Vehicle Routing Problems (VRPs)

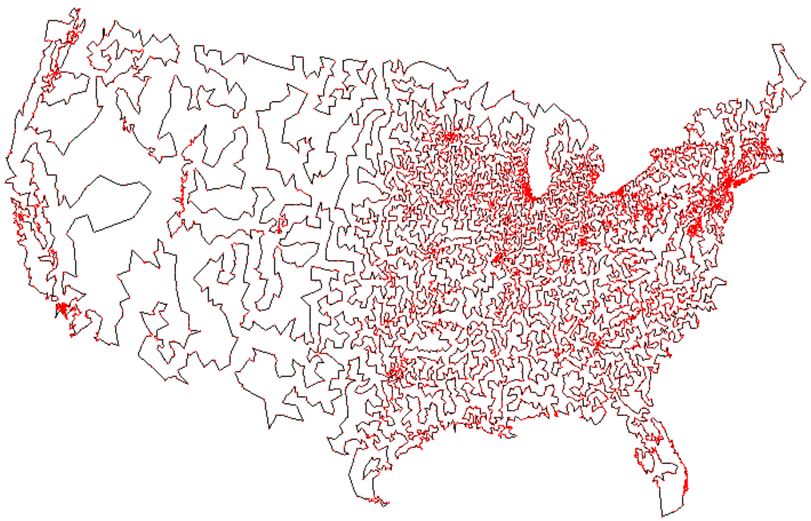
A survey

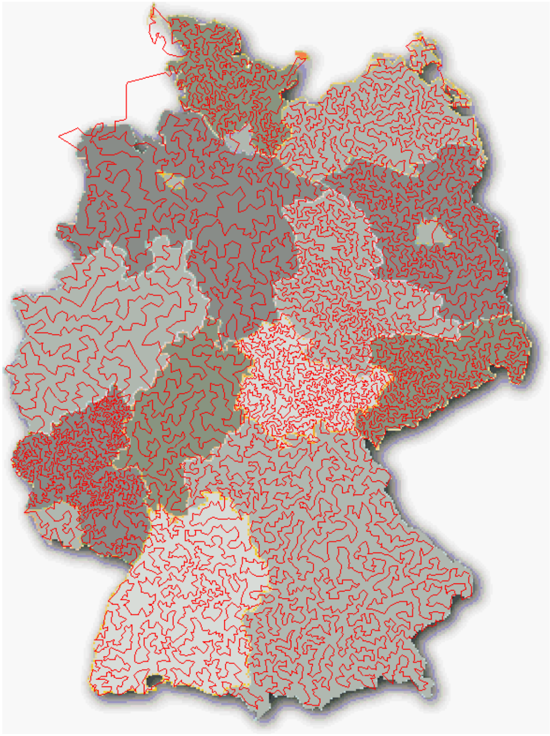
V-D. Cung & N. Teypaz

Traveling Salesman Problem (TSP) : a “simple” VRP

- The most well-known problem in Combinatorial Optimization
- http://www.math.uwaterloo.ca/tsp/
- The basic idea is to have a salesman traveling through N cities of a country by visiting each city once and only once.
- Number of possible solutions (N-1)!/2
  - From a given city, we have N-1 choice for the 2nd, etc.
  - Since the tour is symmetric, we need half of the solutions.

Somes examples
usa13509 (1998), d15112 (2001)
sw24978 (2004)





TSP formulation

$$z_{IP} = \min \quad \sum_{e \in E} c_e \, x_e \tag{1}$$

$$\text{sachant que} \quad \sum_{e \in \delta(i)} x_e = 2 \qquad \forall i \in V \tag{2}$$

$$\sum_{e \in E(S)} x_e \leq |S| - 1 \quad \forall S \subset V \; tq \; 2 \leq |S| \leq |V| - 1 \tag{3}$$

$$x_e \in \{0, 1\} \qquad \forall e \in E \tag{4}$$

(1) Minimize the cost of the tour
(2) Constraints on the adjacency degree over each vertex of the graph
(3) Constraints on subtour elimination
(4) Constraints on the binary variables

