

DTM to decide  $\{0^{2^n} \mid n \geq 0\}$ , the powers of 2 in unary notation (Example 3.7 in the book)

- Keep dividing by 2 and it never becomes an odd integer until 1.
- Implement division by 2 by crossing off every other 0.
- The number of remaining 0's on the tape is an even integer until 1 is reached.

Algorithm

"\_" denotes the blank symbol.

start in  $q_1$

blank 1st 0 to mark the leftmost cell and enter  $q_2$

L: let  $k$  = the # of 0's on the tape, including the leftmost "\_"

scan the tape to right, alternating between  $q_3$  and  $q_4$  to cross off every other 0

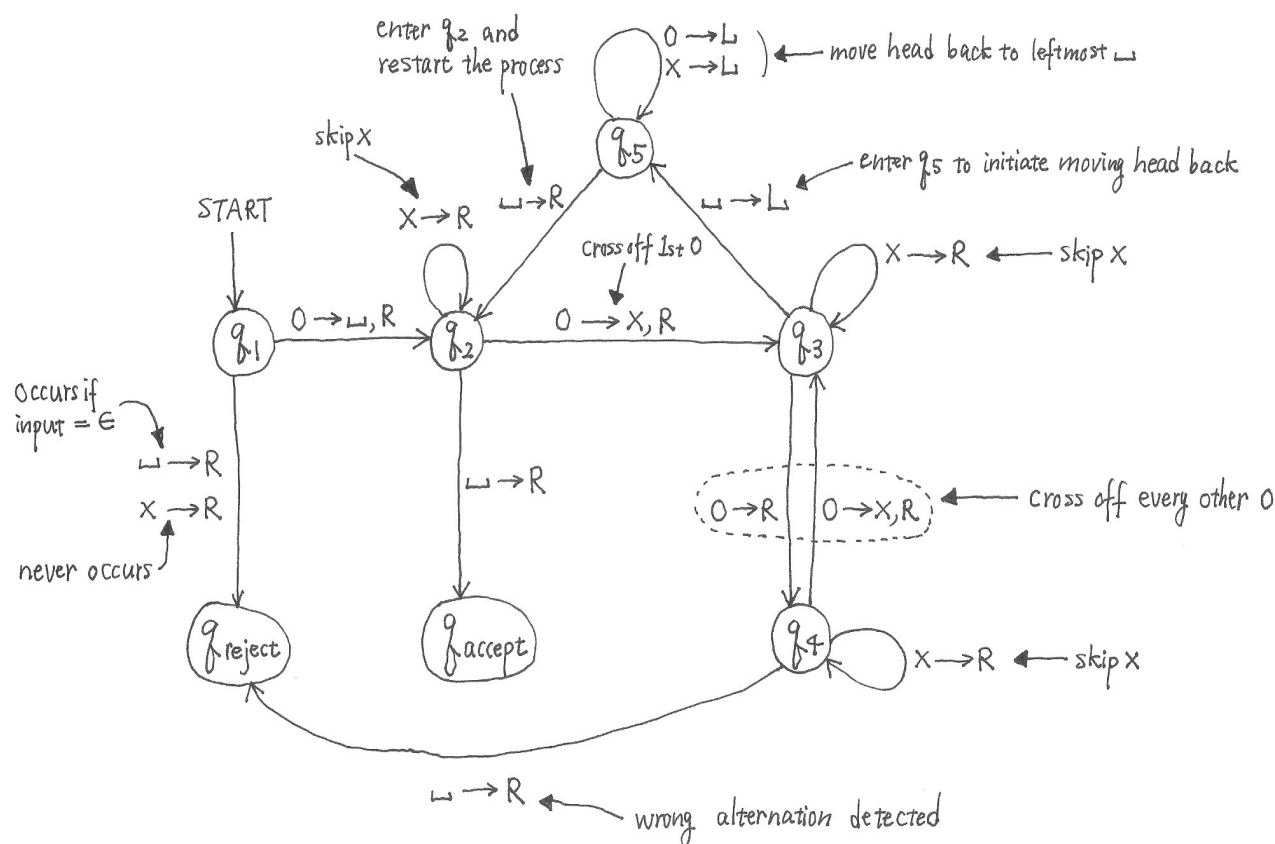
if no 0 is found in the scan, **accept**

if  $k$  is an odd integer  $> 1$ , "\_" will be found on  $q_4$  and **reject**

otherwise "\_" will be found on  $q_3$  and continue

enter  $q_5$  and move the head back to the leftmost "\_"

enter  $q_2$  and goto L



$$\Sigma = \{0\}$$

$$\Gamma = \{0, X, \sqcup\}$$

The following is the transition sequence on the input 000000.

$q_1$  000000 |—

\_q200000 |—  
\_xq30000 |—  
\_x0q4000 |—  
\_x0xq300 |—  
\_x0x0q40 |—  
\_x0x0xq3\_ |—  
\_x0x0q5x |—  
\_x0xq50x |—  
\_x0q5x0x |—  
\_xq50x0x |—  
\_q5x0x0x |—  
q5\_x0x0x |—  
\_q2x0x0x |—  
\_xq20x0x |—  
\_xxq3x0x |—  
\_xxxq30x |—  
\_xxx0q4x |—  
\_xxx0xq4\_ |—  
\_xxx0x\_qreject—