Forget to remember
Remember to forget

VLC = Vision Learning Control

Long Short Term Memories and Gated Recurrent Units

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Vision, Learning and Control
University of Southampton

Many of the images and animations used here were made by Adam Prügel-Bennett.

Recap: An RNN is just a recursive function invocation

- $\boldsymbol{y}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{c}(t-1)|\boldsymbol{W})$
- and the state $\boldsymbol{c}(t) = \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{c}(t-1)|\boldsymbol{W})$
- If the output $\boldsymbol{y}(t)$ depends on the input $\boldsymbol{x}(t-2)$, then prediction will be

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- If $\boldsymbol{y}(t)$ depends on the input $\boldsymbol{x}(t-\tau)$ then the back-propagated signal will be proportional to $a^{\tau-1}$
- This either vanishes or explodes when τ becomes large

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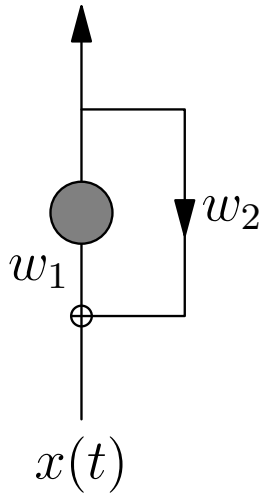

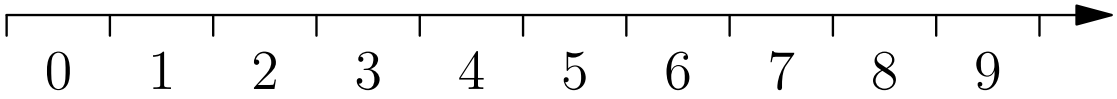

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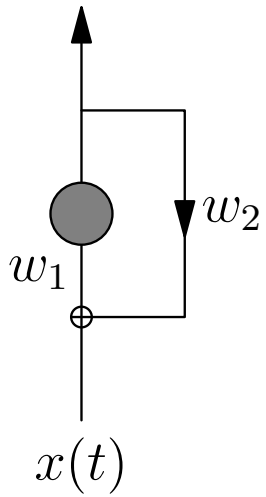

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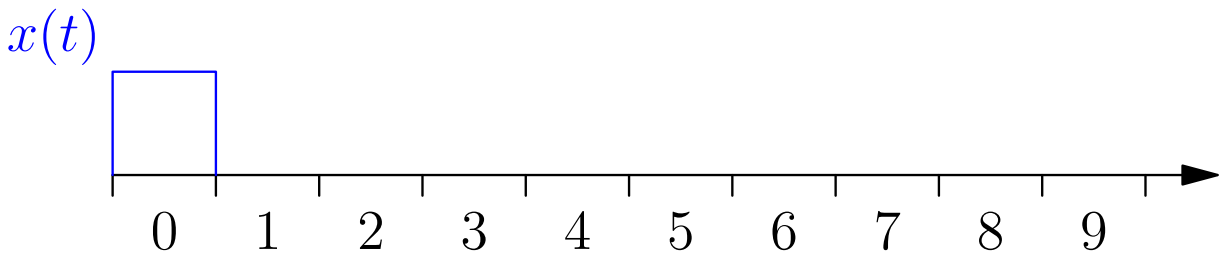

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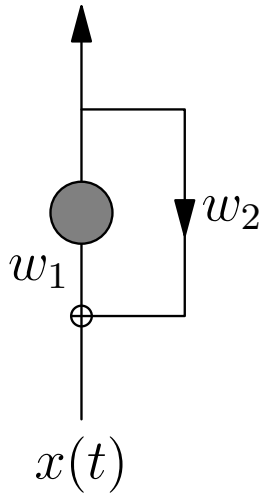

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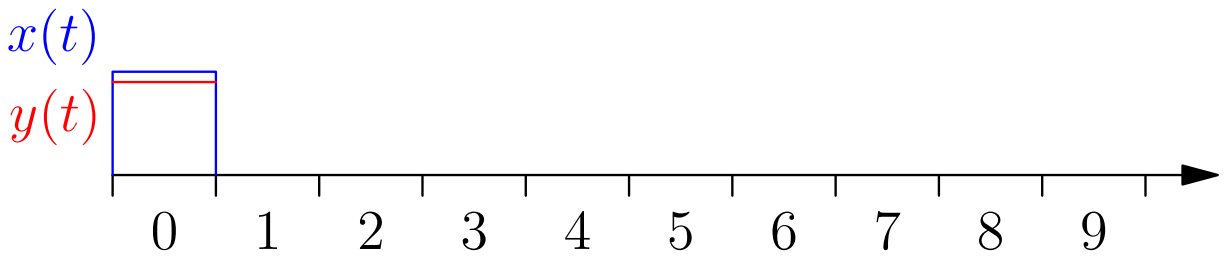

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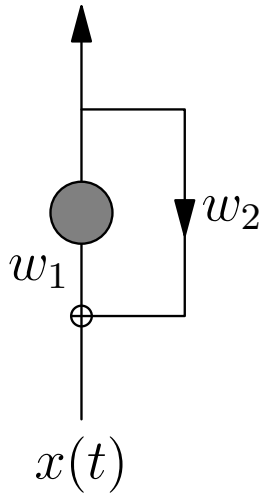

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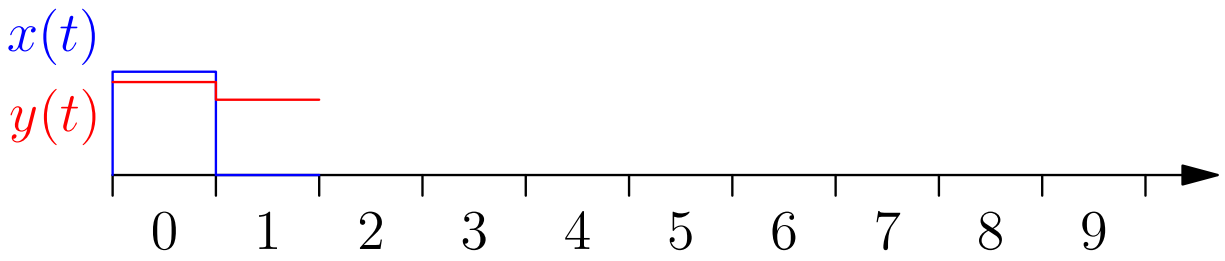

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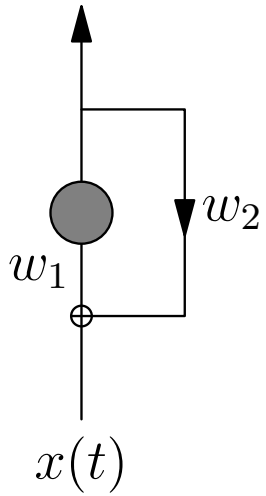

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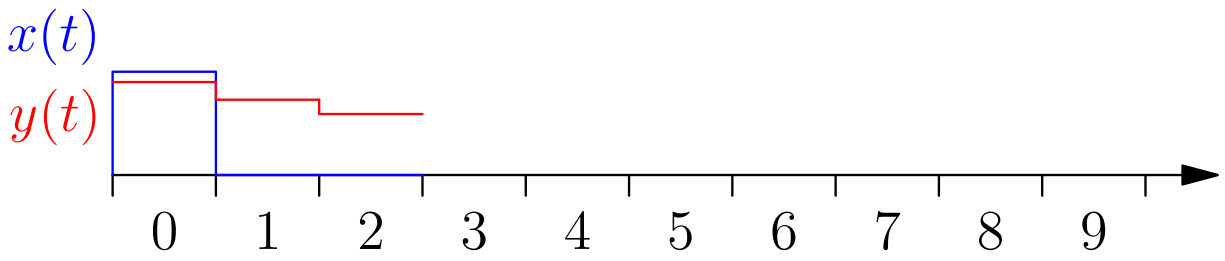

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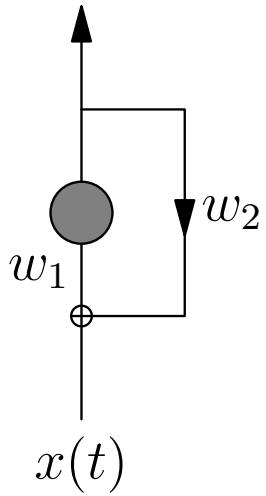

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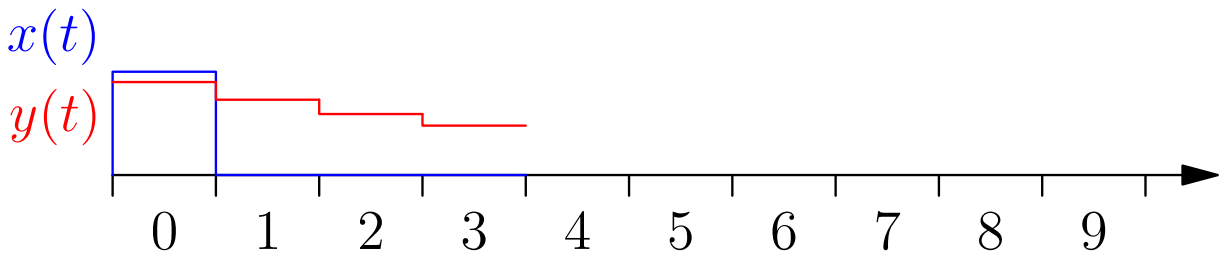

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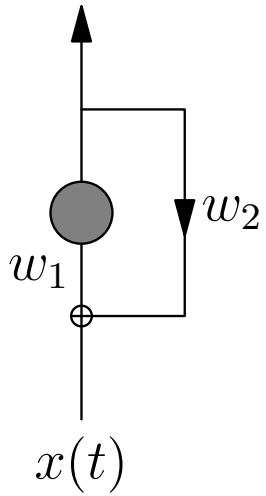

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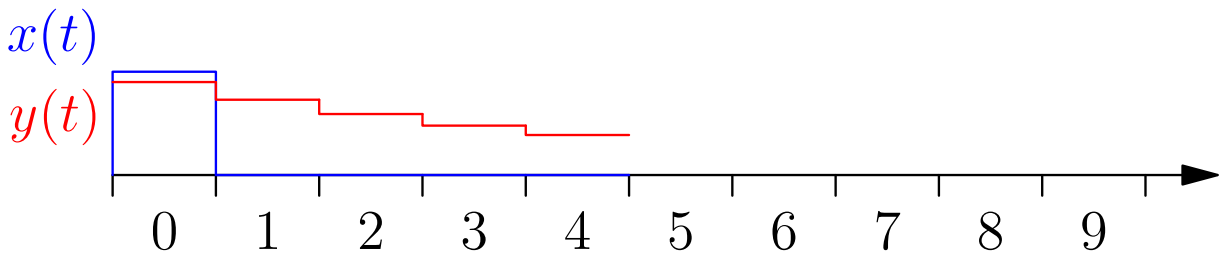

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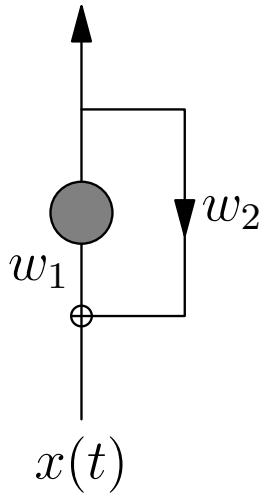

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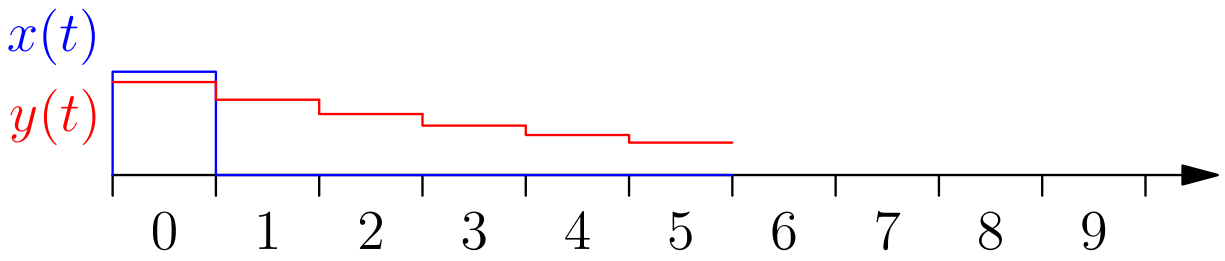

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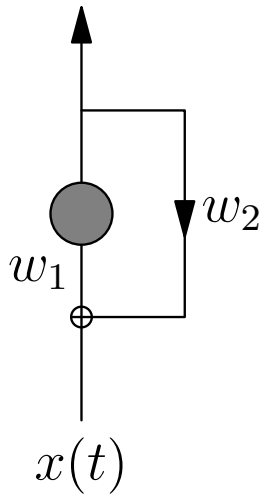

