LAST NAME:

FIRST NAME: Solution

Problem 1 [**20 points**] Let L_1 be the language accepted by the finite automaton given on Figure 1.

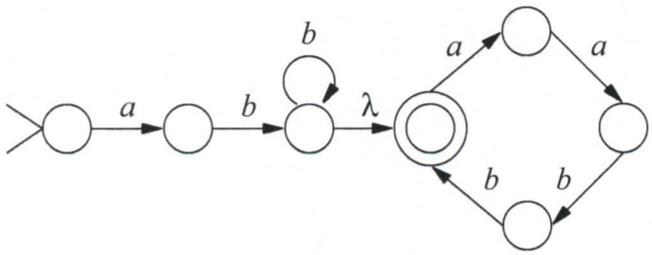


Figure 1: Automaton that accepts language L_1

Let L_2 be the language defined by the regular expression:

$$(\boldsymbol{abb})^* \, (\boldsymbol{aa} \cup \boldsymbol{bb})^*$$

Each row in the table below corresponds to a string, say s, which is given in the first column. Each column (after the first) corresponds to a claim about the string s, say $P(s)$, which is given on on top of the column.

Answer:

s	$s \in L_1$	$s \in L_2$	$s \in L_1L_2$	$s \in L_1^*$
λ	0	1	0	1
aa	0	1	0	0
ab	1	0	1	1
bb	0	1	0	0
abb	1	1	1	1
$aabb$	0	1	0	0
$abaa$	0	0	1	0
$abab$	0	0	0	1
$abbb$	1	0	1	1
$bbaa$	0	1	0	0
$ababb$	0	0	1	1
$abbaa$	0	1	1	0
$abbab$	0	0	0	0
$abbbb$	1	1	1	1
$ababbb$	0	0	0	1
$abbabb$	0	1	1	1
$ababbbb$	0	0	1	1
$abbaabb$	1	1	1	1
$abbbbaa$	0	1	1	0
$abbbbaabb$	1	1	1	1

Your task is to fill the empty cells by writing into each cell one of the following two symbols:

1 0

where 1 in the row s and column $P(s)$ means that $P(s)$ is **true** for s, while 0 in the row s and column $P(s)$ means that $P(s)$ is **false** for s.

There are 20 strings and 80 cells to fill, hence there are 80 credit tokens on offer. Every correctly placed symbol earns (+1) token, and every incorrectly placed symbol earns (−1) token. In other words, every incorrect answer cancels the credit earned by one correct answer. Empty cells neither earn not lose credit.

If your total score is negative, you receive zero points on this problem. Otherwise, you receive a number of points proportional to your score, after rounding up to the nearest integer: 80 tokens = 20 points.

Note: The regular expression for L_1 is: $ab\,b^*(aabb)^*$

LAST NAME:

FIRST NAME: Solution

Problem 1 [**20 points**] Let L_1 be the language defined by the regular expression:

$$(ab)^* \, (cb)^* \, (a \cup c)^*$$

Let L_2 be the language accepted by the finite automaton given on Figure 2.

Each row in the table below corresponds to a string, say s, which is given in the first column. Each column (after the first) corresponds to a claim about the string s, say $P(s)$, which is given on on top of the column.

Your task is to fill the empty cells by writing into each cell one of the following two symbols:

1 **0**

where 1 in the row s and column $P(s)$ means that $P(s)$ is **true** for s, while 0 in the row s and column $P(s)$ means that $P(s)$ is **false** for s.

Answer:

s	$s \in L_1$	$s \in L_2$	$s \in L_1L_2$	$s \in L_2^*$
λ	1	1	1	1
a	1	1	1	1
b	0	1	1	1
c	1	1	1	1
aa	1	0	1	1
ab	1	1	1	1
ac	1	0	1	1
ba	0	1	1	1
bb	0	1	1	1
bc	0	0	0	1
aaa	1	0	1	1
aba	1	1	1	1
bbb	0	1	1	1
cbc	1	1	1	1
$abaa$	1	0	1	1
$abac$	1	0	1	1
$cbcc$	1	0	1	1
$cccb$	0	0	0	1
$cbcab$	0	0	0	1
$cbcbab$	0	0	0	1

There are 20 strings and 80 cells to fill, hence there are 80 credit tokens on offer. Every correctly placed symbol earns (+1) token, and every incorrectly placed symbol earns (−1) token. In other words, every incorrect answer cancels the credit earned by one correct answer. Empty cells neither earn not lose credit.

If your total score is negative, you receive zero points on this problem. Otherwise, you receive a number of points proportional to your score, after rounding up to the nearest integer: 80 tokens = 20 points.