TSP numerical example

$$(c_e) = \begin{pmatrix} - & 30 & 26 & 50 & 40 \\ & - & 24 & 40 & 50 \\ & & - & 24 & 26 \\ & & & - & 30 \\ & & & & - \end{pmatrix}$$

Un greedy algorithm for an UB

- Choose a first city, 1 for instance.
- Take systematically the next city in the numerical order as the next city to visit (with out making a subtour)
- One solution : (1,2,3,4,5)
- Upper Bound = 30+24+24+30+40 = 148

Combinatorial relaxation for a LB (1/3)

- For example, we can delete the subtour constraints which are very difficult to satisfied.
- We solve then a linear assignment problem (LAP): each city is visited once and only once.
- LAP could be viewed as a simple transpotation problem !

CR: LAP formulation (2/3)

CR : Solving using Excel (3/3)

Centres de vente						
Usines	1	2	3	4	5	Prod
1	999	30	26	50	40	1
2	30	999	24	40	50	1
3	26	24	999	24	26	1
4	50	40	24	999	30	1
5	40	50	26	30	999	1
Demandes	1	1	1	1	1	
Variables	0	1	0	0	0	1
	1	0	0	0	0	1
	0	0	0	1	0	1
	0	0	0	0	1	1
	0	0	1	0	0	1
	1	1	1	1	1	
Objectif	140					

Optimal solution methods

- Using better relaxations such as Lagrangean Relaxtion to prove that the LB=UB=148.
- If any LB < UB then use a Branch-and-Bound algorithm combined with good bounding and cutting techniques to visit solutions within [LB, UB] to find the optimal solution.

The VRPs

- CVRP
- VRPTW
- VRP with Backhauls
- Split Delivery
- VRP with Pick-up and Delivery
- VRP with Satellite Facilities
- Stochastic VRP
- Period VRP
- Dynamic VRP

Two links among others:
http://or.dei.unibo.it/library/vrplib-vehicle-routing-problem-library
http://www.bernabe.dorronsoro.es/vrp/

Some basics

- Data
  - Several clients
    - To deliver or/and to collect
  - One or several depots
  - A number of vehicles (fixed or unlimited)

- Objectives
  - Minimize the sum of the lengths of the tours with or without fixed costs for the vehicles used.
  - Minimize the costs depending on the weight of the load and the length of a tour (ILOG's presentation)

Mathematical formulation (1/3)

- A unique visit at a client

  (a client is in only one tour)

$$\forall i \in N, \qquad \sum_{j \in N_i^+ \cup M^-} \sum_{v \in V} x_{ij}^v = 1$$

$$\forall i \in N, \forall v \in V, \qquad \sum_{j \in N_i^- \cup M^+} x_{ji}^v - \sum_{j \in N_i^+ \cup M^-} x_{ij}^v = 0$$

- Departure from and arrival at the depot

$$\forall v \in V, \qquad \sum_{j \in N_{s_v}^+ \cup M^-} x_{s_v j}^v = 1$$

$$\forall v \in V, \qquad \sum_{j \in N_{f_v}^- \cup M^+} x_{j f_v}^v = 1$$

Mathematical formulation (2/3)

- Les sous tours (sous tournées sans dépôt)

$$\forall v \in V, \forall S \subset N; 2 \leq |S| \leq |N| - 2, \quad \sum_{i \in N} \sum_{j \in N} x_{ij}^{v} \leq |S| - 1$$

- Capacité des véhicules

$$\forall v \in V, \quad \sum_{i \in N} d_i \sum_{j \in N_i^{+} \cup M^{-}} x_{ij}^{v} \leq C$$

Mathematical formulation (3/3)

$$Minimiser: \sum_{v \in V} \sum_{(i,j) \in U} c_{ij}^v x_{ij}^v$$

$$\forall v \in V, \quad \sum_{j \in N_{s_v}^+ \cup M^-} x_{s_v j}^v = 1$$

$$\forall i \in N, \quad \sum_{j \in N_i^+ \cup M^-} \sum_{v \in V} x_{ij}^v = 1$$

$$\forall i \in N, \forall v \in V, \quad \sum_{j \in N_i^- \cup M^+} x_{ji}^v - \sum_{j \in N_i^+ \cup M^-} x_{ij}^v = 0$$

