz_Historical	-0.06332594	-0.05206159	-0.03322455	-0.12868014	-0.09528483	-0.06014195
Markowitz_Gaussian	-0.07506154	-0.06089486	-0.04589220	-0.09369878	-0.07631649	-0.05825569
Markowitz_Modified	-0.09219023	-0.05955509	-0.03403592	-0.23263476	-0.13246668	-0.04378387
CCM_Historical	-0.10563785	-0.08429003	-0.05479650	-0.18141963	-0.14008146	-0.09780882
CCM_Gaussian	-0.10734276	-0.09102521	-0.07320633	-0.13347367	-0.11407108	-0.09298586
CCM_Modified	-0.12281788	-0.09441014	-0.06760720	-0.29916046	-0.18645597	-0.09693519
SIM_Historical	-0.03163351	-0.02481193	-0.01681346	-0.07349376	-0.05227868	-0.02009634
SIM_Gaussian	-0.05384718	-0.03810089	-0.01926258	-0.06746708	-0.04788973	-0.02431718
SIM_Modified	-0.02446752	-0.01409957	0.01928876	-0.02148095	-0.01409957	0.21108938
MFM_Historical	-0.03325319	-0.02594962	-0.01894062	-0.07609909	-0.05383655	-0.01852899
MFM_Gaussian	-0.05387550	-0.03879610	-0.01783131	-0.06755845	-0.04875560	-0.02257043
MFM_Modified	-0.02969346	-0.01609193	0.01853455	-0.02420812	-0.01609193	0.40469342

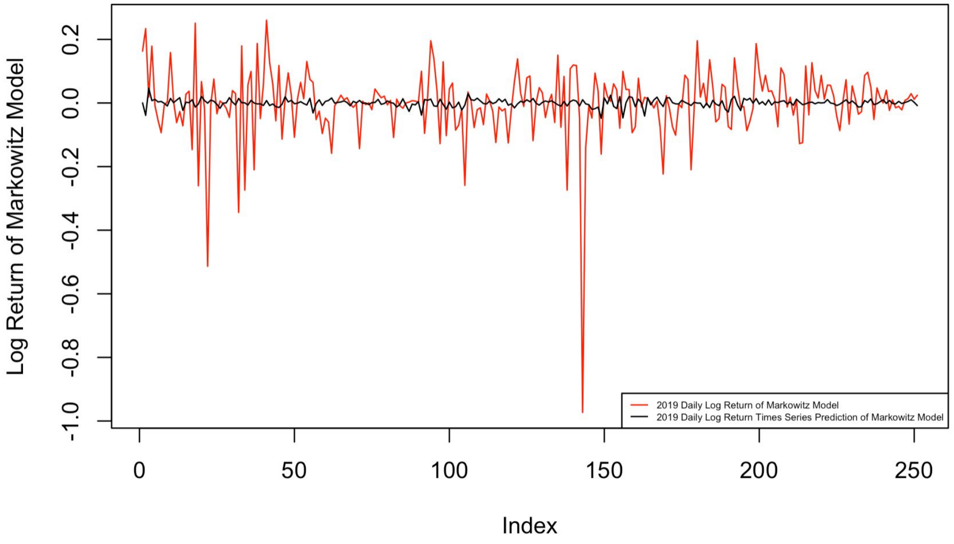
- Historical: The quantile of the period negative return for which exists long history of returns
- Gaussian: Normal distribution N(u,σ) for the return series
- Modified: Take higher moments of non-normal distributions through Cornish Fisher expansion

