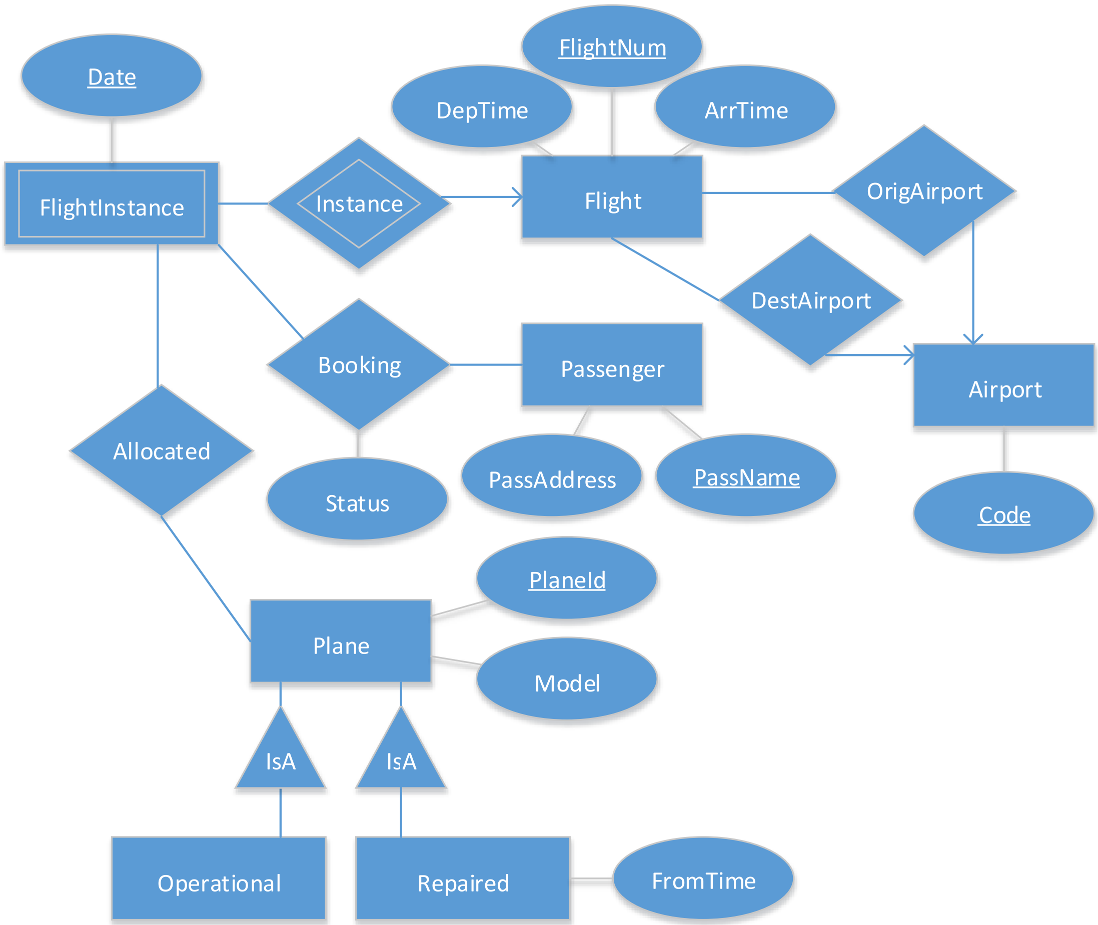
Problem 1



Problem 2

According to the textbook, the definition of "Satisfying the BCNF rule" is:

The left side of every non-trivial FD must contain a key.

1.

This schema is not in BCNF.

We rename Course as "C", rename Teacher as "T", and rename Hour as "H", rename Room as "R", rename Student as "S". The original relation is {C T H R S}

From HT->R, HR->C, we get HST->HSCTR, so {HST} is the key of this schema. Consider the FD C->T, the lhs of the FD is not a superkey(It does not contain the key) of this schema, so this schema is not in BCNF.

2.

Lossless join decomposition into BCNF：

For {C, T, H, R, S}

The key is {H,S,T}

C->T violates the BCNF rule,

The R1 = C deviation = {C, T};

The R2 = C UNION (R-R1) = C UNION (H,R,S).

For R1, FD is C->T;

According to the textbook, the definition of "Satisfying the BCNF rule" is:
The left side of every non-trivial FD must contain a key.
The key of R1 is C.
R1 is in BCNF rule;
For R2,
Since FD C->T is in R1, so there is no T in the R2;
But it is lossless join, T is still existed in the other FDs, so if R1 is {C, T}
R2 is {C, H, R, S, T}.
The Key of R2 is HST.
FD HR->C and HS->R and HT->R is R2's FD.
From the FD HR->C,
HR is not a superkey, so HR->C violates the BCNF.
Decompose for the HR->C;
So, R21={H, R, C}, R22={H, T, S, R}
For R21, The FD is HR->C
The key is HR, R1 satisfies the BCNF rule;
For R22 = {H,R,S},
The key is HS, the FD is HS->R and HT->R;
R2 satisfies the BCNF rule.
So, in the end, it is:
R1= {C,T}
R21={H,R,C}
R22={H, T, S, R}
With the FD C->T, H,R->C, H, S->R, H, T->R;
It preserves the given functional dependencies.

Problem 3
1.
No.
From B->A, we know $A^+ = \{A\}$.
B $\notin$ A^+, so A->B doesn't follow from B->A.

2.
Yes.
From C->A, $C^+ = \{C\ A\}$; from AC->B, $C^+ = \{C\ A\ B\}$.
B$\in$$C^+$, so C->B follow from AC->B and C->A.

3.
No.
From AB->C, we know $A^+ = \{A\}$.
C $\notin$ A^+, so A->C doesn't follow from AB->C.

4.
No.

From A->C and A->B, we know A^+ is {A B C}

D $\notin$ A^+, so A->D doesn't follow from A->C and A->B.