ECO3011 Intermediate Microeconomic Theory: Tutorial 3

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Outline

1. Special Examples of Substitution and Income Effect
   - Perfect Substitutes (covered in lecture notes, ommitted)
   - Perfect Complements
   - Quasilinear Preferences
2. Mathematical Treatment of Slutsky Equation
   - Slutsky Identity
   - Deduction by Rates of Change
   - Deduction by Calculus
3. Suggestions about Leaning This Course

Recap: Slutsky Decomposition

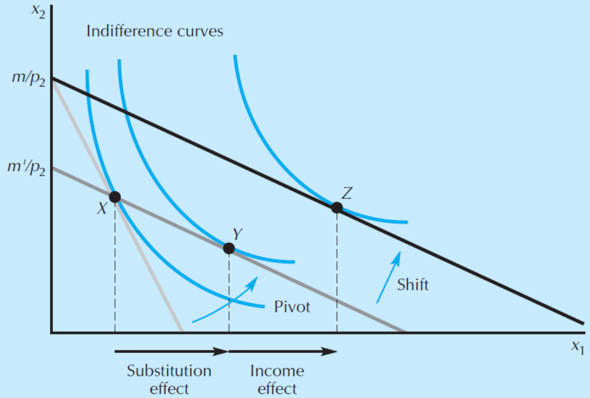
Rough definitions of the two effects:

- Substitution effect: the change in demand due to the change in the rate of exchange between the two goods
- Income effect: the change in demand due to having more purchasing power

Break the price movement into two steps:

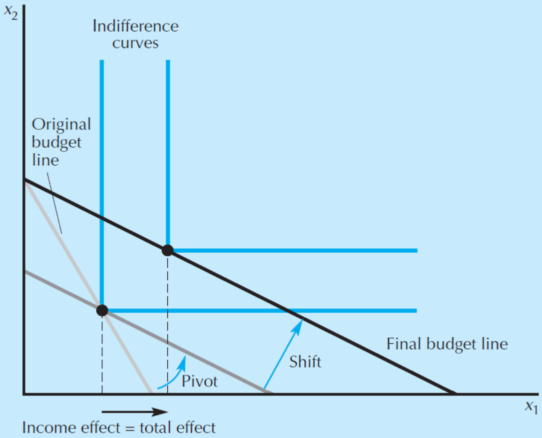
- Substitution effect: let the relative prices change and adjust money income holding purchasing power constant (*pivot* the budget line around the *original* demanded bundle)
- Income effect: let purchasing power adjust while holding the relative prices constant (*shift* the pivoted line out to the *new* demanded bundle)

Recap: Slutsky Decomposition



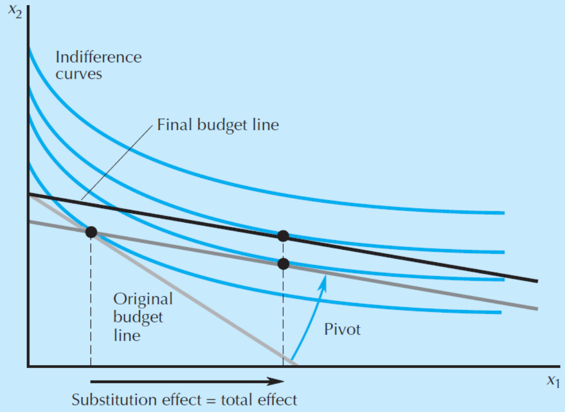
Substitution effect and income effect. The pivot gives the substitution effect, and the shift gives the income effect.

Perfect Complements



Perfect complements. Slutsky decomposition with perfect complements.

Quasilinear preferences



Quasilinear preferences. In the case of quasilinear preferences, the entire change in demand is due to the substitution effect.

The sign of substitution and income effect

- Income effect can be positive or negative, depending on whether the good is a normal good or an inferior good.
- Substitution effect is always negative, at least non-positive, by the fact that the change in demand due to the substitution effect always moves opposite to the change in price (see the detailed proof in the textbook p.142).

Q: Suppose that preferences are concave. Is it still the case that the substitution effect is negative?

