$\mathbf{R}^+$ = the set of nonnegative real numbers
$\mathbf{N} = \{ 0, 1, 2, 3, \dots \}$

Let f, g, h: $\mathbf{N} \rightarrow \mathbf{R}^+$.

- $f(n) = O(g(n))$ if $\exists c, n_0 \in \mathbf{N}$ s.t. $\forall n \geq n_0, f(n) \leq c\, g(n)$. The growth rate of f(n) is less than or equal to that of g(n).
- $f(n) = o(g(n))$ if $\lim_{n\rightarrow\infty}[f(n)/g(n)] = 0$. The growth rate of f(n) is less than that of g(n).
- $f(n) = \Theta(g(n))$ if $\exists c, c', n_0 \in \mathbf{N}$ s.t. $\forall n \geq n_0, c\, g(n) \leq f(n) \leq c'\, g(n)$. The growth rate of f(n) is equal to that of g(n).
- $f(n) = g(O(h(n)))$ if $\exists$ h': $\mathbf{N} \rightarrow \mathbf{R}^+$ s.t. $f(n) = g(h'(n))$ and $h'(n) = O(h(n))$.

Fact 1 If $\lim_{n\rightarrow\infty}[f(n)/g(n)] = c \geq 0$, $f(n) = O(g(n))$.

Fact 2 If $\lim_{n\rightarrow\infty}[f(n)/g(n)] = c > 0$, $f(n) = \Theta(g(n))$.

Fact 3 If $f(n) = o(g(n))$, $f(n) = O(g(n))$.

Fact 4 If $a_k > 0$, $a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0 = \Theta(n^k)$.

Fact 5 For all real numbers $a, b > 1$, $\log_a n = \Theta(\log_b n)$.

Fact 6 For all real numbers a, b,

- $n^a = o(n^b)$ iff $a < b$
- $n^a = O(n^b)$ iff $a \leq b$
- $n^a = \Theta(n^b)$ iff $a = b$

Fact 7 For all real numbers $0 < a < b$, $a^n = o(b^n)$.

Fact 8 $(\log_a)^k < n < n \log_a n < n^2 < n^3 < n^4 < \cdots < 2^n < 3^n < 4^n < \cdots$ ($f(n) < g(n)$ abbreviates $f(n) = o(g(n))$.)

Fact 9 For all real numbers $a, b > 1$, $a^n = b^{O(n)}$.

Proof of Fact 9 $a^n = (b^{\log_b a})^n = b^{(\log_b a)\cdot n}$, and $(\log_b a)\cdot n = O(n)$.