➔ **Single Index Model using Modified method** has relatively lowest VaR and ES

Return Prediction--Markowitz



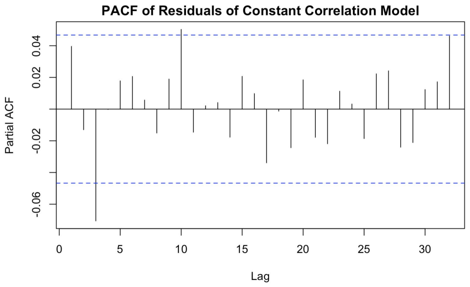


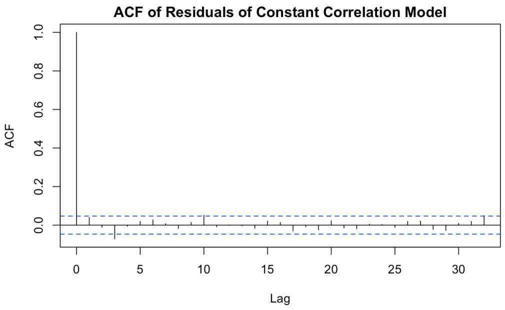


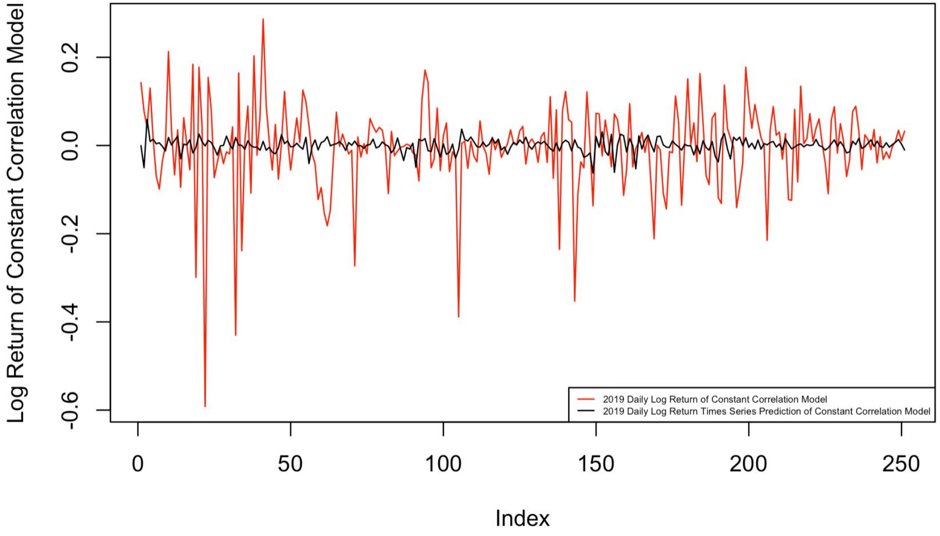
Factor Model:
logreturn = 9.277e-03 - 2.732e-12 * Volume + 1.469 * Market.Return - 3.480e-04 * Risk.free.Rate + ε

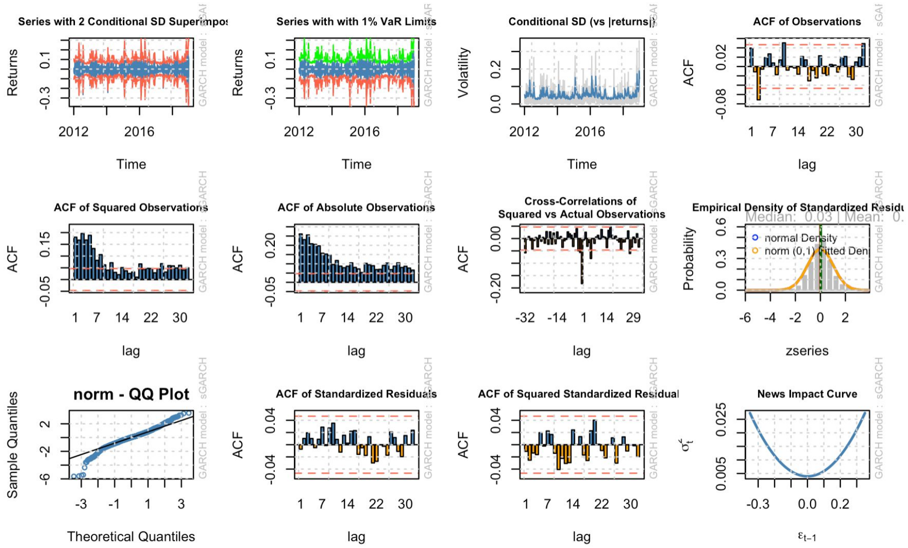
Residuals: ARMA(7,7) + GARCH(1,1)
Adjusted R-squared: 0.1042

