

replace the lower of your scores on exam 1 and 2. if it is higher.

## parse Trees

→ Last time it was mentioned that. Each nonterminal of a grammar represents a set of  
 grammar = context-free grammar = BNF specification. (in this course)

→ finite sequences of terminals.

The set of sequences represented by the starting nonterminal is called the language of / generated by the grammar. [This is regarded as the set of sequences that is defined by the grammar]

①  $u_f p l_{\text{eff}} = i p \cdot f$

224. 1293.

$$\textcircled{2} \quad ip^2 = d \overset{\textcircled{2}}{ip} d \overset{\textcircled{3}}{.}$$
$$f' = d \mid df$$
$$d :: = 0 / 1 / 2 / 3 / \dots / 9$$

"1" means "the left side is the same as in the previous production"

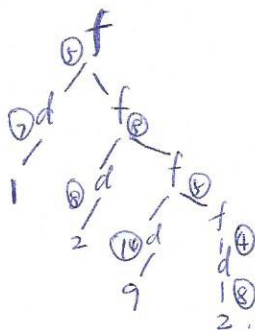
the set of sequence of terminals that is represented by a nonterminal  $N$  is the set of all sequence of terminals that have parse trees rooted ~~and~~ at  $N$ .

important

important  
If the root of a parse tree is not specified. then it should be assumed that the root is the starting nonterminal.

Here is a parse tree that proves:

1 2 9 2 Ef



A parse tree rooted at a nonterminal  $N$  is an ordered rooted tree with the following properties:

1. The root is  $N$
2. each leaf is either a terminal or the symbol  $\langle \text{empty} \rangle$ ;  
 $\langle \text{empty} \rangle$  leaf has no siblings.
3. each ~~terminal~~ node (i.e. each internal node) is a nonterminal, non-leaf.

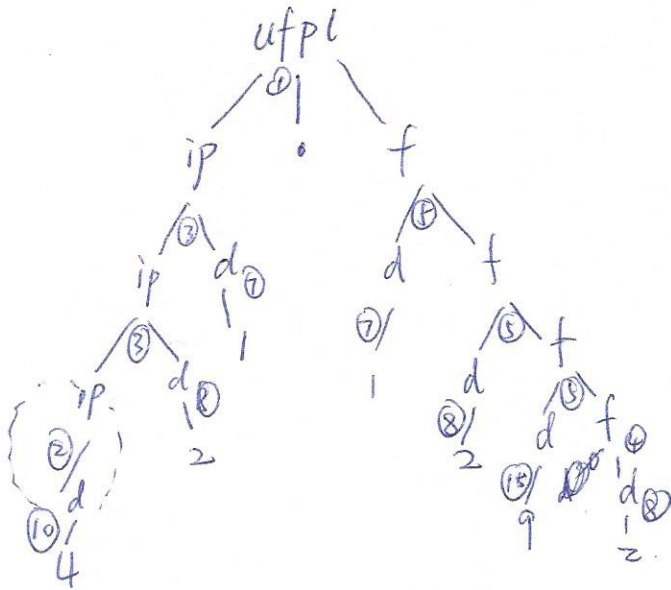
4. the sequence of children of each nonleaf node is the right side of a production whose left side is that node.

A parse tree whose sequence of non-empty leaves is  $t_1 t_2 \dots t_k$  is called a parse tree of  $t_1 t_2 \dots t_k$  or a parse tree that generates  $t_1 t_2 \dots t_k$ .

Here is a parse tree that parses

42101292 ∈ Ufpl

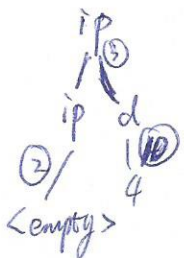
[ie.  $421, 1292 \in$  the language of the above grammar].



Suppose we change production ② to  $ip ::= \langle \text{empty} \rangle$

Q1 Is the above tree still a correct parse tree? No

A correct parse tree of  $421.1292$  should have the following subtree instead of



Extended BNF (EBNF)

$$\text{EBNF} = \text{BNF} + \underbrace{(\dots)^+}_{\text{that mean parentheses}}$$

that mean parentheses

brackets  
braces  
"0" or more times  
brackets, and braces  
parentheses, in BNF has no special meaning; they are just ordinary characters  
means "is optional".