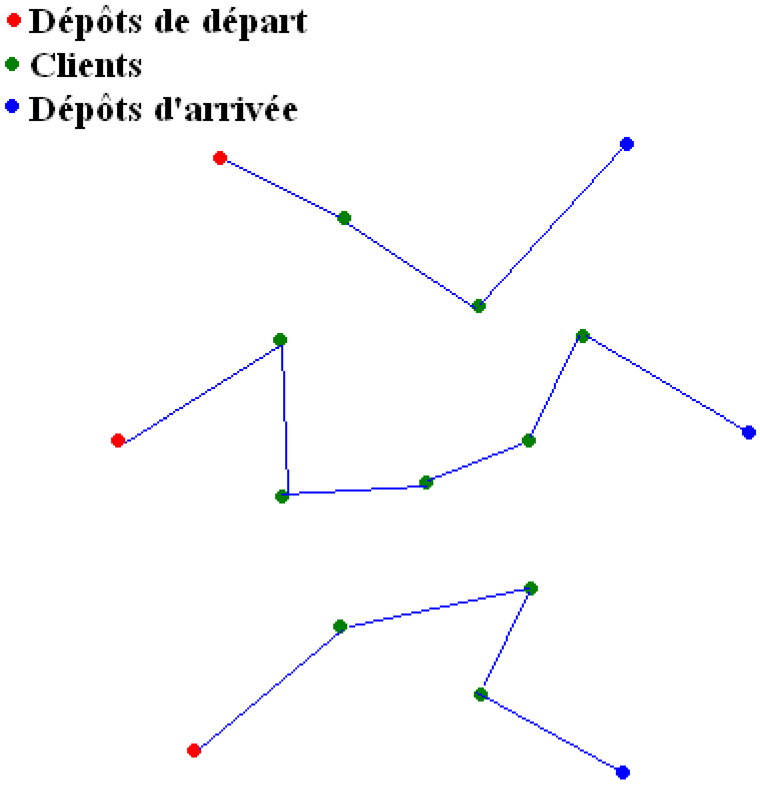
$$\forall v \in V, \quad \sum_{j \in N_{f_v}^- \cup M^+} x_{j f_v}^v = 1$$

$$\forall v \in V, \forall S \subset N; 2 \leq |S| \leq |N| - 2, \quad \sum_{i \in S} \sum_{j \in S} x_{ij}^v \leq |S| - 1$$

$$\forall k \in K, \forall (i,j) \in U, \quad x_{ij}^v \in \{0, 1\}$$

$$\forall v \in V, \quad \sum_{i \in N} d_i \sum_{j \in N_i^+ \cup M^-} x_{ij}^v \leq C$$

Example



Solving methods

The most employed for different VRPs

- Tabu Search
  - Neighborhood:
    - 2-opt, 3-opt, exchange, transfer, ejection chains,

- Algorithme Clarke et Wright
    - Merging 2 tours

CVRP

- Load capacity of a vehicle limited
- Mono- (Multi-)product [Naudin 2003]
  - Constraints on each product

VRP TW

- Time windows for the visits

  [Naudin 2003, Solomon et al. 2000]

  - Flexible case (delays/latenesses are penalized)
  - Strict case (delays/latenesses are not allowed)

