

Profiling Driver Behavior for Personalized Insurance Pricing and Maximal Profit

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Abstract—Profiling driver behaviors and designing appropriate pricing models are essential for auto insurance companies to gain profits and attract customers (drivers). The existing approaches either rely on static demographic information like age, or model only coarse-grained driving behaviors. They are therefore ineffective to yield accurate risk predictions over time for appropriate pricing, resulting in profit decline or even financial loss. Moreover, existing pricing strategies seldom take profit maximization into consideration, especially under the enterprise constraints. The recent growth of vehicle telematics data (vehicle sensing data) brings new opportunities to auto insurance industry, because of its sheer size and fine-grained mobility for profiling drivers. But, how to fuse these sparse, inconsistent and heterogeneous data is still not well addressed. To tackle these problems, we propose a unified PPP (Profile-Price-Profit) framework, working on the real-world large-scale vehicle telematics data and insurance data. PPP profiles drivers' fine-grained behaviors by considering various driving features from the trajectory perspective. Then, to predict drivers' risk probabilities, PPP leverages the group-level insight and categorizes drivers' different temporal risk change patterns into groups by ensemble learning. Next, the pricing model in PPP incorporates both the demographic analysis and the mobility factors of driving risk and mileage, to generate personalized insurance price for supporting flexible premium periods. Finally, the maximal profit problem proves to be NP-Complete. Then, an efficient heuristic-based dynamic programming is proposed. Extensive experimental results demonstrated that, PPP effectively predicts the driver's risk and outperforms the current company's pricing strategy (in industry) and the state-of-the-art approach. PPP also achieves near the maximal profit (difference by only 3%) for the company, and lowers the total price for the drivers.

Index Terms—Driver behavior profiling, personalized insurance pricing, company profit, trajectory data mining

I. INTRODUCTION

Insurance is designed to protect the people and things we value most. Among it, auto insurance is one important category that provides financial protection against damages and liability resulting from car accidents. How to profile driver behaviors and devise pricing models, plays an essential role for insurance companies to gain profits and attract customers (drivers).

There has been considerable research on understanding the driver behavior and auto insurance pricing in the last decade. Traditional approaches, e.g., generalized linear models [1], [2], rely on drivers' static demographic information (e.g., age, gender and vehicle type) to compute the insurance price, but

TABLE I: Drivers' claim statistics.

Driver Type	Driver Percentage	Accident Percentage
Claim count = 0	72%	0%
Claim count = 1	11%	20%
Claim count ≥ 2	17%	80%

usually neglect the driving risk. Usage-Based Insurance (UBI) [3] based methods [4], such as Pay-As-You-Drive model [5] and Pay-How-You-Drive model [6], are introduced to model the driver mobility factors like time, mileage and speed for improving insurance pricing.

However, the above solutions have the following drawbacks. 1) They are able to model only coarse-grained driving behaviors, resulting in inappropriate pricing, and incurring potential profit decline or even financial loss. As reported in 2016, over 70% auto insurance companies in China were in financial loss [7]. Table I shows the real-world claim data offered by a mainstream insurance company (due to the privacy concern, we omit the name) for the year 2016. Note that 17% of the drivers cause 80% of the accidents and claim indemnity. These drivers' risk behaviors necessitate further investigation at a finer-grained level. 2) The existing approaches cannot capture the time-variant driving risk. According to the survey conducted by the same company mentioned above, the number of overlapping accident-involved drivers between 2016 and 2017 is only about 3% (2.8% between 2015 and 2016). This indicates that driver risk behaviors often change over time. Thus, capturing the temporal risk patterns is crucial to build an accurate pricing model, which is required by the company as personalized and flexible. 3) Traditional models fail to link driver behaviors and pricing models with the ultimate goal of maximizing company profits, especially under the real-world enterprise constraints.

Recently, the rapid development of telematics [2], [8] in auto insurance industry has enabled to collect large amounts of fine-grained mobility data, like vehicle speed, acceleration, engine speed and so on, to better profile drivers' risk for pricing. With these telematics data, traditional methods [9] are usually leveraged to compute the insurance price, e.g., Pay-How-You-Drive model [10]. Although the mass of new telematics data has great potential to model driving behaviors more accurately and improve the granularity of risk prediction, it also poses new research challenges. First, telematics data

TABLE II: OBD data description (Driving state variables are determined by domain experts).

Driving Variable	Driving State Variable	Description
Vehicle Angular Velocity (in Radian Per Second (rad/s))	Sharp Turn	Vehicle angular velocity $\geq 30 rad/s$
	Lane Change	$10 rad/s \leq \text{vehicle angular velocity} < 30 rad/s$
Cool Liquid Temperature	Low Temperature	Cool liquid temperature lower than the normal lower bound value
	High Temperature	Cool liquid temperature higher than the normal upper bound value
Acceleration (in Meter Per Second Squared (m/s^2))	Abnormal Acceleration	Acceleration $\geq 1.8 m/s^2$
	Abnormal Deceleration	Acceleration $\leq -1.8 m/s^2$
Engine Speed (in Rounds Per Minute (RPM))	Engine High RPM Warning	Engine speed higher than the default upper bound value of the vehicle
	Abnormal RPM Increase	Engine speed increases sharply in a short time (usually five seconds)
Speed	Vehicle Speeding	Speed higher than the road speed limit after matching with road types by GPS

is sparse and inconsistent. Due to privacy concerns, drivers may be unwilling to or only share short-period data. So, such data sparsity in time stream makes it very difficult to train a reliable risk prediction model for each individual. Alternatively, one may suggest aggregating all drivers' data to train one single model. However, the generated model could suffer from data inconsistency because different drivers often have different driving behaviors. Second, telematics data is essentially distinct from other traditional data like UBI data. How to effectively fuse these heterogeneous data sources for more accurate pricing remains a difficult task.

To tackle the above drawbacks and challenges, we propose a unified PPP framework that offers a personalized pricing and profit maximization solution based on fine-grained driver behavior profiling results. In essence, we aim to address *three inter-related subtasks*: **1. Driver Behavior Profiling for Risk Prediction**. To better model driver behaviors and risks, we fuse heterogeneous data, namely vehicle telematics data collected from the popular On-Board Diagnostic (OBD) [11] devices, and UBI data [3] that includes information such as the driver's demographic, insurance and claim indemnity data. Trajectory-based features are first extracted to model the temporal risk change pattern for each individual. To alleviate the data sparsity and inconsistency problem, motivated by the group-level insight, power-law-based ensemble learning is performed to categorize drivers' different temporal behavior patterns within groups, enabling more reliable predictions of driver risk probability. **2. Personalized Insurance Pricing**. We propose a novel personalized pricing model that incorporates not only the demographic analysis, but also the mobility factors of the driver's risk probability and traveled mileage. The generated pricing model is thus adaptive to personal risk behaviors and can support various premium periods (the time length that the insurance covers). **3. Company Profit Maximization**. We also propose a practical solution for maximizing company profit, under the enterprise constraints (e.g., the total insurance payment). The maximal profit problem is formulated as a driver selection problem, and proves to be NP-Complete by reducing it to the well-known 0-1 Knapsack problem [12]. With real-world heuristics, a constrained dynamic programming algorithm is developed to reduce the search space and solve the problem efficiently. The main contributions are summarized as follows.

