* Producer theory:

$$\max_y \ \pi(y) = R(y) - C(y) \xrightarrow{FOC} MR(y) = MC(y)$$

- Competitive & Monopolistic:

  $\Rightarrow$ choose my own optimal level of output y

$$\begin{cases} C: \max_y \pi(y) = P \cdot y - C(y) \xrightarrow{FOC} P = MC(y) \quad (P \geq AC(y)) \\ M: \max_y \pi(y) = P(y) \cdot y - C(y) \xrightarrow{FOC} P\left(1 + \frac{1}{\varepsilon(p)}\right) = MC(y) \end{cases}$$

Remark: (monopolistic, assume MC is constant)

$$D(p) = Ap^{-b}, \quad \varepsilon = -b \rightarrow P\left(1+\frac{1}{\varepsilon}\right) = MC$$

$$p^* = \frac{MC}{1+\frac{1}{\varepsilon}} \text{ which is a constant}$$

$$\left.\begin{array}{l} D(p) = a - bp, \quad \varepsilon = \dfrac{-bp}{a-bp} \\ D(p) = (p+a)^{-b}, \quad \varepsilon = \dfrac{-bp}{p+a} \end{array}\right\} \rightarrow P\left(1+\frac{1}{\varepsilon(p)}\right) = MC$$

$$p^* = f(a,b) \text{ for some } f(\cdot)$$

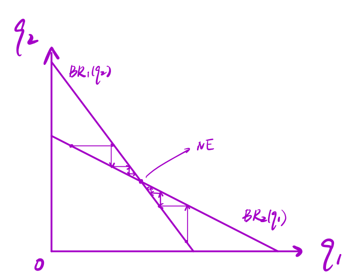
- Oligopoly:

  $\Rightarrow$ choose my y_1 depending on others' behavior y_2, vise versa

  $\Rightarrow$ mutual best response (NE)

$$\begin{cases} \text{Cournot: } \max_{y_1} \pi_1(y_1,y_2) = P(y_1,y_2)\cdot y_1 - C_1(y_1) \\ \text{Cartel: } \max_{y_1,y_2} \pi^M(y_1,y_2) = P(y_1,y_2)(y_1+y_2) - C_1(y_1) - C_2(y_2) \\ \text{Stackelberg: } \max_{y_1} \pi_1(y_1) = P(y_1, BR_2(y_1))\, y_1 - C_1(y_1) \quad (y_1 \text{ leader}) \\ \vdots \end{cases}$$

A graph illustration for Cournot NE:



Ex 1.

$y=4p \iff p=\frac{1}{4}y$

4. A firm has a supply function given by $S(p) = 4p$. Its fixed costs are 100. If the price changes from 10 to 20, what is the change in its profits?

Method 1:

$p = MC(y)$

$\Rightarrow MC(y)=\frac{1}{4}y, \quad C_v(y)=\frac{1}{8}y^2$

$\Rightarrow C(y)=C_v(y)+F=\frac{1}{8}y^2+100$

$\pi = p\cdot y - C(y) = \frac{1}{4}y^2-\frac{1}{8}y^2-100$

$= \frac{1}{8}y^2-100 = 2p^2-100$

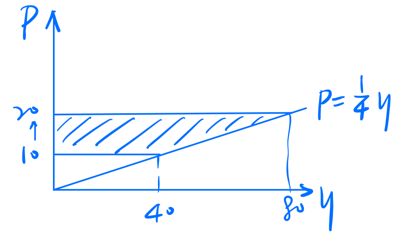
$\Delta\pi = 2(20^2-10^2)=600$

Method 2:

$\pi = py - C_v(y) - F$

$PS = py - C_v(y)$

$\Rightarrow \pi = PS - F$



$\Delta\pi = \Delta PS = \frac{1}{2}\times 10\times(40+80) = 600$

Ex 2.

5. If the long-run cost function is $c(y) = y^2 + 1$, what is the long-run supply curve of the firm?

$\pi = p\cdot y - c(y)$

competitive $\begin{cases} p = MC(y) \iff \text{FOC} \quad (MR=MC) \\ p \geq AC(y) \iff py - c(y) \geq 0 \end{cases}$

$c(y) = y^2+1$

$MC(y) = 2y \quad \Rightarrow \quad p = 2y \quad\quad y=\frac{p}{2}$

$AC(y) = \frac{y^2+1}{y} = y+\frac{1}{y} \quad \Rightarrow \quad p = 2y \geq y+\frac{1}{y}$

$y \geq 1$

$p \geq 2$

or $p \geq \min AC(y)$

$= \min(y+\frac{1}{y})$

$= 2$

LRSC: $y = \begin{cases} \frac{p}{2}, & p \geq 2 \\ 0, & 0 < p < 2 \end{cases}$

Ex 3.

