Let $G := (V, E)$ be a bridgless undirected cubic planar graph.

Lemma 0.1. If G is cubic, then $|V|$ is even.

Proof. $2|E| = \sum_{v \in V} d(v) = 3|V|$
As $|E|$ must be integer, it follows that we must be able o divide $|V|$ by 2. □

Lemma 0.2. If G is cubic, planar and bridgless, then G has a chromatic index of 3.

Consider the graph $G' := (V', E')$, obtained from G by removing an edge xy, adding a vertex z, and adding edges xz and yz.

Lemma 0.3. G' has a chromatic index of 4.

Proof. As $\Delta(G') = 3$; we prove the $\chi'(G') \leq 4$ thanks to Vizing's theorem. Now consider a matching M of the edges of E'. as $|V'| = |V| + 1$ is odd, each matching can cover at most $|V|$ vertices, so it can at most contains $\frac{|V|}{2}$ edges. However, we have $|E'| = \frac{3}{2}|V| + 1$ edges, so we must have $\chi'(G') \geq 4$. □