Review Exercises for Midterm Exam

The following exercises should help your preparations for the midterm exam, but do not necessarily guarantee you will pass it. Since answers to them are straightforward or can be found in your class notes and/or the textbook, solutions will not be posted here.

1. All questions in Assignment #1 and #2.

Turing Machines and RAMs

2. A DTM is specified by a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$. Describe each of the 7 components.

3.

a. Exercise 3.1 in the book
b. Exercise 3.5 in the book
c. Exercise 3.8 (a) in the book

4. Consider the DTM to decide $\{ 0^k 1^k \mid k \geq 0 \}$ shown in Question 2 here. Give the transition sequence on each of the following input strings:

a. 01
b. 010
c. 011

5. Define a decider DTM.

6. Illustrate the idea used for a simulation of multitape DTMs by 1-tape DTMs.

7.

a. Give a precise definition of transition functions of NTMs. How do they differ from the transition functions of DTMs?
b. Give a precise definition of acceptance and rejection of the input string by an NTM.

8. Illustrate the idea used for a simulation of NTMs by DTMs. Explain why a depth-first search of the computation tree of an NTM does not work.

9. Consider the NTM to decide $\{ wab^i : i \geq 0 \} \cup \{ wac^j : j \geq 0 \}$ shown in Question 3 here. Give the computation tree of configurations of this NTM on each of the following input strings:

a. a
b. acacc
c. bcabc

10. This question concerns the simulation of RAMs by 5-tape DTMs described in class. Describe how to simulate each of the following RAM instructions:

- ADD =20
- MULT 20
- LOAD 20

11. Consider a 1-tape DTM with $\Gamma = \{ a_1, \ldots , a_m, \_ \}$, $Q = \{ q_1, \ldots , q_n \}$ where "_" denotes the blank symbol and q_1 is the start state. Describe how this DTM's configurations are simulated by a RAM. Also, illustrate the idea of how its transition function is simulated.

Time Complexity, Verification TMs, P, NP, NP-Complete

12.

a. Exercise 7.1 in the book
b. Exercise 7.2 in the book
c. Give a precise definition of the worst-case time complexity functions of DTMs that halt on all input strings.
d. Give a precise definition of the worst-case time complexity functions of NTMs that halt on all input strings.

13. Explain why any 1-tape DTM or 1-tape NTM, M, with the worst-case time complexity function $W_M(n)$ never reads more than $W_M(n)+1$ tape cells and never writes on more than $W_M(n)$ tape cells.

14.

a. Give a definition of verification TMs.
b. Give a definition of: input string w is verified by a certificate c.
c. Explain why the length of a proposed certificate c for a verification TM, V, can be limited by $|c| \leq W_V(n)+1$ where $n = |w|$.

15. Consider Theorem VtoN. Define an NTM that simulates a given verifier V.

16. Consider Theorem NtoV.

a. The verifier V uses an encoding of the computation branches of a given NTM. Describe this encoding method.
b. Define a verifier V that simulates a given NTM.

17. Give a precise definition of each of the following:

a. the class of languages P
b. the class of languages NP
c. mapping reduction $A \leq_m B$
d. polynomial-time reduction $A \leq_p B$
e. the class of languages NPC

18. Show that all of the following problems are in NP:

a. HAMPATH
b. SUBSET-SUM
c. SAT
d. CLIQUE
e. VERTEX-COVER described on page 312 (page 284 in 2nd edition)
f. 3COLOR described in Problem 7.29 on page 325 (Problem 7.27 on page 297 in 2nd edition)
g. the final exam scheduling problem described in Problem 7.31 on page 326 (Problem 7.29 on page 297 in 2nd edition)
h. SOLITAIRE described in Problem 7.33 on page 326 (Problem 7.31 on page 298 in 2nd edition)

19. Determine if each of the following is true or false. If it is true, prove it.

a. $P \subseteq NP$.
b. $NPC \subseteq NP$.
c. $P \cap NP = \varnothing$
d. Let N be any decider NTM with $W_N(n) \geq n$ for $\forall n \in \mathbf{N}$. Then N can be simulated by a DTM, M, with $W_M(n) = 2^{O(W_N(n))}$.
e. If $A \leq_p B$ and $B \in P$, then $A \in P$.

20.

a. Prove the transitivity of $\leq_p$.
b. Prove: if $A \in NPC$, $B \in NP$, and $A \leq_p B$, then $B \in NPC$.

21. Consider the theorem that SAT is NP-complete and its proof we studied in class.

a. Informally but concisely describe the structure and role of the $n^k \times n^k$ tableaus.
b. The Boolean formula ϕ_w constructed from a string w by the reduction has four components: $\phi_w = \phi_{cell} \wedge \phi_{start} \wedge \phi_{move} \wedge \phi_{accept}$. Informally but concisely describe the property that each of the four components ensures.
c. Describe an 2×3 window used in the proof.
d. Define a legal window.
e. Describe how legal windows are used in ϕ_{move}.

22. Informally and concisely, give two reasons for the significance of NP-complete problems.