ECO4010 Tutorial 9

1. Two high-tech firms (1 and 2) are considering a joint venture. Each firm i can invest in a novel technology and can choose a level of investment $x_i \in [0, 5]$at a cost of $c_i(x_i) = \frac{x_i^2}{4}$ (think of x_i as how many hours to train employees or how much capital to spend for R & D labs). The revenue of each firm depends on both its investment and the other firm's investment. In particular if firms i and j choose x_i and x_j , respectively, then the gross revenue to firm i is

$$R(x_i, x_j) = \begin{cases} 0, & x_i < 1 \\ 2, & x_i \geq 1, x_j < 2 \\ x_i x_j, & x_i \geq 1, x_j \geq 2 \end{cases} \tag{1}$$

   What is the BR for firm i? What are the NE?

2. Consumers are located uniformly along a linear city of length 1. Each consumer wants to buy one unit of good from one existing firm. The transportation cost for the consumer is proportional to the distance to the firm from which he buys. The law prohibits any form of competition through price or service (other than location), so consumers go to the nearest firm. A firm's utility is equal to the number of its customers. Firm located at the same location get the same number of customers.

   - (a) There are two firms, and they choose their locations simultaneously. Show that there exists a unique pure strategy Nash equilibrium.
   - (b) Show that with three firms there exists no pure-strategy equilibrium.

3. Consider first-price auction with 2 players and $v_2 < v_1$. Find a MSNE.