CS 722 Spring 2012
Homework Assignment #1
Solutions

1. Consider the DTM that decides { 0^{2^n} | n ≥ 0 } we studied in class (Example 3.7 in the book). Give the transition sequence on the input 000000.

   "_" denotes the blank symbol.

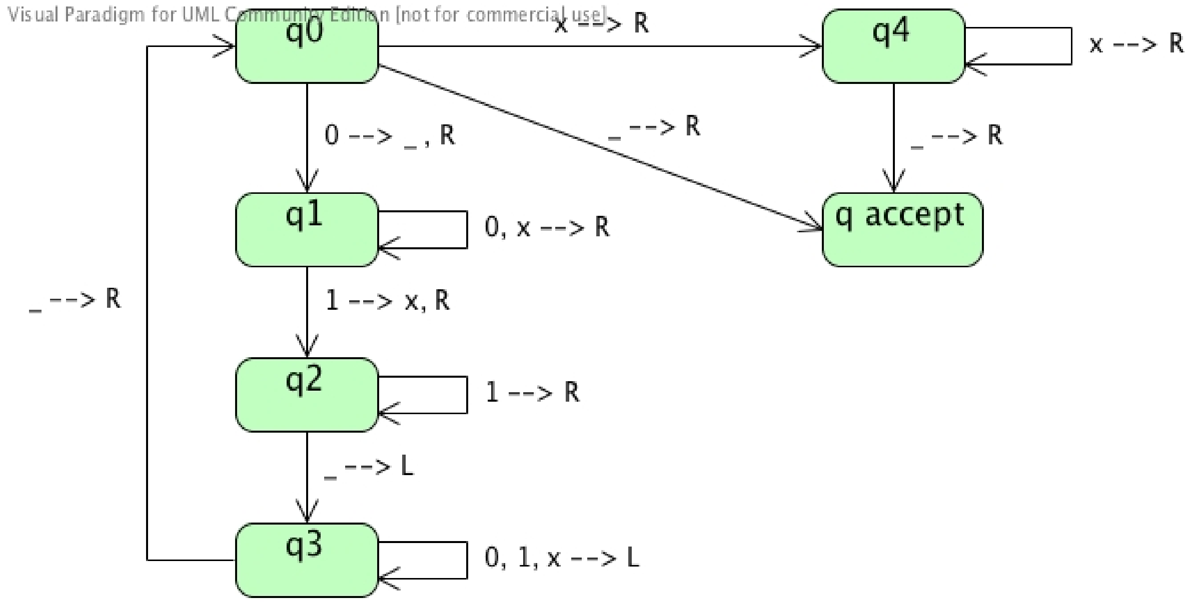
   $q_1$000000 |–
   _$q_2$00000 |–
   _x$q_3$0000 |–
   _x0$q_4$000 |–
   _x0x$q_3$00 |–
   _x0x0$q_4$0 |–
   _x0x0xq_3_ |–
   _x0x0q_5x |–
   _x0x$q_5$0x |–
   _x0q_5x0x |–
   _x$q_5$0x0x |–
   _q_5x0x0x |–
   q_5_x0x0x |–
   _q_2x0x0x |–
   _x$q_2$0x0x |–
   _xxq_3x0x |–
   _xxx$q_3$0x |–
   _xxx0q_4x |–
   xxx0xq_4 |–
   xxx0xq_{reject}_

2. Consider the DTM algorithm that decides { 0^k1^k | k ≥ 0 } we studied in class (M_1 in §7.1 of the book). Design an actual 1-tape DTM implementing this algorithm and give its state-transition diagram.

   The following is one example DTM implementing the algorithm.

   "_" denotes the blank symbol. Σ = { 0, 1 }, Γ = { 0, 1, x, _ }. The start state is q_0. "_" is used to cross off 0 and "x" is used to cross off 1. For brevity of the diagram, the state q_{reject} and all transitions leading to q_{reject} are omitted; any transition δ(q, s) not shown in the diagram is understood to be δ(q, s) = (q_{reject}, s, R).

   The DTM crosses off the first 0 and enters q_1, and stays in q_1 by reading the remaining 0s and the x's representing the 1's already crossed off in the previous steps. Then it crosses off the first 1 and enters q_2, and stays in q_2 by reading the remaining 1's. The DTM enters q_3 upon reading "_", and moves the head back until it reads "_", upon which it reenters q_0 and repeats the process. If the input is 0^k1^k, the DTM will reach the configuration $\_^k q_0 x^k$ and enter q_4 leading to q_{accept}. The checking for an occurrence of 0 after a 1 is incorporated into the first scan.

