Supplementary Material: View Planning for High-Fidelity Reconstruction of Dynamic Actor using Flying Camera

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In the appendix, we will first discuss the geometry meaning of the PPA metric and then further explain the math behind it.

1 Geometry Meaning of PPA

As defined in Sec. III-A, we use the projection length in the image plane of the 3D segment, or in other words, the ratio between the area of a 3D patch and its projection in the image plane, as illustrated in the Fig. 1.

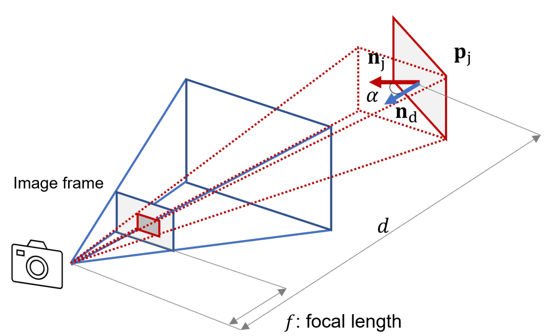

Figure 1: **Geometry meaning of PPA values**. A patch (colored in red) is projected into the image plane (colored in blue. f stands for the focal length of a camera.

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$$
\begin{aligned}
\mathbf{ppa}\left(\mathbf{x}_d, \mathbf{x}_j\right) &= \frac{A_{img}}{A_j} = \frac{A_{img}}{A_{norm}} \frac{A_{norm}}{A_j} \\
&= \frac{f}{d(\mathbf{p}_d, \mathbf{p}_j)} \cos(\alpha(\mathbf{n}_d, \mathbf{n}_j)) \\
&= \left(\frac{\cos(\alpha(\mathbf{n}_d, \mathbf{n}_j))}{d(\mathbf{p}_d, \mathbf{p}_j)} \right) f
\end{aligned} \tag{1}
$$

Since the focal length of a camera is a constant, we define the PPA function as the Eq. 2.

2 Jacobian of PPA

In Sec. IV, we propose the view planning algorithm based on optimizing the PPA functions. The underlying assumption is that the camera's orientation is de-coupled with its position. As stated in the definition of PPA Eq.2 and the corresponding jacobian Eq. 5. The entire equation could be written as follows.

$$
\begin{aligned}
&\frac{\partial}{\partial \hat{\mathbf{p}}_d'} \sum_j \mathbf{ppa}\left(\hat{\mathbf{x}}_d', \hat{\mathbf{x}}_j\right) \\
&= \frac{\partial}{\partial \hat{\mathbf{p}}_d'} \sum_j \frac{\cos(\alpha(\hat{\mathbf{n}}_d', \hat{\mathbf{n}}_j))}{d(\hat{\mathbf{p}}_d', \hat{\mathbf{p}}_j)} \\
&= \frac{\partial}{\partial \hat{\mathbf{p}}_d'} \sum_j \left[\frac{\hat{\mathbf{n}}_d' \cdot \hat{\mathbf{n}}_j}{\|\hat{\mathbf{n}}_d'\| \cdot \|\hat{\mathbf{n}}_j\|} \frac{1}{\|\hat{\mathbf{p}}_d' - \hat{\mathbf{p}}_j\|} \right] \\
&= \sum_j (\hat{\mathbf{n}}_d' \cdot \hat{\mathbf{n}}_j) \frac{\partial}{\partial \hat{\mathbf{p}}_d'} \frac{1}{\|\hat{\mathbf{p}}_d' - \hat{\mathbf{p}}_j\|} \\
&= \sum_j (\hat{\mathbf{n}}_d' \cdot \hat{\mathbf{n}}_j) \frac{-1}{\|\hat{\mathbf{p}}_d' - \hat{\mathbf{p}}_j\|^2} \frac{\partial}{\partial \hat{\mathbf{p}}_d'} \|\hat{\mathbf{p}}_d' - \hat{\mathbf{p}}_j\| \\
&= \sum_j (\hat{\mathbf{n}}_d' \cdot \hat{\mathbf{n}}_j) \frac{-1}{\|\hat{\mathbf{p}}_d' - \hat{\mathbf{p}}_j\|^2} \frac{\hat{\mathbf{p}}_d' - \hat{\mathbf{p}}_j}{\|\hat{\mathbf{p}}_d' - \hat{\mathbf{p}}_j\|} \\
&= -\sum_j \frac{\hat{\mathbf{n}}_d' \cdot \hat{\mathbf{n}}_j}{\|\hat{\mathbf{p}}_d' - \hat{\mathbf{p}}_j\|^3} \cdot \left(\hat{\mathbf{p}}_d' - \hat{\mathbf{p}}_j \right)
\end{aligned} \tag{2}
$$

