**Analysis of Algorithms (CSCI 323) and Graduate Algorithms I (CSCI 700)**

**Spring 2016 - Homework #1\***

**(Due at the beginning of class on 2/10/2016)\*\***

1. Derive a formula for the *sum of squares* **12 + 22 + 32 + … + n2**. Hint: assume the formula is a polynomial of degree 3, i.e. an3 + bn2 + cn + d, and use the cases of n=0, n=1, n=2, and n=3 to determine its coefficients.
2. Using mathematical (weak) induction, prove that the formula obtained in the previous question works for all cases n ≥ 0. (If you were unable to derive the formula above, find the formula on-line so you can still do the proof.)
3. Derive a formula for the *arithmetic series* **a + (a + d) + (a + 2d) + … + (a + nd)**. Hint: find the sum of the a’s, then find the sum of the other terms using the formula for 1 + 2 + 3 + … + n.
4. Derive a formula for the *geometric series* **g(n) = c + cr + cr2 + ... + crn**. Hint: subtract g(n) from r\*g(n) and watch all but two of the terms cancel.
5. Recall that 0, 1, 1, 2 are the first four terms of the *Fibonacci Sequence*, defined as f(0) = 0, f(1) = 1, f(n) = f(n-1) + f(n-2). However, as discussed in class, it is equally possible that any given set of numbers may be a different sequence and not the “obvious” one. Find a polynomial p(n) of degree 3 – see question 1 for the format – such that p(0) = 0, p(1) = 1, p(2) = 1, p(3) = 2, and then use it to compute p(4). How does it compare to f(4) of the Fibonacci Sequence?
6. Rank these functions in increasing order of asymptotic growth: (log n)2, n!, 2(2^n), ne, en, 2(ln n), (√2)n, 2n, n√2, log log n, (n log n)2
7. Suppose that f(n) = o(g(n)). Prove that there are infinitely many functions h1(n), h2(n), etc. such that f(n) = o(h1(n)) = o(h2(n)) = … = o(g(n)). [All the o’s in this question are little-ohs.] Hint: how can you construct a new function that has an order of growth strictly between two other orders of growth?

\* There are only seven questions in this homework, not eight as stated in Announcements #2.

\*\* Only if you will not be able to attend class on the due date, submit your solutions - *before 6:00 p.m. on the due date* - to the instructor at LT.CS320@yahoo.com