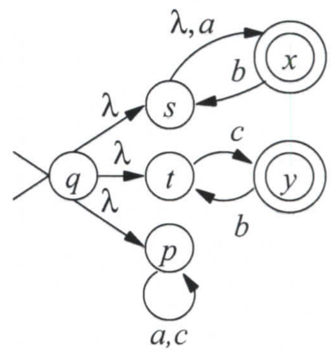


Figure 2: Automaton for language L_2

Note: The regular expression for L_2 is:

$$(\lambda \cup a)(b(\lambda \cup a))^* \cup c(bc)^*$$

LAST NAME:

FIRST NAME: Solution

Problem 2 [**20 points**] Let L_1 be the language accepted by the following context free grammar.

$G = (V, \Sigma, P, S)$, where $\Sigma = \{a, b, c, d, e\}$,
$V = \{S, A, B, D\}$, and the production set P is:

$$
\begin{aligned}
S &\to AB \\
A &\to aAb \mid D \\
D &\to cDd \mid e \\
B &\to aBa \mid e
\end{aligned}
$$

Let L_2 be the language defined by the regular expression:

$$(\boldsymbol{ac})^* \, (\boldsymbol{ee})^* \, (\boldsymbol{db})^* \, (\boldsymbol{a})^* \, \boldsymbol{e} \, (\boldsymbol{a})^*$$

Each row in the table below corresponds to a string, say s, which is given in the first column. Each column (after the first) corresponds to a claim about the string s, say $P(s)$, which is given on on top of the column.

Your task is to fill the empty cells by writing into each cell one of the following two symbols:

1 0

where 1 in the row s and column $P(s)$ means that $P(s)$ is **true** for s, while 0 in the row s and column $P(s)$ means that $P(s)$ is **false** for s.

There are 20 strings and 80 cells to fill, hence there are 80 credit tokens on offer. Every correctly placed symbol earns (+1) token, and every incorrectly placed symbol earns (−1) token. In other words, every incorrect answer cancels the credit earned by one correct answer. Empty cells neither earn not lose credit.

If your total score is negative, you receive zero points on this problem. Otherwise, you receive a number of points proportional to your score, after rounding up to the nearest integer: 80 tokens = 20 points.

Answer:

s	$s \in L_1$	$s \in L_2$	$s \in L_1L_2$	$s \in L_1^*$
λ	0	0	0	1
ee	1	0	0	1
aea	0	1	0	0
eee	0	0	1	0
$eaea$	1	0	0	1
$eeee$	0	0	0	1
$aaeaa$	0	1	0	0
$acedb$	0	0	0	0
$aeaee$	0	0	0	0
$eaeae$	0	0	1	0
$eeaea$	0	0	1	0
$acedbe$	1	0	0	1
$aeaaea$	0	0	0	0
$eaeaee$	0	0	0	0
$acedbee$	0	0	1	0
$eaeaeee$	0	0	1	0
$aaccbaab$	0	0	0	0
$acceddbe$	1	0	0	1
$acacedbdbe$	0	0	0	0
$acceddbaea$	0	0	0	0

Note: The template for L_1 is: $a^n c^k e d^k b^n a^j e a^j$

Problem 2 [**20 points**] Let L_1 be the language accepted by the following context free grammar.

$G = (V, \Sigma, P, S)$, where $\Sigma = \{a, b, c\}$, $V = \{S, A\}$, and the production set P is:

$$S \to aSbb \mid aA$$
$$A \to ccAaa \mid b$$

Let L_2 be the language defined by the regular expression:

$$a^* \, (bb)^* \, (cc)^* \, b \, (aa)^* \, (bb)^* \, b$$

Each row in the table below corresponds to a string, say s, which is given in the first column. Each column (after the first) corresponds to a claim about the string s, say $P(s)$, which is given on on top of the column.

LAST NAME:

FIRST NAME: Solution

Your task is to fill the empty cells by writing into each cell one of the following two symbols:

1 **0**

where 1 in the row s and column $P(s)$ means that $P(s)$ is **true** for s, while 0 in the row s and column $P(s)$ means that $P(s)$ is **false** for s.

There are 20 strings and 80 cells to fill, hence there are 80 credit tokens on offer. Every correctly placed symbol earns (+1) token, and every incorrectly placed symbol earns (−1) token. In other words, every incorrect answer cancels the credit earned by one correct answer. Empty cells neither earn not lose credit.

If your total score is negative, you receive zero points on this problem. Otherwise, you receive a number of points proportional to your score, after rounding up to the nearest integer: 80 tokens = 20 points.