Mathematical Treatment of Slutsky Equation

Representations

Let p_1' be the new price of the good x_1. Let m' be the amount of money income that will just make the original consumption bundle affordable, associated with the hypothetical pivoted budget line. Then we obtain three budget lines and three corresponding optimal bundles:

Original budget line:	$p_1 x_1 + p_2 x_2 = m$	(1)	$\Rightarrow x_1(p_1, m)$
Hypothetical budget line:	$p_1' x_1 + p_2 x_2 = m'$	(2)	$\Rightarrow x_1(p_1', m')$
New budget line:	$p_1' x_1 + p_2 x_2 = m$	(3)	$\Rightarrow x_1(p_1', m)$

Mathematical Treatment of Slutsky Equation

Representations (conti.)

Since the original bundle (x_1, x_2) is affordable at both budget lines (2) and (1), we have:

$$m' = p_1' x_1 + p_2 x_2$$

$$m = p_1 x_1 + p_2 x_2$$

Subtracting the second equation from the first gives:

$$m' - m = x_1(p_1' - p_1)$$

Let $\Delta p_1 = p_1' - p_1$ represent the change in price 1, and $\Delta m = m' - m$ represent the change in income necessary to make the old bundle just affordable, we have:

$$\Delta m = x_1 \Delta p_1$$

Note: The change in income and the change in price will always move in the same direction.

Mathematical Treatment of Slutsky Equation

Expression for substitution effect

The substitution effect, Δx_1^s, is the change in the demand for good 1 when the price of good 1 changes to p_1' and money income changes to m':

$$\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m)$$

Expression for income effect

The income effect, Δx_1^n, is the change in the demand for good 1 when we change income from m' to m, holding the price of good 1 fixed at p_1':

$$\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m')$$

Slutsky identity

The total change in demand

The total change in demand, Δx_1, is the change in demand due to the change in price holding income constant:

$$\Delta x_1 = x_1(p_1', m) - x_1(p_1, m)$$

The change can be decomposed by:

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n = [x_1(p_1', m') - x_1(p_1, m)] + [x_1(p_1', m) - x_1(p_1', m')]$$

which is identical to the above equation after simplification. This equation is called the **Slutsky identity**, which algebraically describe the substitution effect and income effect. We can determine the sign of the total effect as long as we know about the signs of Δx_1^s and Δx_1^n.

Example

Suppose that the consumer has a demand function for milk of the form:

$$x_1 = 10 + \frac{m}{10p_1}$$

Originally his income is \$120 per week, and the price of milk is \$3 per quart. Now the price of milk falls to \$2 per quart. What is the total change in demand? What are the substitution effect and income effect?

Deduction by Rates of Change

Slutsky equation in rates of change

When we express the Slutsky identity in terms of rates of change instead of absolute value, it is convenient to define Δx_1^m to be the negative of the income effect:

$$\Delta x_1^m = x_1(p_1', m') - x_1(p_1', m) = -\Delta x_1^n$$

so Δx_1^m is consistent with the order in the representation $\Delta m = m' - m$. The Slutsky identity becomes:

$$\Delta x_1 = \Delta x_1^s - \Delta x_1^m$$

Divide each side by Δp_1:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1}$$

Deduction by Rates of Change

Slutsky equation in rates of change (conti.)

Since the second term on the right hand side refers to income change, the denominator should also be replaced by an income change. Recall that $\Delta m = x_1 \Delta p_1$, $\Delta p_1 = \frac{\Delta m}{x_1}$. We then substitute the Δp_1 in the last term and obtain:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1$$

which is the Slutsky identity in terms of rates of change. The income effect is composed of two parts: how demand changes as income changes and the original level of demand.

Use of the Slutsky equation

True or False: (?)

- With other variables being fixed, the demand for a Giffen good falls when income increases.
- With other variables being fixed, the demand for a normal good falls when its price increases.

Deduction by Rates of Change

Adding the endowment income effect

Now consider the situation where the money income changes as the value of the endowment changes. Since the income is defined as $m = p_1 w_1 + p_2 w_2$, we have $\frac{\Delta m}{\Delta p_1} = w_1$. Thus,

$$endowment\ income\ effect = \frac{\Delta x_1^w}{\Delta p_1} = \frac{\Delta x_1^w}{\Delta m}\frac{\Delta m}{\Delta p_1} = \frac{\Delta x_1^m}{\Delta m}\frac{\Delta m}{\Delta p_1} = \frac{\Delta x_1^m}{\Delta m} w_1$$

Adding this new term to the original equation above, we obtain an updated Slutsky equation:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + (w_1 - x_1)\frac{\Delta x_1^m}{\Delta m}$$

Use of the Slutsky equation (conti.)