- Early arrivals are not scarsely penalized

VRP with Backhauls

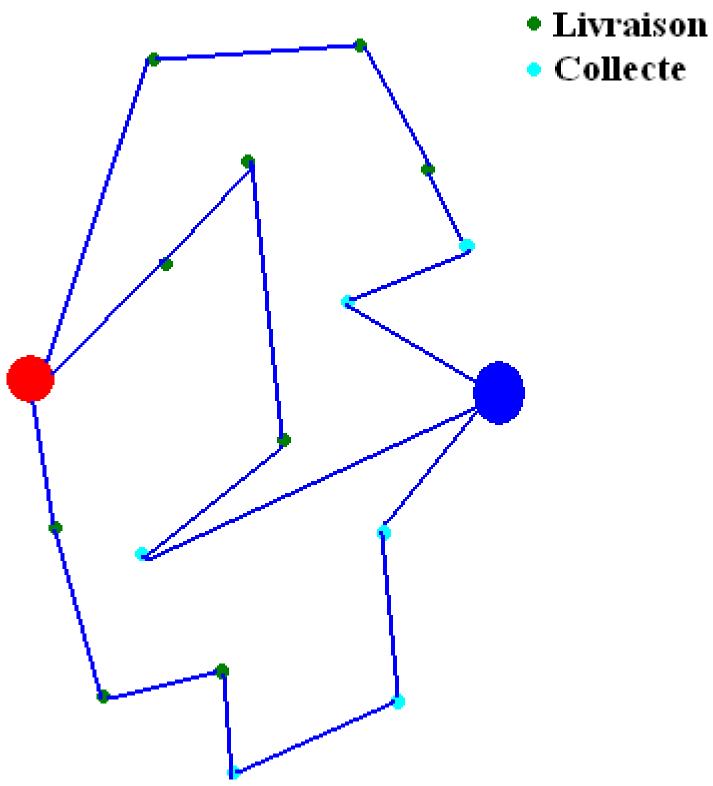
- Tours must respect an visiting order

  [Solomon et al. 2000, Jacobs-Blecha & Goetschalkx]

  - Deliveries must be done first

  - Collections can begin only when deliveries are completed

Example



Split Delivery VRP

- Relaxation of CVRP
  - [Dror & Trudeau 1994, Ho & Haughland 2004]

  - A client is visited at least once
  - Deliveries are splitted in several parts

Use cases:

  - Deliveries of air conditionners with installations
  - Food distribution for cattle in a ranch [Dror & Trudeau 1994]