$\pi > 0 \iff$ payment for licence

7. A New York City cab operator appears to be making positive profits in the long run after carefully accounting for the operating and labor costs. Does this violate the competitive model? Why or why not?

No.

LR: zero economic profits

If not, the operator forgot to include all the costs, specifically, the payment to the operating license.

$\Rightarrow$ economic rent (market value of factors that prevent entry)

Ex 4.

1. The market demand curve for heroin is said to be highly inelastic. ① Heroin supply is also said to be monopolized by the Mafia, which we assume to be interested in maximizing profits. ② Are these two statements consistent?

No.

$$\pi = p(y)\cdot y - c(y)$$

$$MR = MC$$

$$p'(y)\cdot y + p(y) = MC(y)$$

$$p(y)\left[1+\frac{dp(y)}{dy}\cdot\frac{y}{p(y)}\right] = MC(y)$$

$$p(y)\left[1+\frac{1}{\varepsilon(y)}\right] = MC(y)$$

$$MR = \underbrace{p(y)\left[1-\frac{1}{|\varepsilon(y)|}\right]} = MC(y) \quad \text{(monopolistic)}$$

statement ①: $|\varepsilon| < 1$

statement ②: $p(y^*)\left[1-\frac{1}{|\varepsilon(y^*)|}\right] = MC(y^*)$

From ①, $\frac{1}{|\varepsilon|} > 1$, $1-\frac{1}{|\varepsilon|} < 0$, $MR < 0 < MC$, thus ② never holds.

$\Rightarrow$ ① & ② inconsistent

Ex 5.

3. The monopolist faces a demand curve given by $D(p) = 10p^{-3}$. Its cost function is $c(y) = 2y$. What is its optimal level of output and price?

Method 1: $D(p) = 10p^{-3} \Rightarrow p(y) = 10^{\frac{1}{3}} y^{-\frac{1}{3}}$

$$R(y) = p(y)\cdot y = 10^{\frac{1}{3}} y^{\frac{2}{3}}$$

$$MR = 10^{\frac{1}{3}}\cdot\frac{2}{3}\cdot y^{-\frac{1}{3}}, \quad MC = 2$$

By $MR = MC$, $10^{\frac{1}{3}}\frac{2}{3} y^{-\frac{1}{3}} = 2$

$$10^{-1} y = 3^{-3}$$

$$y^* = \frac{10}{27}$$

$$p^* = 10^{\frac{1}{3}}\left(\frac{10}{27}\right)^{-\frac{1}{3}} = 3$$

Method 2: $D(p) = 10p^{-3}$

$\Rightarrow$ constant elasticity: $\varepsilon = \frac{p}{y}\cdot\frac{dy}{dp} = \frac{p}{10p^{-3}}\cdot 10(-3)\cdot p^{-4} = -3$

By $MR = p\left(1+\frac{1}{\varepsilon}\right) = MC$

$$p(1-\tfrac{1}{3}) = 2$$
$$p^* = 3$$
$$y^* = D(p^*) = 10 \cdot 3^{-3} = \frac{10}{27}$$

Ex 6.

4. If $D(p) = 100/p$ and $c(y) = y^2$, what is the optimal level of output of the monopolist? (Be careful.)

$$D(p) = 100\,p^{-1} \Rightarrow \varepsilon = -1 \quad \text{unit elastic}$$
$$MR = p(1+\tfrac{1}{\varepsilon}) = p(1-1) = 0$$
$$MC = 2y > 0 \quad \text{since } y = \frac{100}{p} > 0$$
$$\Rightarrow MR < MC \text{ always holds}, \quad \nexists\, y^*$$
In fact, $\pi = 100 - y^2$. As $y \to 0$, $\pi \to 100$.

Ex 7.

