1. Prove that a weakly dominant strategy $s_i' \in S_i$ is a strictly dominant strategy iff $argmax_{s_i} u_i(s_i, s_{-i})$ is a singleton for all $s_{-i} \in S_{-i}$.

Pf: "$\Leftarrow$": Suppose $\arg\max_{s_i} u_i(s_i, s_{-i})$ is a singleton for $\forall s_{-i} \in S_{-i}$

Given $s_i' \in S_i$ is a weakly dominant strategy, i.e.,

$$u_i(s_i', s_{-i}) \geq u_i(s_i, s_{-i}) \text{ for } \forall s_i \in S_i, \forall s_{-i} \in S_{-i}$$

$\Rightarrow s_i' \in \arg\max_{s_i} u_i(s_i, s_{-i})$

and for $\forall \hat{s}_i \neq s_i'$, $\hat{s}_i \notin \arg\max_{s_i} u_i(s_i, s_{-i})$

$\Rightarrow u(s_i', s_{-i}) = \max\{u_i(s_i, s_{-i})\} > u_i(\hat{s}_i, s_{-i})$ for $\forall \hat{s}_i \neq s_i'$, $\forall s_{-i} \in S_{-i}$

By def, s_i' is a strictly dominant strategy.

"$\Rightarrow$": Suppose not, i.e.,

$\exists s_{-i}$ s.t. $\arg\max_{s_i} u_i(s_i, s_{-i})$ is not a singleton

$\Rightarrow$ Given $s_i' \in \arg\max_{s_i} u_i(s_i, s_{-i})$,

$\exists \hat{s}_i \neq s_i'$ s.t. $\hat{s}_i \in \arg\max_{s_i} u_i(s_i, s_{-i})$

$\Rightarrow u_i(s_i', s_{-i}) = u(\hat{s}_i, s_{-i})$

$\Rightarrow$ By def. s_i' is not a strictly dominant strategy.

※

2. Consider the following modification of a two-bidder second-price sealed-bid auction. Bidder 2 receives an advantage as follows: If bidder 2's bid is at least 80% of bidder 1's bid, then bidder 2 wins and pays 80% of bidder 1's bid. If bidder 2's bid is less than 80% of bidder 1's bid, then bidder 1 wins and pays 1.25 times bidder 2's bid. Suppose bidder i values the object being sold at vi, i = 1, 2. Prove that it is a weakly dominant strategy for each bidder to bid his or her valuation.

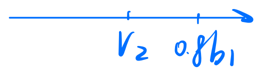
pf: $b_2 \geq 0.8 b_1$ $(1.25 b_2 \geq b_1)$ $\begin{cases} u_1 = 0 \\ u_2 = V_2 - 0.8 b_1 \end{cases}$

$b_2 < 0.8 b_1$ $(1.25 b_2 < b_1)$ $\begin{cases} u_1 = V_1 - 1.25 b_2 \\ u_2 = 0 \end{cases}$

For bidder 2:

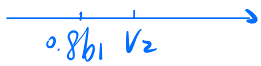
1) $b_2 = V_2$ is always weakly better

if $0.8 b_1 > V_2$



$\begin{cases} ① \ b_2 \geq 0.8 b_1 \Rightarrow \text{win} \\ \quad u_2 = V_2 - 0.8 b_1 < 0 \\ ② \ b_2 < 0.8 b_1 \Rightarrow \text{lose} \\ \quad u_2 = 0 \end{cases}$

if $0.8 b_1 \leq V_2$

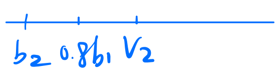


$\begin{cases} ① \ b_2 < 0.8 b_1 \Rightarrow \text{lose} \\ \quad u_2 = 0 \\ ② \ b_2 \geq 0.8 b_1 \Rightarrow \text{win} \\ \quad u_2 = V_2 - 0.8 b_1 \geq 0 \end{cases}$

2) $b_2 = V_2$ is sometimes strictly better

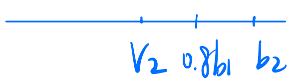
if $b_2 < V_2 \Rightarrow \exists b_1$ s.t. $b_2 < 0.8 b_1 < V_2$, $u_2 = 0$

while $b_2 = V_2$ yields $u_2 > 0$



if $b_2 > V_2 \Rightarrow \exists b_1$ s.t. $V_2 < 0.8 b_1 < V_2$. $u_2 = V_2 - 0.8 b_1 < 0$

while $b_2 = V_2$ yields $u_2 = 0$



For bidder 1, the argument is similar.
(compare $1.25b_2$ with V_1 in step 1 and
compare b_1 with V_1 in step 2)

Therefore, $b_i = V_i$, $i = 1, 2$ is the WDS.