- To the best of our knowledge, we are the first to comprehensively profile driver behaviors from heterogeneous data over time, which not only captures fine-grained driving behaviors, but also provides group-level insights

of temporal risk change patterns with Power Law models.

- We propose a *novel* insurance pricing model, which is *personalized* and *flexible* by incorporating both mileage and driving risk from the mobility perspective.
- We formulate the maximal profit problem and prove it to be NP-Complete. An efficient heuristic-based dynamic programming algorithm is also provided.
- We conduct extensive experiments on the real-world large-scale OBD and UBI data to verify the effectiveness of PPP. By comparing with the industrial and state-of-the-art approach, PPP achieves the approximate maximal profit, lowers the total price for the drivers and receives positive reviews from domain experts for promising applications.

II. PRELIMINARIES

A. Data Description

OBD [11] is an advanced plug-in device, to record a vehicle's various sensing data in real-time. Each OBD record x is defined as a tuple $\langle u_x, t_x, l_x, \phi_x \rangle$ where: (1) u_x is the driver id; (2) t_x is the corresponding timestamp (in second); (3) l_x is a two-dimensional vector representing the longitude and latitude where x is created; (4) ϕ_x is a fourteen-dimensional vector (in Table II), consisting of five driving variables (real-valued number) and nine driving state variables (0-1 valued number).

UBI [3] data, z , records the driving history and insurance information of a vehicle, including the traveled mileage, the past insurance price, the driver's demographic information (e.g., age, gender and driving experience/years) and the time, location and claim indemnity of the driver's per traffic accident.

B. Problem Description

In this paper, we solve the following three problems:

- (1) Given a driver set with N drivers, $U = \{u_1, u_2, \dots, u_N\}$, and their OBD data in the past H weeks, $X = \{x_i\}$, how to predict a driver u 's future w -th week risk probability \hat{p}_u^{H+w} ?
- (2) Considering a driver u 's future risk \hat{p}_u^{H+w} and UBI data z , how to calculate this driver's insurance price r_u ?
- (3) With the driver set U , the upper bound of the insurance company's risk \mathcal{P} and insurance payment \mathcal{C} , how to generate the maximal insurance profit D ?

III. PROFILE-PRICE-PROFIT (PPP) FRAMEWORK

A. PPP Framework Overview

To solve the aforementioned problems, we propose a unified PPP (Profile-Price-Profit) framework comprising of three major models, as shown in Fig. 1. Specifically, the **Driver Behavior**

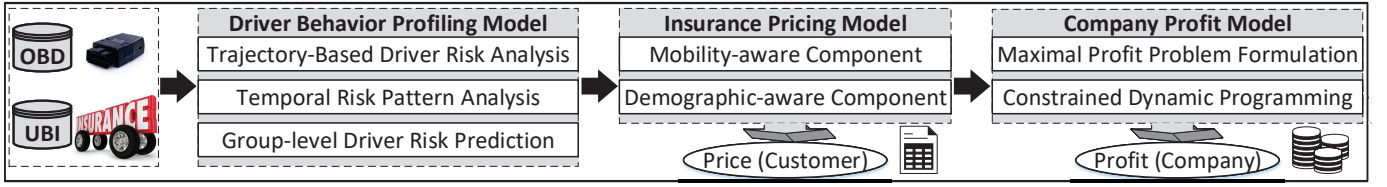


Fig. 1: The overview of PPP framework.

Profiling Model analyzes trajectory-based driver risk from OBD and UBI data, models temporal risk change patterns in time series, and proposes a group-level solution to predict future risk. The **Insurance Pricing Model** generates a driver's insurance price based on mobility factors extracted from the driver behavior profiling model, and demographic information. The **Company Profit Model** tackles the profit maximization problem, which is proved to be NP-Complete, and provides a heuristic-based dynamic programming solution.

B. Driver Behavior Profiling Model

In this subsection, we present how to predict a driver's future driving risk. First in Section III-B1, we predict a driver's risk from trajectories [13] to capture the mobility pattern. Then in Section III-B2, considering the time-varying property of trajectories [14], we partition the data by week, and fit the behavior in the time stream through the Power Law pattern. Finally, in Section III-B3, to overcome the challenges of data sparsity and data inconsistency in the risk prediction, we find the group-level insight, and propose an iterative refinement algorithm solution by ensemble learning.

1) *Trajectory-based Driver Risk Analysis*: Intuitively, from the mobility perspective, a driver's driving risk can be captured from his/her OBD records. Considering the large volume of the historical OBD data (i.e., a sequence of chronologically ordered OBD records $S = x_1x_2...x_n$), we model the risk by *trajectory* [13], which is a set of records to capture a driver's behavior, defined as (where the speed of an OBD record x is denoted as v_x):

DEFINITION 1 (TRAJECTORY). Given a driver's OBD record sequence $S = x_1x_2...x_n$ and a time gap $\Delta t > 0$, a subsequence $S' = x_ix_{i+1}...x_{i+k}$ is a trajectory of S if S' satisfies: (1) $v_{x_{i-1}} = 0, v_{x_i} > 0, v_{x_{i+k}} > 0, v_{x_{i+k+1}} = 0$; (2) if there exists a subsequence $S'' = x_jx_{j+1}...x_{j+g} \in S'$, where $\forall 0 \leq q \leq g, v_{x_{j+q}} = 0$, then, $t_{x_{j+g}} - t_{x_j} \leq \Delta t$; (3) there are no longer subsequences in S that contain S' and satisfy condition (1)(2).

Leveraging the speed and time constraints of a trajectory, we can extract reliable and informative record sequences for more efficient and effective study. At the same time, since a trajectory can either be safe (having no accidents) or dangerous (having accidents), we use a 0-1 valued variable y_S to denote a trajectory's label. It is set by querying a driver's UBI data: if there is a UBI accident record during the trajectory period, $y_S = 1$; otherwise, $y_S = 0$.