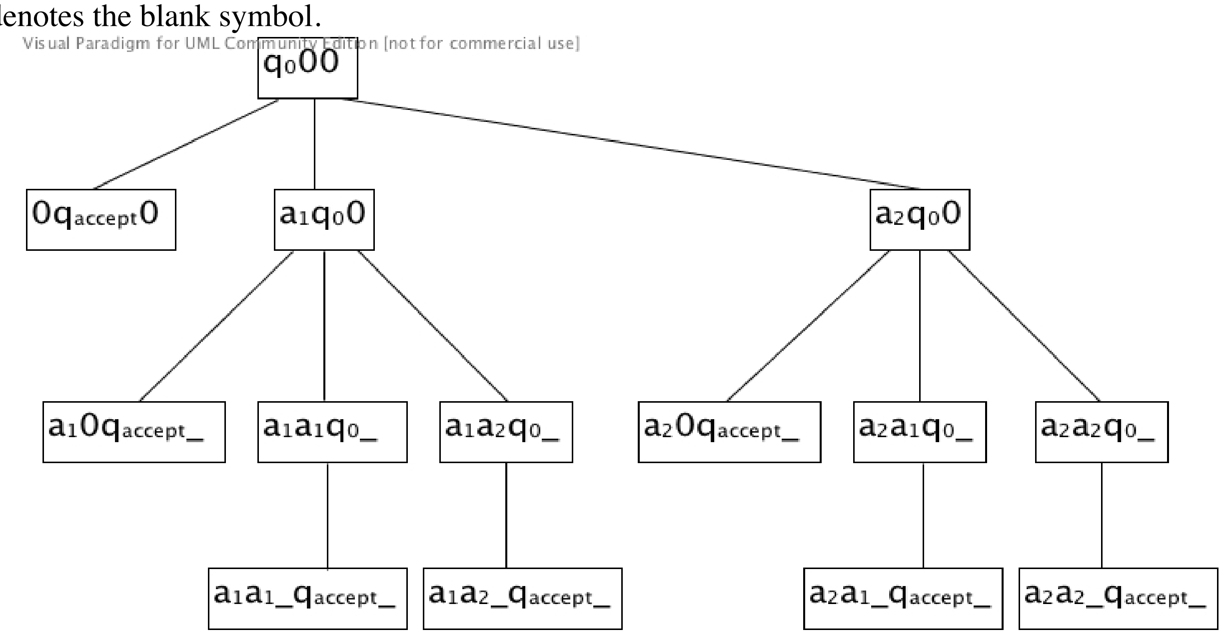


3. Consider the NTM defined by $\Sigma = \{ 0 \}$, $\Gamma = \{ 0, a_1, a_2, \_ \}$ ("_" denotes the blank symbol), $Q = \{ q_0, q_{accept} \}$, and

   $\delta(q_0, 0) = \{ (q_0, a_1, R), (q_0, a_2, R), (q_{accept}, 0, R) \}$
   $\delta(q_0, \_) = \{ (q_{accept}, \_, R) \}$

   Given an input string 0^n, $n \geq 0$, this NTM nondeterministically writes a string $x \in \{a_1, a_2\}^*$ such that $0 \leq |x| \leq n$. Give the computation tree of configurations starting with $q_0 00$ at the root.

   "_" denotes the blank symbol.



4. Give a RAM program to compute the function $f(m, n) = m + (m+1) + \cdots + (n-1) + n$, where m, n are integers such that $m \leq n$, initially given on the input tape.

   The following is one example RAM program.

```
read r1; // read m
read r2; // read n
```

```
r3 = r1+1; // r3 = m+1
while ( r2-r3 ≥ 0 )
{
    r1 = r1+r3;
    r3 = r3+1;
}
write r1;

      Read 1       // r1 = m
      Read 2       // r2 = n
      Load 1
      Add =1
      Store 3      // r3 = r1+1
loop: Load 3
      Sub 2
      Jgtz out     // if r3-r2 > 0 then jump to "out"
      Load 1
      Add 3
      Store 1      // r1 = r1+r3
      Load 3
      Add =1
      Store 3      // r3 = r3+1
      Jump loop
out:  Write 1      // write r1
      Halt
```

5. This question concerns the simulation of RAMs by the 5-tape DTM's described in class. Consider the simulation process of the following RAM instruction stream:

   Load =2, Store 3, Load =3, Store 2, Load =4, Store 1, Load 2, Mult 1, Add 3, Store 4

   Show the contents of tape 1 (simulating the RAM memory) and tape 2 (simulating the accumulator r_0) after the execution of each instruction.

   For brevity, integers in unary notation on the tape are abbreviated by decimal notation.

   Load =2

   tape 1: empty
   tape 2: 2

   Store 3

   tape 1: ##3#2##
   tape 2: 2

   Load =3

   tape 1: ##3#2##
   tape 2: 3

   Store 2

   tape 1: ##3#2##2#3##
   tape 2: 3

   Load =4

tape 1: ##3#2##2#3##
tape 2: 4

Store 1

tape 1: ##3#2##2#3##1#4##
tape 2: 4

Load 2

tape 1: ##3#2##2#3##1#4##
tape 2: 3

Mult 1

tape 1: ##3#2##2#3##1#4##
tape 2: 12

Add 3

tape 1: ##3#2##2#3##1#4##
tape 2: 14

Store 4

tape 1: ##3#2##2#3##1#4###4#14##
tape 2: 14