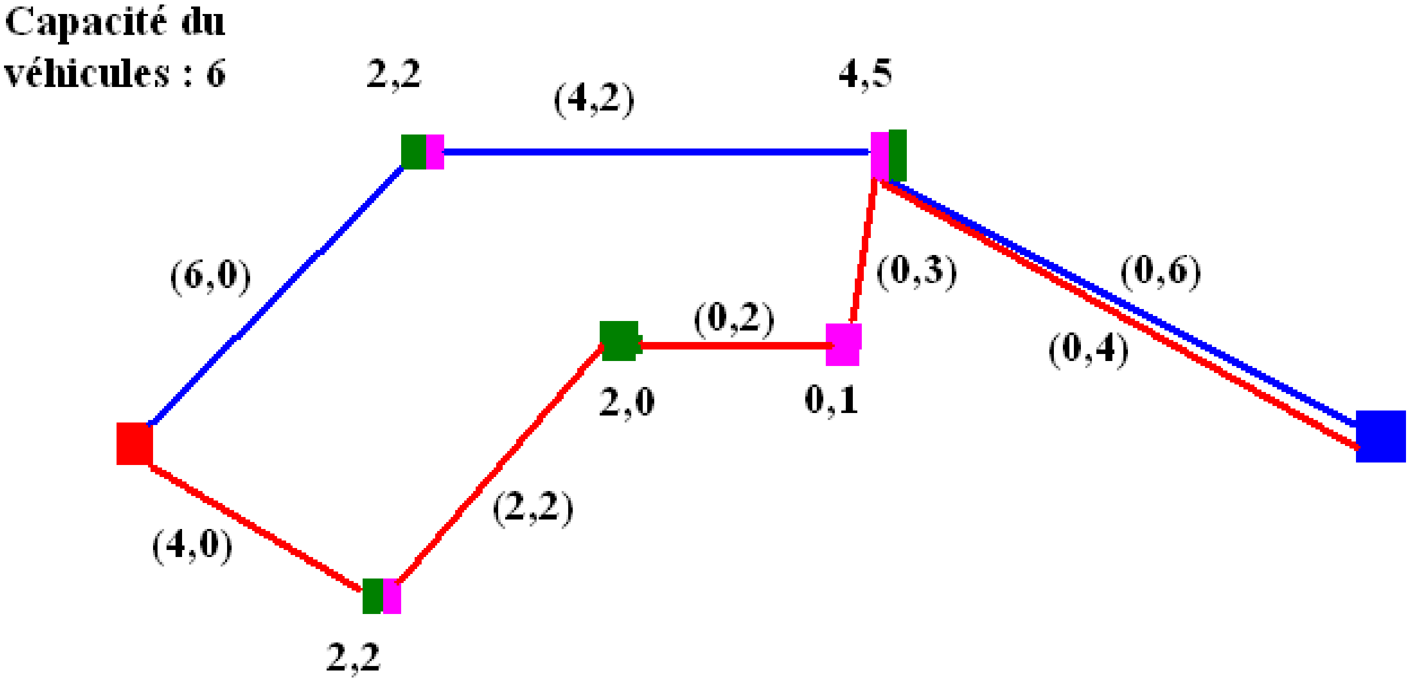
VRP with Pick-up and Delivery

- A tour is constituted of deliveries and collections without orders (≠ Backhauls)
- Deliveries and collections are splittable (in unit)

Use cases

- School bus transportation [Mosheiov 1997]
- Goods distributions and garbage collections [Righini 2000]

Example

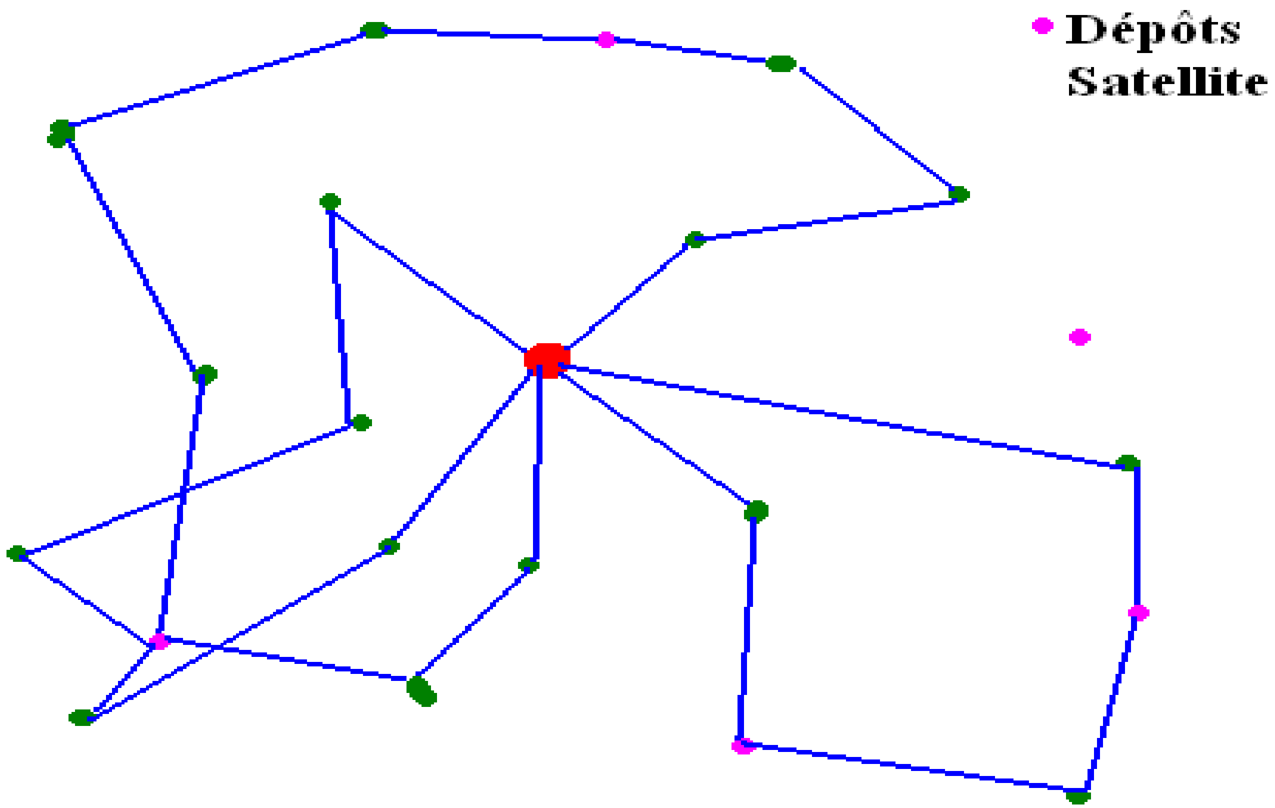


VRP with Satellite Facilities

- Extra depots to refill the vehicle during the tour
  - Limited accesses to the depots (maximum nomber of visits)
- Start and end tours at the central depot(s)