Similarly, given $\|\hat{\mathbf{n}}_d\| = \|\hat{\mathbf{n}}_j\| = 1$, we have

$$
\begin{aligned}
&\frac{\partial}{\partial \hat{\mathbf{n}}_d^{'}} \sum_j \mathbf{ppa}\left(\hat{\mathbf{x}}_d^{'}, \hat{\mathbf{x}}_j\right) \\
&= \frac{\partial}{\partial \hat{\mathbf{n}}_d^{'}} \sum_j \left(\frac{\hat{\mathbf{n}}_d^{'} \cdot \hat{\mathbf{n}}_j}{\|\hat{\mathbf{n}}_d^{'}\| \cdot \|\hat{\mathbf{n}}_j\|} \frac{1}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|} \right) \\
&= \sum_j \frac{1}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|} \frac{\partial}{\partial \hat{\mathbf{n}}_d^{'}} \left(\frac{\hat{\mathbf{n}}_d^{'} \cdot \hat{\mathbf{n}}_j}{\|\hat{\mathbf{n}}_d^{'}\| \cdot \|\hat{\mathbf{n}}_j\|} \right) \\
&= \sum_j \frac{1}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|} \frac{\hat{\mathbf{n}}_j - (\hat{\mathbf{n}}_d^{'} \cdot \hat{\mathbf{n}}_j) \frac{\hat{\mathbf{n}}_d^{'}}{\|\hat{\mathbf{n}}_d^{'}\|} \cdot \|\hat{\mathbf{n}}_j\|}{\|\hat{\mathbf{n}}_d^{'}\|^2 \cdot \|\hat{\mathbf{n}}_j\|^2} \\
&= \sum_j \frac{\hat{\mathbf{n}}_j - (\hat{\mathbf{n}}_d \cdot \hat{\mathbf{n}}_j) \cdot \hat{\mathbf{n}}_d}{\|\hat{\mathbf{p}}_d - \hat{\mathbf{p}}_j\|}
\end{aligned} \tag{3}
$$

However, for a single-actor case, since the drone camera is relatively far from the actor patch, we can simplify the problem by constraining the camera orientation targeting the actor, i.e.,

$$
\mathbf{n_d} = \frac{\mathbf{p_a} - \mathbf{p_d}}{\|\mathbf{p_a} - \mathbf{p_d}\|} \tag{4}
$$

With such constraints, we could rewrite our PPA functions and corresponding Jacobian matrix as below.

$$
\begin{aligned}
\mathbf{ppa}\left(\hat{\mathbf{x}}_d^{'}, \hat{\mathbf{x}}_j\right) &= \frac{\cos(\alpha(\hat{\mathbf{n}}_d^{'}, \hat{\mathbf{n}}_j))}{d(\hat{\mathbf{p}}_d^{'}, \hat{\mathbf{p}}_j)} \\
&= \frac{\hat{\mathbf{n}}_d^{'} \cdot \hat{\mathbf{n}}_j}{\|\hat{\mathbf{n}}_d^{'}\| \cdot \|\hat{\mathbf{n}}_j\|} \frac{1}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|} \\
&= \frac{(\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j) \cdot \hat{\mathbf{n}}_j}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\| \cdot \|\hat{\mathbf{n}}_d^{'}\| \cdot \|\hat{\mathbf{n}}_j\|} \frac{1}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|} \\
&= \frac{(\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j) \cdot \hat{\mathbf{n}}_j}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|^2}
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial \hat{\mathbf{p}}_d^{'}} \sum_j \mathbf{ppa}\left(\hat{\mathbf{x}}_d^{'}, \hat{\mathbf{x}}_j\right) \\
&= \sum_j \frac{\partial}{\partial \hat{\mathbf{p}}_d^{'}} \frac{(\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j) \cdot \hat{\mathbf{n}}_j}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|^2} \\
&= \sum_j \frac{\hat{\mathbf{n}}_j \cdot (\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j)^2 - ((\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j) \cdot \hat{\mathbf{n}}_j) \cdot 2(\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j)}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|^4} \\
&= \sum_j \frac{\hat{\mathbf{n}}_j \cdot \|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\| - \left(\frac{\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|} \cdot \hat{\mathbf{n}}_j\right) \cdot 2(\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j)}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|^3} \\
&= \sum_j \frac{\hat{\mathbf{n}}_j \cdot d - 2\cos(\alpha)(\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j)}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|^3} \\
&= \sum_j \frac{\hat{\mathbf{n}}_j \cdot \frac{d}{2\cos(\alpha)} - (\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j)}{\|\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j\|^3}
\end{aligned}
\tag{6}
$$

The result is consistent with the geometry meaning. In Fig. 2, we show the contour map of the PPA values with a single patch in a 2D situation. The contour of a certain PPA value $\mathbf{ppa} = t$ is a circle considering the equation $d = \cos(\alpha)t$. As drawn in Fig. 2, the drone is located on the outside circle. The gradient of PPA values is the tangent direction which is pointing at the circle's center, as drawn in Fig. 2, and can be described as $\hat{\mathbf{n}}_j \cdot \frac{d}{2\cos(\alpha)} - (\hat{\mathbf{p}}_d^{'} - \hat{\mathbf{p}}_j)$.

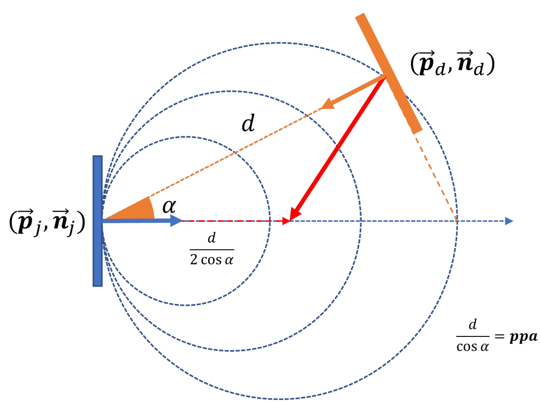

Figure 2: **Contour map of PPA values in 2D**. One actor patch is colored by blue, and the drone pose is colored by orange. PPA values' contour map is plotted by the blue dash. The optimal direction to maximize PPA value is drawn in red.