Return Prediction--Constant Correlation





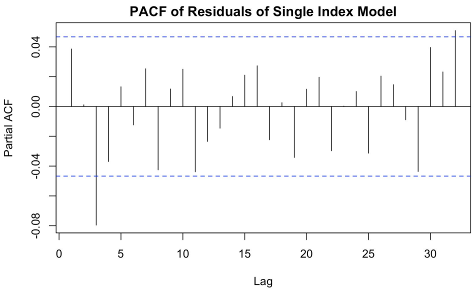


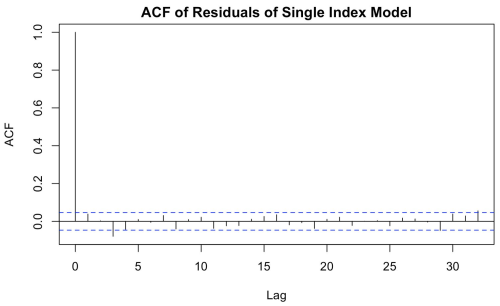


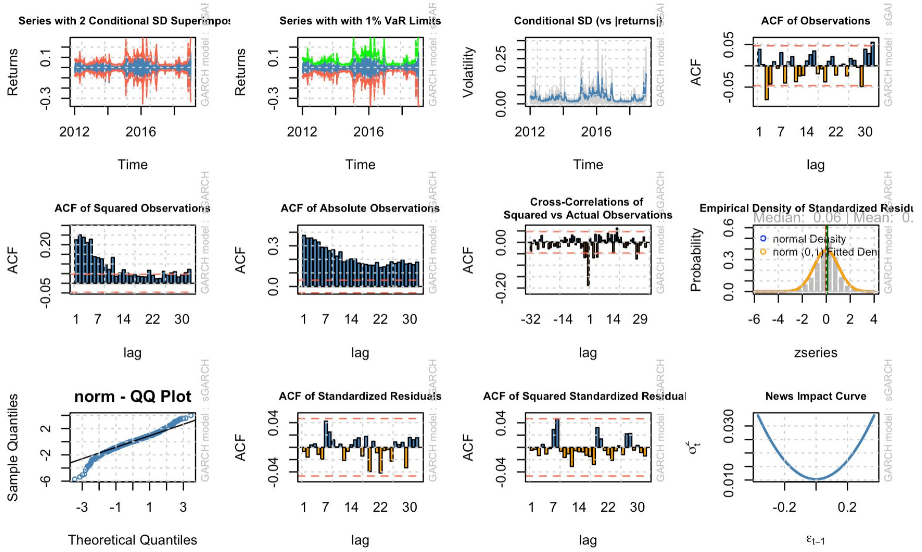
Factor Model:
logreturn = 1.371e-02 - 4.005e-12 * Volume + 1.888 * Market.Return - 5.683e-04 * Risk.free.Rate + ε

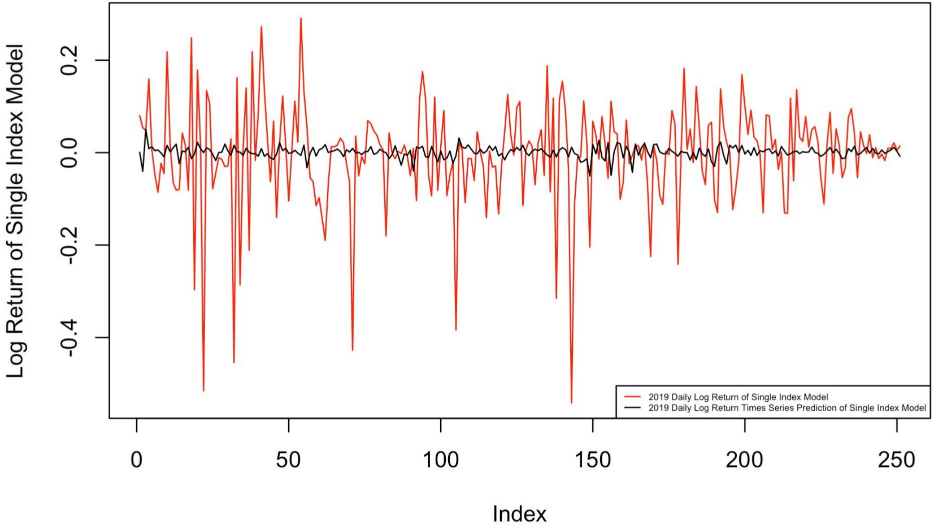
Residuals: ARMA(3,3) + GARCH(1,1)
Adjusted R-squared: 0.216