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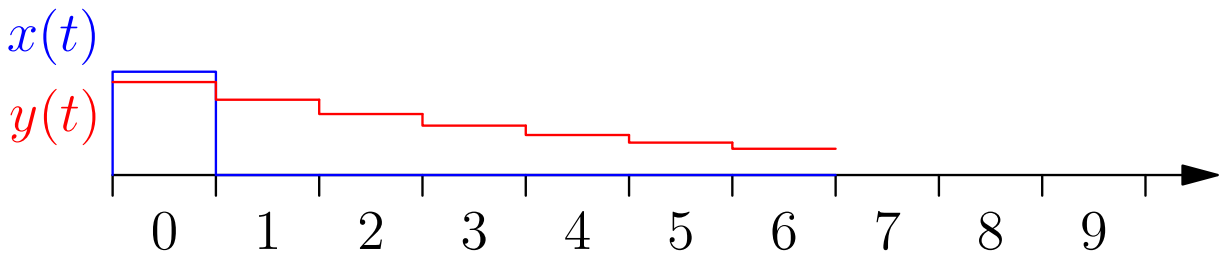

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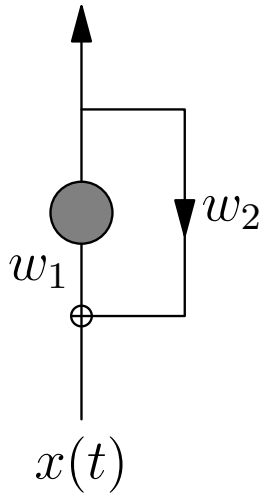

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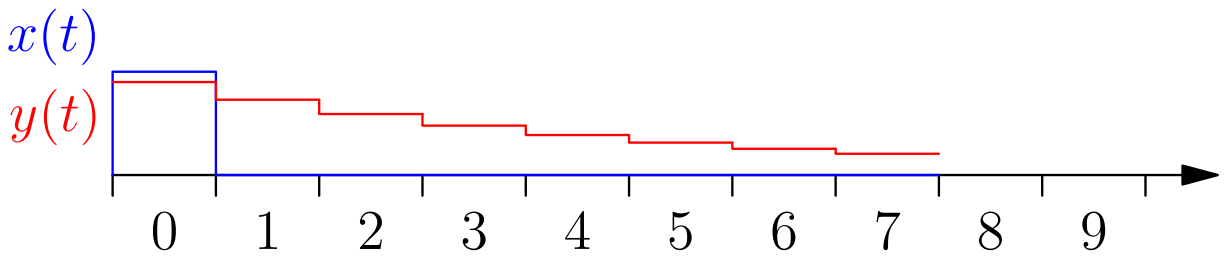

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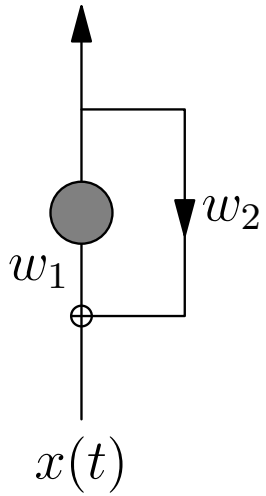

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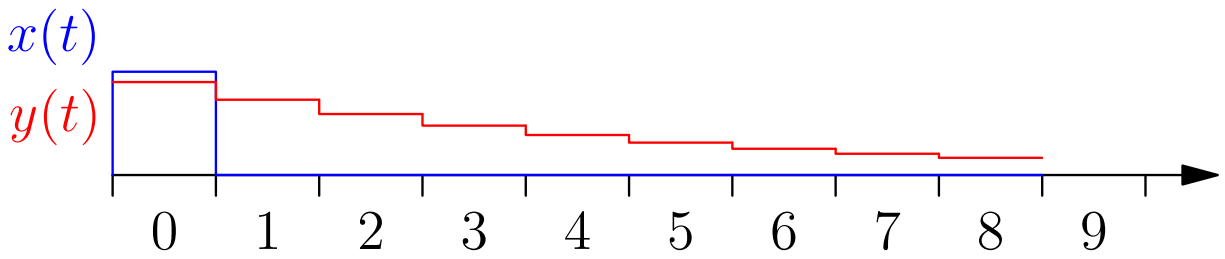

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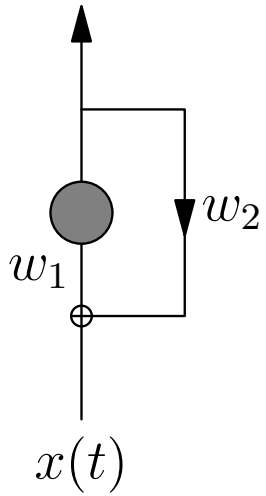

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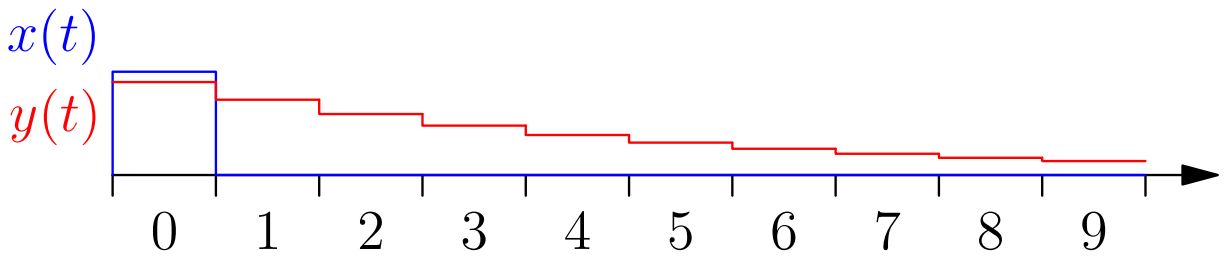

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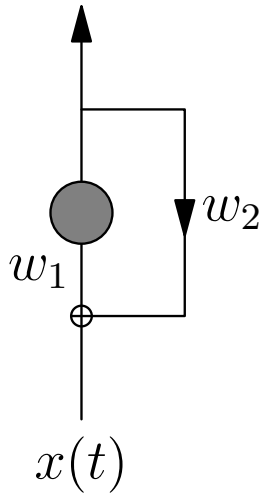

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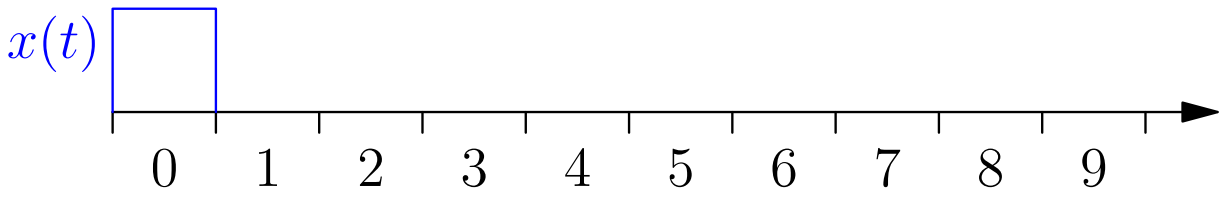

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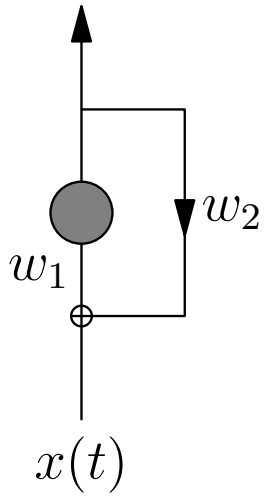

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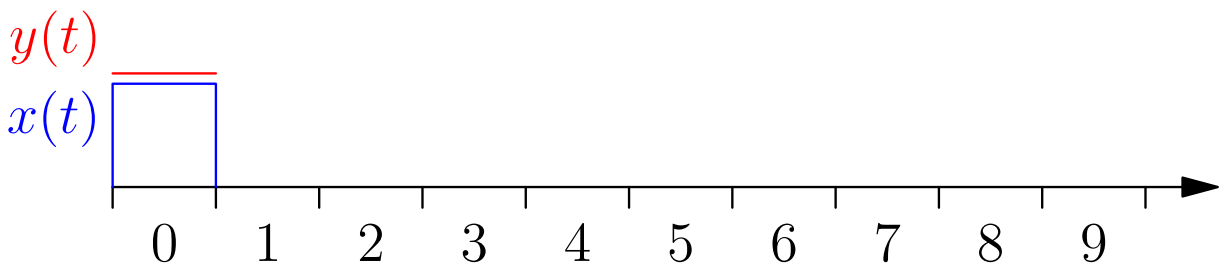

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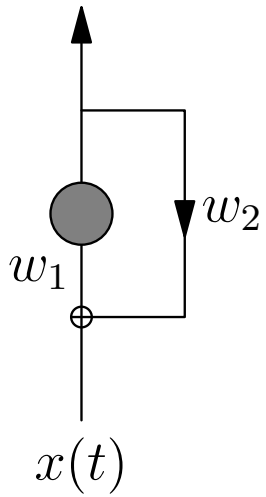

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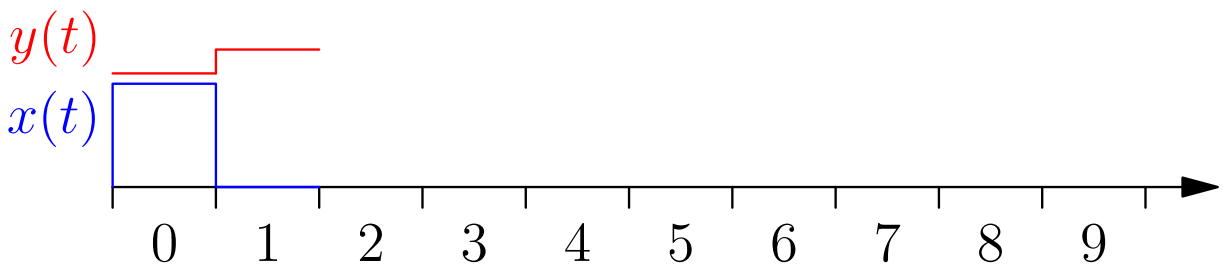

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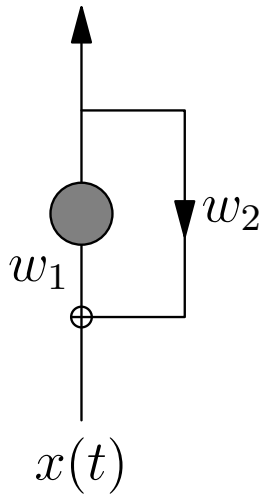

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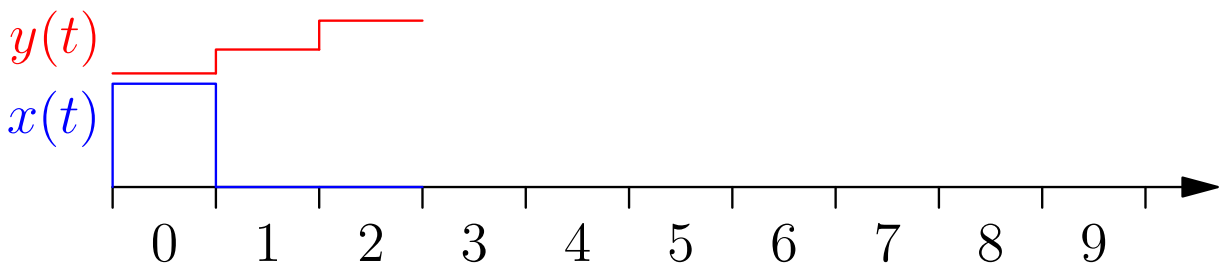

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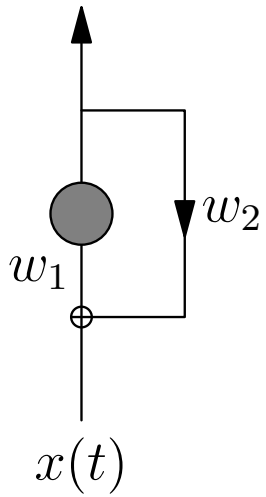

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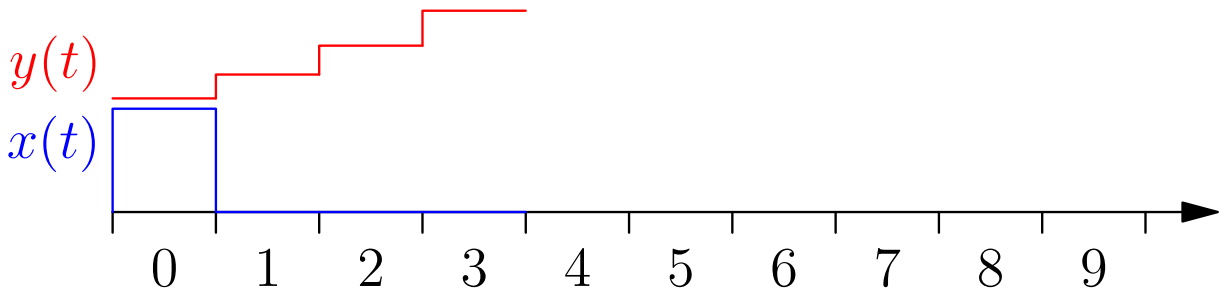

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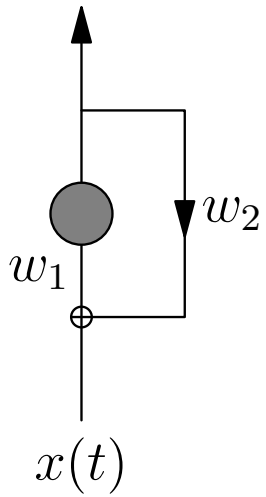