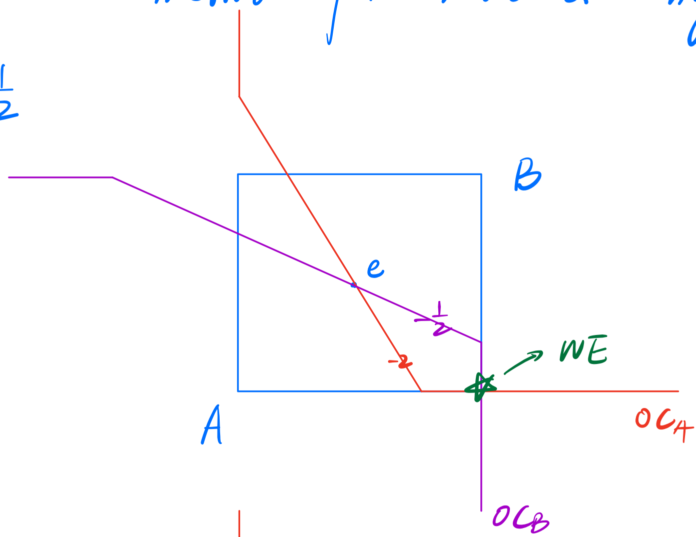
#

Tut 6:

2. In a two-consumer (Ms. A and Mr. B), two-commodity, x_1 and x_2, pure exchange economy, A's preference is represented by the utility function: $U^A(x_1^A, x_2^A) = 2x_1^A + x_2^A$. Consumer B's preference is represented by the following utility function: $U^B(x_1^B, x_2^B) = \alpha x_1^B + x_2^B$ where x_j^i denotes the quantity of commodity $j \in \{1, 2\}$ consumed by consumer $i \in \{A, B\}$. Assume the total quantities available in the economy of commodity x_1, denoted by $\overline{x_1}$, and of commodity x_2, denoted by $\overline{x_2}$, are identical. Further, assume that the endowment allocation is such that A owns half of the entire quantity available of commodity 1, $\overline{x_1}/2$, and half of the quantity available of commodity 2, $\overline{x_2}/2$. Find the set of Walrasian equilirbium prices and allocations in this economy when $\alpha = 1/2$, $\alpha = 4$, respectively.

An alternative method for Tut 6 Q2 by drawing OC:

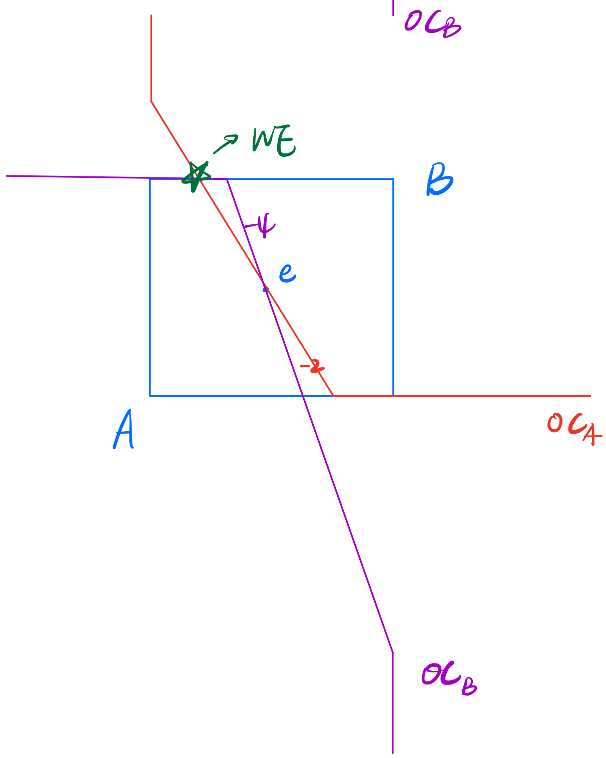
① $\alpha = \frac{1}{2}$



$X^A = (\overline{x_1}, 0)$

$X^B = (0, \overline{x_2})$

② $\alpha = 4$



$X^A = (\frac{1}{4}\overline{x_1}, \overline{x_2})$

$X^B = (\frac{3}{4}\overline{x_1}, 0)$