Answer:

s	$s \in L_1$	$s \in L_2$	$s \in L_1L_2$	$s \in L_1^*$
λ	0	0	0	1
aa	0	0	0	0
ab	1	0	0	1
bb	0	1	0	0
$aabb$	0	1	0	0
$aabbb$	1	0	0	1
$ababab$	0	0	0	1
$accaab$	0	0	0	0
$accbaa$	1	0	0	1
$ababbbb$	0	0	1	0
$aaabbbbb$	1	0	0	1
$aabbccaa$	0	0	0	0
$aaccaabb$	0	0	0	0
$aaccbaab$	0	1	0	0
$aaccbaabb$	1	0	0	0
$abbaccaab$	0	0	0	0
$abbaccbba$	0	0	0	0
$aaabbccbaa$	0	0	0	0
$aabbbaabbb$	0	1	0	1
$aaccbaabbbb$	0	0	1	0

Note: The template for L_1 is: $a^{n+1} c^{2k} b a^{2k} b^{2n}$

LAST NAME:

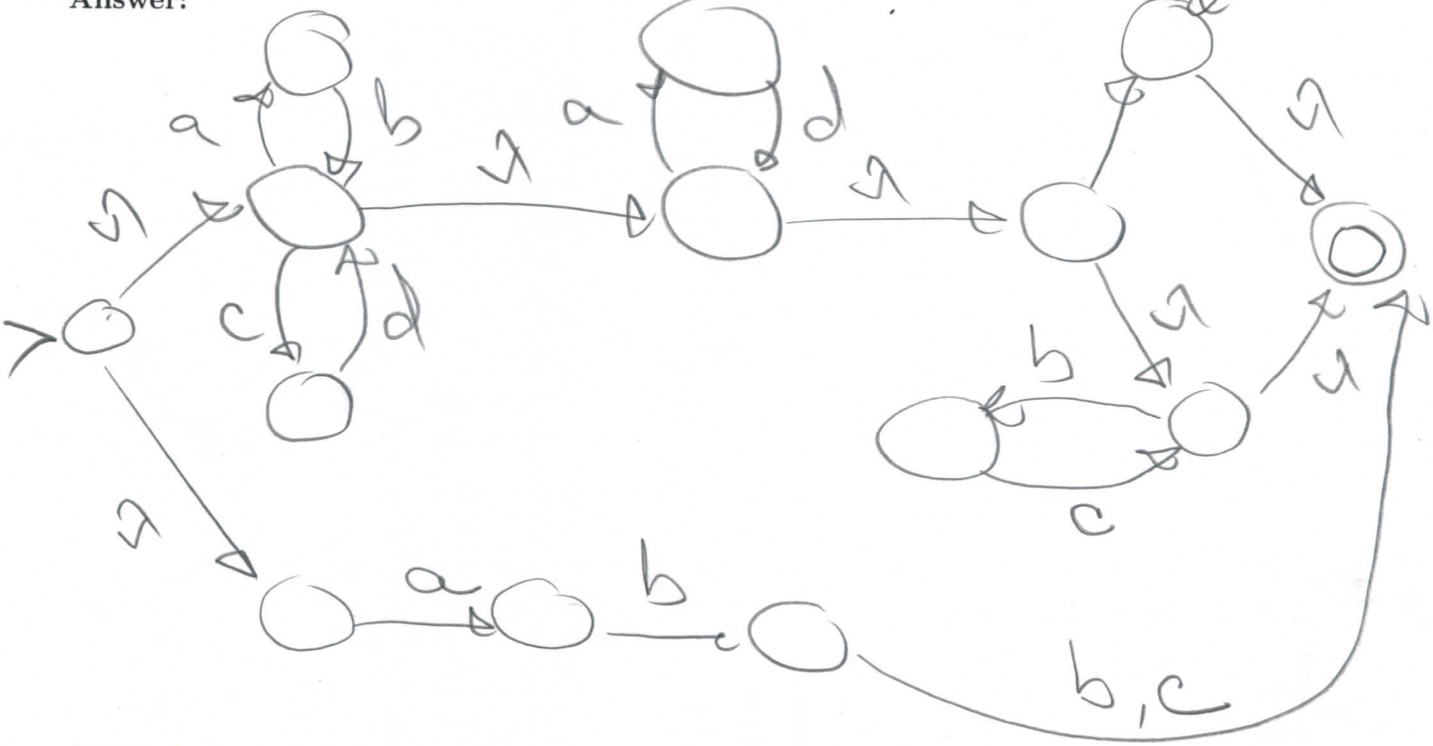
FIRST NAME: Solution

Problem 3 [**20 points**] Let L be the language defined by the regular expression:

$$(ab \cup cd)^* \, (ad)^* \, (b^* \cup (bc)^*) \; \cup \; ab\,(b \cup c)$$

(a) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, prove it.