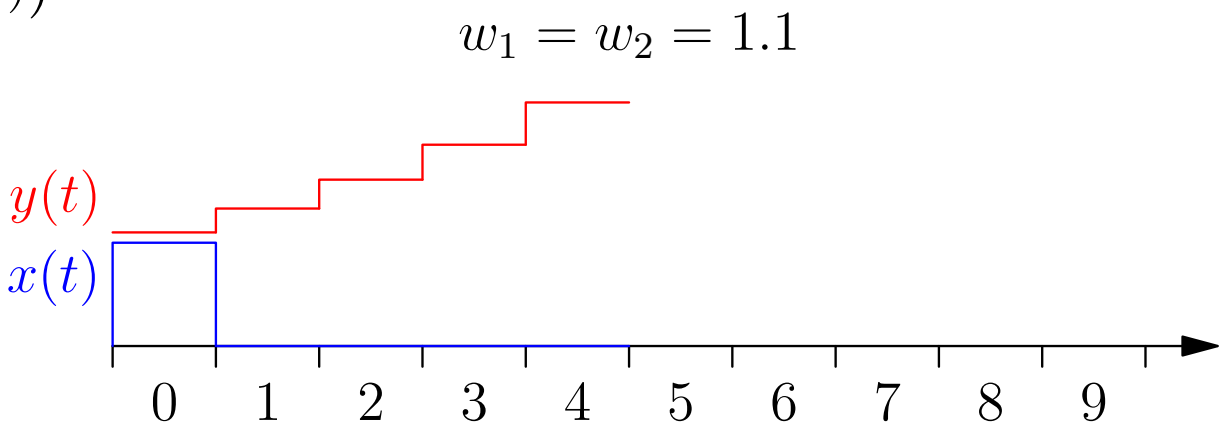

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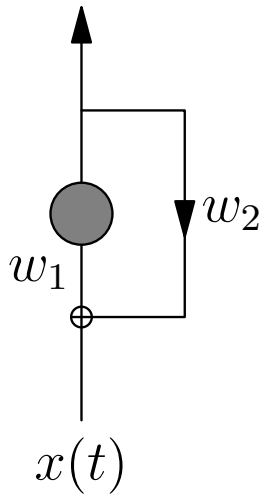

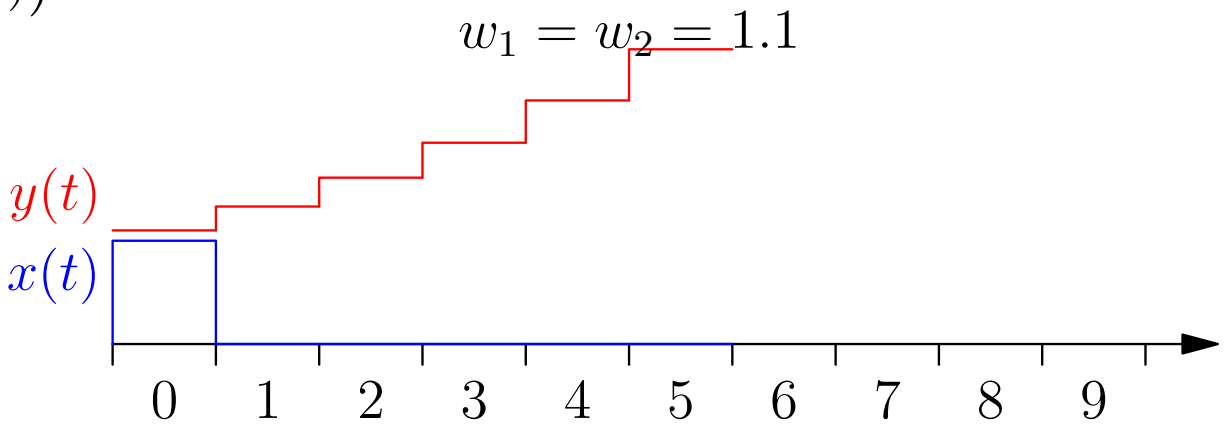

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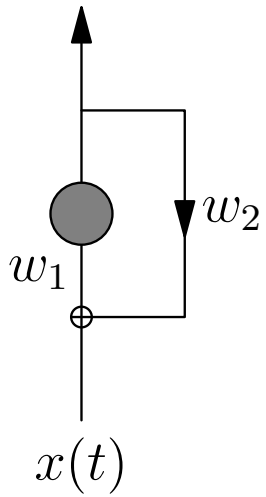

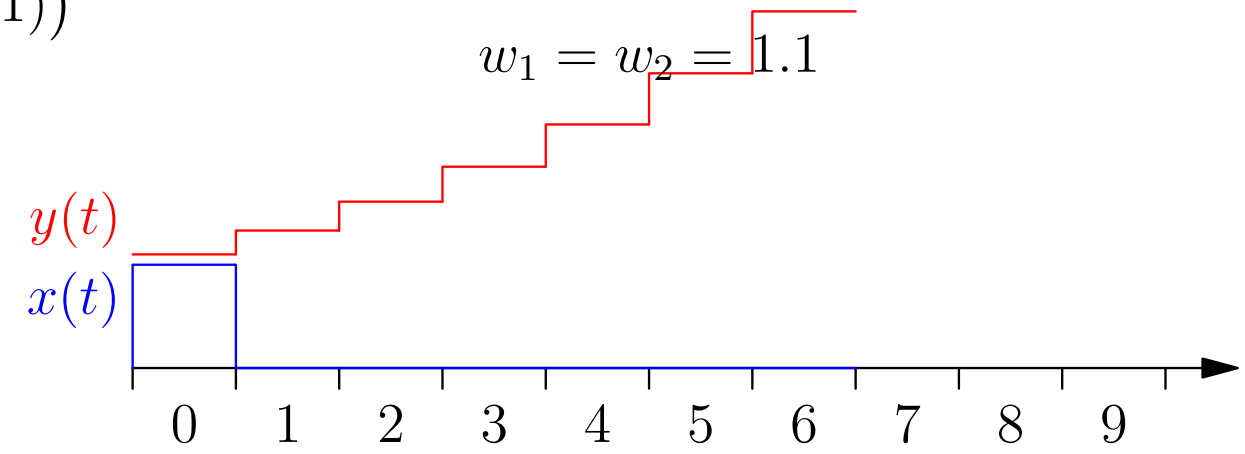

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- Sigmoid functions naturally saturate at 0 and 1

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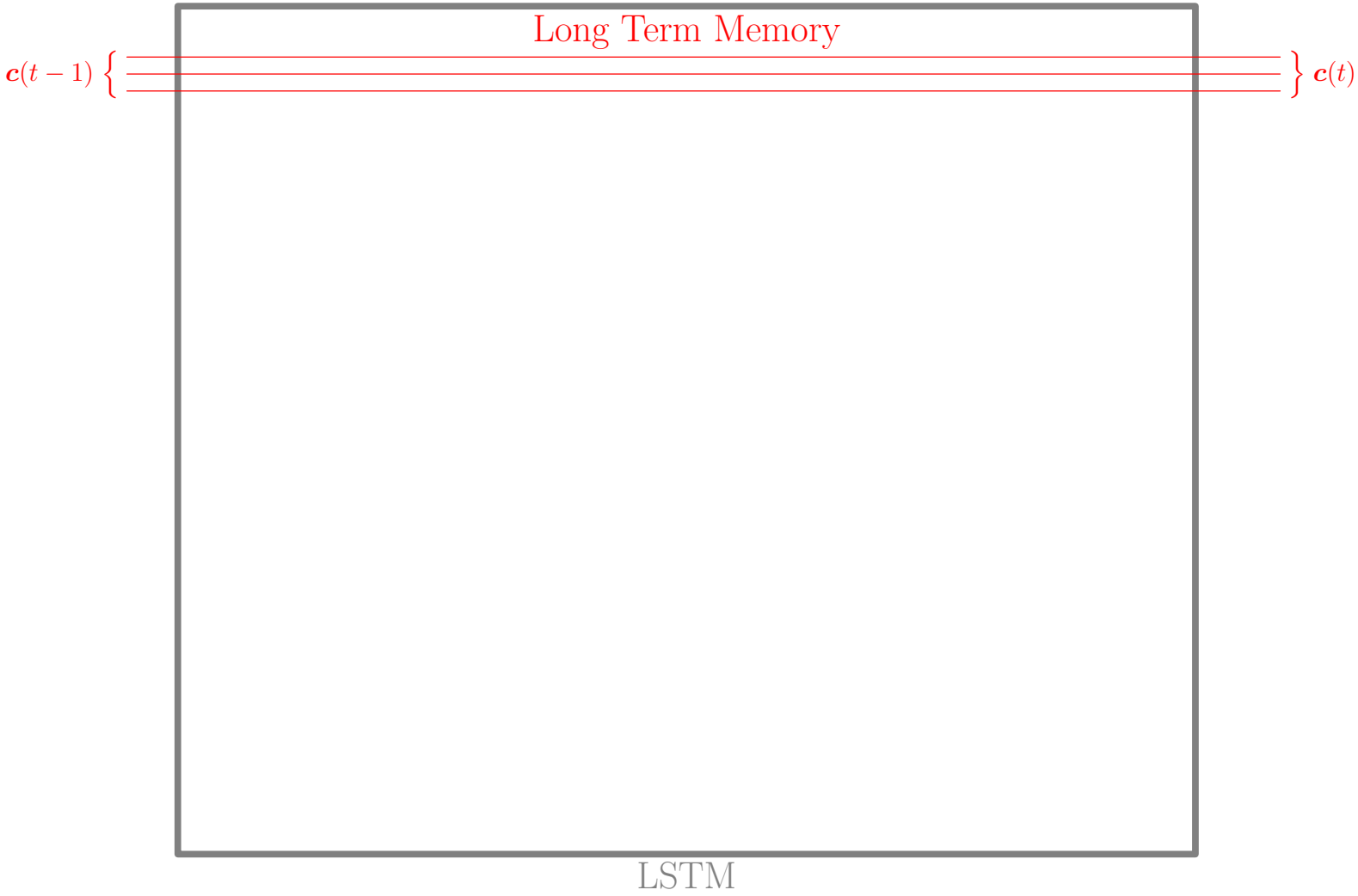

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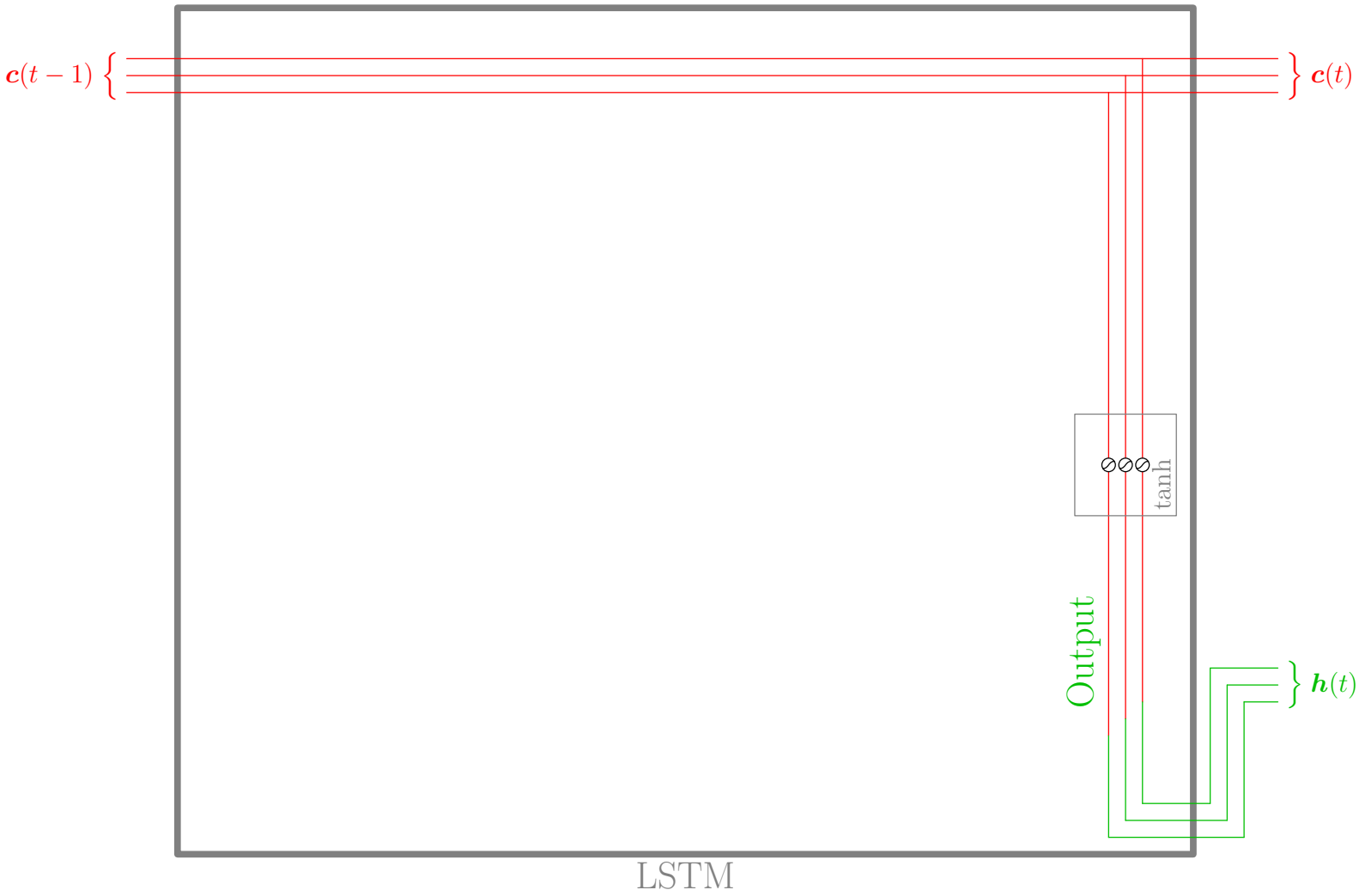