$MC = c$ $\qquad$ $p(1-\frac{1}{3}) = c$ ✗

5. A monopolist with constant marginal cost is producing where $|\epsilon| = 3$. The government imposes a quantity tax of \$6 per unit of output. If the demand curve facing the monopolist is linear, how much does the price rise?

$$MC' = c + t$$
$$\Delta MC = t$$

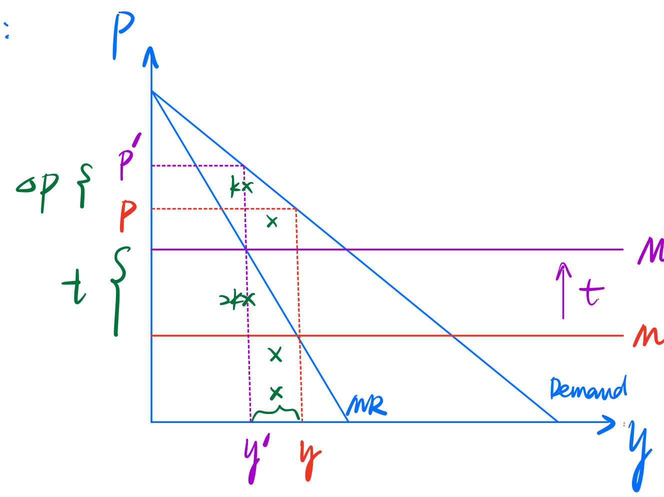
Method 1: Suppose $D(p) = a - bp$
$$p = \frac{a}{b} - \frac{1}{b}\cdot y$$
$$R(y) = \frac{a}{b}y - \frac{1}{b}y^2$$
$$MR = \frac{a}{b} - \frac{2}{b}y$$
$$\Rightarrow \frac{a}{b} - \frac{2}{b}y = MC \qquad y = \frac{a - b\cdot MC}{2} \qquad p = \frac{a + b\cdot MC}{2b} = \frac{MC}{2} + \frac{a}{2b}$$
$$\Delta p = \frac{\Delta MC}{2} = \frac{t}{2} = \frac{6}{2} = 3$$

Method 2: Suppose $D(p) = a - bp$. $\qquad \varepsilon(p) = \dfrac{-bp}{a-bp}$
$$MR = MC$$
$$p(1+\tfrac{1}{\varepsilon(p)}) = MC$$
$$p(1 - \tfrac{a-bp}{bp}) = MC$$
$$2bp - a = b\cdot MC$$

$$p = \frac{MC}{2} + \frac{a}{2b}$$

$$\Delta p = \frac{\Delta MC}{2} = \frac{t}{2} = \frac{6}{2} = 3$$

Method 3:



$$\frac{\Delta p}{t} = \frac{kx}{2kx} = \frac{1}{2}$$

$$\Delta p = \frac{t}{2} = 3$$

* property of linear demand curve in monoplistic market:

slope of MR curve = 2 × slope of demand curve

Ex 8. 6. What is the answer to the above question if the demand curve facing the monopolist has constant elasticity? $\varepsilon = -3$

$$p\left(1+\frac{1}{\varepsilon}\right) = MC$$

$$p = \frac{MC}{1+\frac{1}{\varepsilon}}$$

$$\Delta p = \frac{\Delta MC}{1+\frac{1}{\varepsilon}} = \frac{t}{1+\frac{1}{\varepsilon}} = \frac{6}{1-\frac{1}{3}} = 9$$

Ex 9. 7. If the demand curve facing the monopolist has a constant elasticity of 2, then what will be the monopolist's markup on marginal cost? $\varepsilon = -2$

$$p(y^*) = \frac{MC(y^*)}{1+\frac{1}{\varepsilon}} = \frac{MC(y^*)}{1-\frac{1}{2}} = 2MC(y^*)$$

$$\text{markup} = p(y^*) - MC(y^*) = MC(y^*)$$

Ex 10. 8. The government is considering subsidizing the marginal costs of the monopolist described in the question above. What level of subsidy should the government choose if it wants the monopolist to produce the socially optimal amount of output? (y')

$\varepsilon = -2$

Socially optimal $\Rightarrow$ competitive $\Rightarrow p(y') = MC(y')$

Suppose the subsidy is t:

$$MR(y') = MC(y') - t$$

$$p(y')\left(1+\frac{1}{\varepsilon}\right) = MC(y') - t$$

$$\frac{1}{2} p(y') = MC(y') - t$$

$$t = MC(y') - \frac{1}{2}p(y') = MC(y') - \frac{1}{2}MC(y') = \frac{1}{2}MC(y')$$

Ex 11. 10. True or false? Imposing a quantity tax on a monopolist will always cause the market price to increase by the amount of the tax.

False.

$$p(y^*)\left(1+\frac{1}{\varepsilon(y^*)}\right) = MC(y^*) \Rightarrow p(y^*) = \frac{MC(y^*)}{1+\frac{1}{\varepsilon(y^*)}}$$

$$p(y')\left(1+\frac{1}{\varepsilon(y')}\right) = MC(y') + t \Rightarrow p(y') = \frac{MC(y')+t}{1+\frac{1}{\varepsilon(y')}}$$

Since $MR(y^*) = MC(y^*)$, $MR(y^*) < MC(y^*) + t$

the firm will reduce output, i.e., $y^* > y'$

By the negative slope of demand curve, $p(y^*) < p(y')$.