Given a driver u 's one trajectory $S = x_1x_2...x_n$, we aim to predict the risk probability of the trajectory p_S , that is, the probability of having an accident or not. This probability depends on many factors, such as driving variables of speed

and driving state variables of vehicle speeding. According to domain experts, such information can be encoded into a feature vector $\Phi_S \in \mathbb{R}^{14}$, which, without loss of generality, utilizes the whole fourteen driving factors (in ϕ_x with details in Table II) in OBD records by time ratio. Due to the space limit, we take the driving factor of speed v as an example to present the time ratio computation as:

$$\bar{v} = \frac{\sum_{1 \leq i \leq n-1} (v_{x_i} + v_{x_{i+1}})(t_{x_{i+1}} - t_{x_i})}{t_{x_n} - t_{x_1}}, \quad (1)$$

where \bar{v} is the time ratio of speed. Through this feature extraction, Φ_S captures trajectory-level distinctive information instead of focusing on individual OBD record. Then, the trajectory risk probability prediction is defined as follows:

$$p_S = p(y_S = 1 | \Phi_S). \quad (2)$$

This formulates the prediction problem to a typical binary classification problem. We employ two widely-used methods: Logistic Regression (LR) [15] and Gradient Boosted Decision Tree (GBDT) [16] in terms of Precision (PRE), Recall (REC), F1 score (F1), Accuracy (ACC) and Area Under Curve of ROC (AUC). As shown in Fig. 2, GBDT performs better than LR. Therefore, we choose GBDT in our framework. Note that PPP is open for other classification methods to plug in.

In practice, a driver can generate massive trajectories in a given period. To evaluate the driver's risk, we empirically average the risk probability of these trajectories. Meanwhile, to capture the driving property in both weekday (work) and weekend (rest), we set the time period as a week. Formally, in week h , driver u can generate a set of trajectories $\mathbb{S}_h = \{S_1, S_2, \dots, S_{|\mathbb{S}_h|}\}$ from week h 's OBD data $\mathcal{X}_h = \{x_i\}$, and output the risk probability p_u^h :

$$p_u^h = \frac{1}{|\mathbb{S}_h|} \sum_{S \in \mathbb{S}_h} p_S. \quad (3)$$

2) Temporal Risk Pattern Analysis:

a) *Risk Probability Matrix Generation*: It is known that the properties of historical trajectory data may vary over time [14]. Motivated by this observation, we partition the trajectories based on the temporal dimension of week (i.e., the same duration as the aforementioned period in Section III-B1), within which the trajectories are gathered for predicting the risk. This setting also meets the practical requirements of the insurance company with all drivers' past long-period OBD data. That is, given driver u 's H -week OBD records X , we first split X by week into $\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_H$. Then, in week h , we have driver u 's risk probability p_u^h from \mathcal{X}_h by Eq. 3. Next, with H -week data, the driver can produce his/her risk probability changing

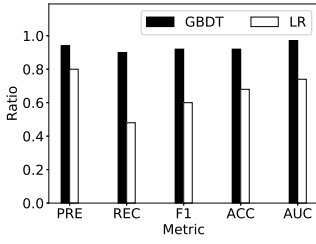


Fig. 2: Trajectory risk probability prediction.

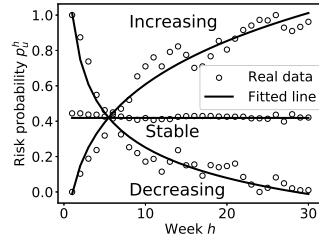


Fig. 3: Three typical temporal risk probability patterns.

pattern: a vector $\mathbf{p}_u = [\mathbf{p}_u^1, \mathbf{p}_u^2, \dots, \mathbf{p}_u^H]$. Finally, considering the whole driver set $U = \{u_1, u_2, \dots, u_N\}$, we can create all drivers' risk probability matrix $\mathbf{P} = (\mathbf{p}_{u_n}^h)_{N \times H}$.

b) *Power-law-based Risk Probability Changing Pattern:*

After generating the risk probability matrix \mathbf{P} , we plot it, as shown in Fig. 3 (For better illustration and understanding, if the risk probability is increasing or decreasing, we normalize the driver's changing pattern to between zero and one). There are three typical behavior changing patterns (i.e., increasing, stable and decreasing). Besides, the increasing and decreasing patterns have a distinctly long-tail distribution. While, the stable pattern is nearly unchanged. On the other hand, *Power Law* is a typical and widely-used distribution and changing pattern in nature such as human mobility modeling [17]–[19] and it can also express the long-tail distribution. Moreover, by fitting the data with Power Law in Fig. 3, we can see that Power Law can effectively match and denote the three behavior changing patterns. Thus, based on these observations, we build the **insight** that Power Law $F(\cdot)$,

$$\mathbf{p}_u^h = F(h) = kh^b, \quad (4)$$

where k and b are Power Law parameters, can be used as the basic function to model the behavior changing pattern. The appealing *advantages* of Power Law can be listed as:

- **Unification:** In Eq. 4, when $b > 0/b = 0/b < 0$, Power Law can separately denote the increasing/stable/decreasing trend with just one unified formula format.
- **Succinctness:** Power Law only uses two parameters (i.e., k and b). Besides, after log-log likelihood processing, it is turned into a linear regression with simple and effective solutions for the value of k and b .
- **Effectiveness:** Power Law can capture the temporal patterns of different groups precisely, as shown in Fig. 3. Experiment results in Section IV-B shows that it can also outperform other compared methods.

3) *Group-level Driver Risk Prediction:*

a) *Group-level Insight:* Given the whole drivers' risk probability matrix, it is nontrivial to predict a driver's future risk by Power Law due to the two aforementioned challenges of data sparsity and data inconsistency (in Section I). To tackle these challenges, we propose a *group-level* Power Law model. The insight is that the whole drivers' risk probability changing patterns usually have multiple groups (e.g., stable and decreasing), and within the same group the drivers share the behavior changing trend. Then, we can utilize the data within

the same group to train one Power Law model to overcome data sparsity problem. Meanwhile, by training the data in different groups, we can have multiple Power Law models rather than only one model to alleviate the data inconsistency problem.

Meanwhile, to achieve effective Power Law modeling and driver grouping, we find that the two sub-tasks can mutually enhance each other: (1) better Power Law modeling serves as useful knowledge, which contributes to inferring the group a driver belongs to; (2) better driver grouping provides better within-group data consistency, which promotes to offer more trusty Power Law models.

This observation motivates us to develop an iterative refinement algorithm, called *power-law-based ensemble learning*, where we alternate between Power Law modeling and driver grouping, for generating better group-level Power Laws.

b) *Power-law-based Ensemble Learning:* We propose the power-law-based ensemble learning for depicting different-group changing patterns. First, we introduce the iterative refinement algorithm for ensemble learning. Then we present the procedures for the Power Law modeling and driver grouping in the algorithm. Finally, we prove the convergence of this algorithm for the time complexity analysis.

(1) **The Iterative Refinement Algorithm.** The algorithm conducts the following steps:

Step 1. Initialization: With the driver set U , we set $\mathcal{G} = \{1, 2, \dots, G\}$ to be the G underlying risk probability changing pattern groups.