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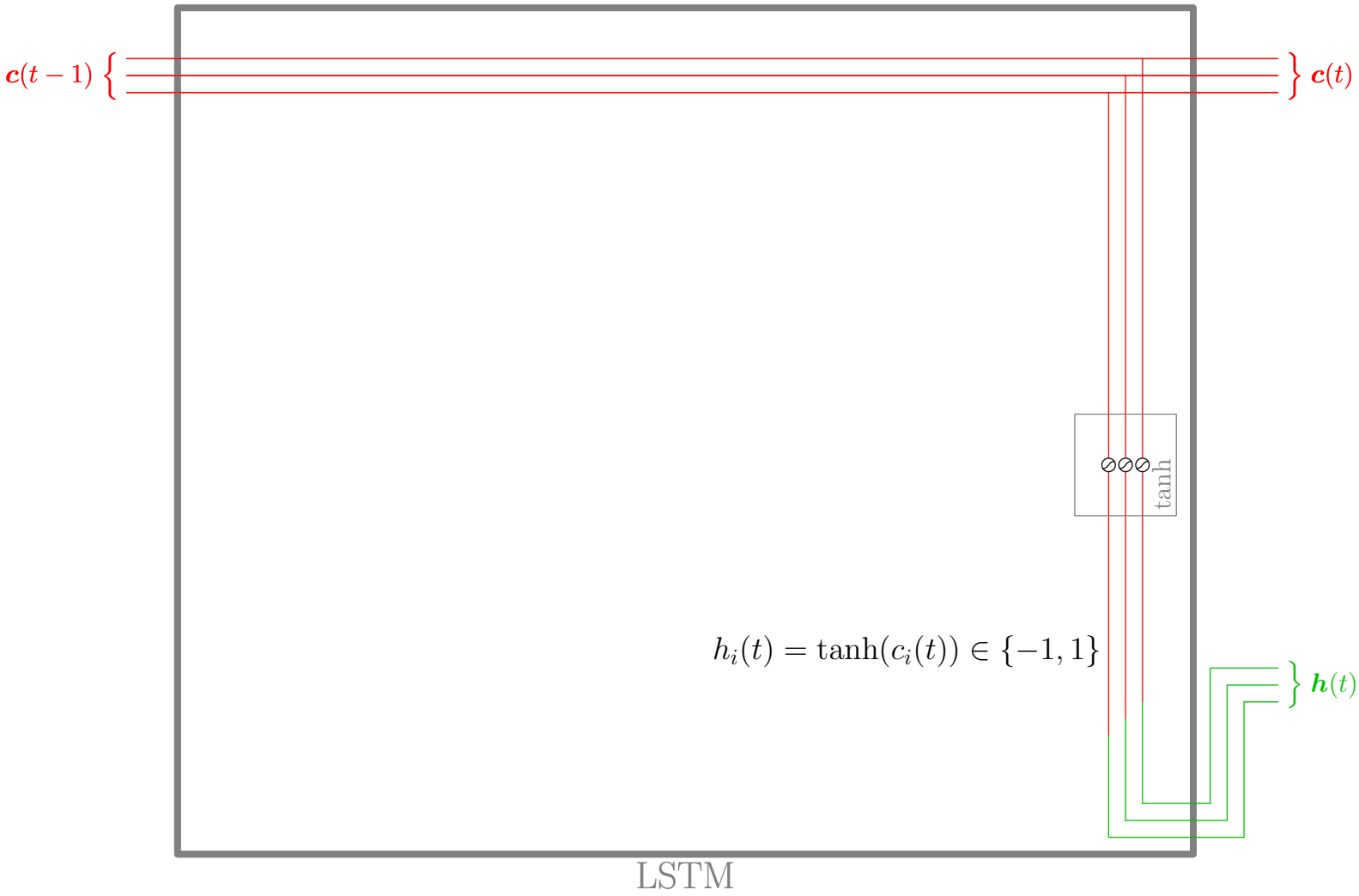

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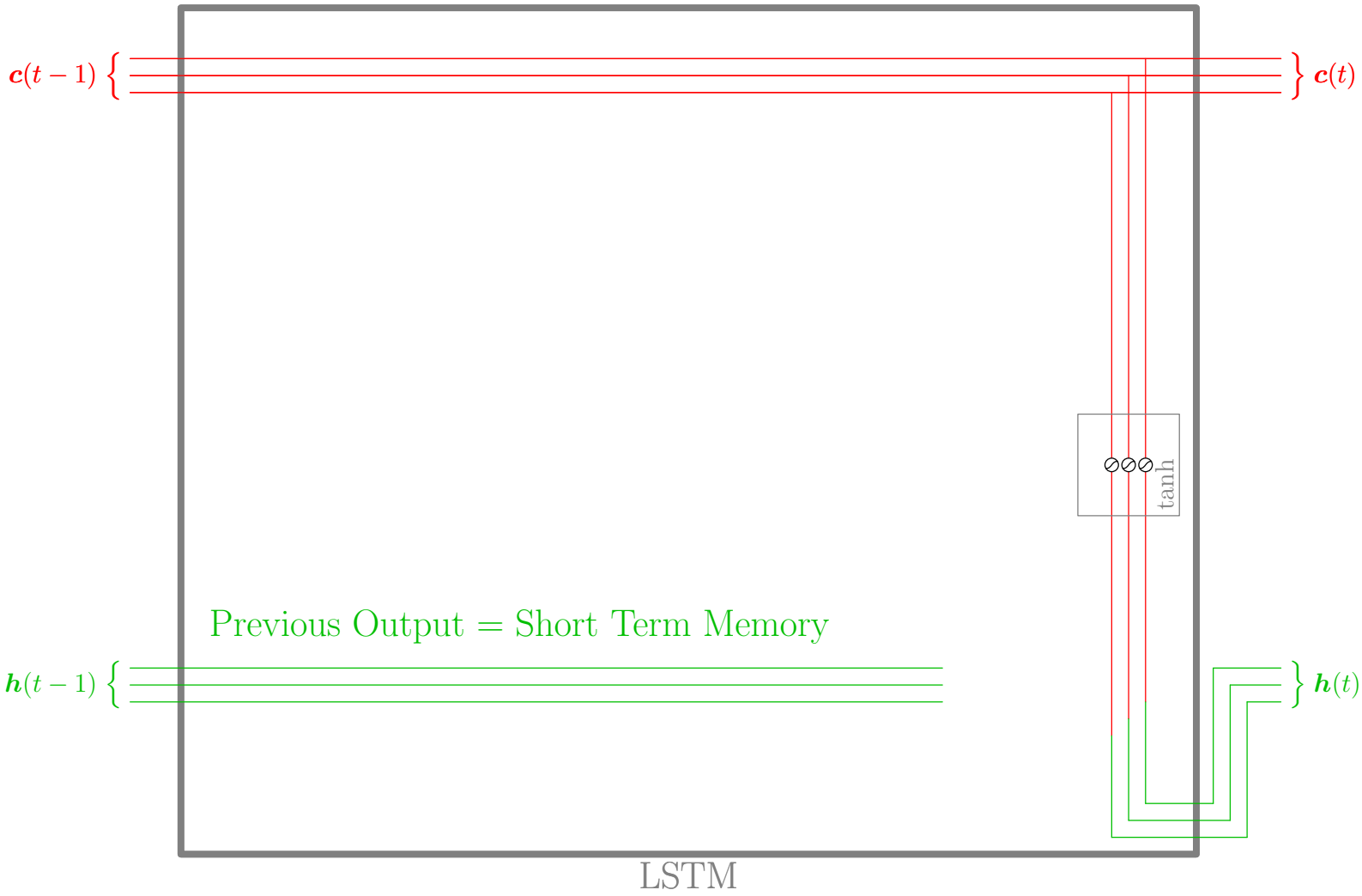

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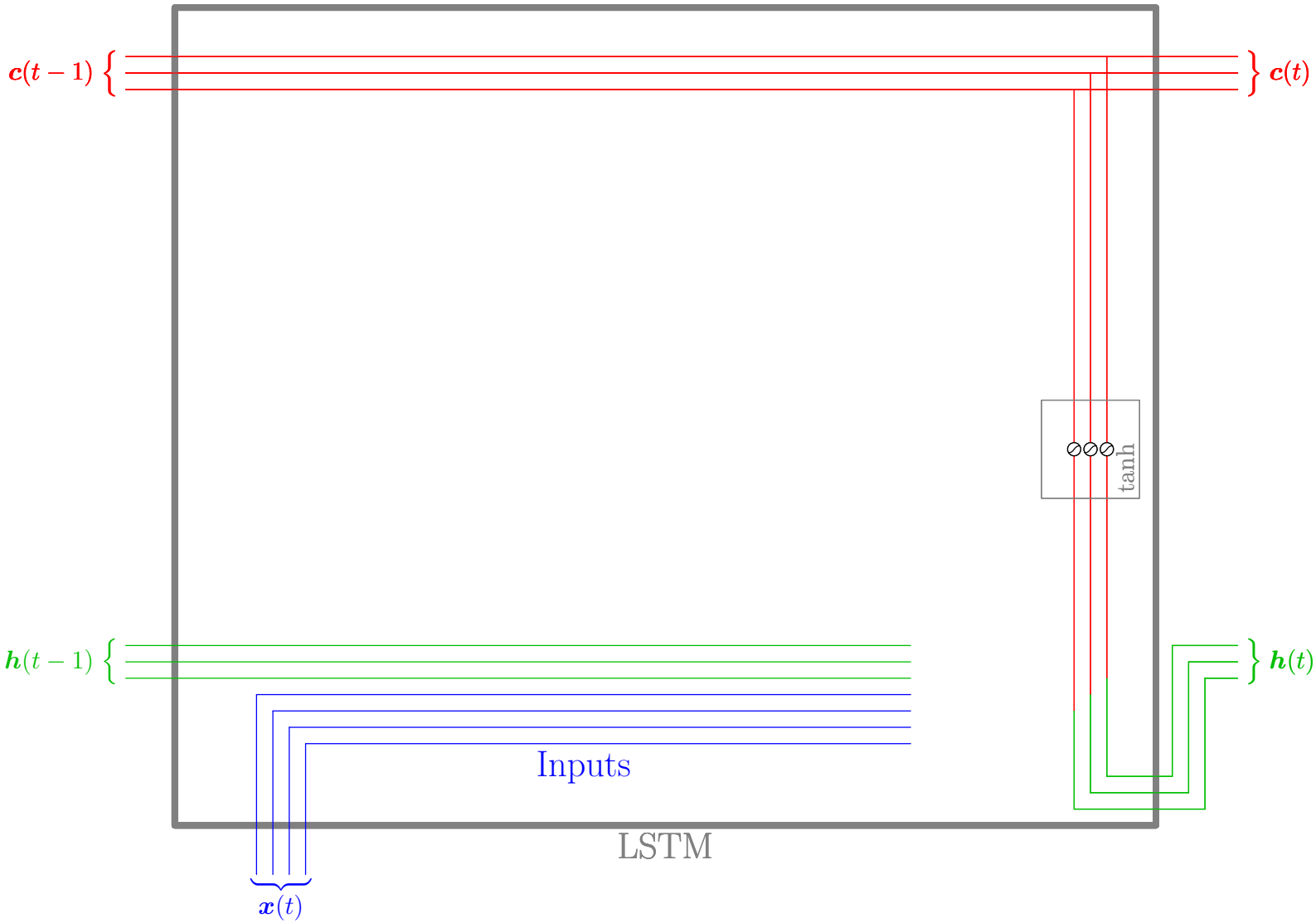

