Social Network Analysis for American Football Games

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DATA DESCRIPTION

The file football.gml contains the network of American football games between Division IA colleges during regular season Fall 2000, as compiled by M. Girvan and M. Newman. The nodes have values that indicate to which conferences they belong.

1. SETUP

Load the package we need to use.

2. LOAD DATA AND SET UP ATTRIBUTE

Load the football.gml into the working environment and add some attributes to the nodes

```
wd <- "~/Desktop/GWU/SocialNetwork/Project02/football"
setwd(wd)
football<-read.graph("football.gml",format=c("gml"))
clo = closeness(football)
btw = betweenness(football)
ev_obj = evcent(football)
eigen = ev_obj$vector
V(football)$Degree.Centrality = degree(football)
V(football)$Closeness.Centrality = clo
V(football)$Betweenness.Centrality = btw
V(football)$Eigenvalue.Centrality =eigen
V(football)$id <- V(football)
V(football)$university <- V(football)$label
V(football)$label <- V(football)</pre>
```

3. SRTUCTURE OF THE NETWORK

We will show density, average centrality of the social network here

```
## Closeness Degree Betweenness Eigen
## 1 0.003502799 10.66087 85.96522 0.7186437
```

From the result, we can see that the density of the network is about 5.33, and the average closeness of the network is really low, which is only 0.01. Though we know the meaning of all the centrality measures in gengeral, we don't know the specific meaning of these centrality measurements in this sceaning. After doing some research about the rules of American Football Competitions, we conclude the meaning of these measurements below:

Degree = How many times does one university have competition with other universities?

Betweenness = Measurement of the winning times for one university

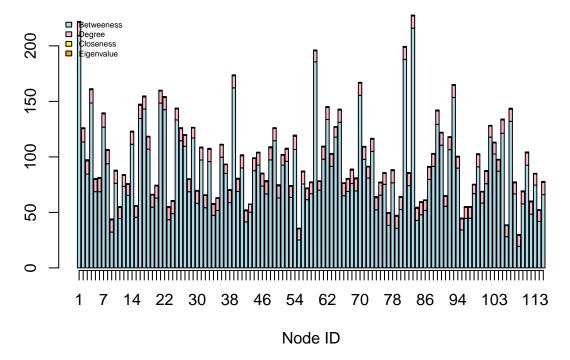
Closeness = 1/ average shortest path of one university node

Eigen = Power of influence of your neighbour university node

We will explain why the average closeness is so low later.

4. PLOT THE CENTRALITY FOR EACH NODE

```
g = football
bet <- 0
for (i in 1:num_nodes){
    #print(i)
    bet[i] <- V(g)[i]$Betweenness.Centrality
}
deg <- 0
for (i in 1:num_nodes){
    deg[i] <- V(g)[i]$Degree.Centrality
}
clo <- 0
for (i in 1:num_nodes){
    clo <- 0
for (i in 1:num_nodes){
    clo <- V(g)[i]$Closeness.Centrality</pre>
```

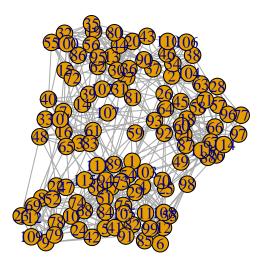


From the barplot, we can see that the betweeness centrality of node 85 is highest.

Node 85 is Oklahomo University, which belongs to Big Twelve Conference

5. SHOW THE NETWORK

plot(football,layout=layout.fruchterman.reingold)

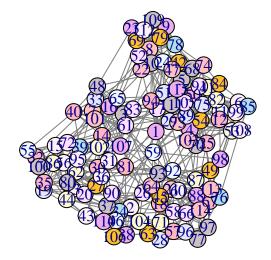


This plots shows the whole network of the football data. Since different node belongs to different conference, we will define the color of vertice based on the value attribute of nodes, which contains the information of conference the node belongs to.

```
colrs <- c("lavender", "lavenderblush", "lavenderblush3", "lemonchiffon",</pre>
            "lightblue2", "lightpink",
            "plum2", "rosybrown1", "thistle1", "aliceblue", "goldenrod1", "gainsboro")
V(football)$color <- colrs[V(football)$value]</pre>
```