Answer:



(b) Write a complete formal definition of a context-free grammar that generates L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, A, B, D, E, F\}$

$\Sigma = \{a, b, c, d\}$

P: $S \to ABD \mid abb \mid abc$

$A \to \lambda \mid AA \mid ab \mid cd$

$B \to \lambda \mid BB \mid ad$

$D \to E \mid F$

$E \to \lambda \mid EE \mid b$

$F \to \lambda \mid FF \mid bc$

LAST NAME:

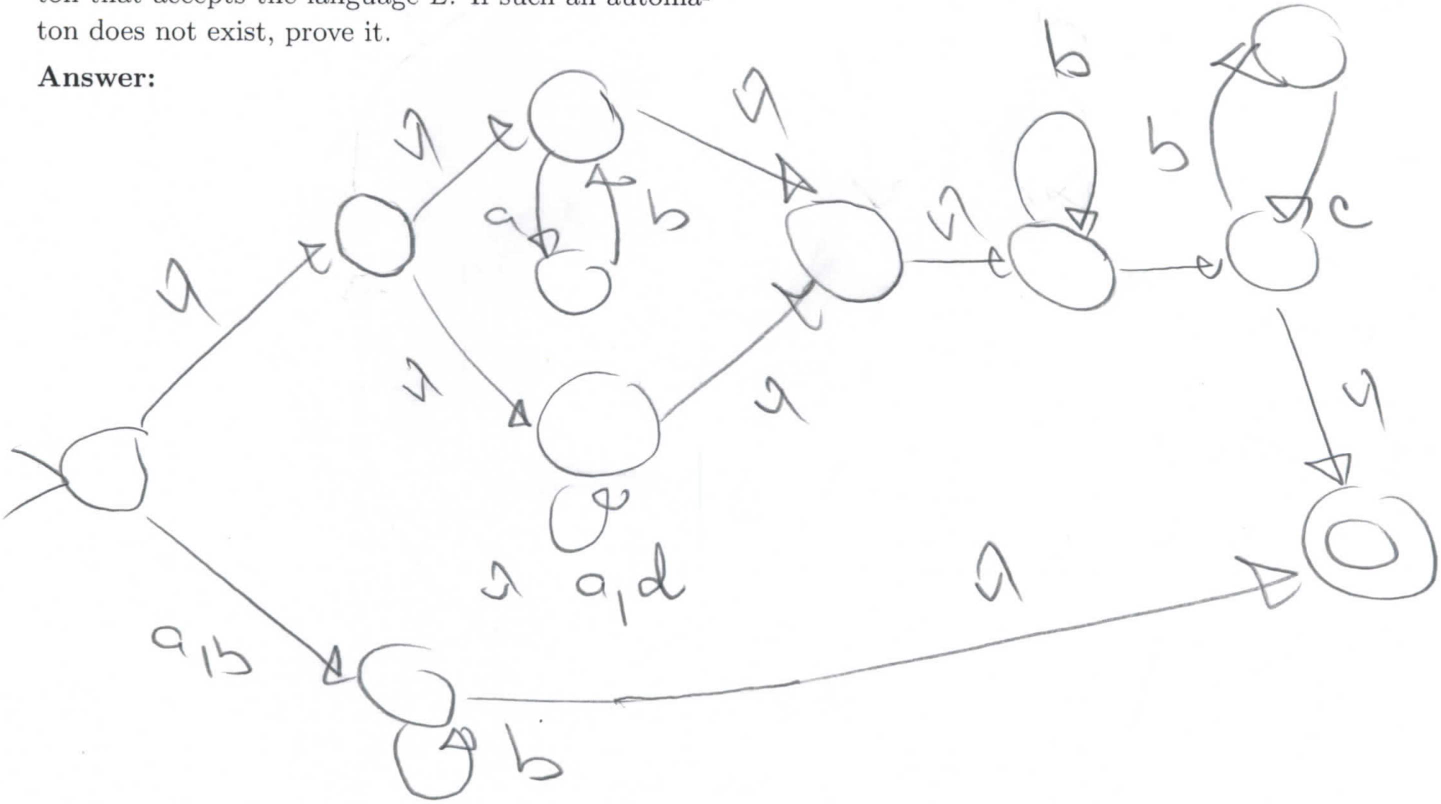
FIRST NAME: Solution

Problem 3 [**20 points**] Let L be the language defined by the regular expression:

$$((ab)^* \cup (a \cup d)^*)\, b^*\, (bc)^* \cup (a \cup b)\, b^*$$

(a) Draw a state-transition graph of a finite automaton that accepts the language L. If such an automaton does not exist, prove it.

Answer:



(b) Write a complete formal definition of a context-free grammar that generates L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, A, B, D, F, H\}$

$\Sigma = \{a, b, c, d\}$

P: $S \to ABD \mid aB \mid bB$

$A \to FH$

$F \to \lambda \mid FF \mid ab$

$H \to \lambda \mid HH \mid a \mid d$

$B \to \lambda \mid BB \mid b$

$D \to \lambda \mid DD \mid bc$

LAST NAME:

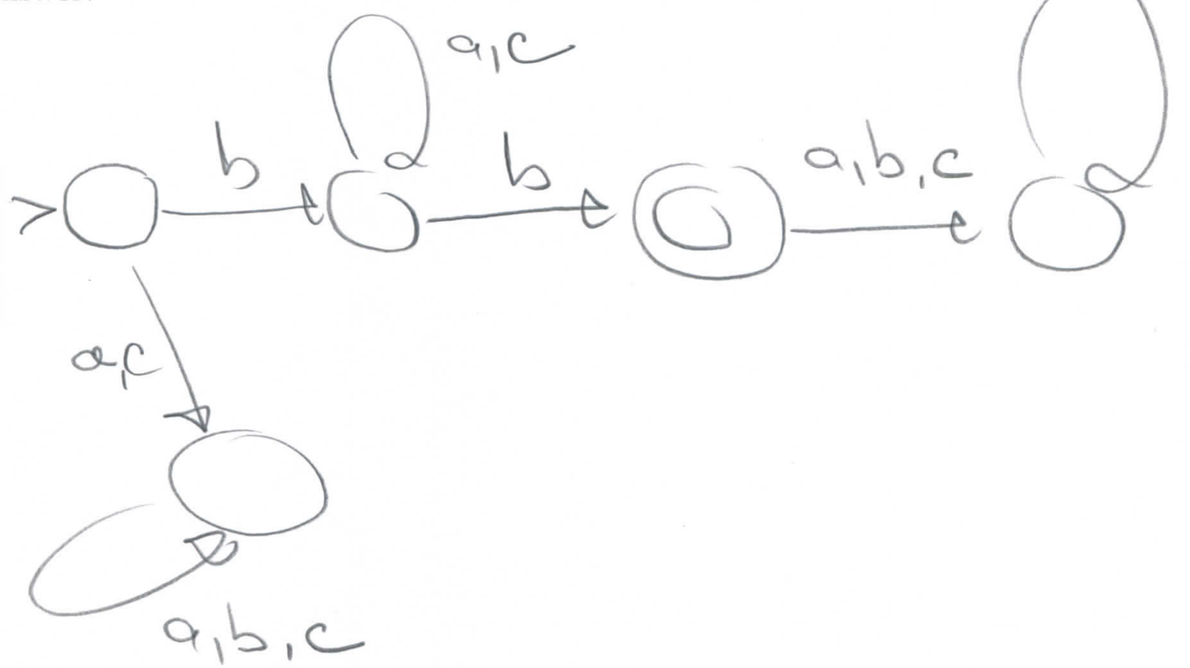
FIRST NAME: Solution

Problem 4 [**20 points**] Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c\}$ that contain exactly two b's, such that one of these two b's is the first letter of the string, while the other b is the last letter of the string.

Let L_2 be the set of exactly those strings over the alphabet a, b, c that do not belong to L_1.

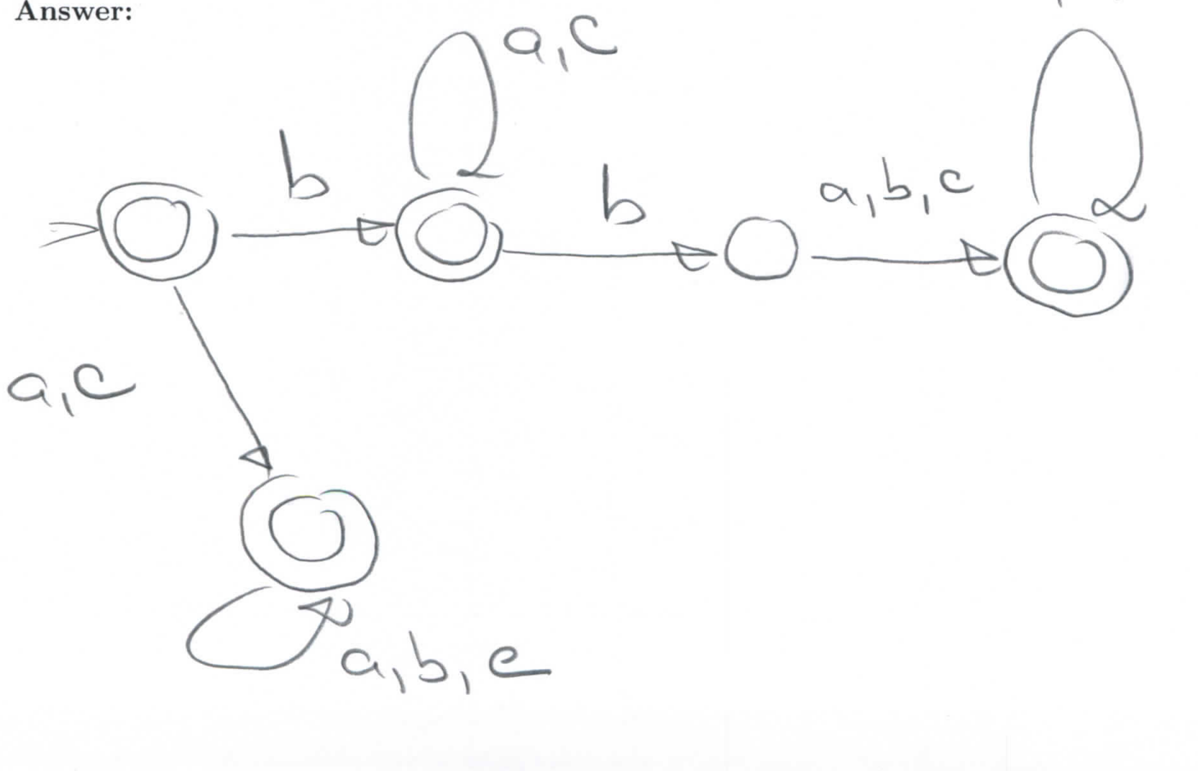
(a) Draw a state-transition graph of a finite automaton that accepts the language L_1. If such an automaton does not exist, prove it.