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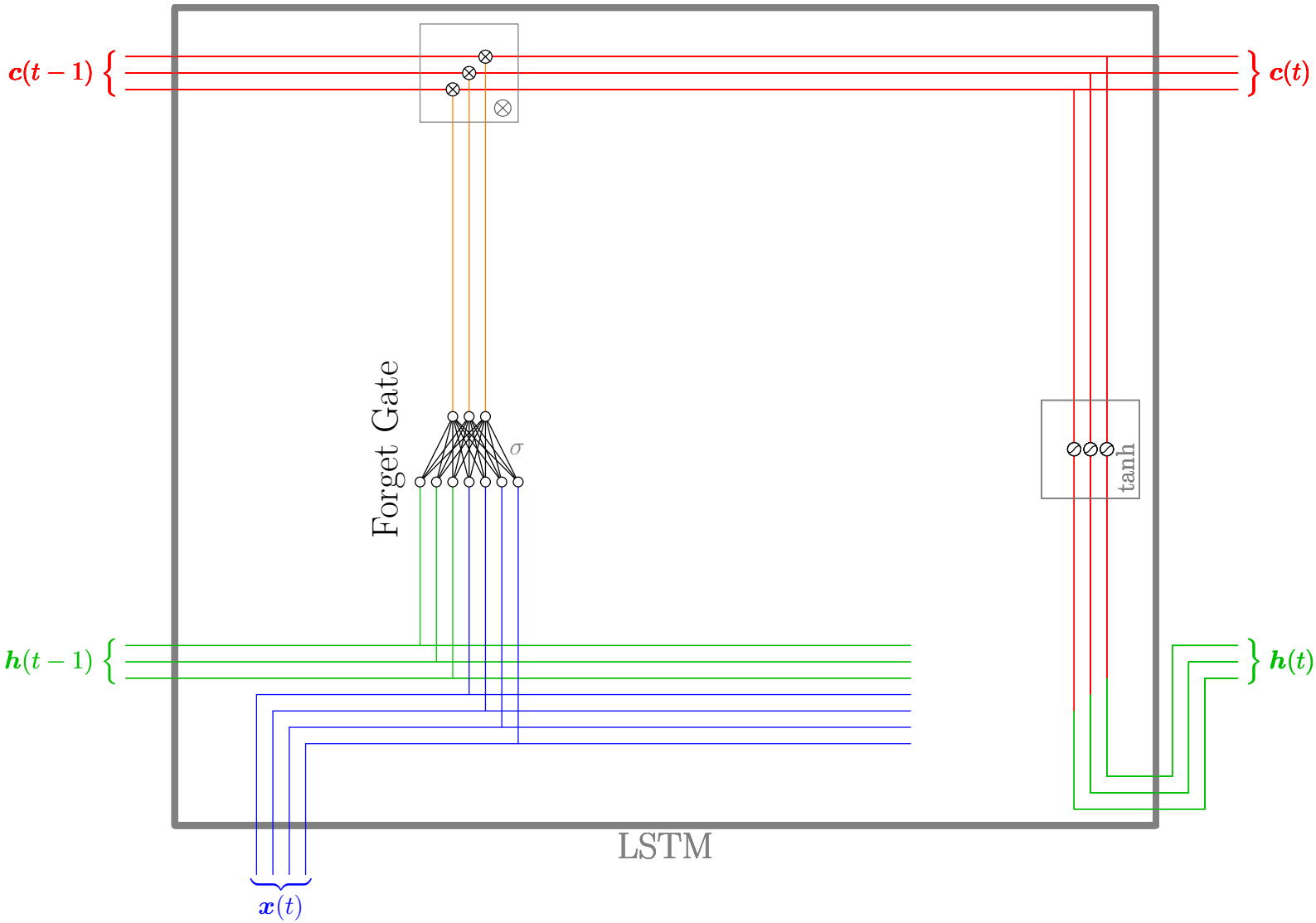

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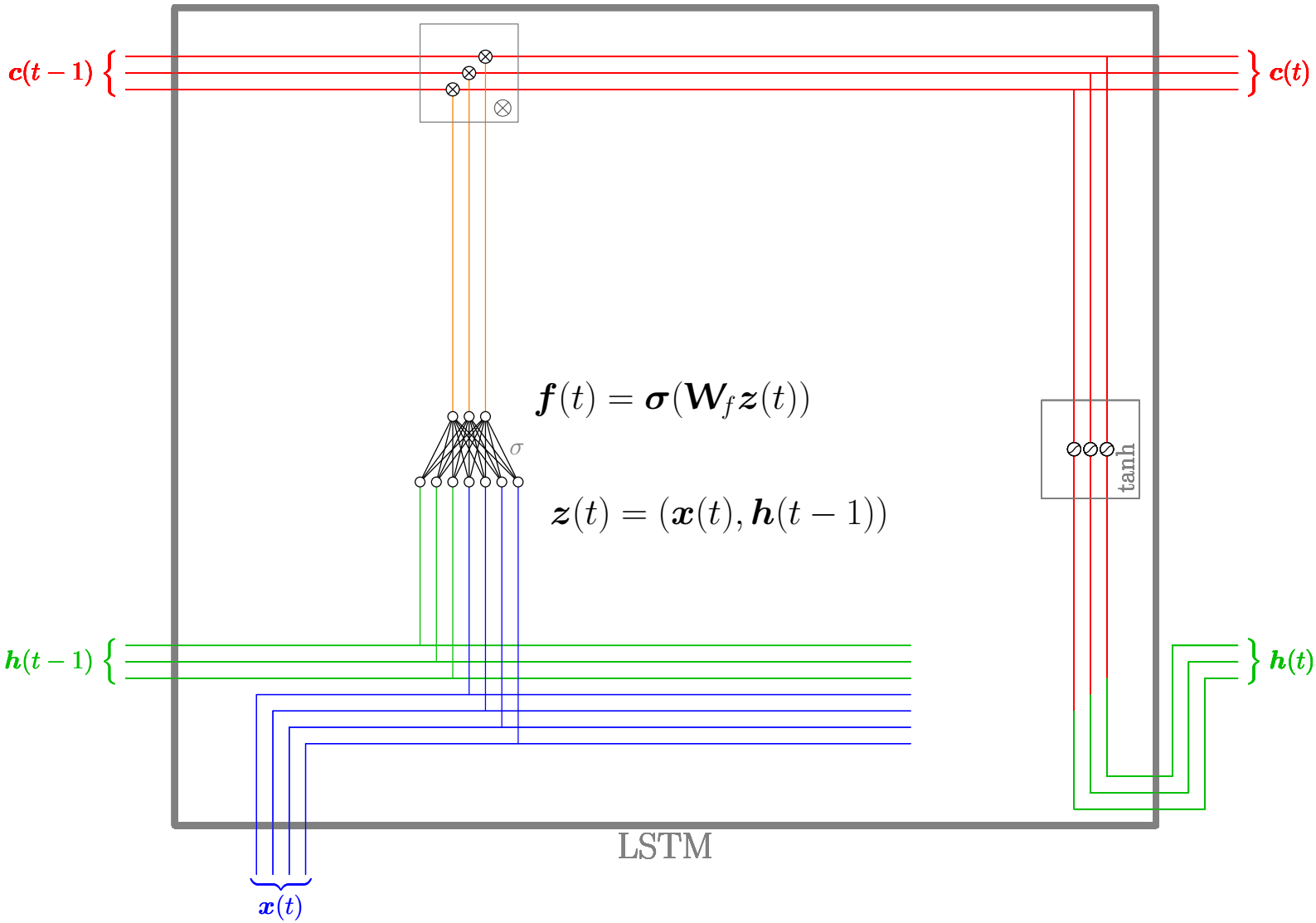

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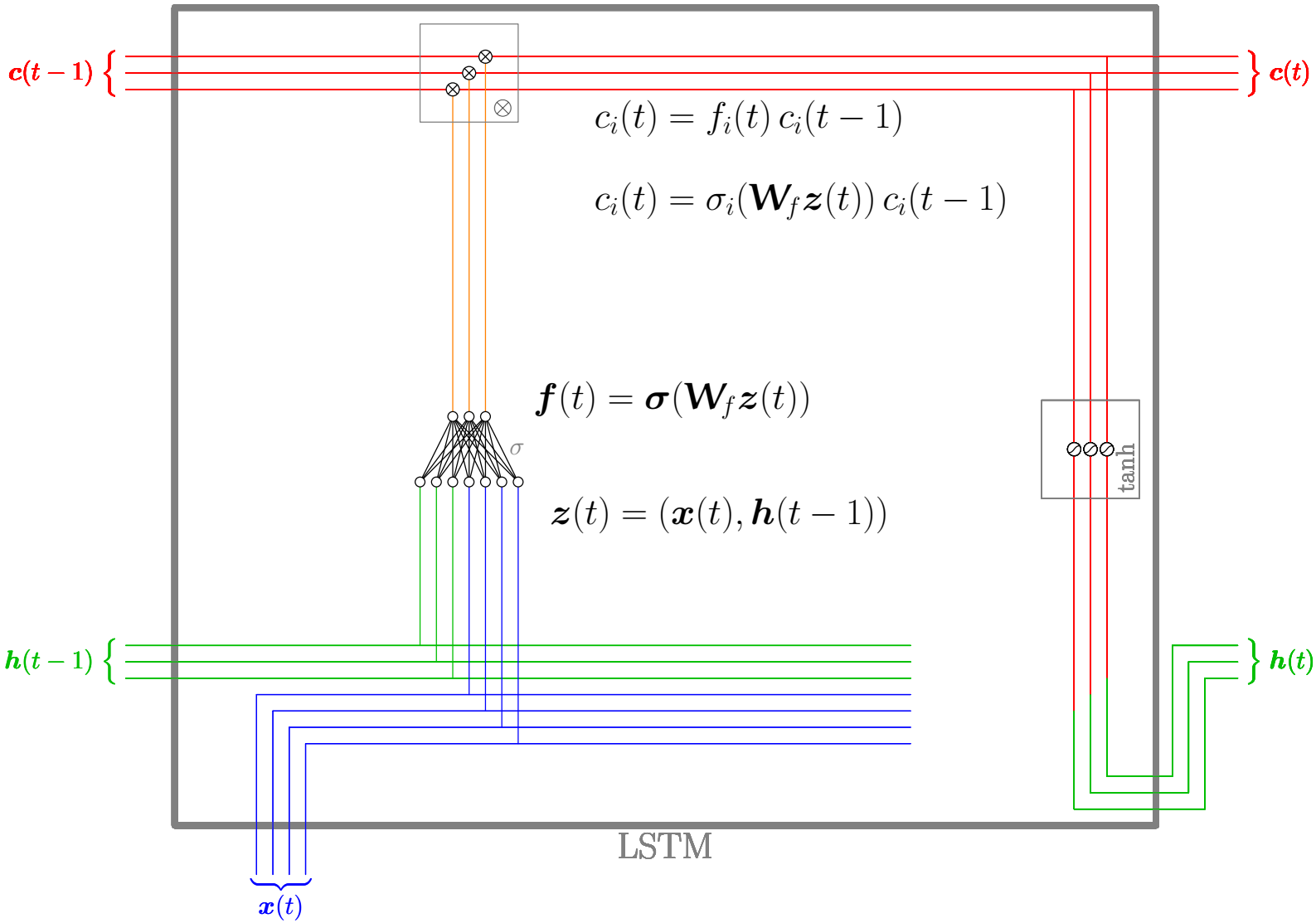

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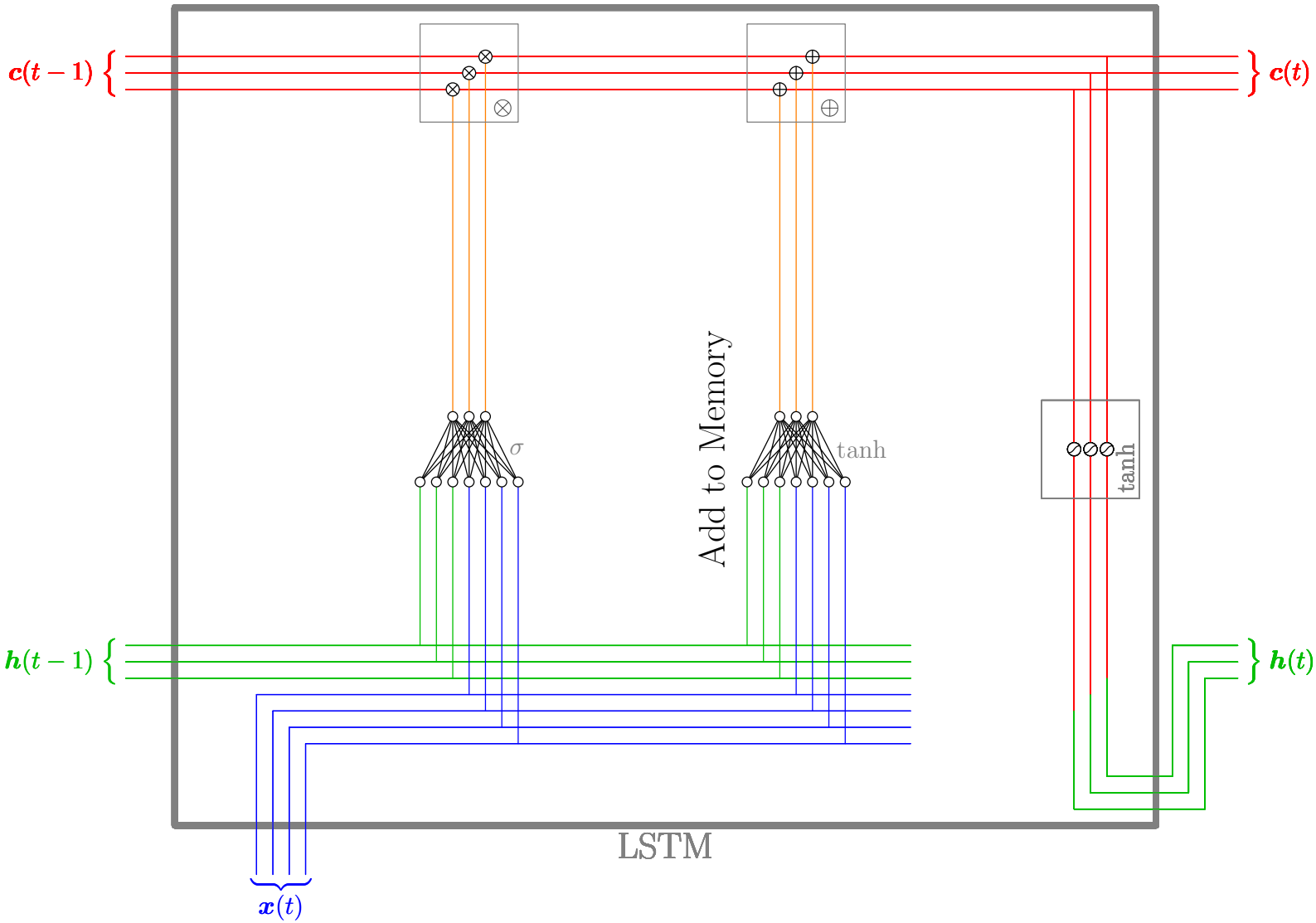