Warning in vattrs[[name]][index] <- value: number of items to replace is ## not a multiple of replacement length

```
plot(football, layout = layout.fruchterman.reingold,
vertex.color = V(football)$color, edge.color = grey(0.5), edge.arrow.mode = "-")
legend(x=-1.5, y=-1.1, c("Atlantic Coast", "Big East", "Big Ten", "Big Twelve",
                     "Conference USA", "Independents", "Mid-American",
                     "Mountain West", "Pacific Ten", "Southeastern",
                     "Sun Belt", "Western Athletic"), pch=21,
   col="#777777", pt.bg=colrs, pt.cex=1, cex=.8, bty="n", ncol=4)
```



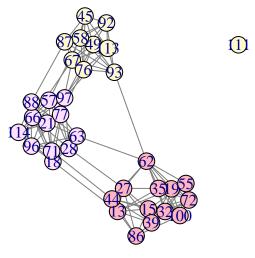
- Atlantic Coast
- Big East
- Big Ten
- Big Twelve
- Mid–American
- Southeaste
- Conference USA
 Mountain West
- Sun Belt

- Independents
- Pacific Ten
- Western At

From this plot we can have a basic understanding of the conference. But since there are so many nodes in the plot, which makes it a little difficult to see the distribution of the conferences, we will then remove some of the conferences and keep three of them to explain why the closeness of the network is so low.

WHY THE AVERAGE CLOSENESS IS SO LOW?

```
newfootball = football
for (i in c(0,1,2,3,5,7,8,10,11)){
    newfootball <- delete.vertices(newfootball, V(newfootball)[V(newfootball)$value == i])
}
V(newfootball)$color <- colrs[V(newfootball)$value]
plot(newfootball, layout = layout.fruchterman.reingold,edge.color = grey(0.5), edge.arrow.mode = "-")</pre>
```



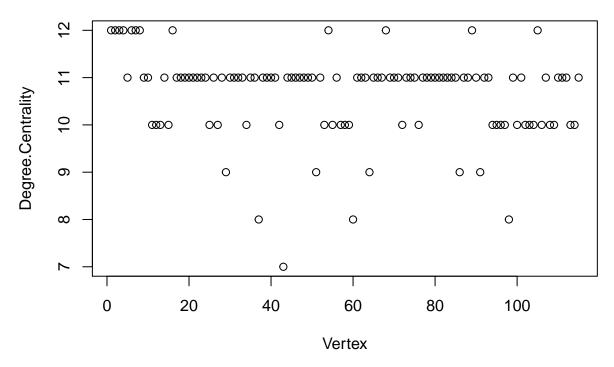
From this plot, we can see that basically nodes in one conference will form an area. The nodes in the center of this area will only connect with the nodes in the outer area. This situation is consistent with the rule for American Football Competition that teams in one conference compete to each other first and then the winner will compete with the teams in other conference.

We can take the node 19 in the pink area as an example. If node 19 wants to connect with node 49, it must connect to node 62 at first, then connect to 93 and 58. The logic is same for every node in the network. Therefore, the shortest average path for every node gets higher, then the closeness will get smaller and smaller.

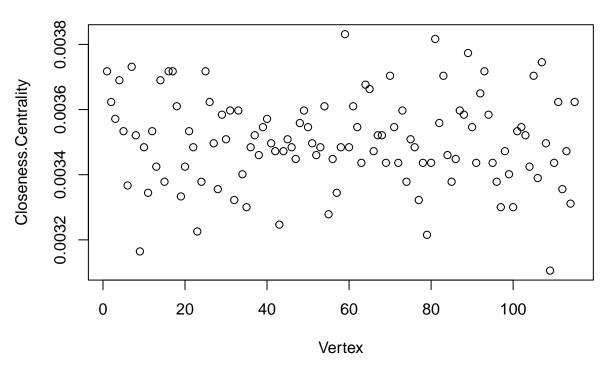
WHAT IS THE DISTRIBUTION OF DIFFERENT CENTRALITY?

```
g=football
closeness_g = closeness(g)
btw_g = betweenness(g)
ev_obj_g = evcent(g)
eigen_g = ev_obj_g$vector
V(g)$Degree.Centrality = degree(g)
V(g)$Closeness.Centrality = closeness_g
```

