ECO4010 Tutorial 5

1. Use examples to see FOSD and SOSD.

   (a) p is a lottery of winning 2 and 3 dollars with probability $\frac{1}{4}$, respectively and winning 5 dollars with probability $\frac{1}{2}$; q is an even randomization between 1 and 4 dollars. Show that p FOSD q.

   (b) p is an even randomization between 2 and 3 dollars; q is a lottery of winning 1, 2, 3, and 4 dollars with probability $\frac{1}{4}$, respectively. Show that p SOSD q.

2. The definition of first-order stochastic dominance is that p FOSD q if every expected-utility maximizer with nondecreasing u weakly prefers p to q, i.e., $\forall u$ *s.t.* $u'(x) \geq 0$, $\int_{\underline{x}}^{\bar{x}} u(x)\mathrm{d}F_p(x) \geq \int_{\underline{x}}^{\bar{x}} u(x)\mathrm{d}F_q(x)$.

   (a) Show that $F_p(x) \leq F_q(x)$ for $\forall x$ is an equivalent definition of FOSD.

   (b) Show that if p FOSD q, then the mean of x under p is larger than or equal to that under q. [Hint: Use the definition of FOSD]

   (c) Show that the converse of (b) is not true, i.e., provide an example where the mean of x under p is larger than or equal to that under q, but p does not FOSD q.

3. (Optional) The definition of second-order stochastic dominance is that p SOSD q if every expected-utility maximizer with nondecreasing and concave u weakly prefers p to q, i.e., $\forall u$ *s.t.* $u'(x) \geq 0$ and $u''(x) \leq 0$, $\int_{\underline{x}}^{\bar{x}} u(x)\mathrm{d}F_p(x) \geq \int_{\underline{x}}^{\bar{x}} u(x)\mathrm{d}F_q(x)$. Assume $E_p = E_q$, i.e., p and q have the same expected value. Rothschild and Stiglitz (1970) showed that there are two alternative definitions of SOSD, as displayed in the lecture note, and proved the equivalence of the three definitions.

   (a) Show that $\int_{\underline{x}}^{r} F_p(x)\mathrm{d}x \leq \int_{\underline{x}}^{r} F_q(x)\mathrm{d}x$ for $\forall r \geq 0$ is an equivalent definition of SOSD.

   (b) Show that mean-preserving spread implies SOSD. We say F_q is a mean-preserving spread of F_p if $x_q \overset{d}{=} x_p + \epsilon$ for some $x_p \sim F_p$, $x_q \sim F_q$ and ϵ such that $E[\epsilon|x_p] = 0$ for every x_p. (The other direction is omitted and you may refer to the paper *Increasing risk: I. A definition* for details)

4. (Fun) If you intend to apply for Duke's MAE program in your application season next year, you might encounter a question that asks you to comment critically on an academic article. The following is one of the three articles and is related to Microeconomic theory. After learning *Expected Utility*, you should have enough background knowledge to understand its main idea. You may read it for fun.

   Sarver, Todd, 2008, "Anticipating regret: why fewer options may be better," Econometrica 76(2): 263-305.