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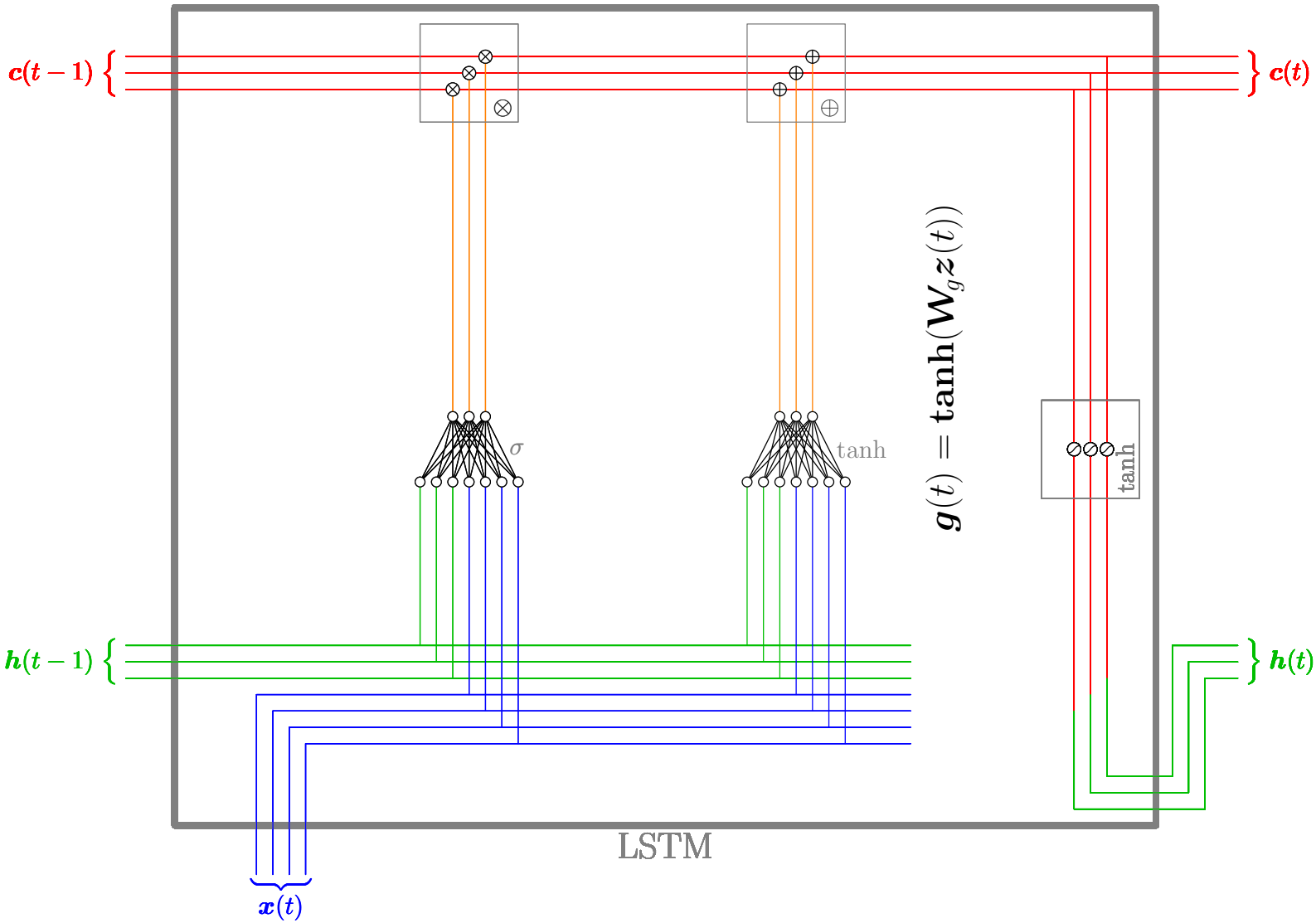

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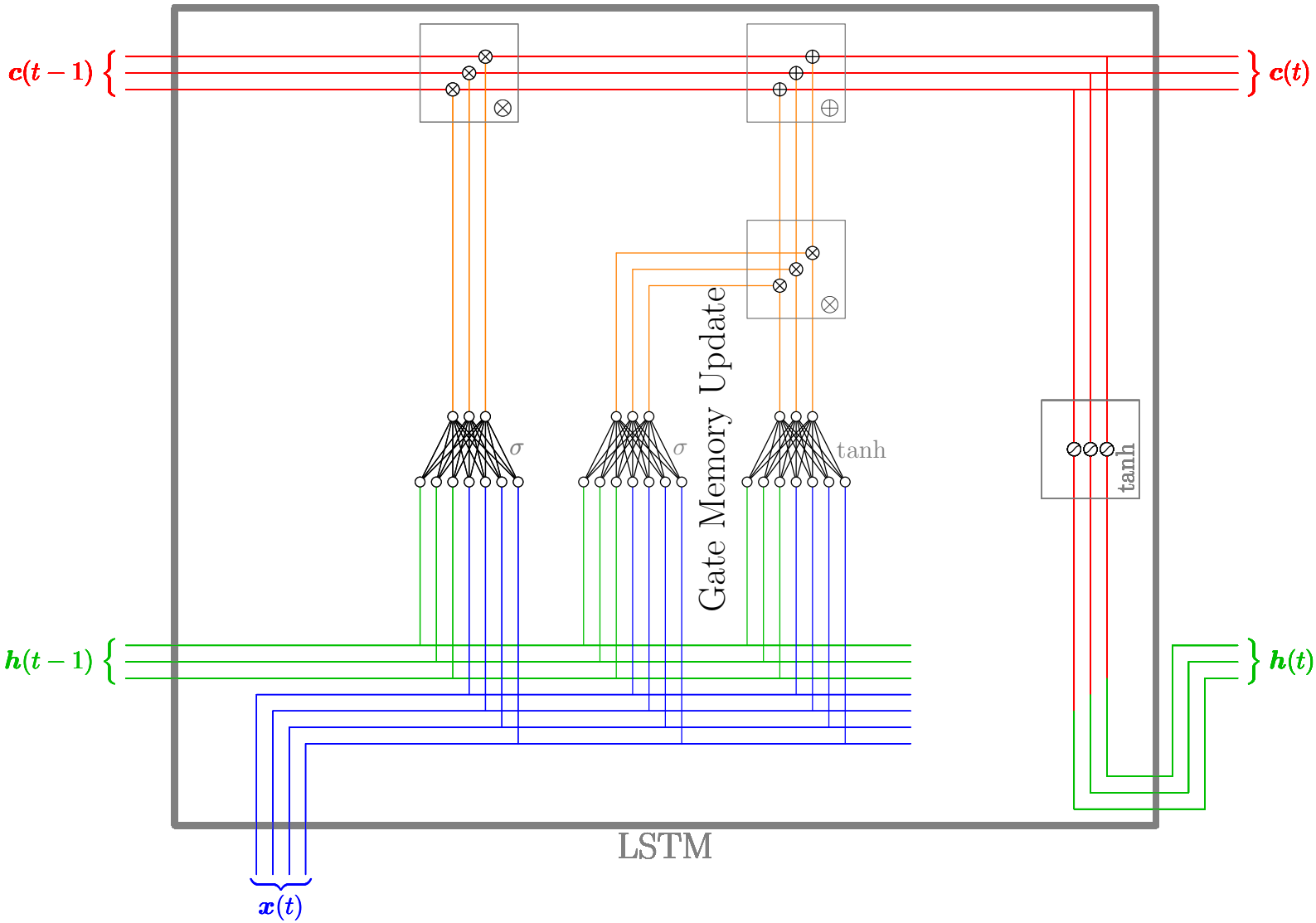

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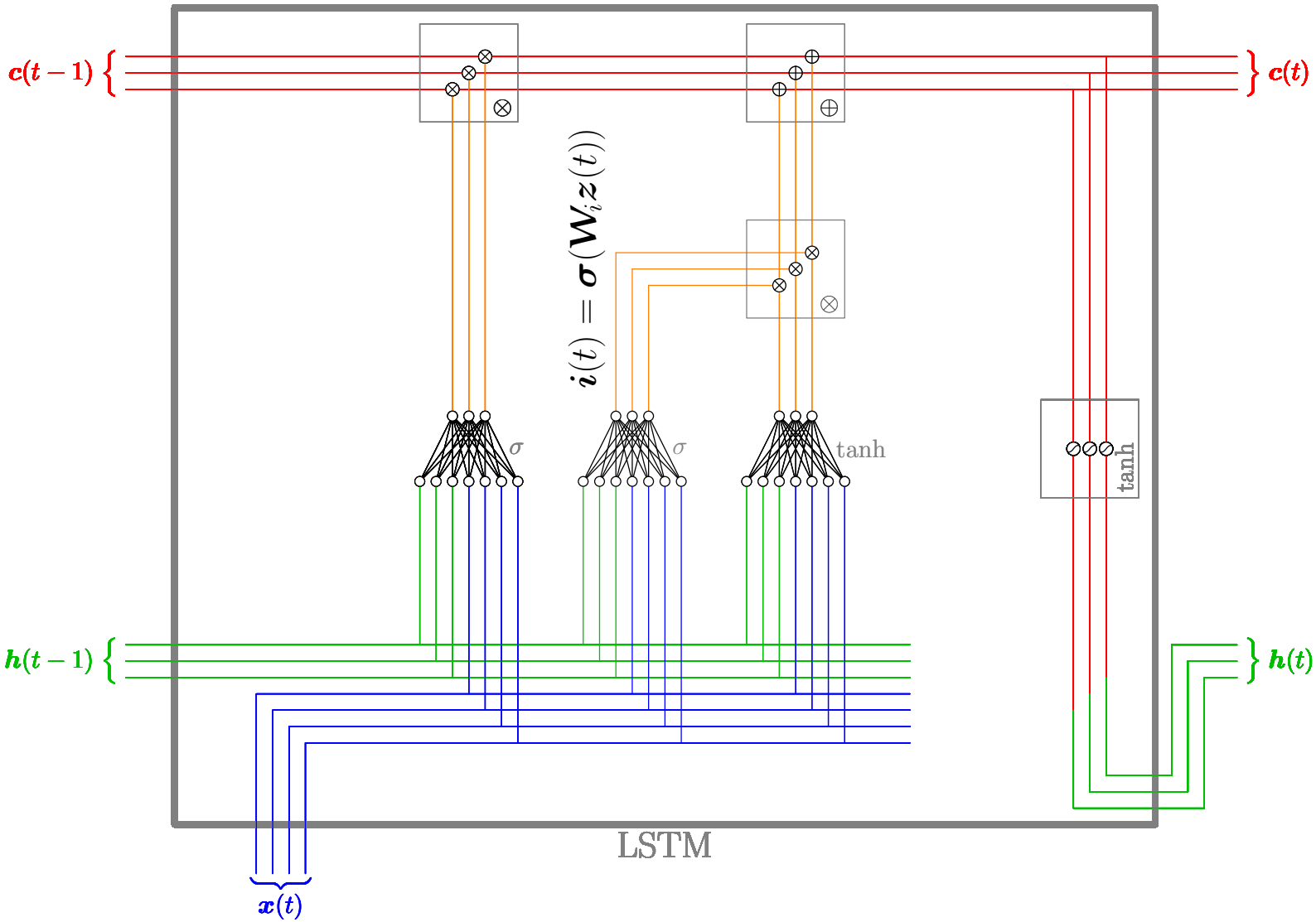