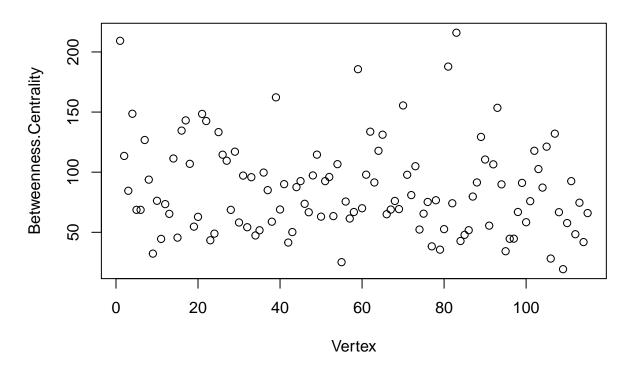
The distribution of Degree Centrality



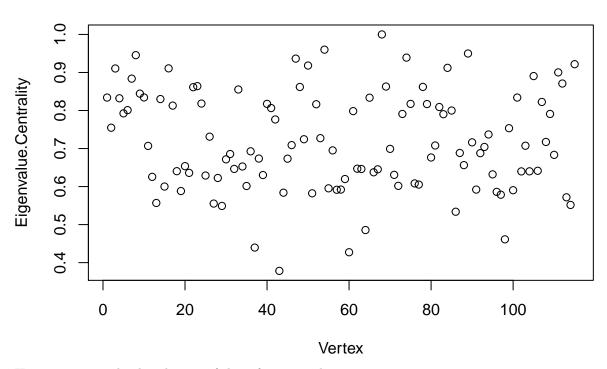
The distribution of Closeness.Centrality



The distribution of Betweenness.Centrality



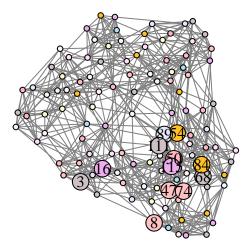
The distribution of Eigenvalue.Centrality



Here we can see the distribution of these four centralities.

WHAT DOES EGIEN CENTRALITY MEAN?

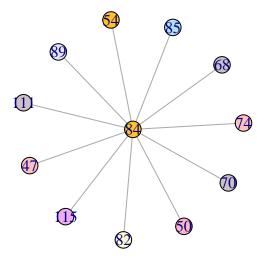
As we mentioned before, eigen centrality means the power of the neighbor nodes. Now, we will have a more detailed look at the eigen centrality. From the plot of distribution of Egien Centrality avove, we can find that only a few nodes have eigen centrality above 0.9. So, we will make the nodes with eigen centrality higher than 0.9 look bigger.



The nodes in bigger size are the nodes with eigen centrality higher than 0.9. We will take node 84 as an example.

Therefore, we need to see a subnetwork that only contains node 84 and the nodes connected to node 84.

```
e = E(football)[from(84)]
sub = subgraph.edges(football, as_ids(e))
plot(sub)
```



Here, we can see that node 84 is connected to node 85, so we think maybe the influence power is related to the winning times which is betweeness centrality.

Therefore, we will explore the relationship between the eigen centrality and the average betweeness of the neighbor nodes.

First, we will define a new attribute:avg_bet for V(football)

```
for (i in 1:115){
  e = E(football)[from(i)]
  sub = subgraph.edges(football, as_ids(e))
  node_num = vcount(sub)-1
  sum_bet = 0
```

```
list = V(sub)$id
for (node in list){
   bet = V(football)[node]$Betweenness.Centrality
   sum_bet = bet+sum_bet
}
sum_bet = sum_bet - V(football)[i]$Betweenness.Centrality
avg_bet= sum_bet/node_num
V(football)[i]$avg_bet = avg_bet
}
V(football)$avg_bet
```

```
##
    [1] 80.87474 86.71277 105.96873 89.19635 99.13833 72.19918 99.19464
##
    [8]
        79.37477 70.79980 86.23675 79.28835 100.38129 86.63935
                                                                 95.67982
##
   [15]
        85.99830 91.51181 97.72408 90.02512 75.43011 74.08667
                                                                 80.03423
##
   [22] 71.82297 71.61699 84.78724 90.94055 86.26905 86.52581
                                                                 81.01006
##
   [29] 90.96375 100.04265 87.25059 69.80940 103.43502 91.62326
                                                                 69.63947
   [36] 91.40870 91.66933 86.37902 77.23699 115.91147 86.21018
##
                                                                 95.55978
##
   [43] 78.03970 82.54560 83.62388 83.01471 83.35304 96.30402
                                                                 86.25850
##
   [50] 80.96500 92.53768 69.80183 90.05886 80.08289 78.59325 76.28932
##
  [57] 74.09442 92.18870 103.51309 87.79730 98.37157 78.58433 81.20505
   [64] 85.53833 96.72592 96.11908 91.21940 82.86168 74.04457
##
                                                                 87.42271
##
   [71] 80.91221 78.85058 85.65841
                                     79.91215
                                              93.04367 90.42611
                                                                 73.92284
##
   [78] 79.14638 67.05522 84.79846 78.94713 96.75169 83.83482
                                                                 84.55501
   [85] 78.34158 99.71166 102.82509 85.33989 90.36692 76.74987 103.43745
##
   [92] 103.82886 98.76593 121.91475 101.93691 81.22378 81.13047
                                                                 95.11386
   [99]
        74.19365 68.41069 111.58239 87.54649 83.79434 80.47131
                                                                 90.78123
## [106] 94.25878 107.32932 82.84209 75.04925 81.94234 94.12114 73.83286
## [113] 85.64744 82.43550 97.34687
```