Answer:



(b) Draw a state-transition graph of a finite automaton that accepts the language L_2. If such an automaton does not exist, prove it.

Answer:



LAST NAME:

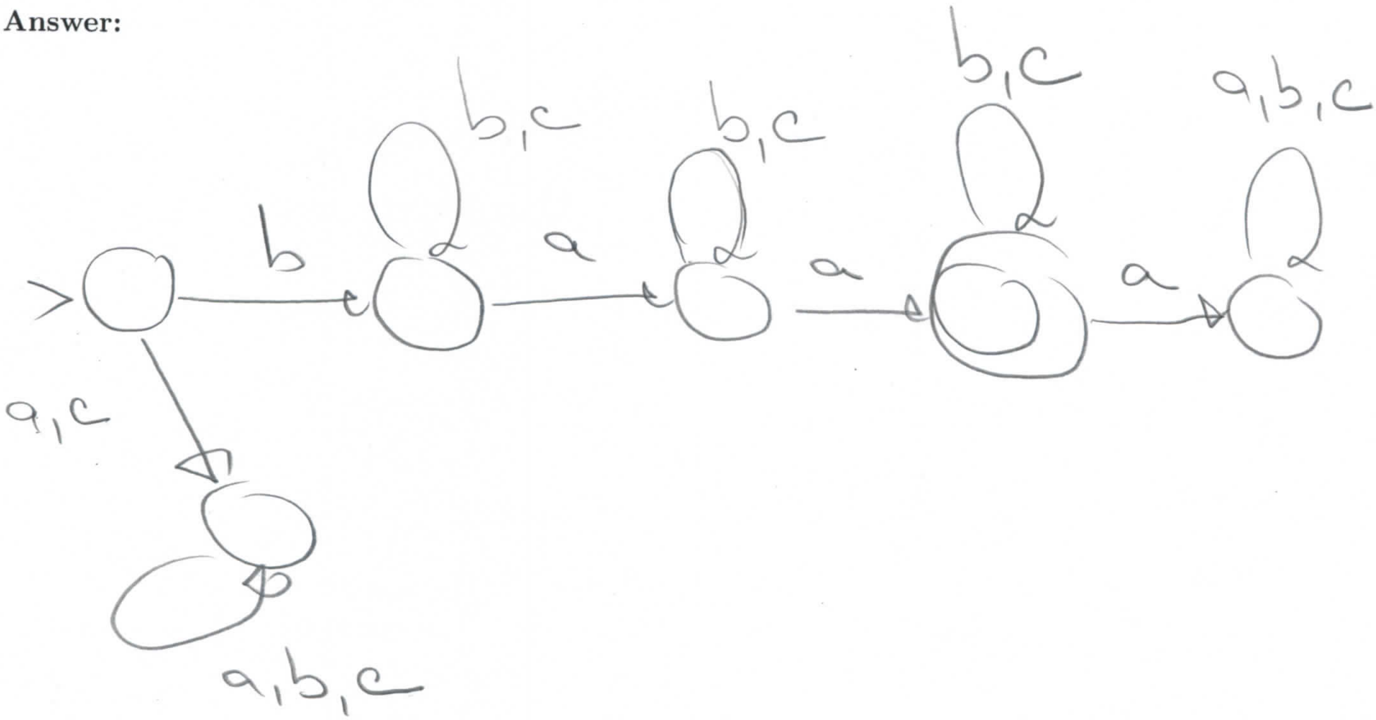
FIRST NAME: Solution

Problem 4 [**20 points**] Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c\}$ that begin with b and contain exactly two a's.

Let L_2 be the set of exactly those strings over the alphabet a, b, c that do not belong to L_1.

