CS 722 Fall 2016
Homework Assignment #2
Due: in class on 10/18/16, Tuesday

Only hard copies will be accepted. Late submissions and email submissions will not be accepted. If you have not been able to solve a problem completely, you may show the work you have done for partial points. Observe course policies in solving assignment problems.

1. Consider the NTM in Question 3 here. Derive the exact formula of the worst-case time complexity function, $W_M(n)$, of this NTM. Justify your answer.

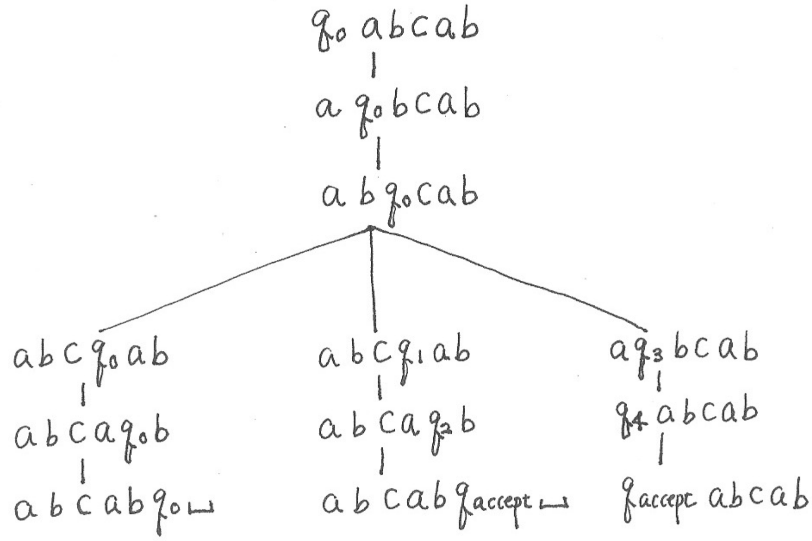
2. This question concerns Theorem VtoN applied to the NTM in Question 3 here. Consider the following verifier V for the language $L = \{ wab^i : i \geq 0 \} \cup \{ wac^j : j \geq 0 \}$ where $\Sigma = \{ a, b, c \}$, $w \in \Sigma^*$. A proposed certificate encodes a pair of integers $<n_1, n_2>$.

   step 1: Read n_1. Move the head back to the leftmost cell on tape. Scan the initial n_1 symbols on tape (skip w).
   step 2: Check if the current tape symbol is "a"; if it is not, reject.
   step 3: Read n_2.
   If $n_2 = 0$, check if "a" is followed by b^i, $i \geq 0$; if so, accept, o.w. reject.
   If $n_2 = 1$, check if "a" is followed by c^j, $j \geq 0$; if so, accept, o.w. reject.

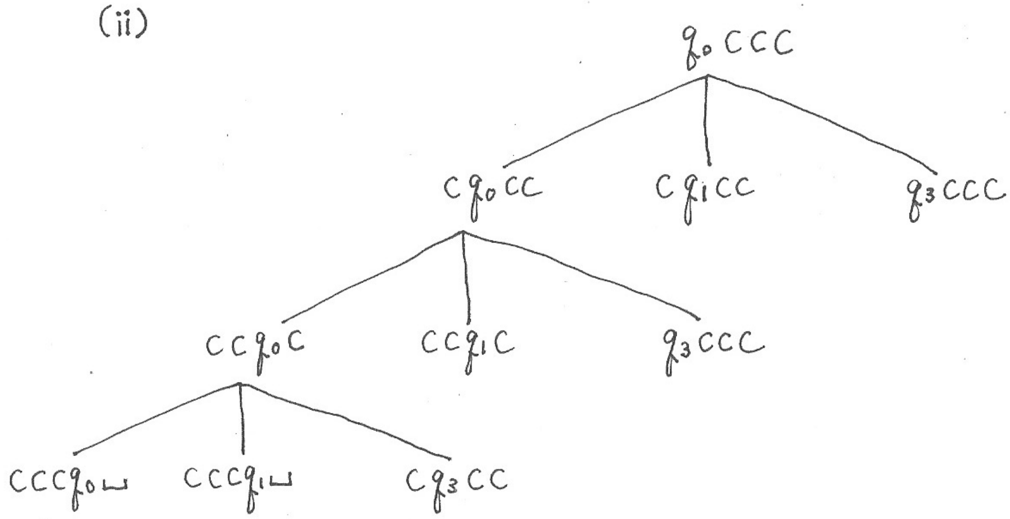
   (a) Prove that V verifies L.
   (b) Describe the operation of an NTM simulating V.

3. This question concerns two examples of the access-path encoding of computation trees' branches used in Theorem NtoV. Consider the NTM in Question 2 here, and the following computation trees of this NTM on the input strings "abcab" and "ccc":

(i)



(ii)



In this NTM, $\delta(q_0, c) = \{(q_0, c, R), (q_1, c, R), (q_3, c, L)\}$. Sequentially number these three nondeterministic transitions by 1, 2, 3.

a. Give the access-path strings that encode the branches of the above computation trees.
b. Which of these strings are the certificates for "abcab", if any?
c. Which of these strings are the certificates for "ccc", if any?

4. Show that the following problems are in NP. You don't have to show they are NP-complete.

a. LPATH in Problem 7.21 on page 324 (Problem 7.20 in the 2nd edition).
b. DOUBLE-SAT in Problem 7.22 on page 324 (Problem 7.21 in the 2nd edition).
c. ≠SAT in Problem 7.26 on page 324 (Problem 7.24 in the 2nd edition).
d. MAX-CUT in Problem 7.27 on page 325 (Problem 7.25 in the 2nd edition).
e. SET-SPLITTING in Problem 7.30 on page 326 (Problem 7.28 in the 2nd edition).

5. Prove each of the following:

   a. If $A, B \in NP$, then $A \cup B \in NP$.
   b. If $A, B \in NP$, then $A \cdot B \in NP$, where $A \cdot B = \{ xy \mid x \in A \text{ and } y \in B \}$.

6. Consider the polynomial-time reduction used to prove $3SAT \leq_p CLIQUE$. Let:

$$\phi = (x_1 \vee \neg x_2 \vee \neg x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee \neg x_2)$$

   Here, $\neg x_i$ is the negation of x_i.

   a. Give (G_ϕ, k) constructed from ϕ by the reduction.
   b. Give one satisfying assignment A for ϕ, and a corresponding k-clique in G_ϕ produced by the proof.
   c. Give one k-clique in G_ϕ that is distinct from the one you gave in (b), and give a corresponding assignment for ϕ produced by the proof.