Now, we can see the average betweenness centrality for all the neighbor nodes of one node.

Then, we will do a linear regression for the avg_bet and eigen for every node.

```
avg_bet = V(football)$avg_bet
node_eigen = V(football)$Eigenvalue.Centrality
slr <- lm(node_eigen~avg_bet)
summary(slr)</pre>
```

```
##
## Call:
## lm(formula = node_eigen ~ avg_bet)
##
## Residuals:
##
       Min
                  1Q
                      Median
## -0.33466 -0.09416 -0.01671 0.10343 0.28396
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
                                     6.521 2.03e-09 ***
## (Intercept) 0.663089
                          0.101680
## avg bet
              0.000639
                          0.001161
                                     0.550
                                              0.583
```

```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1308 on 113 degrees of freedom
## Multiple R-squared: 0.002673, Adjusted R-squared: -0.006153
## F-statistic: 0.3029 on 1 and 113 DF, p-value: 0.5832
```

We can see though p-value is around 0.5, so we cannot reasonably sure that there is relationship between avg_bet and node_eigen.

So, we will define a new attribute: avg_deg for V(football)

```
for (i in 1:115){
    e = E(football)[from(i)]
    sub = subgraph.edges(football, as_ids(e))
    node_num = vcount(sub)-1
    sum_deg = 0
    list = V(sub)$id
    for (node in list){
        bet = V(football)[node]$Degree.Centrality
            sum_deg = bet+sum_deg
    }
    sum_deg = sum_deg - V(football)[i]$Degree.Centrality
    avg_deg= sum_deg/node_num
    V(football)[i]$avg_deg = avg_deg
}
V(football)$avg_deg
```

```
##
     [1] 10.750000 10.666667 11.166667 10.750000 10.818182 10.333333 10.916667
     [8] 11.000000 10.818182 10.909091 10.800000 10.200000 10.300000 11.090909
##
## [15] 10.500000 11.000000 11.090909 10.545455 10.000000 10.454545 10.454545
   [22] 11.000000 11.000000 10.818182 10.300000 10.818182 10.300000 10.545455
## [29] 10.222222 10.454545 10.636364 10.545455 11.181818 11.000000 10.090909
## [36] 10.909091 10.375000 10.454545 10.181818 11.090909 10.818182 11.200000
## [43] 10.428571 9.909091 10.727273 10.818182 11.272727 11.272727 10.818182
   [50] 11.272727 10.333333 10.636364 11.100000 11.083333 10.700000 10.818182
## [57] 10.700000 10.400000 10.100000 9.750000 10.818182 10.636364 10.727273
## [64] 9.666667 11.000000 10.636364 10.454545 11.166667 11.000000 10.272727
## [71] 10.636364 10.700000 10.818182 11.363636 11.000000 10.700000 10.272727
   [78] 11.000000 10.636364 10.818182 10.545455 11.000000 10.909091 11.272727
## [85] 10.818182 10.666667 10.636364 10.636364 11.000000 10.818182 10.555556
## [92] 10.727273 10.818182 11.100000 10.900000 10.800000 10.700000 10.125000
## [99] 10.454545 10.500000 11.090909 10.900000 10.700000 10.800000 10.916667
## [106] 10.900000 11.181818 10.900000 11.100000 10.636364 11.181818 11.000000
## [113] 10.100000 10.300000 11.363636
```

Now, we can see the average betweenness centrallity for all the neighbor nodes of one node.

