Big Programming Exercise

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Natural logarithm

The natural logarithm of a real positive number x using the integral representation is:

$$\ln x = \int_1^x \frac{1}{t} dt \tag{1}$$

To reduce the logarithm of an arbitrary positive number to the logarithm of a number in the range $1 \leqslant x < 2$ using the formulae:

$$\ln x = -\ln \frac{1}{x} \tag{2}$$

$$\ln x = \ln 2 + \ln \frac{x}{2} \tag{3}$$

We divide $(0, +\infty)$ into 4 intervals:$(0,\frac{1}{2})$, $(\frac{1}{2}, 1)$, (1,2) and $(2, +\infty)$.
if $x \in (0,\frac{1}{2})$ then use formulae $\ln x = -\ln \frac{1}{x}$ to change x to $x \in (2, +\infty)$ then use formulae $\ln x = \ln 2 + \ln \frac{x}{2}$ several times until change $x \in$ (1,2). for example:

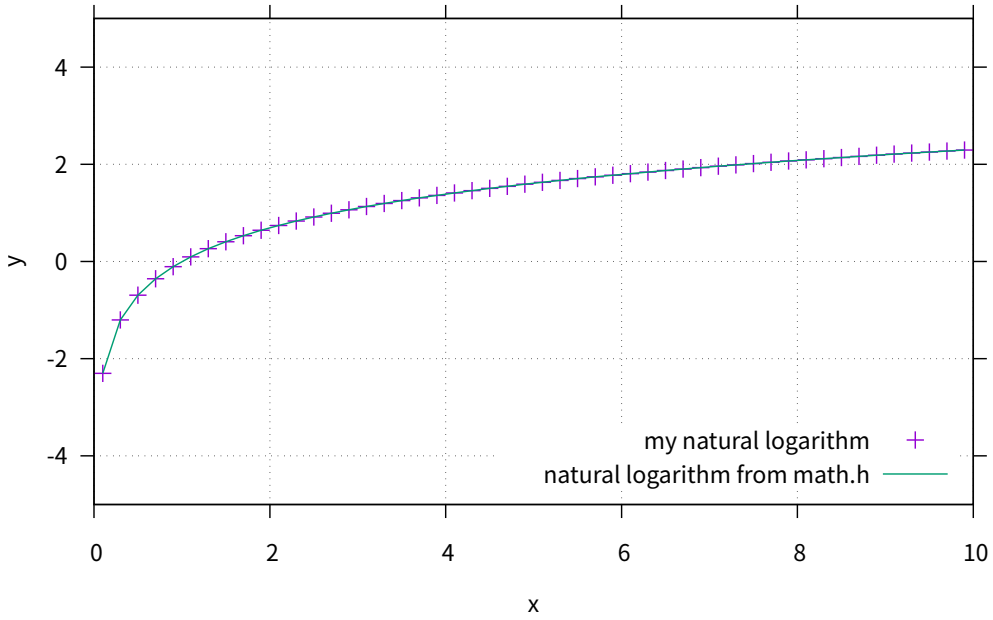
$$\ln 10 = \ln 2 + \ln \frac{10}{2} = 2\ln 2 + \ln \frac{10}{2^2} = 3\ln 2 + \ln \frac{10}{2^3} \tag{4}$$

else if $x \in (\frac{1}{2}, 1)$ then use formulae $\ln x = -\ln \frac{1}{x}$ to change x to $x \in$ (1,2).
else if $x \in (2, +\infty)$ then use formulae $\ln x = \ln 2 + \ln \frac{x}{2}$ several times until change $x \in$ (1,2).

This is a numerical integration problem can be solved by using the GNU Scientific Library (GSL). First provide to function to integrate gsl_function as $\frac{1}{x}$, then i use adaptive Gauss-Kronrod integrator fucntion gsl_integration_qag with 61 point Gauss-Kronrod rules to calculate the numerical integration,

the lower bound a is 1 and the upper bound b is x, the desired absolute and relative error limits is 1e-8, the maximum number of subintervals is 1e3. Then in the main function, x ranges from 0.1 to 10.0 with step 0.2 was caculated, and print into a file called data, then use gnuplot with plot.gpi to plot the final result and compared with natural logarithm function from <math.h>.

Plot



As shown on the figure, the green curve is the result of natural logarithm function from `math.h`, the points with marker '+' is my natural logarithm with numerical integration, it's clear that they matches very well.

References

Galassi, M., et al. "GNU Scientific Library Reference Manual, ISBN 0954612078." Library available online at http://www.gnu.org/software/gsl (2015).