Ex:  $e ::= (+|-)t (+|-)t$  is a EBNF rule that is equivalent to the following 4 BNF productions

$$e ::= +t+t \mid -t+t \mid +t-t \mid -t-t$$

$e ::= [+|-]t [(+|-)t]$  is equivalent to 9 BNF productions:  $[\alpha] = \alpha \mid \langle \text{empty} \rangle$

$$e ::= t \mid t+t \mid t-t \mid +t \mid +t+t \mid +t-t \mid -t \mid -t+t \mid -t-t$$

$e ::= [+|-]t \{ (+|-)t \}$  is equivalent to infinitely many production, including:

$$e ::= t+t+t-t-t-t+t$$

$$e ::= -t-t+t+t+t+t-t-t$$

$$e ::= t$$

EBNF specifications can always be rewritten as a finite set of BNF productions.

One way to do this is to replace each occurrence of  $(\dots)$ ,  $[\dots]$ , and  $\{\dots\}$  with a new nonterminal, which you must define.

$$(\dots) \Rightarrow N \text{ where } N ::= \dots$$

$$[\dots] \Rightarrow N \text{ where } N ::= \dots \mid \langle \text{empty} \rangle$$

$$\{\dots\} \Rightarrow N \text{ where } N ::= \langle \text{empty} \rangle \mid N \dots$$

$$\{\alpha \mid \beta \mid \gamma\} \Rightarrow \langle \text{empty} \rangle \mid N(\alpha \mid \beta \mid \gamma) \Rightarrow N ::= \langle \text{empty} \rangle \mid N\alpha \mid N\beta \mid N\gamma$$

Work from <sup>the</sup> inside outwards

$$e ::= \underbrace{[+|-]}_{\text{option-sign}} t \underbrace{\{ (+|-)t \}}_{\text{rest}} \Rightarrow e ::= \text{opt-sign} + \text{rest}$$

$$\text{opt-sign} ::= + \mid - \mid \langle \text{empty} \rangle$$

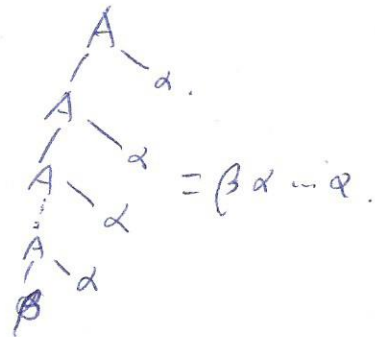
$$\text{op} ::= + \mid - \quad \text{rest} ::= \langle \text{empty} \rangle \mid \text{rest op } t$$



Useful Equivalences

$$A ::= A\alpha/\beta \equiv A ::= \beta\{\alpha\}$$

$$A ::= \alpha A/\beta \equiv A ::= \{\alpha\}\beta$$



Ex: A seq of one or more  $\langle \text{stmt} \rangle$ s in which consecutive  $\langle \text{stmt} \rangle$ s are separated by a semicolon.

EBNF  $\langle \text{stmt-seq} \rangle ::= \langle \text{stmt} \rangle \{ ; \langle \text{stmt} \rangle \}$  (~~BNF~~ BNF:  $A ::= A\alpha/\beta$ ).

$$\langle \text{stmt-seq} \rangle ::= \langle \text{stmt-seq} \rangle ; \langle \text{stmt} \rangle \mid \langle \text{stmt} \rangle$$

seq of one or more d's

$$ip ::= d \{ d \} \Rightarrow ip ::= ip d / d.$$

$$f ::= \{ d \} d \Rightarrow f ::= d f$$

$$f ::= \{ d \} d \Rightarrow f ::= d f / d.$$

$$e ::= [+|-] t \{ (+|-) t \}$$

$$\Rightarrow e ::= e (+|-) t \mid [+|-] t$$

$$\Rightarrow e ::= e + t \mid e - t \mid + t \mid - t \mid t$$