Then, we will do a multiple regression for the avg_bet, avg_deg and eigen for every node.

```
avg bet = V(football)$avg bet
avg_deg = V(football)$avg_deg
node_eigen = V(football)$Eigenvalue.Centrality
mr <- lm(node_eigen~avg_bet+avg_deg)</pre>
summary(mr)
##
## Call:
## lm(formula = node_eigen ~ avg_bet + avg_deg)
## Residuals:
##
       Min
               1Q Median
                                 3Q
## -0.26498 -0.05158 0.00283 0.05224 0.17792
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.3774784 0.2342875 -10.15 <2e-16 ***
## avg_bet -0.0014365 0.0007366 -1.95 0.0537 .
             ## avg_deg
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.08113 on 112 degrees of freedom
## Multiple R-squared: 0.6195, Adjusted R-squared: 0.6127
## F-statistic: 91.19 on 2 and 112 DF, p-value: < 2.2e-16
```

Therefore, we can be reasonably sure that eigen centrality is something related to average degree and average betweenness of all the neighbor nodes. However, we can see that the p-value for avg_bet is higher than 0.05. So, we will use eta square to see how much proportion of the variance can be uniquely explained by every independent variable.

```
require(heplots)

## Loading required package: heplots

## Loading required package: car

etasq(mr,anova=TRUE,partial=FALSE)

## Anova Table (Type II tests)

## Response: node_eigen

## eta^2 Sum Sq Df F value Pr(>F)

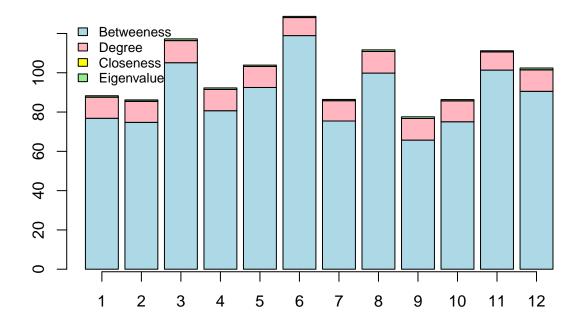
## avg bet 0.01279 0.02503 1 3.8025 0.05368 .
```

From the result above, we can conclude that there is only 0.1279 of the variance can be explained by avg_bet.

So, average degree of the neighbor nodes is the one that really matters for the eigen centrality.

WHICH CONFERENCE IS MORE ACTIVE?

```
g = football
bet <- 0
for (i in 0:11){
        \texttt{bet[i+1]} \leftarrow \texttt{sum(V(g)[value==i]} \\ \texttt{Betweenness.Centrality)} / \texttt{length(V(g)[value==i]} \\ \texttt{Betweenness.Centrality}) / \texttt{length(V(g)[value==i]} \\ \texttt{length(V(g)[value==i]} \\
}
deg <- 0
for (i in 0:11){
        deg[i+1] <- sum(V(g)[value==i]$Degree.Centrality)/length(V(g)[value==i]$Degree.Centrality)
clo <- 0
for (i in 0:11){
        clo[i+1] <- sum(V(g)[value==i]$Closeness.Centrality)/length(V(g)[value==i]$Closeness.Centrality)</pre>
}
eig <- 0
for (i in 0:11){
        eig[i+1] <- sum(V(g)[value==i]$Eigenvalue.Centrality)/length(V(g)[value==i]$Eigenvalue.Centrality)
}
        matrix <- rbind(bet,deg,clo,eig)</pre>
        matrix <- as.matrix(matrix)</pre>
        xlabbb <-seq(0:11)
        bar = barplot(matrix,col=c("lightblue","lightpink","yellow","lightgreen"))
        axis(1, at=bar, labels=xlabbb)
        legend("topleft", c("Betweeness","Degree", "Closeness", "Eigenvalue"), cex=0.8, bty="n", fill = c("ligh
```



Here, we can see that conference five (Independents) has the highest Betweeness. Considering the competition rule of American Football, we think this conference win many of other conferences, and then compete with other conferences. Also, the winner of this conference maybe came to the last round of the competition. Following this logic, we take a look at the betweeness of every university node in the conference five.

```
V(football)[value==5]$id

## [1] 37 43 81 83 91

V(football)[value==5]$Betweenness.Centrality

## [1] 85.06001 50.17220 187.82634 215.98577 55.52058
```

We can see that node 83 has a really high betweeness (top 5 nodes with high Betweenness.Centrality). We think this node should be the winner in this conference.