(a) $\forall u \in U$, generate a random initial membership vector $\mathbf{m}_u = [\mathbf{m}_u^g | g \in \mathcal{G}, \|\mathbf{m}_u\|_1 = 1]$, where \mathbf{m}_u^g indicates the probability that driver u belongs to group g .

(b) $\forall g \in \mathcal{G}$, initialize a random Power Law F_g . The ensemble of Power Laws is denoted as $\mathcal{F} = \{F_g | g \in \mathcal{G}\}$.

Step 2. Power Law Modeling: $\forall g \in \mathcal{G}$, use the membership vectors to reweight all drivers' risk probability matrix so that the weight of driver u 's risk probability changing pattern vector \mathbf{p}_u is set or proportional to \mathbf{m}_u^g . Then refine F_g to produce a new ensemble, ${}^{new}\mathcal{F} = \{{}^{new}F_g | g \in \mathcal{G}\}$.

Step 3. Driver Grouping: $\forall u \in U$, use ${}^{new}\mathcal{F}$ to update driver u 's membership vector \mathbf{m}_u so that the g -th dimension is the posterior probability that driver u belongs to group g , specifically, ${}^{new}\mathbf{m}_u^g = p(g|u; {}^{new}\mathcal{F})$.

Step 4. Iteration: Check the convergence by using the log-likelihood of all drivers' risk probability changing pattern matrix. If the convergence criterion is not met, then update by:

$$\forall g, F_g \leftarrow {}^{new}F_g; \forall u, \mathbf{m}_u \leftarrow {}^{new}\mathbf{m}_u;$$

and turn to Step 2.

(2) **Power Law Modeling.** In this step, the task is to use the weighted risk probability data to train a new Power Law ${}^{new}F_g$ for group g . Due to each driver's starting point of Power Law is different (i.e., $\mathbf{p}_u^1 = F(1) = k(1^b) = k$ where k is different because of individual differences), we set a common b and individual k_u for driver u . These form the Power Law parameters $\theta = \{b, k_1, k_2, \dots, k_N\}$ for group g . Formally, we construct group g 's membership vector with whole drivers as

$M = [M_i | M_i = \mathbf{m}_{u_i}^g, u_i \in \{u_1, u_2, \dots, u_N\}]$, and compute the objective function with the risk probability matrix P :

$$\theta = \arg \min_{\theta} \sum_{i=1}^N \sum_{j=1}^H (k_{ij}^b - P_{ij})^2 M_i, \quad (5)$$

using Levenberg-Marquardt [20] method to minimize the error and obtain the parameters for Power Law $^{new}F_g$.

(3) Driver Grouping. After the new ensemble of Power Laws $^{new}\mathcal{F}$ is generated, we utilize it to softly allocate or assign each driver u into the G groups. Specifically, we derive the posterior probability ($p(g|u; ^{new}\mathcal{F})$) that driver u belongs to group g , for updating the membership vector \mathbf{m}_u . First, we use the Bayes' theorem to formulate the probability as follows:

$$p(g|u; ^{new}\mathcal{F}) \propto p(g)p(u|g; ^{new}\mathcal{F}),$$

where $p(g)$ is estimated from membership vectors as:

$$p(g) = \frac{1}{|U|} \sum_{u \in U} \mathbf{m}_u^g. \quad (6)$$

Meanwhile, $p(u|g; ^{new}\mathcal{F})$ represents the probability of observing driver u 's risk probability changing pattern \mathbf{p}_u , given group g with $^{new}\mathcal{F}$. It can be computed from the Euclidean distance error with softmax function [21] as:

$$p(u|g; ^{new}\mathcal{F}) = \frac{e^{\frac{1}{\|^{new}F_g(\mathbf{h}) - \mathbf{p}_u\|_2}}}{\sum_{i=1}^G e^{\frac{1}{\|^{new}F_i(\mathbf{h}) - \mathbf{p}_u\|_2}}}, \mathbf{h} = [1, 2, \dots, H]. \quad (7)$$

Finally, the new membership vector $^{new}\mathbf{m}_u$ is obtained as $^{new}\mathbf{m}_u^g = p(g|u; ^{new}\mathcal{F})$.

(4) Time Complexity Analysis. Now, we prove the convergence of power-law-based ensemble learning.

Lemma 1. *The iterative refinement algorithm converges.*

Proof. The iterative algorithm can be reduced into an EM algorithm where Driver Grouping is equivalent to the E-Step and Power Law Modeling is equivalent to the M-Step. Following the typical EM convergence proof [22], in our algorithm, we have: (1) the log-likelihood $l(\mathcal{F}) = \sum_{u \in U} \log \sum_{g \in G} p(u, g; \mathcal{F}) = \sum_{u \in U} \log \sum_{g \in G} \mathbf{m}_u^g \frac{p(u, g; \mathcal{F})}{\mathbf{m}_u^g}$ satisfies the Jensen's inequality, where the equality is guaranteed to hold because \mathbf{m}_u^g is set to $p(g|u; \mathcal{F})$ for a constant $p(u, g; \mathcal{F})/\mathbf{m}_u^g$. (2) in each iteration, the constructed lower bound in the objective function (Eq. 5) is typically convex to have a global optimum. (3) the algorithm uses the previous parameters to initialize the current Power Law ensemble, which guarantees $p(u, g; \mathcal{F})$ to be non-decreasing (i.e., $p(u, g; \mathcal{F}) \leq p(u, g; ^{new}\mathcal{F})$). With the three settings, the total likelihood is non-decreasing (i.e., $l(\mathcal{F}) \leq l(^{new}\mathcal{F})$) after iterations [22]. So, we proved the convergence. ■

c) *Driver's Future Risk Probability Prediction:* After generating the Power Law ensemble \mathcal{F} and driver u 's corresponding membership vector \mathbf{m}_u , we can quantify driver u 's future w -th week's group-level risk probability \hat{p}_u^{H+w} , by involving the membership and Power Laws together as:

$$\hat{p}_u^{H+w} = \sum_{g=1}^G \mathbf{m}_u^g F_g(H+w). \quad (8)$$

C. Insurance Pricing Model

In this subsection, we present the insurance pricing model, which is the aggregation of: (1) Mobility-aware component λ_u , a novel pricing part depending on the driver's mobility in mileage and driving risk; (2) Demographic-aware component o_u , computed by the driver's demographic information according to the current insurance policy.

1) *Mobility-aware Component:* We first formally describe how to compute the mobility-aware component; then present the insight for the computation process. Finally, we conclude the benefit from this component.