$\Rightarrow \Delta p > 0$, price must increase.

However, $\Delta p = p(y') - p(y^*) = \dfrac{MC(y')+t}{1+\frac{1}{\varepsilon(y')}} - \dfrac{MC(y^*)}{1+\frac{1}{\varepsilon(y^*)}} \overset{?}{=} t$

$\Rightarrow$ depending on the property of demand curve $(\varepsilon(y))$.

$\Rightarrow \Delta p$ may $>$, $<$, or $=$ t. (see Ex. 7 as an example)

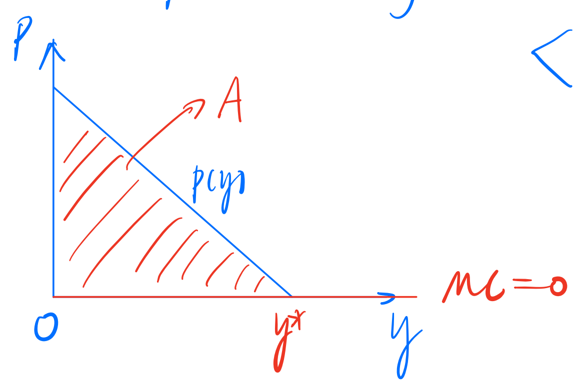
Ex 12.

3. Suppose that the amusement park owner can practice perfect first-degree price discrimination by charging a different price for each ride. Assume that all rides have zero marginal cost and all consumers have the same tastes. Will the monopolist do better charging for rides and setting a zero price for admission, or better by charging for admission and setting a zero price for rides?

Two-part tariffs : $p_1 + p_2 x$

$p_1 = 0, \; p_2 > 0$

$p_1 > 0, \; p_2 = 0$



① Charging ride, free admission :

perfect first-degree price discrimination $\Rightarrow$ set $p_y = WTP$ for each unit of ride

$R(y) = PS = A, \quad CS = 0$

Since $MC = 0$, $C(y) = 0$, $TC = F$

$\pi = R(y) - C(y) = A - F$

② Charging admission, free ride :

Set $p_{ad} = A$, $\pi = A - F$

same, indifferent

Ex 13.

2. Consider a cartel in which each firm has identical and constant marginal costs. If the cartel maximizes total industry profits, what does this imply about the division of output between the firms?

Suppose there are n firms, $Y = \sum_{i=1}^{n} y_i$

$$\max_{y_i} \; p(Y) \cdot Y - \sum_{i=1}^{n} c(y_i)$$

FOC: $\frac{\partial \pi}{\partial y_i} = p'(Y) \cdot Y + p(Y) - c'(y_i) = 0$ for $\forall i$

Since the n firms have identical and constant MC,

$MR = MC_1 = MC_2 = \cdots = MC_n$ always holds

$\Rightarrow$ Can be arbitrary division of output between firms

Ex 14.

4. Suppose there are n identical firms in a Cournot equilibrium. Show that the absolute value of the elasticity of the market demand curve must be greater than $1/n$. (Hint: in the case of a monopolist, $n = 1$, and this simply says that a monopolist operates at an elastic part of the demand curve. Apply the logic that we used to establish that fact to this problem.)

Suppose there are n firms, $Y = \sum_{i=1}^{n} y_i$

$$\max_{y_i} \; p(Y)\cdot y_i - C(y_i)$$

FOC: $p'(Y)\cdot y_i + p(Y) - MC(y_i) = 0$

$$p(Y)\cdot\left[1 + \frac{dp(Y)}{dY}\cdot\frac{Y}{p(Y)}\cdot\frac{y_i}{Y}\right] = MC(y_i)$$

Define $s_i = \dfrac{y_i}{Y}$ as the market share

$$\Rightarrow p(Y)\cdot\left[1 + \frac{s_i}{\varepsilon(Y)}\right] = MC(y_i) > 0$$

$$1 + \frac{s_i}{\varepsilon} > 0$$

Since all firms are identical, $s_i = \dfrac{1}{n}$

$$1 + \frac{1}{n\varepsilon} > 0$$

$$1 - \frac{1}{n|\varepsilon|} > 0$$

$$|\varepsilon| > \frac{1}{n}$$