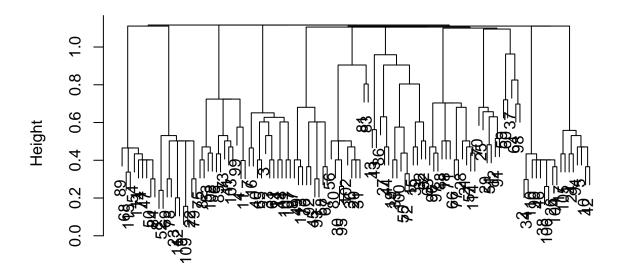
WHAT DOES BLOCK MEAN IN OUR MODEL

First, let us take a look at our cluster, community and block model.

```
matrix_row_to_col <- get.adjacency(football)
matrix_row_to_col <-as.matrix(matrix_row_to_col)
matrix_col_to_row <- t(matrix_row_to_col)
matrix <- rbind(matrix_row_to_col,matrix_col_to_row )
cors <- cor(matrix_row_to_col)</pre>
```

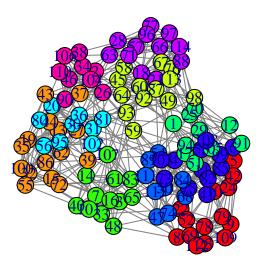
```
dissimilarity <- 1 - cors
dist <- as.dist(dissimilarity)
hclust <- hclust(dist)
plot(hclust)</pre>
```

Cluster Dendrogram



dist hclust (*, "complete")

This is what cluster looks like.



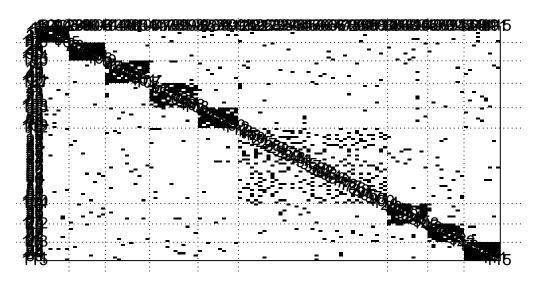
From the community plot, we can see that there are nine blocks

```
matrix_row_to_col <- get.adjacency(football)
matrix_row_to_col <-as.matrix(matrix_row_to_col)
matrix_col_to_row <- t(matrix_row_to_col)
matrix <- rbind(matrix_row_to_col,matrix_col_to_row )
cors <- cor(matrix_row_to_col)
dissimilarity <- 1 - cors
dist <- as.dist(dissimilarity)
hclust <- hclust(dist)
num_clusters = 9
clusters <- cutree(hclust, k = num_clusters)
cluster_cor_mat <- clusterCorr(cors,clusters)
gcor(cluster_cor_mat, cors)</pre>
```

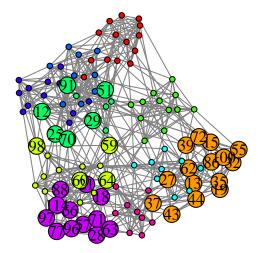
[1] 0.7978018

```
mean <- mean(matrix_row_to_col)
blockmodel <- blockmodel(matrix_row_to_col, clusters)
plot(blockmodel,main="blockmodel")</pre>
```

Relation - 1



```
V(football)$block = clusters
V(football)$size = 5
V(football)[V(football)$block == 6]$size = 15
V(football)$label = NA
V(football)[V(football)$block == 6]$label = V(football)[V(football)$block == 6]$id
plot(football, layout = layout.fruchterman.reingold, vertex.size = V(football)$size,
    vertex.color = V(football)$color,vertex.label = V(football)$label,
    vertex.label.color="black",edge.color = grey(0.5),edge.arrow.mode = "-")
```



Here we can see the block model plot of our social network.

Form the plot of block model, we can see that there is sparse interaction in block 6. Based on the definition of block, we can know that nodes in one block have similar position and role in the whole network. Therefore, we assume that the block 6 contains nodes which lose in the first round competition in every conference. To explore this, we visualize the nodes in block 6. We can see that the nodes in block 6 are distributed across all the conferences, which can prove our assumption to some extent.