Given driver u 's future w -th week risk probability (i.e., \hat{p}_u^{H+w} in Eq. 8) and past H -week traveled mileage a_u , we quantify driver u 's future W weeks' overall risk probability \tilde{p}_u and mileage \tilde{a}_u as:

$$\tilde{p}_u = \frac{1}{W} \sum_{w=1}^W \hat{p}_u^{H+w}, \quad \tilde{a}_u = \frac{W}{H} a_u. \quad (9)$$

Then, we have the mobility-aware component λ_u as:

$$\lambda_u = \tilde{p}_u \cdot (\eta \tilde{a}_u), \quad (10)$$

where η is a per-mile customer-tolerance cost, to charge a customer (driver). (In the pricing model, η is a hyper parameter. We will discuss its setting in more detail in Section IV-E.)

Insight. The rationale behind the component are twofold. First, the mileage is one of the most significant factors in conventional UBI pricing model [23]. Second, according to domain experts, drivers are willing to accept that the insurance cost is distributed averagely by the mileage. Therefore, η 's physical meaning encodes this finding, because η multiplied by mileage \tilde{a}_u denotes the charging for the driver's total mileage. Then, to capture the driver's driving risk in mobility, the pre-computed amount $\eta \tilde{a}_u$ multiplied by the risk probability \tilde{p}_u is able to comprehensively give the pricing model with this mobility-aware aim. Finally, the arithmetic product function incorporates mileage and driving risk factors for pricing in an interpretable way.

The attractive benefit of the mobility-aware component is:

- **Personalized:** With driver u 's different \tilde{p}_u and \tilde{a}_u , *personalized* pricing is obtained.
- **Flexible:** Considering \tilde{p}_u and \tilde{a}_u are dependent on the driver's preferred insurance premium period (i.e., the number of future weeks, W), the pricing model is able to support *flexible* premium period.

2) *Demographic-aware Component:* According to the insurance policy, each auto insurance package should include a basic insurance cost to provide an essential protection for a driver. Following this requirement, we employ the demographic-aware component o_u to compute this cost. In practice, this component is pre-computed and served as the base part of the final price.

Specifically, with a driver's UBI records, we have a demographic variable set to basically describe the driver and his/her vehicle, including *gender, age, vehicle price, driving experience/years, vehicle type, marital status*. Then, o_u utilizes

these typical variables and the generalized linear model [1] to generate the insurance price. All the coefficients are usually set according to the government's insurance regulation [1].

Finally, we output the total insurance price r_u as:

$$r_u = \lambda_u + o_u. \quad (11)$$

D. Company Profit Model

In this subsection, we first formulate the profit maximization problem under the real-world constraints and prove its NP-Completeness, then propose a dynamic programming solution with the real-world heuristic.

1) *Maximal Profit Problem Formulation:* Given a driver set U with each driver u 's past H -week claim record set $\mathbf{c}_u = \{c_u^1, c_u^2, \dots, c_u^{|c|}\}$, and $|c_u| \geq 0$, where c_u^i is the i -th claim indemnity and future W -week predicted risk probability \tilde{p}_u , promoted total insurance price r_u . The profit problem aims to discover a subset of drivers $U' \subseteq U$, that follows three criteria: (1) risk probability constraint, (2) insurance payment constraint, and (3) maximal profit.

(1) **risk probability constraint.** According to the government's regulation on the insurance companies, the risk probability of having at least one accident for the whole insured should not exceed a probability limit \mathcal{P} , to avoid the business operating risk as:

$$1 - \prod_{u \in U'} (1 - \tilde{p}_u) \leq \mathcal{P} \Leftrightarrow \sum_{u \in U'} \ln \frac{1}{1 - \tilde{p}_u} \leq \ln \frac{1}{1 - \mathcal{P}}. \quad (12)$$

(2) **insurance payment constraint.** As the insurance policy requires, the company should prepare for the worst case, where the whole insured (drivers) are to be claimed at the same time/period. Then, the maximal total claim payment amount should be below the insurance payment (budget) \mathcal{C} as:

$$\sum_{u \in U'} \mathbb{I}(|c_u| > 0) \max(c_u) + \mathbb{I}(|c_u| = 0) \beta \leq \mathcal{C}, \quad (13)$$

where $\mathbb{I}(\cdot)$ is the indicator function and β is the standard claim for the drivers who have no claim records (i.e., $|c_u| = 0$) and usually set as the average of the whole claim payment amount. While, for the claimed driver, the worst case is the maximal amount of his/her past claim indemnity.

(3) **maximal profit.** The goal of company is to achieve the maximal profit. For driver u , his/her resulting profit d_u is computed as:

$$d_u = r_u - \alpha r_u - \hat{C}_u, \quad (14)$$

where α is the *operating expenditure ratio* cost for the driver (e.g., the advertising and employee wages). It is generally a standard default value with $\alpha = 40\%$; \hat{C}_u denotes the predicted claim payment for driver u , by considering whether the driver has claim records ($|c_u| > 0$) or not ($|c_u| = 0$), as:

$$\hat{C}_u = \begin{cases} (\frac{|c_u|}{H} W) \cdot (\frac{1}{|c_u|} \sum_{i=1}^{|c_u|} c_u^i) \Leftrightarrow \frac{W}{H} \sum_{i=1}^{|c_u|} c_u^i & |c_u| > 0 \\ \beta \tilde{p}_u & |c_u| = 0 \end{cases} \quad (15)$$

Then, the overall profit of the driver set U' , denoted as D , is the aggregation as:

$$D = \sum_{u \in U'} d_u. \quad (16)$$

Thus, the maximal profit problem is formally presented as an integer programming problem of finding a driver subset U' from the whole driver set U as:

$$\max D, \text{ s.t. } \begin{cases} \sum_{u \in U'} \ln \frac{1}{1 - \tilde{p}_u} \leq \ln \frac{1}{1 - \mathcal{P}} \\ \sum_{u \in U'} \mathbb{I}(|c_u| > 0) \max(c_u) + \mathbb{I}(|c_u| = 0) \beta \leq \mathcal{C} \end{cases} \quad (17)$$

The problem is NP-Complete as proven below:

Lemma 2. (NP-Complete). *Given the risk probability and the insurance payment constraints, finding a set of drivers to maximize the profit is NP-Complete.*

Proof. This problem is equivalent to the 0-1 Knapsack problem. Specifically, each driver is an item, with an item weight (i.e., risk probability), an item volume (i.e., insurance payment) and an item value (i.e., profit). Then, the set of selected drivers, U' , is viewed as a knapsack with a fixed weight \mathcal{P} (i.e., risk probability constraint) and a fixed volume \mathcal{C} (i.e., insurance payment constraint) to get the maximal value. Thus, our problem is the 0-1 Knapsack problem according to its definition, which completes the proof of NP-Complete. ■

Since the maximal profit problem is NP-Complete, it is impossible to find the optimal driver set in polynomial time. In this paper, we propose an efficient heuristic-based dynamic programming solution.