Return Prediction--Single Index Model





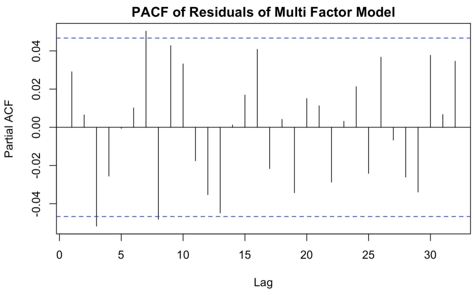


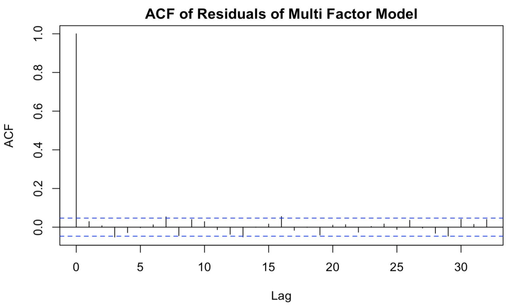


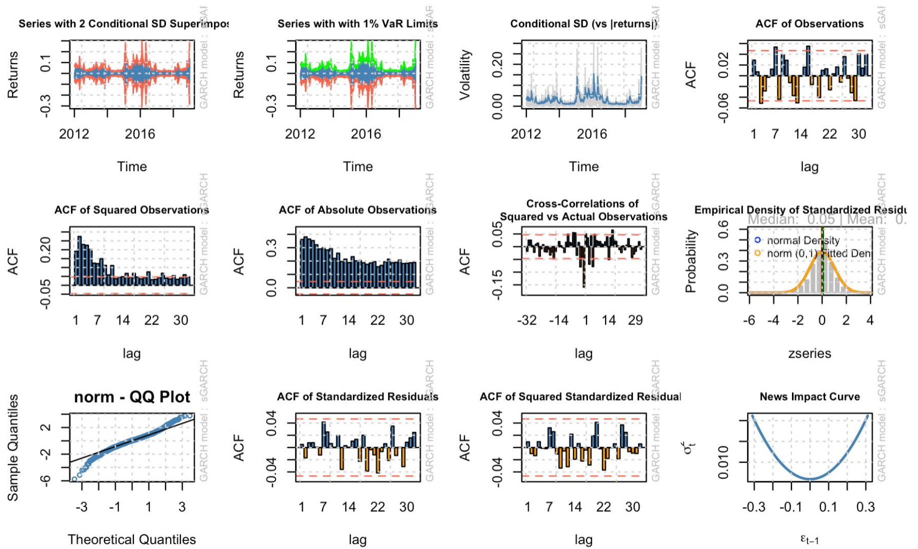
Factor Model:
logreturn = 1.005e-02 - 3.007e-12 * Volume + 1.555 * Market.Return - 3.561e-05 * Risk.free.Rate + ε

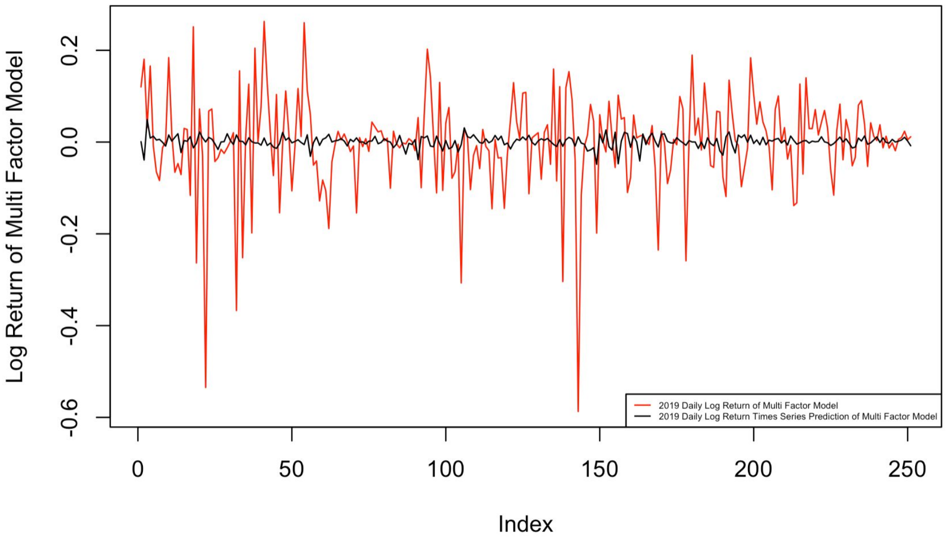
Residuals: ARMA(3,3) + GARCH(1,1)
Adjusted R-squared: 0.198

Return Prediction--Multi Factor Model









Factor Model:

Logreturn = 7.917e-03 - 2.349e-12 * Volume + 1.511 * Market.Return - 4.840e-05 * Risk.free.Rate + ε

Residuals: ARMA(3,3) + GARCH(1,1)

Adjusted R-squared: 0.2503

Thank you!