Use case: Fuel or article retail distribution [Dror et al. 1997]

Example



VRP with multiple use of vehicles (multi trip VRP)

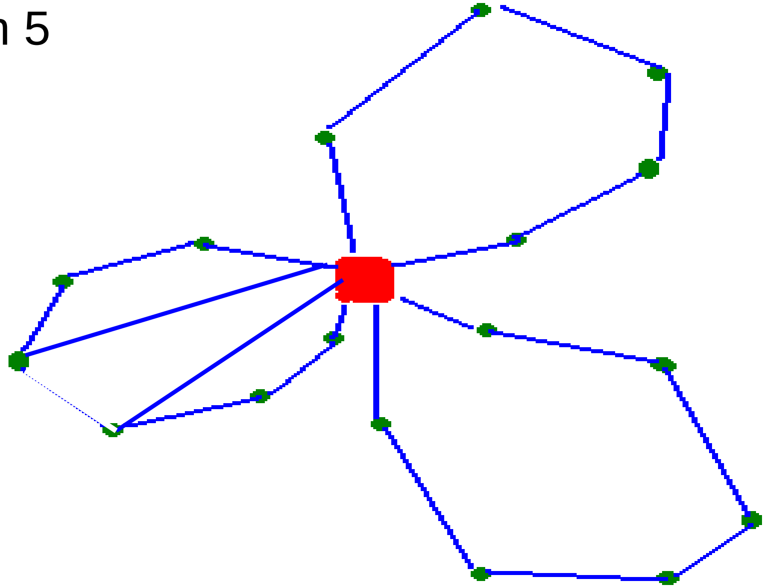
- A same vehicle can be used in several tours
- Objective function
  - A cost for the use of the vehicle and a cost for the tour
- Respect the labor code
  - Working hours of a day
  - Breaks

Use case: Product distribution of a biscuit factory to the retailers [Mercer, Brandao 1997]

Example

Optimal VRP :

3 tours of length 5



Optimal MT VRP

2 vehicles : 4 tours

2 of length 5

2 of length 3

Stochastic VRP

- Random components
  - Travel time on the arcs
  - Clients demands (the most studied)
  - Presence of the clients

- Stochastic programming : 2 steps
  - An *a priori* solution
  - An alternative policy
    - Example: go back to the depot when arriving at a node the demand cannot be satisfied [Laporte & Louveaux 1997]

Stochastic VRP
Mathematical formulation

2 approaches [Stewart Jr & Golden 1982]

- Chance Constrained Program (CCP)
  - Classical objective
  - Constraints on the failed probabilities

$$\forall v \in V \quad Pr\{ \sum_{(i,j) \in U} d_i x_{ij}^v \leq C \} \geq 1 - \alpha$$

- Stochastic Program with Recourse (SPR)
  - Minimize costs of the corrective actions and the set of tours