2) *Constrained Dynamic Programming:* **Heuristic:** By viewing the accident record survey from the same anonymous insurance company (mentioned in Section I), not all the risk driving behaviors trigger accidents due to the narrow overlapping accident-involved drivers between two adjacent years (3%). But the drivers having many claim records have relatively high risk probability. Based on the above observation, when computing the maximal profit, we can set a threshold ρ to remove these drivers with more than ρ claim records by rejecting their insurance request. This constraint can avoid the profit decline caused by the possible future accidents as much as possible.

Then, the constrained dynamic programming algorithm is proposed with the following steps:

Step 1. Given a driver set U , we remove a driver u if driver u 's past claim record set \mathbf{c}_u satisfies $|c_u| \geq \rho$. It finally generates a new driver set \hat{U} ;

Step 2. From \hat{U} , we use dynamic programming to solve the formulated 0-1 Knapsack problem, which outputs the selected driver subset U' and the maximal profit D .

IV. EXPERIMENT

In this section, we conduct extensive experiments to evaluate PPP. By default, the parameters used in experiments are: $G = 3$, $\mathcal{P} = 40\%$, $\mathcal{C} = 4 \times 1e5$, $\eta = 0.78$, $\rho = 2$. The detailed investigation of these parameters are described in Section IV-E.

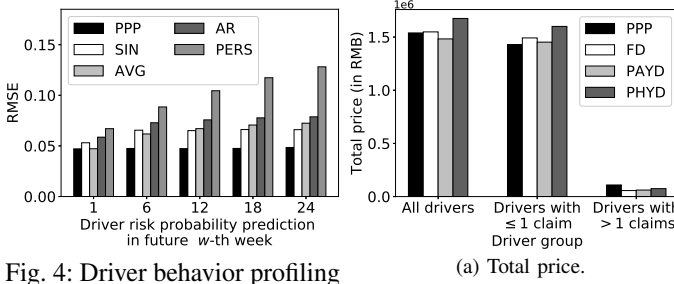


Fig. 4: Driver behavior profiling model evaluation.

A. Dataset

Our experiments are based on two real-world datasets. First, the OBD data includes 400 driver's driving records during 2016.8.22–2017.3.27 for nearly 30 weeks, provided by a major OBD company in China. After grouping the records by driver id and extracting trajectories (Trajectory time constraint Δt (in Definition 1) is set by domain experts as four minutes, which provides a trajectory with enough information to analyze. Meanwhile, the filtering of noisy data record is conducted with a heuristic-based outlier detection method [13] through speed information.), we obtain: 400 drivers, 190,928 trajectories, average trajectory time 25.83 min and average trajectory distance 14.23 km. Second, the UBI data includes the insurance records of the corresponding drivers in the OBD dataset, ranging from 2016 to 2017. Besides the aforementioned statistics in Table I, among the claimed drivers, the demographic factor statistics is: (1) Gender: there are 11% female drivers and 89% male drivers. (2) Age: age of 20-30, 30-40 and 40-50 caused 25%, 33% and 28% accidents. While, age over 50 reported 14% accidents. (3) Driving experience/years: 40% accidents were created by driving years less than 3 year, 34% were recored by years between 4 and 6. The rest were generated by driving years greater than 7. The contribution of these demographic factors is also evaluated in Section IV-C.

B. Driver Behavior Profiling Model Evaluation

We examine the driver behavior profiling model in PPP, by predicting driver u 's future w -th week's risk probability \hat{p}_u^{H+w} , given the driver's past H -week risk probability changing pattern vector $\mathbf{p}_u = [\mathbf{p}_u^1, \mathbf{p}_u^2, \dots, \mathbf{p}_u^H]$. The evaluation metric is Root Mean Square Error (RMSE) as $RMSE = \sqrt{(\sum_{u \in U} (\mathbf{p}_u^{H+w} - \hat{p}_u^{H+w})^2) / |U|}$.

To compare with PPP, we implemented the following baseline models: (1) Average (AVG) uses the average of risk probability in past H weeks to predict the future probability. (2) Autoregressive Model (AR) [24] trains autoregressive models with the past H weeks' risk probability for prediction. (3) Single Model (SIN) trains one Power Law using the whole data. (4) Personal Model (PERS) trains a personalized Power Law for each driver. (In experiment, $H = 6$, $w = 1, 6, 12, 18, 24$).

As shown in Fig. 4, PPP significantly beats all the baselines for different w -th week. Comparing the performance of PPP and SIN, we find that the prediction RMSE of PPP is about 23.61% better on average. It indicates that there are indeed multiple groups of drivers that have various driving behaviors.

Fig. 5: Insurance pricing model evaluation.

So, PPP can deploy ensemble learning to effectively distinguish different group patterns for better performance. Meanwhile, we find that PPP performs better than PERS on average of 50.01%. The reason is that given the short-period data, PERS cannot effectively train and learn the Power Law parameters, suffering from the data sparsity problem. Thus, it justifies the necessity to consider some data as a group to find the general pattern. Besides, PPP beats AVG and AR, possibly because AVG and AR pay more attention to the data without considering the changing pattern and Power Law behind the data.

C. Insurance Pricing Model Evaluation

In this subsection, we evaluate the insurance pricing model in PPP under the following enterprise scenario: based on the drivers' past 6-week OBD data and past 1-year UBI data, we compute driver u 's insurance price r_u for the premium period of future 24 weeks. Then, we evaluate the results in metrics of: (1) *Total price*, that is, by summing up all drivers' insurance price as $\sum_u r_u$; (2) *Validated profit rate*. It is defined as the ratio of the validated profit to the total price. Specifically, with all drivers' future 24-week ground truth claim indemnity in UBI data, we have the total claim indemnity R_{gt} . Then, the validated profit rate during the future 24 weeks is $(\sum_u r_u - \alpha \sum_u r_u - R_{gt}) / \sum_u r_u$, where α is the operating expenditure ratio cost as mentioned in Eq. 14.

Meanwhile, the compared methods are: (1) Full Demographic-aware Model (FD). It is a baseline according to the insurance company's current deployed strategy from the industry perspective. FD only considers the driver's full demographic factors including vehicle usage, vehicle seat number, past claim/accident records and so on, which are more than the variable scope in the demographic-aware component of PPP framework. Besides, the coefficients [1] in FD are mainly based on the company's empirical data. (2) Pay-As-You-Drive (PAYD) [5] and (3) Pay-How-You-Drive (PHYD) [6] are two state-of-the-art methods, with focus on per-mile/per-minute information and speed/location data respectively to profile driver behaviors for pricing. The parameters of the two models have been carefully tuned to the best performance together with the guidance of the domain experts from insurance companies.

