Learn Latent Representations

Autoencoders and Self-supervised Learning

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Low Dimensional Representations

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- In the last lecture this was particularly evident when when we looked at embedding models like word2vec which explictly try to capture relationships in the data in a low dimensional 'latent' space.

Self-supervised Learning

Yann LeCun

30 April 2019 ·

I now call it "self-supervised learning", because "unsupervised" is both a loaded and confusing term.

In self-supervised learning, the system learns to predict part of its input from other parts of it input. In other words a portion of the input is used as a supervisory signal to a predictor fed with the remaining portion of the input.

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- Let's now consider a different type self-supervised of task where we want to learn a model that learns to copy its input to its output.

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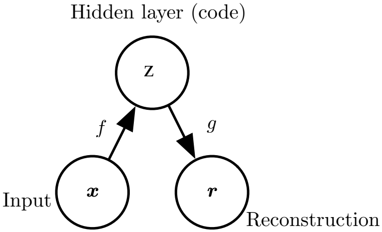

Autoencoder constraints

- Clearly a linear autoencoder with a sufficient number of weights (e.g. if the dimension of $\boldsymbol{h}$ was greater than or equal to that of x) could learn set $g(f(\boldsymbol{x})) = \boldsymbol{x}$ everywhere, but this obviously wouldn't be useful!

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- In practice we apply *restrictions*[1] to stop this happening.
- The objective is to use these restrictions to force the autoencoder to learn useful properties of the data.

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Undercomplete Autoencoders

- Undercomplete autoencoders have $\dim(\boldsymbol{h}) << \dim(\boldsymbol{x})$.
- This forces the encoder to learn a *compressed representation* of the input.
- The representation will capture the most *salient* features of the input data.

Undercomplete Autoencoders — Linear

Consider the single-hidden layer linear autoencoder network given by:

$$h = \mathbf{W}_e \boldsymbol{x} + \boldsymbol{b}_e$$
$$r = \mathbf{W}_d \boldsymbol{h} + \boldsymbol{b}_d$$

where $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{h} \in \mathbb{R}^m$ and $m < n$.

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With the MSE loss, this autoencoder will learn to span the same subspace as PCA for a given set of training data.

Note that the autoencoder weights are not however constrained to be orthogonal (like they would be in PCA)

Undercomplete Autoencoders — deeper and nonlinear

- A linear autoencoder with a single hidden layer learns to map into the same subspace as PCA.

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  - But, if you add more hidden layers with non-linear activations (to either the encoder, decoder or both) you can effectively perform a powerful non-linear generalisation of PCA

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Deep Autoencoders

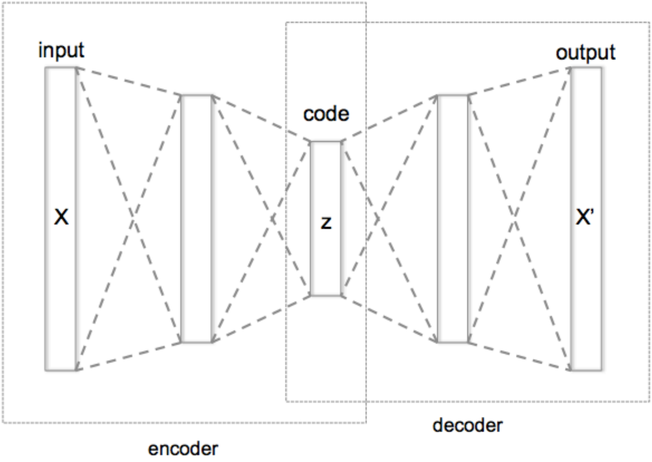

Image taken from wikipedia

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  - The decoder just needs to memorise the training examples so that it can map back from i.

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- There is nothing stopping us using any other kinds of layers though...
- If we're working with image data, where we know that much of the structure is 'local', then using convolutions in both the decoder makes sense

Convolutional Autoencoder

Regularised Autoencoders

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Regularised Autoencoders

- Rather than (necessarily) forcing the hidden vector to have a lower dimensionality than the input, we could instead utilise some form of regularisation to force the network to learn interesting representations...
- Many ways to do this; let's look at two of them:
  - Denoising Autoencoders
  - Sparse Autoencoders

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  - E.g. by adding Gaussian noise.
- The loss is computed between the reconstruction (computed from the noisy input) against the original noise-free data.

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- A popular choice that you've seen before would be to use an l1 penalty $\Omega(\boldsymbol{h} = \lambda \sum_i |h_i|$
  - this of course does have a slight problem... what is the derivative of $y = |x|$ with respect to x at $x = 0$?

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- Semantic segmentation

Beyond Deterministic Autoencoders: Stochastic Encoders and Decoders

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    - Binary $\boldsymbol{x}$ would correspond to a Bernoulli distribution parameterised by sigmoid outputs
    - Discrete (or categorical) $\boldsymbol{x}$ would correspond to a softmax distribution.
- What about the encoder - could we make that output $p(\boldsymbol{h}|\boldsymbol{x})$?