Q: If leisure is an inferior good, what can you say about the slope of the labor supply curve?

Deduction by Calculus

Next we derive the Slutsky equation formally using Calculus.

Slutsky demand function

Given the original consumption bundle (x_1, x_2), we define a **Slutsky demand function** for good 1 at prices (p_1, p_2) as $x_1^s(p_1, p_2, x_1, x_2)$, to measure what consumer would demand when facing prices (p_1, p_2) and having income $m = p_1x_1 + p_2x_2$. At some other prices (p_1', p_2') and income $m' = p_1'x_1 + p_2'x_2$, which is exactly enough to buy the original bundle, the ordinary demand is $x_1(p', m')$. If we adjust the ordinary demand to the previous purchasing power, by the definition of the substitution effect, we can write:

$$x_1^s(p_1, p_2, x_1, x_2) \equiv x_1(p_1, p_2, m)$$

Deduction by Calculus

Formal derivation for Slutsky equation

Differentiating the above identity with respect to p_1, we have:

$$\frac{\partial x_1^s(p_1, p_2, x_1, x_2)}{\partial p_1} = \frac{\partial x_1(p_1, p_2, m)}{\partial p_1} + \frac{\partial x_1(p_1, p_2, m)}{\partial m}\frac{\mathrm{d}m}{\mathrm{d}p_1}$$

$$\frac{\partial x_1^s(p_1, p_2, x_1, x_2)}{\partial p_1} = \frac{\partial x_1(p_1, p_2, m)}{\partial p_1} + \frac{\partial x_1(p_1, p_2, m)}{\partial m}x_1$$

Rearranging we have:

$$\frac{\partial x_1(p_1, p_2, m)}{\partial p_1} = \frac{\partial x_1^s(p_1, p_2, x_1, x_2)}{\partial p_1} - \frac{\partial x_1(p_1, p_2, m)}{\partial m}x_1$$

which is just the form of the Slutsky equation except that we have replaced the Δ's with partial derivative signs.

Deduction by Calculus

Formal derivation for Slutsky equation with endowment

To derive the Slutsky equation with endowment, we hold p_2 fixed and consider $x_1(p_1, m(p_1))$ where $m(p_1) = p_1 w_1 + p_2 w_2$. We take derivative with respect to p_1:

$$\frac{\mathrm{d}x_1(p_1, m)}{\mathrm{d}p_1} = \frac{\partial x_1(p_1, m)}{\partial p_1} + \frac{\partial x_1(p_1, m)}{\partial m} \frac{\mathrm{d}m}{\mathrm{d}p_1} \tag{1}$$

Recall the basic Slustsky equation we have derived in the last slide:

$$\frac{\partial x_1(p_1, m)}{\partial p_1} = \frac{\partial x_1^s(p_1, x_1, x_2)}{\partial p_1} - \frac{\partial x_1(p_1, m)}{\partial m} x_1 \tag{2}$$

And since $m(p_1) = p_1 w_1 + p_2 w_2$, we have:

$$\frac{\mathrm{d}m}{\mathrm{d}p_1} = w_1 \tag{3}$$

Deduction by Calculus

Formal derivation for Slutsky equation with endowment (conti.)

Inserting equation (2) and equation (3) into equation (1), we finally obtain:

$$\frac{\mathrm{d}x_1(p_1, m)}{\mathrm{d}p_1} = \frac{\partial x_1^s(p_1, x_1, x_2)}{\partial p_1} - \frac{\partial x_1(p_1, m)}{\partial m} x_1 + \frac{\partial x_1(p_1, m)}{\partial m} w_1$$

$$= \frac{\partial x_1^s(p_1, x_1, x_2)}{\partial p_1} + (w_1 - x_1)\frac{\partial x_1(p_1, m)}{\partial m}$$

which is the form of the Slutsky equation that we want.

Suggestions

- Read the **textbook** for further details and interesting cases (e.g., Rebating a tax in pp. 148-151 for today's topic).
- Please refer to the **English version**. Get familiar with terminologies in an English context, which is very helpful for your homework and exams.
- **Practice by doing exercises** as much as possible to become proficient in numerous models and their applications.

The End