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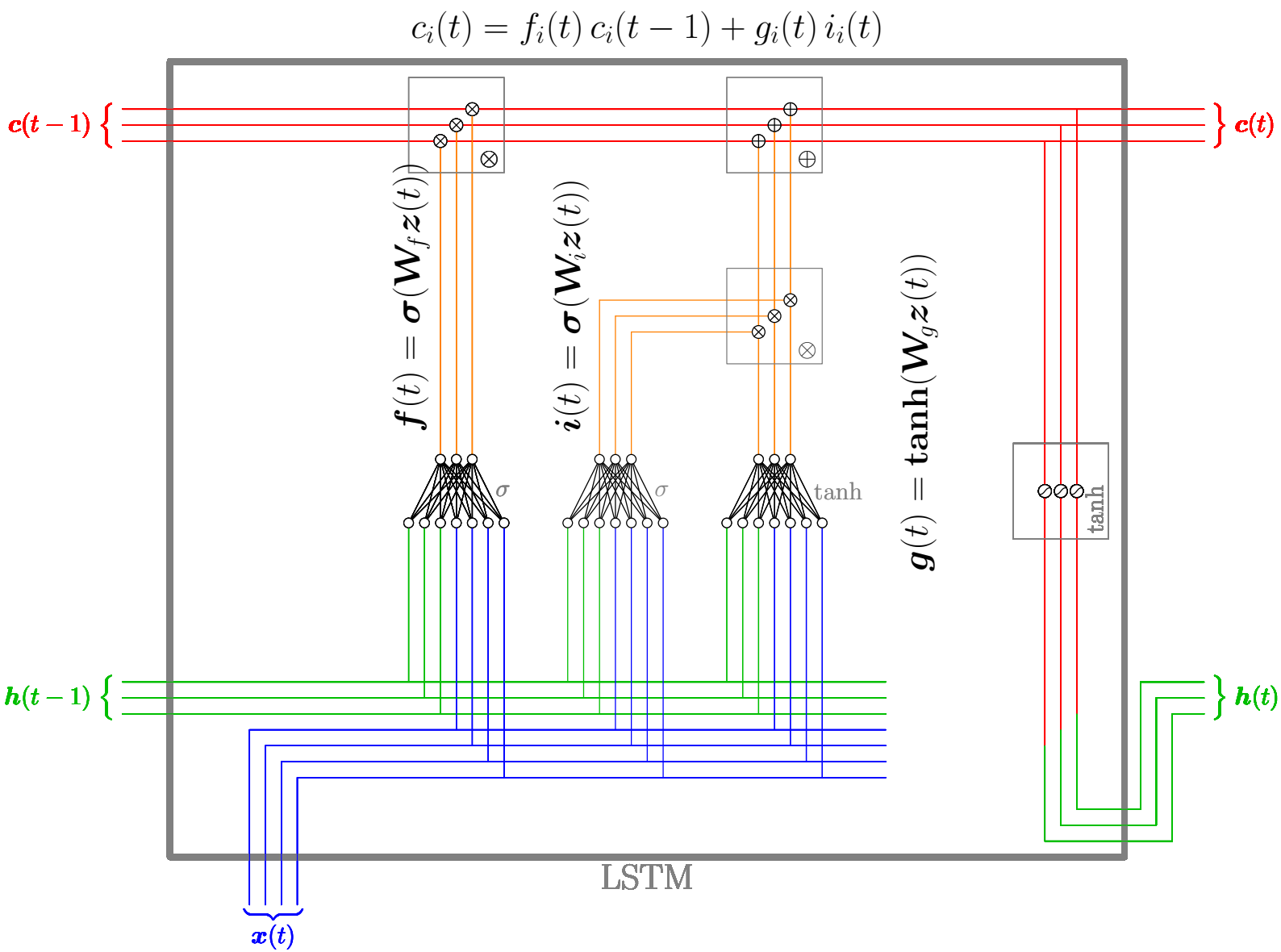

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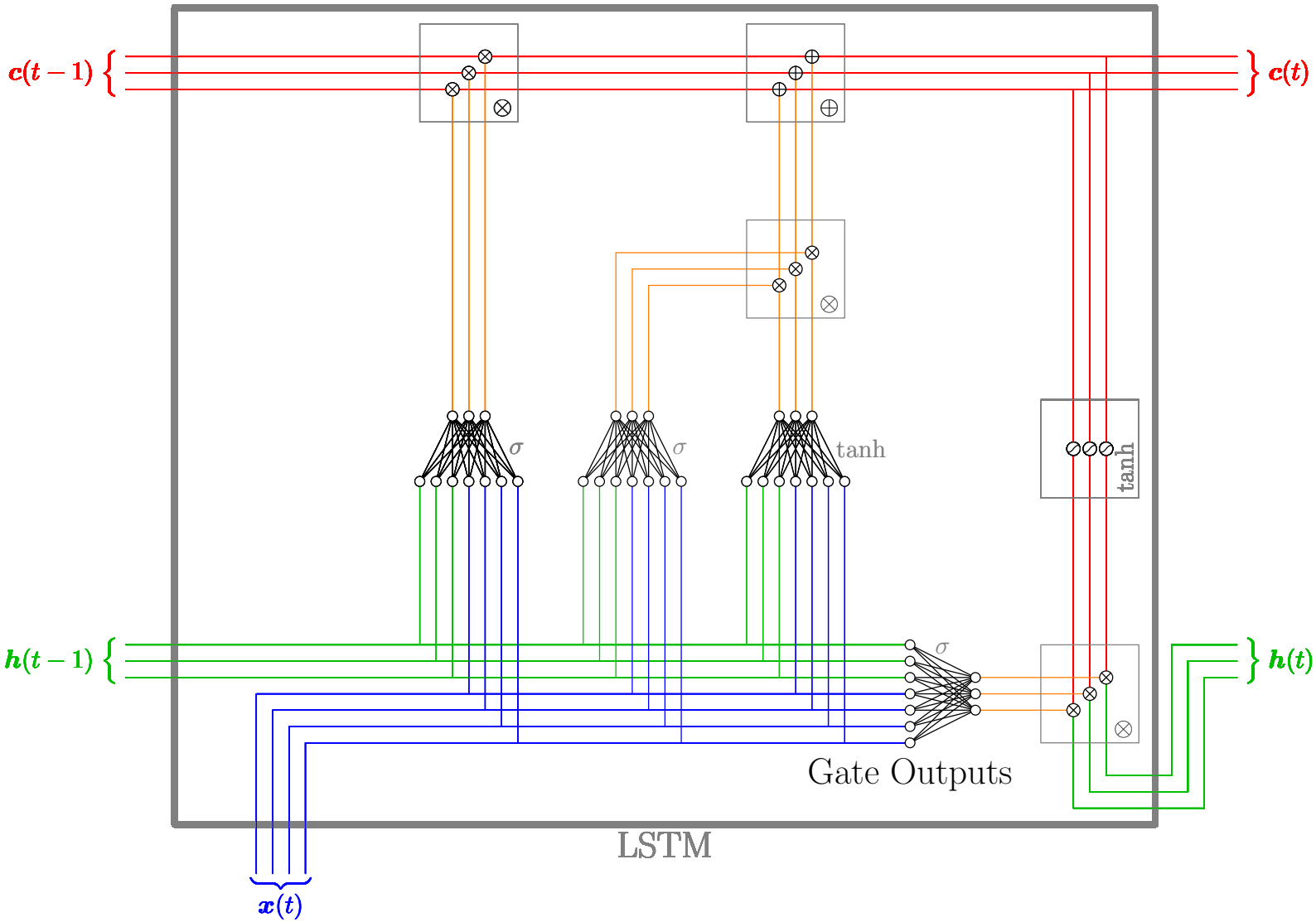

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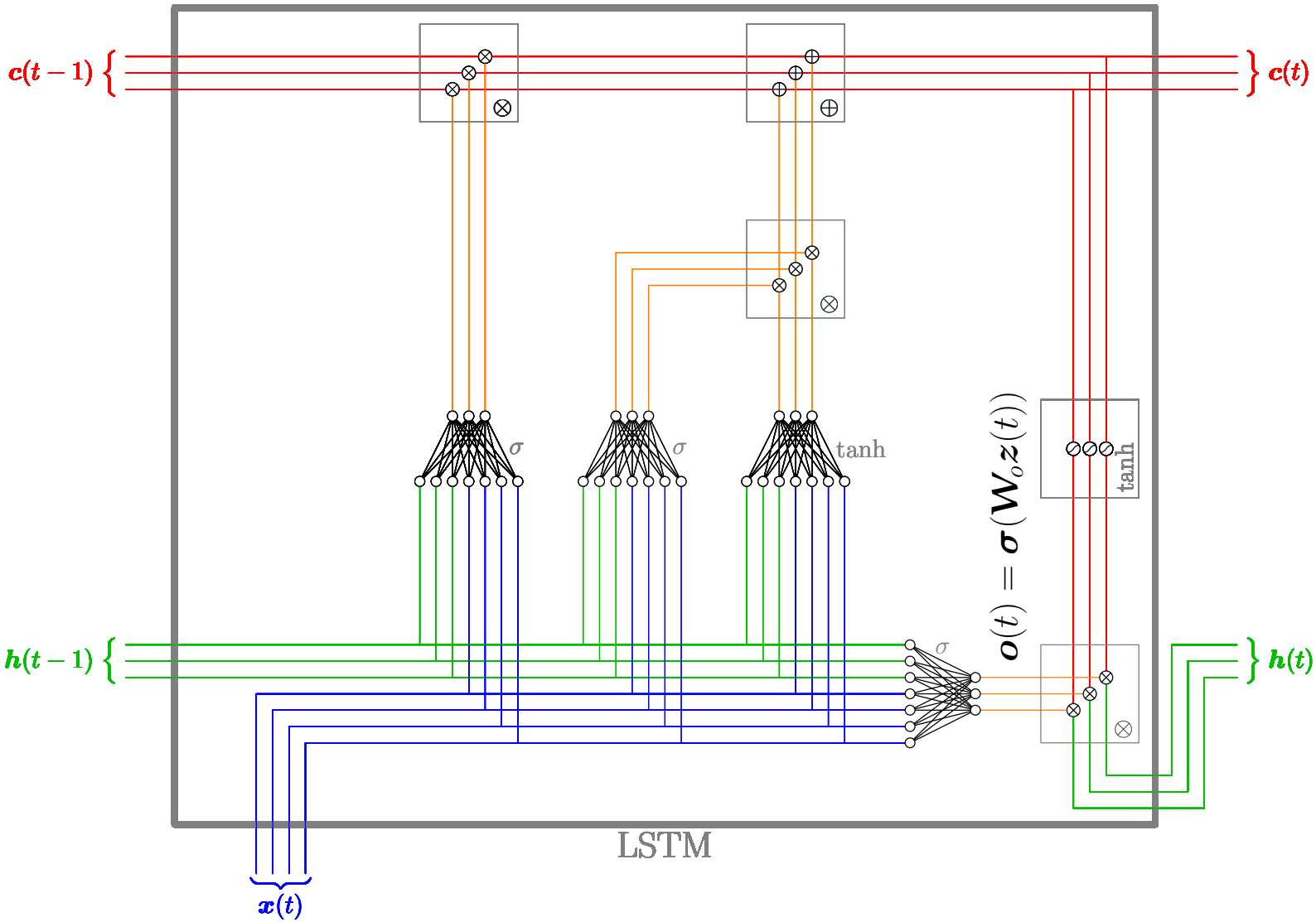

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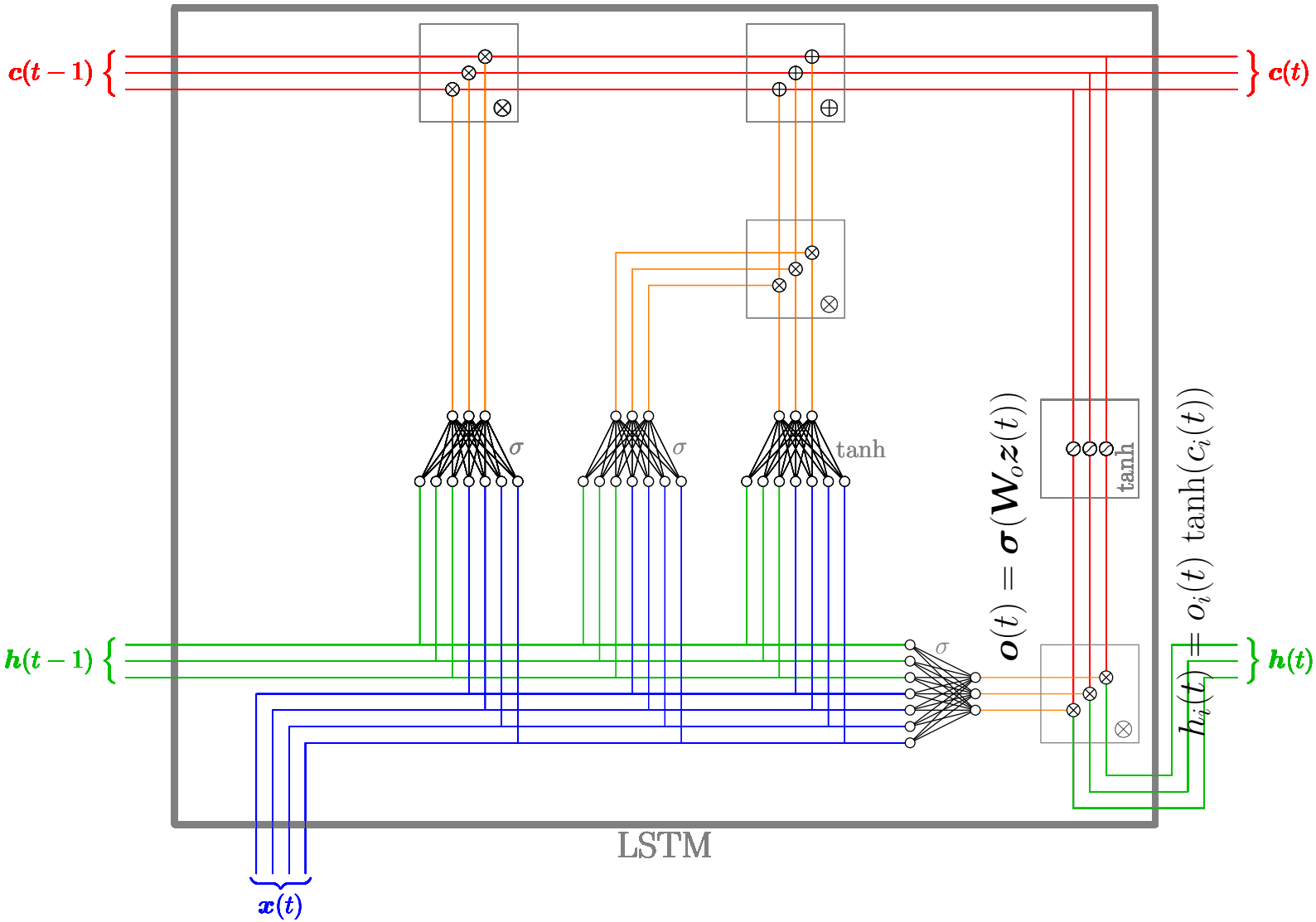