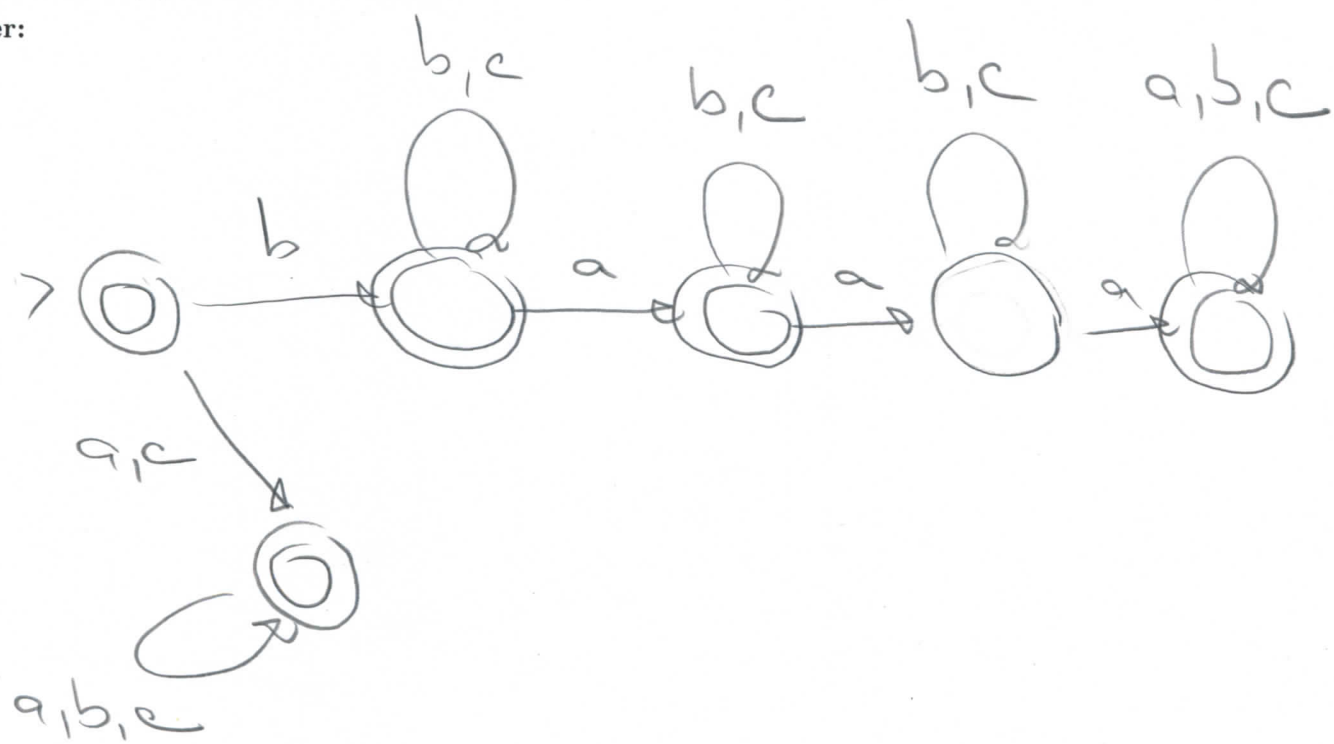
(a) Draw a state-transition graph of a finite automaton that accepts the language L_1. If such an automaton does not exist, prove it.

Answer:



(b) Draw a state-transition graph of a finite automaton that accepts the language L_2. If such an automaton does not exist, prove it.

Answer:



LAST NAME: ______________________

FIRST NAME: Solution

Problem 5 [**20 points**] Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c\}$ that are palindromes with odd length.

Let L_2 be the set of exactly those *non-empty* strings over the alphabet a, b, c that are palindromes with even length.

(a) Write a complete formal definition of a context-free grammar that generates L_1. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, D)$

$V = \{D\}$

$\Sigma = \{a, b, c\}$

P:

$D \to aDa \mid bDb \mid cDc \mid a \mid b \mid c$

(b) Write a complete formal definition of a context-free grammar that generates L_2. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, E)$

$V = \{E\}$

$\Sigma = \{a, b, c\}$

P:

$E \to aEa \mid bEb \mid cEc \mid \Lambda$

(c) Write a complete formal definition of a context-free grammar that generates $(L_1 L_2)^*$. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, D, E\}$

$\Sigma = \{a, b, c\}$

P: $S \to \Lambda \mid SS \mid DE$

$D \to aDa \mid bDb \mid cDc$

$D \to a \mid b \mid c$

$E \to aEa \mid bEb \mid cEc$

$E \to \Lambda$

(d) Write a complete formal definition of a context-free grammar that generates $(L_1^* \cup L_2^*)$. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, D, E, L, R\}$

$\Sigma = \{a, b, c\}$

P: $S \to L \mid R$

$L \to \Lambda \mid LL \mid D$

$R \to \Lambda \mid RR \mid E$

$D \to aDa \mid bDb \mid cDc$

$D \to a \mid b \mid c$

$E \to aEa \mid bEb \mid cEc \mid \Lambda$

LAST NAME:

FIRST NAME: Solution

Problem 5 [**20 points**] Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c\}$ that contain exactly three a's.

Let L_2 be the set of exactly those strings over the alphabet a, b, c that have even length and the substring cc in the middle (i.e., as many letters to the left of the substring cc as there are to the right of it.)