According to the results of total price in Fig. 5(a) and validated profit rate in Fig. 5(b), PPP defeats other methods. Specially, compared to FD, PPP charges less (5% decrease in price) for the low-risk drivers (drivers whose claim count ≤ 1 , according to domain experts), while charges more (93%

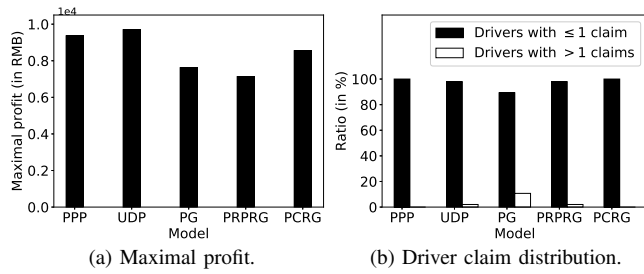


Fig. 6: Company profit model evaluation.

increase in price) for the high-risk drivers (drivers whose claim count > 1 , according to domain experts). Meanwhile, the whole drivers' total price is lower (about 1%). These results show that PPP with fine-grained mobility features better describes drivers' risk for pricing than FD with only using traditional data like accident records. Besides, PPP provides nearly 10% price decrease for low-risk drivers and 43% increase for high-risk drivers than PHYD. Although PAYD has the lowest total price, its price charges less for the high-risk drivers, which easily leads to financial loss (-3.25% profit rate among all drivers). These comparison results mean that, PPP efficiently detects the high-risk drivers to generate higher price. Therefore, PPP greatly decreases the financial loss from the high-risk drivers (with average 41% profit rate increase compared to other methods). On the other hand, PPP generates lower price for the low-risk drivers, making it more attractive for the drivers to reduce their insurance costs. Furthermore, the companies still have high probability to earn money from such low-risk drivers' lower price (54.93% profit rate). It is a win-win game.

Pricing Component Contribution. We further examine the mobility-aware/demographic-aware pricing component contribution in the pricing model of PPP, by averaging the ratio of the price from the mobility-aware λ_u (demographic-aware o_u) component to the total price r_u in the driver set U (i.e., $(\sum_{u \in U} \lambda_u / r_u) / |U|$ and $(\sum_{u \in U} o_u / r_u) / |U|$).

As shown in Fig. 5(c), mobility-aware component contributes up to 76% in high-risk drivers' price, while for low-risk drivers' price, the contribution of mobility-aware and demographic-aware component is almost the same, near 50%. It means that mobility factors play a significant role in pricing, especially a major part for high-risk drivers. Considering the aforementioned fact that the profit decline usually comes from high-risk drivers, this result justifies the necessity to consider the mobility factors in insurance pricing, especially in high-risk drivers to avoid possible financial loss by higher charging from mobility.

D. Company Profit Model Evaluation

In this subsection, we evaluate the company profit model in PPP with the following business setting. We used the whole drivers' past 6-week OBD and 1-year UBI data to generate the selected driver subset U' and the maximal profit D during the future 24 weeks. We adopt the *driver claim distribution* in the selected driver subset U' (i.e., how drivers with different claim counts distribute in the subset) and *maximal profit* D as the performance metric.

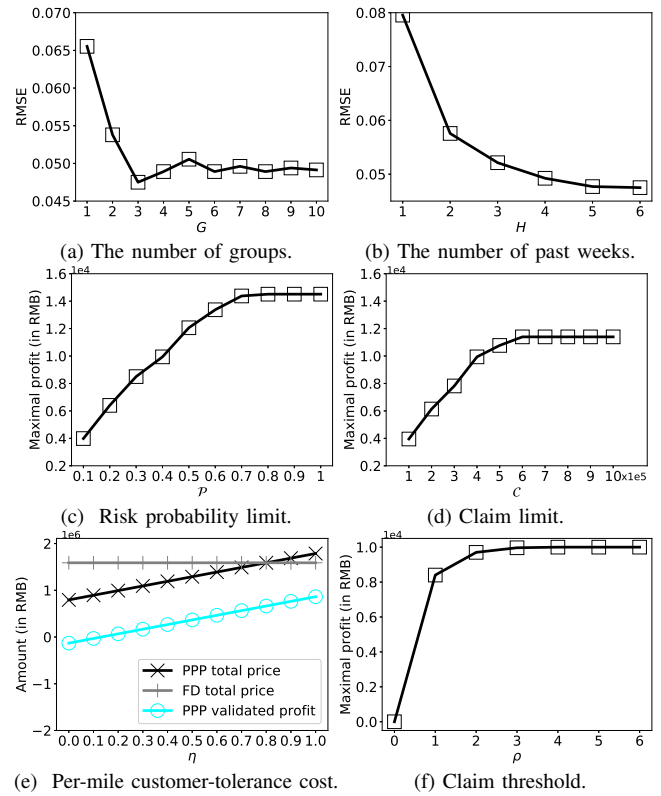


Fig. 7: Effects of parameters.

Meanwhile, due to the NP-Complete property of the formulated 0-1 Knapsack problem [12], the compared baselines are mainly greedy-search-based and dynamic-programming-based as: (1) Profit Greedy (PG): Sort by the individual profit; (2) Profit Risk Probability Ratio Greedy (PRPRG): Sort by the ratio of individual profit to risk probability; (3) Profit Claim Ratio Greedy (PCRG): Sort by the ratio of individual profit to claim; (4) Un-constrained Dynamic Programming (UDP): No constraint to the driver's claim count.

As shown in Fig. 6(a), we find UDP and PPP outperform the greedy-based methods of PG, PRPRG and PCRG in profit. It illustrates the effectiveness of the dynamic programming algorithm. While PPP achieves slightly lower profit than UDP, it is very close to UDP (difference by only 3%).

Besides, Fig. 6(b) shows the driver claim distribution results. Considering the high-risk drivers (drivers whose claim count > 1 , according to domain experts), PG has the highest ratio, while PCRG has no such drivers. The reason is that this greedy search sorting considers the no claim drivers first, similar to PPP. So its profit is the highest among the three greedy-based methods. UDP contains 2.04% such drivers while PPP filters out all such high-risk drivers. As the domain experts interpret, removing the high-risk drivers will not cause the significant decline of the profit.

Additionally, in terms of the validated profit rate (the metric is aforementioned in pricing model evaluation), we examined and reported PPP's resulting profit rate, 58.78%. Compared to the profit rate from the current company's strategy (FD) in Fig. 5(b), 1.49%, PPP generated much higher profit rate.

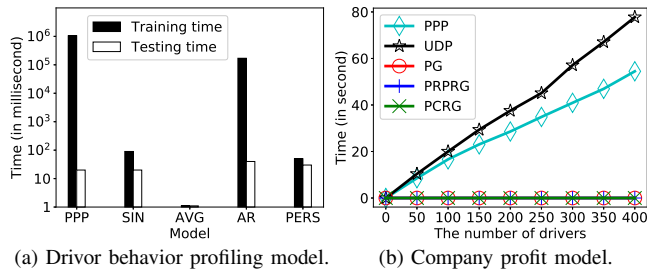


Fig. 8: Running time test.