$$Minimiser : \quad \sum_{(i,j) \in U} c_{ij} x_{ij} + Q(x)$$

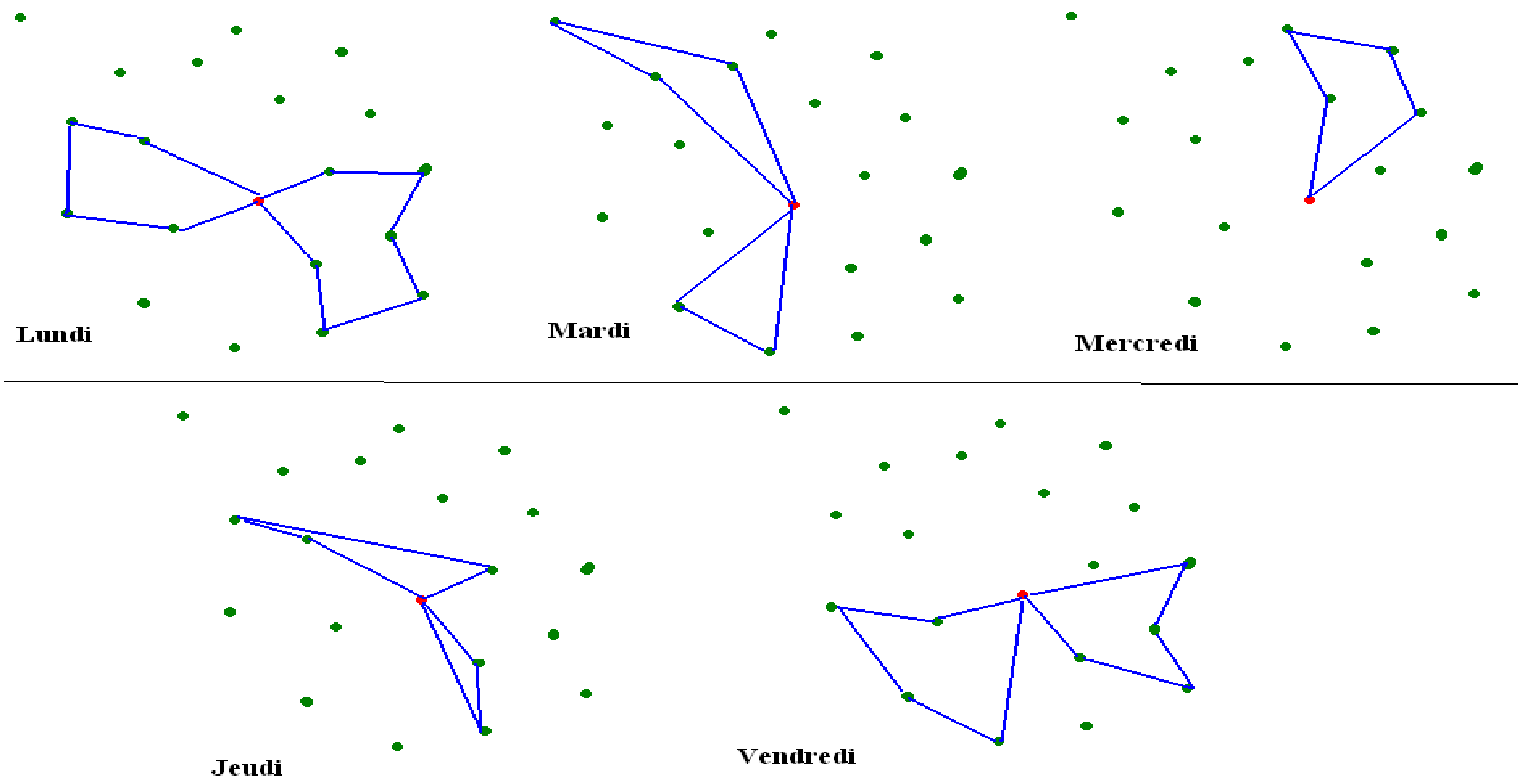
Periodic VRP

- Etablishing a planning of tours
  - Over a period T
  - Clients require a certain number of visits
  - Visit pattern for each client
    - Ex: visit pattern of a week

1	0	0	1	0
1	0	0	0	1
0	1	0	0	1

  - Results
    - A visit planning
    - Daily tours
- Use cases
  - Garbage collections [Beltrami, Bodin 1974], container collections [Baptista et al. 2002]

Example



Dynamic VRP

- Information are not all known
  - New demands have to be taken into account during the execution of the tours
  - Moving speed

- Use cases [Larsen 2001]
  - Repairer problem
  - Taxi or emergency services

Dynamic VRP

- Process information quickly
  - Look for an alternative solution instead of an optimal solution → alpha-competitive algorithms

Use cases:

- « Dispatcher » center
- Permanent contacts with the vehicles:
  - GPS+Cellular phones
- GI-3A project LOSII option

DVRP formulation

- Evolutive formulation [Haghani, Jung 2004] :
  - Updates according to the information
- Solving methods
  - Fast
  - Several executions for the updates

Other related problems

- Combinaisons of VRPs
- Heterogeneous fleet VRP [Taillard 1999]
- LRP: Location routing problem
  - Locate depots
  - Plan the tours
- IRP: Inventory routing problem
  - In Vendor Manage Inventory policy
  - Plan the tours according to the client storage levels