(a) Write a complete formal definition of a context-free grammar that generates L_1. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, T)$

$V = \{T, A\}$

$\Sigma = \{a, b, c\}$

P, $T \to AaAaAaA$

$A \to \lambda \mid AA \mid b \mid c$

(b) Write a complete formal definition of a context-free grammar that generates L_2. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, M)$

$V = \{M, Z\}$

$\Sigma = \{a, b, c\}$

P: $M \to ZMZ \mid cc$

$Z \to a \mid b \mid c$

(c) Write a complete formal definition of a context-free grammar that generates $(L_1L_2 \cup L_2L_1)$. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, T, A, M, Z\}$

$\Sigma = \{a, b, c\}$

$S \to TM \mid MT$

$T \to AaAaAaA$

$A \to \lambda \mid AA \mid b \mid c$

$M \to ZMZ \mid cc$

$Z \to a \mid b \mid c$

(d) Write a complete formal definition of a context-free grammar that generates $(L_1 \cup L_2)^*$. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, T, A, M, Z\}$

$\Sigma = \{a, b, c\}$

$S \to \lambda \mid SS \mid T \mid M$

$T \to AaAaAaA$

$A \to \lambda \mid AA \mid b \mid c$

$M \to ZMZ \mid cc$

$Z \to a \mid b \mid c$

LAST NAME:

FIRST NAME: Solution

Problem 6 [**15 points**] Let L be the set of all strings over the alphabet $\{a, b, c, d, e, g\}$ whose form is:

$$a^{3k+1}b^{k+2}d^{2\ell}c^{2m+1}g^{\ell+3}ee$$

where $k, m, \ell \geq 0$.

Write a complete formal definition of a context-free grammar that generates L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, A, B, E\}$

$\Sigma = \{a, b, c, d, e, g\}$

P: $S \to ABee$

$A \to aaaAb \mid abb$

$B \to ddBg \mid Eggg$

$E \to ccE \mid c$

Problem 7 [**5 points**] Let M be the finite automaton given on Figure 1 (page 2).

(a) Is M deterministic? Prove your answer.

Answer:

No. For instance, it has a λ-transition.

(b) Does there exist a deterministic finite automaton that has fewer than 200 states and is equivalent to M? Prove your answer.

Answer:

Yes. The known algorithm converts a nondeterministic automaton with k states into a deterministic one with $\leq 2^k$ states. M has 7 states, so it has a deterministic equivalent with ≤ 128 states

LAST NAME:

FIRST NAME: Solution

Problem 6 [**15 points**] Let L be the set of all strings over the alphabet $\{a, b, c, d, e, g\}$ whose form is:

$$b^{3k+1} g\, a^{2\ell} c^{2m+1} e^{\ell+3} g\, d^{k+2}$$

where $k, m, \ell \geq 0$.

Write a complete formal definition of a context-free grammar that generates L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, A, B\}$

$\Sigma = \{a, b, c, d, e, g\}$

P:

$S \to bbbSd \mid bgAgdd$

$A \to aaAe \mid Beee$

$B \to ccB \mid c$

Problem 7 [**5 points**] Let M be the finite automaton given on Figure 2 (page 2), and let M_1 be the automaton obtained from M by application of the known algorithm for conversion of a nondeterministic automaton to a deterministic automaton.

(a) Which name will the algorithm assign to the initial state of M_1? Prove your answer.

Answer:

$\{q, s, t, p, x\}$

The initial state of M_1 is the Λ-closure of the initial state q of M.

(b) Does the initial state of M_1 belong to the final states of M_1? Prove your answer.

Answer:

Yes. The initial state of M_1 is $\{q, s, t, p, x\}$, and it contains one initial state: x of the original M.

LAST NAME: Solution

FIRST NAME:

Problem 8 [**20 points**] Let L be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties.

1. if the length of the string is odd then the string is a concatenation, in any order, of an even-length palindrome with an odd-length palindrome;
2. if the length of the string is even then the string is a concatenation of four odd-length palindromes.

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, E, D\}$

$\Sigma = \{a, b, c\}$

$S \to ED \mid DE \mid DDDD$

$E \to aEa \mid bEb \mid cEc \mid \Lambda$

$D \to aDa \mid bDb \mid cDc \mid a \mid b \mid c$

LAST NAME:

FIRST NAME: Solution

Problem 8 [**20 points**] Let L be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties.

1. the length of the string is odd;
2. the first symbol is equal to the middle symbol;
3. the first symbol is different from the last symbol;
4. if the middle symbol is c then the length of the string is greater than 6.

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$V = \{S, A, B, E, Z\}$

$\Sigma = \{a, b, c\}$

$S \to aAb \mid aAc \mid bBa \mid bBc$

$S \to cZZEZZa \mid cZZEZZb$

$A \to ZAZ \mid a$

$B \to ZBZ \mid b$

$E \to ZEZ \mid c$

$Z \to a \mid b \mid c$