Therefore, in application under the enterprise constraints, PPP is more ideal for companies to increase their profit rates, and it is able to obtain the close-to-maximal result within the reduced searching space. Rejecting the possible high-risk drivers brings the significant benefit to insurance companies. Besides, **Domain experts** praise these results and findings in achieving the maximal profit, excited about using the trajectory risk for profiling and pricing.

E. Parameter Study

The effects of the major parameters are shown in Fig. 7:

(1) **The number of groups G** : With G increasing in Fig. 7(a), the prediction performance first decreases to the minimal value with $G = 3$, then slightly increases to the final convergence. The reason for $G = 3$ is that 3 can denote the typical driver behavior patterns of increasing/stable/decreasing. Although more groups can describe more complex behavior patterns, the group difference is very limited. (2) **The number of past weeks H** : From Fig. 7(b), we can see that with more data in the past weeks, the prediction performance becomes better. (3) **Risk probability limit \mathcal{P}** : In Fig. 7(c), it shows when the risk probability limit increases, the profit will first increase then converge. This means that when risk probability is greater than the driver set's innate maximal risk probability limit, PPP can include the whole drivers for searching without considering the risk probability limit. (4) **Claim limit \mathcal{C}** : Meanwhile, the claim limit in Fig. 7(d) shows the similar pattern to risk probability in Fig. 7(c). (5) **Per-mile customer-tolerance cost η** : As shown in Fig. 7(e), PPP total price increases linearly along with η , due to the linear multiplication in the price computing (Eq. 10). To select proper η , according to the insurance company's practical requirements: i) PPP total price should be lower than the current company's total price (i.e., FD total price), because higher price can lead to most drivers' complaints about higher charging, and even worse the number of the insured may decrease. ii) PPP total price should not be too low because lower price can cause the negative validated profit (where, the validated profit rate multiplied by PPP total price (in Section IV-C) is PPP validated profit), which is harmful for the company operation. Based on these requirements and findings, the upper and lower bound of η can be determined by PPP total price equal to FD total price and PPP validated validated profit equal to zero, as in Fig. 7(e). This is a practical method for the insurance company to estimate η for determining the insurance price. (6) **Claim threshold ρ** : In Fig. 7(f) when $\rho = 1$, PPP traverses those

drivers having no claim records. Then, with ρ increasing, it starts to traverse drivers with claims, and finally converges after including all drivers when searching for the maximal profit.

F. Efficiency Study

Now we report the efficiency of the three models in PPP. By examining, the running time of the pricing model does not change much. While, the result of different profiling models is shown in Fig. 8(a), we see that PPP needs more training time due to its iterative refinement algorithm (Detailed time complexity analysis can refer to Section III-B2b). Besides, for different profit models, due to the NP-Complete property, we also record the influence of the number of drivers. As shown in Fig. 8(b), the running time of UDP and PPP increases along with the number of the drivers. Note that, UDP is higher than PPP due to the larger searching space from the whole drivers. For the greedy-based methods, the time cost is almost stable.

V. RELATED WORK

The related work can be grouped into two categories: driver behavior profiling and auto insurance pricing.

In the literature, driver behavior profiling has been widely studied [25]. First, with the sensor equipment in vehicles, researchers use each sensor data tuple to analyze the driving states [26] for profiling. Furthermore, using massive sensor data tuples to generate trajectories for modeling and mining driving behaviors has also been explored [13]. This analysis includes more driving information. However, the previous work, usually using GPS [27], seldom considers the high-dimensional fine-grained driving behaviors, like from OBD [11]. Second, from the driver's perspective, Paefgen et al. used the driving exposure features to classify the accident-involved/free drivers for behavior analysis and profiling [5], and it was extended to multi-class drivers by Guo [28]. But, it is hard for these methods to capture the mobility patterns in the driving behaviors. In addition, social influence [29], location and behavioral data [30], peer and temporal-aware perspective from GPS data [31], statistical analysis [32] and outlier (fraud) detection [33] have also been examined. Different from the previous work, we profile the driver behaviors by not only fine-grained heterogeneous driving features from the trajectory perspective, but also the driver's temporal risk changing patterns. Finally, the previous methods are hard to directly solve the personalized insurance pricing and maximal profit problems in this work.

Traditional auto insurance price is mainly studied in the management, business and transportation domains, where the pricing is decided by the actuarial ratings and empirical experience [2]. Specifically, the risk is quantified based on the driver's basic demographic information for pricing. However, it can be deficit when ignoring the driving risk. Then, with the vehicle sensor equipment and information technology development [9], UBI models and telematics data were introduced to record the driving information and analyze the driving behaviors to set the price [4], like pay-as-you-drive-based [5], [6], methods of per-minute-premium [34] and per-mile-premium [35]. In the price quantification, these models mainly regard the mileage

and time as the driving factors. But, neglecting the driver's rich driving behavior features, they can be ineffective to distinctively differentiate the driving risk, which results in poor price packages for drivers and profit declines for companies. The most relevant works to our study are those pay-how-you-drive-based approaches [6], e.g., behavior-centric-vehicle-insurance-pricing-model [10]. They mainly model the driving behaviors from speed, acceleration and location for pricing. But, few considers the mileage and trajectory-level driving risk together, or the dynamic changing patterns of the driving risk. Unlike the mentioned models, our pricing model quantifies the driving risk over time from massive fine-grained trajectories, which is later combined with the mileage for the personalized and flexible insurance price. Besides, maximal profit problems are also explored in some domains [36]. While, in auto insurance, instead of profiting with the case of Pay-As-You-Drive-based model [37], we discuss the profit maximization under the enterprise constraints.

VI. CONCLUSION AND FUTURE WORK

In this paper, we proposed PPP framework to empower the insurance companies to provide the personalized insurance price and achieve the maximal profit. First, PPP fine-grained profiles the driver behaviors in time stream. Meanwhile, an ensemble learning algorithm is proposed to predict the driving risk by considering the group-level insight. Then, PPP generates personalized insurance price with flexible premium periods. Both the driving behavior and the demographic information are considered. Finally, the maximal profit problem is proved to be NP-Complete and a constrained dynamic programming solution is proposed. PPP is evaluated comprehensively on the real-world large-scale OBD and UBI data. Experimental results demonstrated that, PPP achieves near the maximal profit for the company under the real-world constraints, lowers the total price for the drivers and is highly praised by domain experts.

In the future, we will validate PPP on more data (e.g., more drivers) and introduce the spatial factors (e.g., location and road type) to PPP, which contribute to the driver risk prediction.

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