ECO4010 Tutorial 1

1. Consider a choice problem with choice set $X = \{x, y, z\}$. Consider the following choice structures:

   (a) $(\mathscr{B}_1, C(\cdot))$, in which $\mathscr{B}_1 = \{\{x,y\},\{y,z\},\{x,z\},\{x\},\{y\},\{z\}\}$ and $C(\{x,y\}) = \{x\}, C(\{y,z\}) = \{y\}, C(\{x,z\}) = \{z\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \{z\}$.

   (b) $(\mathscr{B}_2, C(\cdot))$, in which $\mathscr{B}_2 = \{\{x,y,z\},\{x,y\},\{y,z\},\{x,z\},\{x\},\{y\},\{z\}\}$ and $C(\{x,y,z\}) = \{x\}, C(\{x,y\}) = \{x\}, C(\{y,z\}) = \{z\}, C(\{x,z\}) = \{z\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \{z\}$.

   (c) $(\mathscr{B}_3, C(\cdot))$, in which $\mathscr{B}_3 = \{\{x,y,z\},\{x,y\},\{y,z\},\{x,z\},\{x\},\{y\},\{z\}\}$ and $C(\{x,y,z\}) = \{x\}, C(\{x,y\}) = \{x\}, C(\{y,z\}) = \{y\}, C(\{x,z\}) = \{x\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \{z\}$.

   For every choice structure, comment on if the $WARP$ is satisfied and if there exists a rational preference relation $\succsim$ that rationalizes $C(\cdot)$ relative to its $\mathscr{B}$. If such a rationalization is possible, write it down.

2. Define two alternative versions of $WARP$: assume $C(\cdot)$ on $(X, \mathscr{B})$ satisfies

   $$WARP^* \text{ iff for each } A, B \in \mathscr{B}, C(A) \cap B \neq \emptyset \Rightarrow C(B) \cap A \subset C(A)$$

   $$WARP^{**} \text{ iff for each } A, B \in \mathscr{B}, x, y \in A \cap B, x \in C(A), y \notin C(A) \Rightarrow y \notin C(B)$$

   (a) Show that $WARP^*$ is equivalent to $WARP$.

   (b) Show that $WARP^{**}$ is equivalent to $WARP$.

   (Hint: Firstly verify that $WARP$ can be rewritten as: for each $A, B \in \mathscr{B}, x, y \in A \cap B, x \in C(A), y \in C(B) \Rightarrow y \in C(A)$ or $\Rightarrow x \in C(B)$.)

3. Show that if X is finite and $\succsim$ is rational, then $C_{\succsim}(B) \neq \emptyset$ for any $B \in \mathscr{B}$. (Hint: Use induction.)

4. Let X be a finite set with more than $N \geq 1$ elements, $\mathscr{B}$ its non-empty subsets, and $\succsim_1, \succsim_2$ two rational preference relations on X. Suppose that someone follows the following choice procedure: for each $B \in \mathscr{B}$, if B has more than N elements, then $C(B)$ is based on $\succsim_1$; if B has no more than N elements, then $C(B)$ is based on $\succsim_2$. Show that this choice rule violates $WARP$.

5. The *path-invariance* property has the following definition: For every pair $B_1, B_2 \in \mathscr{B}$ such that $B_1 \cup B_2 \in \mathscr{B}$ and $C(B_1) \cup C(B_2) \in \mathscr{B}$, we have $C(B_1 \cup B_2) = C(C(B_1) \cup C(B_2))$, that is, the decision problem can safely be subdivided.

   (a) Show that a choice structure $(\mathscr{B}, C(\cdot))$ for which a rationalizing preference relation $\succsim$ exists satisfies the *path-invariance* property.

   (b) Find examples of choice procedures that do not satisfy this property.

6. Suppose that choice structure $(\mathscr{B}, C(\cdot))$ satisfies $WARP$. In the lecture, the *revealed (at-least-as-good-as) preference relation* $\succsim_C$ is defined by:

   $$x \succsim_C y \Leftrightarrow \exists B \in \mathscr{B} \ s.t. \ x, y \in B \text{ and } x \in C(B)$$

   Consider the following other two possible revealed preferred relations, $\succ^*$ and $\succ^{**}$:

   $$x \succ^* y \Leftrightarrow \exists B \in \mathscr{B} \ s.t. \ x, y \in B, x \in C(B), \text{ and } y \notin C(B)$$

   $$x \succ^{**} y \Leftrightarrow x \succsim_C y \text{ but not } y \succsim_C x$$

   (a) Show that $\succ^*$ and $\succ^{**}$ give the same relation over X; that is, for any $x, y \in X$, $x \succ^* y \Leftrightarrow x \succ^{**} y$. Is this still true if $(\mathscr{B}, C(\cdot))$ does not satisfy $WARP$?

   (b) Must $\succ^*$ be transitive?

   (c) Show that if $\mathscr{B}$ includes all subsets of X up to 3 elements, then $\succ^*$ is transitive.

7. Monotonicity and nonsatiation are two properties of $\succsim$. This exercise investigates the relationship between them. Suppose $\succsim$ is defined on the consumption set $X = R^L_+$. According to MWG, we have the following definitions regarding monotonicity in a slightly different way from the definitions in the lecture:

   $\succsim$ on X is monotone if $x, y \in X, y >> x \Rightarrow y \succ x$

   $\succsim$ on X is strongly monotone if $x, y \in X, y \geq x$ and $y \neq x \Rightarrow y \succ x$

   $\succsim$ on X is weakly monotone if $x, y \in X, y \geq x \Rightarrow y \succsim x$

   where $y >> x$ means that every element of y is greater than every element of x.

   (a) Show that if $\succsim$ is strongly monotone, then it is monotone.

   (b) Show that if $\succsim$ is monotone, then it is locally nonsatiated. (Hint: For $x, y \in R^L_+$, the Euclidean distance between x and y is defined as $||x - y|| = [\sum_{l=1}^{L}(x_l - y_l)^2]^{\frac{1}{2}}$.)

   (c) Draw a convex preference relation that is locally nonsatiated but is not monotone to show that the converse proposition of (b) does not hold, that is, local nonsatiation is a weaker assumption than monotonicity.

   (d) Show that if $\succsim$ is transitive, locally nonsatiated, and weakly monotone, then it is monotone.

8. Let $\succsim$ be a preference relation on a set X. Suppose $\succsim$ is complete and transitive. Recall the definitions of $\succ$ and $\sim$ derived from $\succsim$ in the lecture.

   (a) Show that $\succ$ is:

(1) Irreflexive: $x \succ x$ never holds

(2) Transitive: $x \succ y, y \succ z \Rightarrow x \succ z$

(3) Asymmetric: $x \succ y \Rightarrow y \not\succ x$

(4) Satisfying negative transitivity: $x \not\succ y, y \not\succ z \Rightarrow x \not\succ z$

(b) Show that $\sim$ is:

(1) Reflexive: $x \sim x$ always holds

(2) Transitive: $x \sim y, y \sim z \Rightarrow x \sim z$

(3) Symmetric: $x \sim y \Rightarrow y \sim x$

(c) Define $I(x)$ to be the set of all $y \in X$ for which $y \sim x$. Show that the set $\{I(x)|x \in X\}$ is a partition of X, that is,

(1) $\forall x \in X, I(x) \neq \emptyset$

(2) $\forall x \in X, \exists y \in X$ such that $x \in I(y)$

(3) $\forall x, y \in X,$ either $I(x) = I(y)$ or $I(x) \cap I(y) = \emptyset$