WHICH CNOFERENCE HAS TEAMS THAT COMPETE AMONG ONE ANOTHER?

```
football <- data.matrix(football)
adj = get.adjacency(football)
adj = as.matrix(adj)
value <- V(football)$value
value <- data.frame(value)
label <- V(football)$label
label <- data.frame(label)
df <- cbind(value,label)
foot <- set_vertex_attr(football, "value", index = V(football), df[,1])
foot <- set_vertex_attr(football, "label", index = V(football), df[,2])
foot <- asNetwork(foot)
ergm_fb <- ergm(foot ~ edges)</pre>
```

Evaluating log-likelihood at the estimate.

```
summary(ergm_fb)
```

```
##
## ==========
## Summary of model fit
## ==========
##
## Formula: foot ~ edges
## Iterations: 6 out of 20
##
## Monte Carlo MLE Results:
        Estimate Std. Error MCMC % p-value
##
## edges -2.27144 0.04242
                             0 <1e-04 ***
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
       Null Deviance: 9087 on 6555 degrees of freedom
## Residual Deviance: 4072 on 6554 degrees of freedom
## AIC: 4074 BIC: 4081
                          (Smaller is better.)
ergm_fb2 <- ergm(foot ~ edges + nodematch("value"))</pre>
```

Evaluating log-likelihood at the estimate.

```
summary(ergm_fb2)
```

```
## =========
##
## Formula: foot ~ edges + nodematch("value")
## Iterations: 6 out of 20
##
## Monte Carlo MLE Results:
##
                 Estimate Std. Error MCMC % p-value
## edges
                 -3.27878
                            0.06883
                                        0 <1e-04 ***
## nodematch.value 4.39532
                            0.12259
                                         0 <1e-04 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
##
       Null Deviance: 9087
                           on 6555 degrees of freedom
##
   Residual Deviance: 2467 on 6553 degrees of freedom
##
## AIC: 2471
              BIC: 2484
                           (Smaller is better.)
ergm_fb3 <- ergm(foot ~ edges + nodematch("value", diff=T))</pre>
## Evaluating log-likelihood at the estimate.
summary(ergm_fb3)
##
## ===========
## Summary of model fit
  _____
##
## Formula: foot ~ edges + nodematch("value", diff = T)
## Iterations: 13 out of 20
## Monte Carlo MLE Results:
                     Estimate Std. Error MCMC % p-value
##
                                            0 <1e-04 ***
## edges
                     -3.27878
                                0.06883
                                               0.972
## nodematch.value.0 20.59079 580.70519
## nodematch.value.1 20.34938 583.58969
                                               0.972
                                            0
## nodematch.value.2 4.66507
                              0.34406
                                            0 <1e-04 ***
## nodematch.value.3 4.25961
                              0.28483
                                            0 <1e-04 ***
## nodematch.value.4 4.07371
                              0.32928
                                            0 <1e-04 ***
## nodematch.value.5
                    1.08156
                                               0.306
                               1.05634
                                            0
                              0.24587
## nodematch.value.6
                    3.85860
                                            0 <1e-04 ***
## nodematch.value.7 20.34938 583.58969
                                               0.972
## nodematch.value.8
                    5.35822
                                            0 <1e-04 ***
                               0.47931
## nodematch.value.9
                      4.25961
                                0.28483
                                            0 <1e-04 ***
## nodematch.value.10 3.18347
                                0.44232
                                            0 <1e-04 ***
## nodematch.value.11
                      3.97193
                                0.32363
                                            0 <1e-04 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
##
       Null Deviance: 9087 on 6555 degrees of freedom
## Residual Deviance: 2374 on 6542 degrees of freedom
```

```
##
## AIC: 2400 BIC: 2488 (Smaller is better.)

exp(-2.27144)/(1+exp(-2.27144))
```

```
## [1] 0.09351607
```

First, we do a ergm model to explore the relationship between the probability based on edge formation. We can see that we get a negative edge parameter (p<50%) since the network is rather sparse. The edge parameter (-2.27144) here is the log of the edge odds. The corresponding probability is 0.09351607: exp(-2.27144)/(1+exp(-2.27144)).

To understand that which conference has teams that play more among one another we have to look at the individual coefficient of node match values in following summary fit chart.

We can see that conference eight has the significant p value and highest coefficient, i.e. 5.36, among remaining nodes.

This indicates that team under conference eight has competed more withing among one another in comparison to teams outside its conference.