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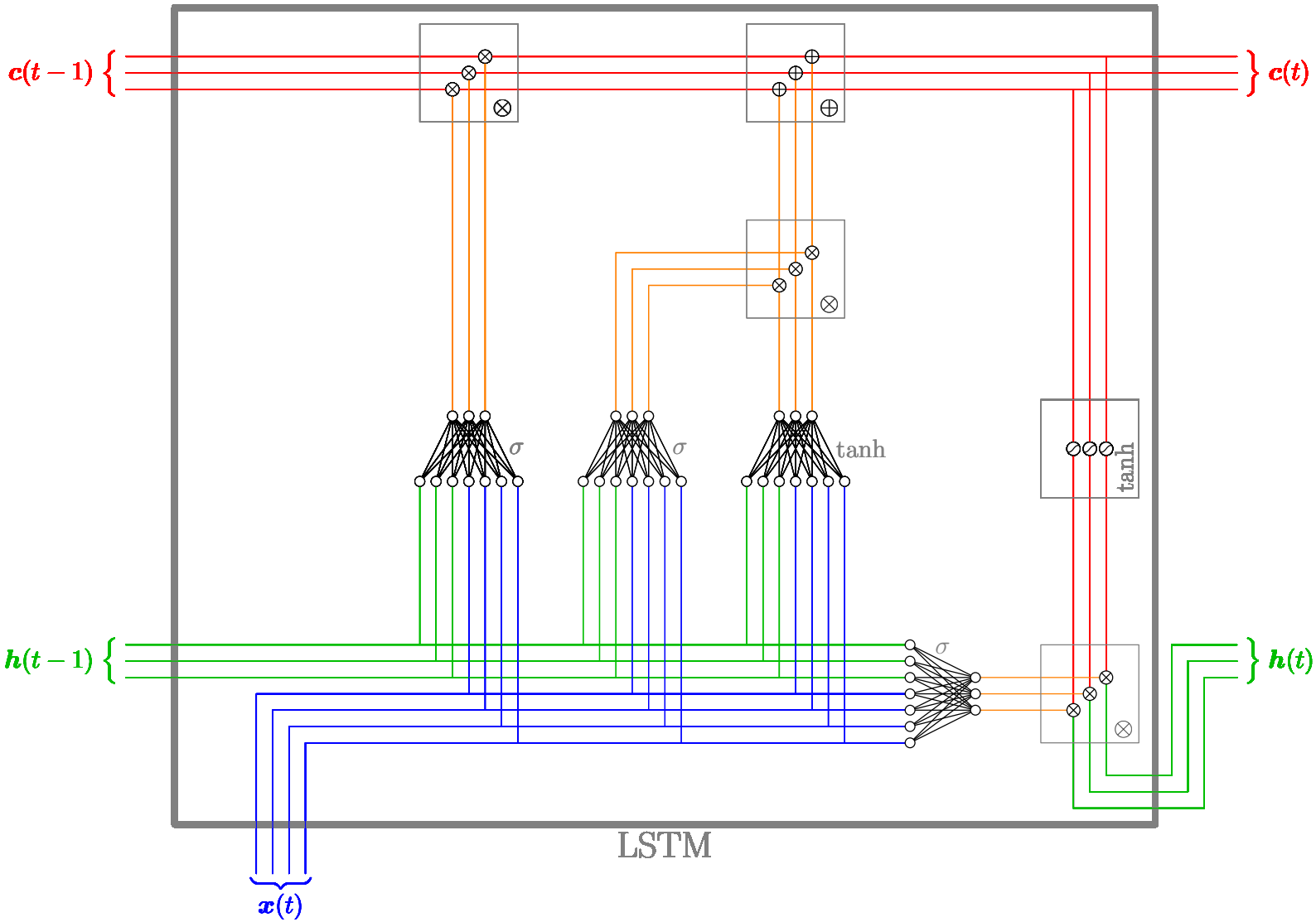

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Training LSTM

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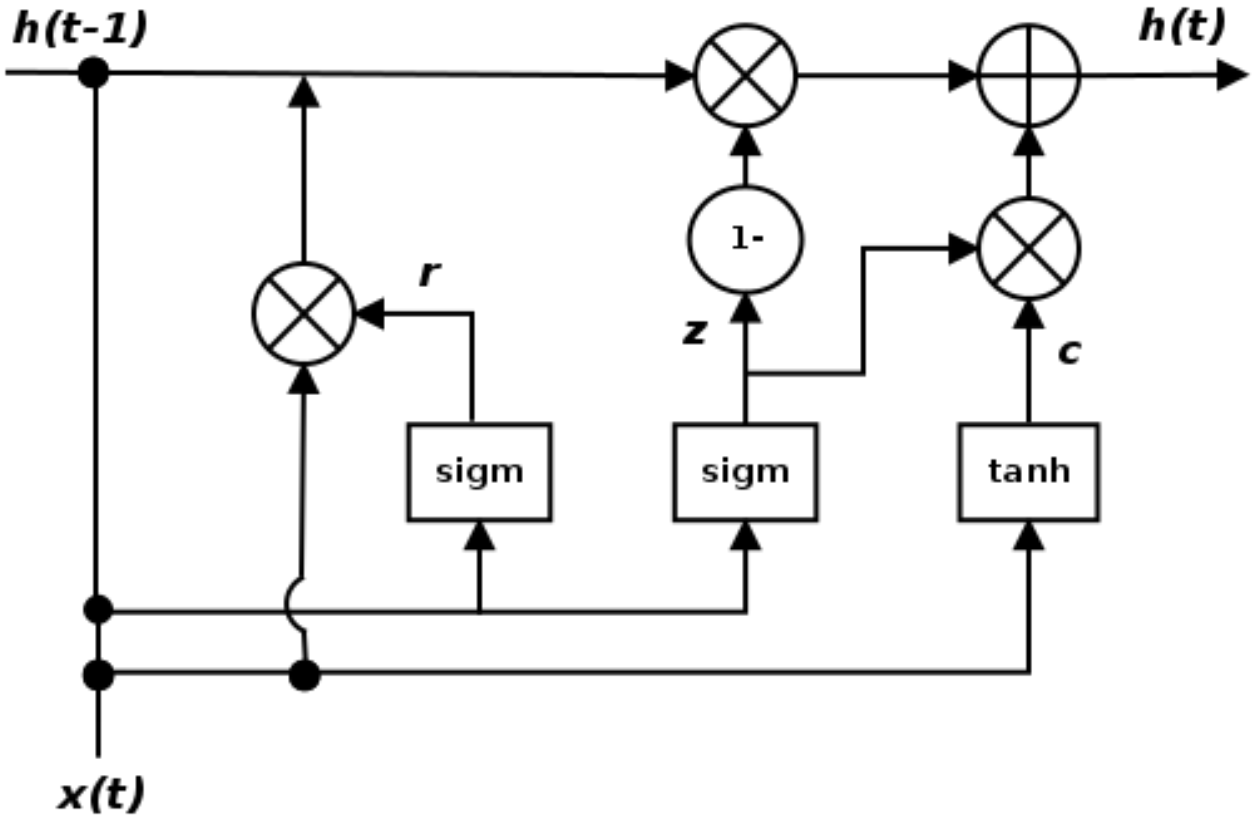
- We can train an LSTM by unwrapping it in time
- Note that it involves four dense layers with sigmoidal (or tanh) outputs
- This means that typically it is very slow to train
- There are a few variants of LSTMs, but all are very similar. The most popular is probably Gated Recurrent Unit (GRU)

LSTM Success Stories

- LSTMs have been used to win many competitions in speech and handwriting recognition
- Major technology companies including Google, Apple, and Microsoft are using LSTMs as fundamental components in new products.
- Google used LSTM for speech recognition on the smartphone, for Google Translate.
- Apple uses LSTM for the "Quicktype" function on the iPhone and for Siri.
- Amazon uses LSTM for Amazon Alexa.
- In 2017, Facebook performed some 4.5 billion automatic translations every day using long short-term memory networks [1]

[1] https://en.wikipedia.org/wiki/Long_short-term_memory

Gated Recurrent Unit (GRU)



Gated Recurrent Unit (GRU)

- x_t: input vector
- h_t: output vector
- z_t: update gate vector
- r_t: reset gate vector
- W, U, and b: parameter matrices and vector
- *sigm* or σ_g is the sigmoid function
- *tanh* or σ_h is the hyperbolic tangent

Gated Recurrent Unit (GRU)

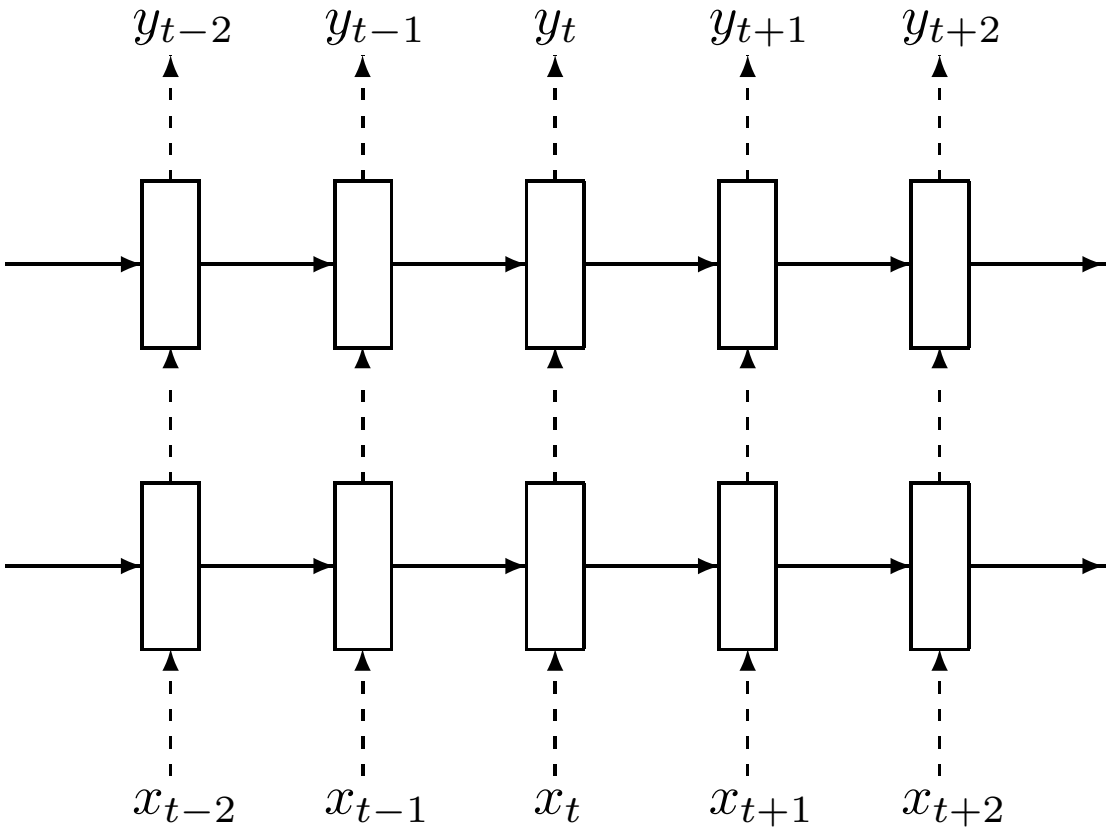
Initially, for $t = 0$, $h_0 = 0$

$$
\begin{aligned}
z_t &= \sigma_g(W_z x_t + U_z h_{t-1} + b_z) \\
r_t &= \sigma_g(W_r x_t + U_r h_{t-1} + b_r) \\
h_t &= (1 - z_t) \odot h_{t-1} + z_t \odot \sigma_h(W_h x_t + U_h(r_t \odot h_{t-1}) + b_h)
\end{aligned}
$$

GRU vs. LSTM?

- GRUs have two gates (reset and update) whereas LSTM has three gates (input/output/forget)
- GRU performance on par with LSTM but computationally more efficient (less complex).
- In general, if you have a very large dataset then LSTMs will likely perform better.
- GRUs are a good choice for smaller datasets.

Regularization in RNNs with LSTM units



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[2] https://arxiv.org/pdf/1409.2329.pdf

Regularization in RNNs with LSTM units

$$\text{LSTM} : h_t^{l-1}, h_{t-1}^l, c_{t-1}^l \rightarrow h_t^l, c_t^l$$

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \text{sigm} \\ \text{sigm} \\ \text{sigm} \\ \tanh \end{pmatrix} T_{2n,4n} \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$

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3

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