Assigned: 24 March 2019

Project #6 – Continuous sampling

EE 511: Fall 2019

Due: Thursday, 31 October 2019 at 16:00. Late penalty: 15% per day before 02 November at 16:00.

1. Generate 1000 samples of the random variable $A = X + Y$ where $X \sim N(1,4)$ and $Y \sim N(2,9)$. Use the Box-Muller method to generate independent random samples from the component normal distributions. Use every sample generated by the Box Muller method (do not throw one of the pair away). Estimate the covariance between X and Y in your 1000 samples. Generate a histogram for A. Overlay the theoretical p.d.f. the histogram. Calculate the sample mean and sample variance for your samples and compare you estimates with the theoretical values.

   The Polar Marsaglia method is an alternate method to generate samples from normal random variables. The method works by choosing a random point in the square $-1 < x < 1$ and $-1 < y < 1$ until $s = x^2 + y^2 < 1$ and then returning $R_1 = x\sqrt{\frac{-2\ln s}{s}}$ and $R_2 = y\sqrt{\frac{-2\ln s}{s}}$. Simulate 1,000,000 pairs of independent samples from a standard normal random variable using the Polar Marsaglia method. Compute the sample mean, sample variance, and covariance between the paired random samples. Repeat the experiment many times and compare the computational time required to generate 1,000,000 pairs of independent samples using the Polar Marsaglia method and the Box-Muller method.

2. Consider sampling from a $Gamma(\theta, 1)$ random variable. If θ is an integer then you can perform the sampling by summing θ different $Exp(1)$ random variables. But if θ is not an integer then it's more difficult. Generate 1000 samples from $Gamma(5.5,1)$ using an accept-reject method (i.e. do not use built-in functions to generate from the gamma distribution). Generate a histogram and overlay the theoretical p.d.f. the histogram. Comment on the acceptance rate and your overall fit.

3. Thick-tailed alpha-stable pdfs have found many applications in physics and engineering where thicker tails can model energetic or impulsive processes. Alpha-stable pdfs are bell curves whose tails get thicker as the parameter α gets smaller for $\alpha \in (0,2]$. The Gaussian pdf has the thinnest tails and corresponds to $\alpha = 2$. There are two main problems with using stable pdfs in physical models. The first is that only a few stable pdfs have a known closed form. These special cases include the symmetric alpha-stable pdfs of the Gaussian ($\alpha = 2$) and the Cauchy or Lorentzian ($\alpha = 1$) as well as the asymmetric Levy ($\alpha = 0.5$). An alpha-stable pdf f has characteristic function

$$\varphi(\omega) = \exp\{ic\omega - \gamma|\omega|^\alpha(1 + i\beta \operatorname{sgn} \omega)\Gamma\}$$

   where

$$\Gamma = \begin{cases} \tan\dfrac{\alpha\pi}{2}, & \alpha \neq 1 \\ -\dfrac{2}{\pi}\ln|\omega|, & \alpha = 1 \end{cases}$$

$i = \sqrt{-1}$, $0 < \alpha \leq 2$, $-1 \leq \beta \leq 1$ and $\gamma > 0$. The parameter α is the characteristic exponent. It is a measure of the tail thickness for symmetric bell curves. The α controls the tail thickness of the resulting bell curve because the bell curve has thicker tails as α falls. The location parameter c is the median of a symmetric stable density. β is a skewness parameter. The density is symmetric about c if $\beta = 0$. The dispersion parameter γ acts like a variance because it controls the width of an alpha-stable bell curve even though such densities have no variance except in the Gaussian case of $\alpha = 2$.

The Chambers-Mallows-Stuck method describes how to generate samples from an arbitrary alpha stable distribution [1]. Use the Proposition 2.1 and Theorem 3.1 from the formulation by Weron [2] to sample from symmetric ($\beta = 0$) alpha stable pdfs using for four different values of α ($\alpha = 0.5$, 1, 1.8, 2.0).

For each value of α produce a histogram and a time series plot. Comment on the sample magnitude as a function of α. Use the Python function `scipy.stats.levy_stable` to overlay the corresponding theoretical alpha-stable pdf on your histogram. Repeat the above procedure assuming a right-skewed alpha stable distribution with $\beta = 0.75$.

You can obtain a copy of the original papers on Canvas under Files:

[1] Chambers, John M., Colin L. Mallows, and B. W. Stuck. "A method for simulating stable random variables." Journal of the american statistical association 71.354 (1976): 340-344.

[2] Weron, Rafał. "On the Chambers-Mallows-Stuck method for simulating skewed stable random variables." Statistics & probability letters 28.2 (1996): 165-171.