Problem 1: Logistics hub location of the SLog company

SLog company is specialized in the transportation of appliances, from the factories to the N wholesalers who are located over all the European Union dominion. But in order to reduce the transportation costs, Slog has decided to transport the appliances in two stages. The first stage consists in transporting products from factories to hubs, then at the hubs, the second stage consists in repacking products by wholesaler and in dispatching them by destination.

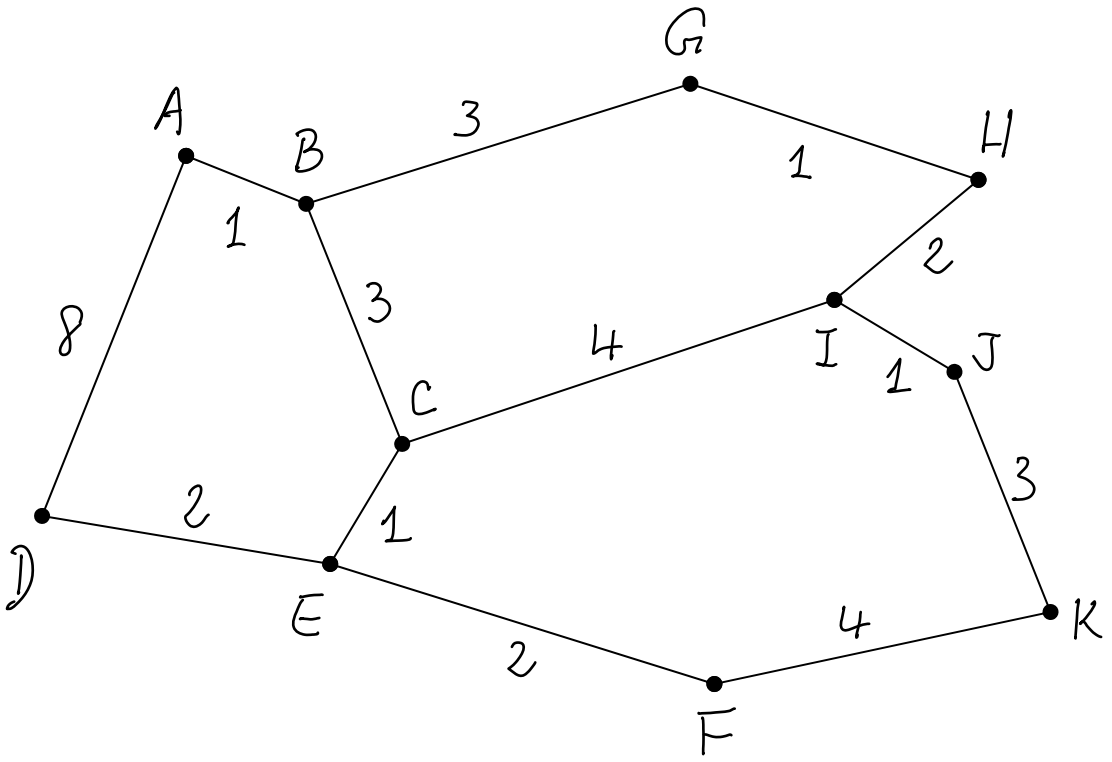
Therefore, the logistics hub location becomes a crucial problem for Slog who has decided to call you to help them as consultant. The objective of SLog is to **minimize only the delivery cost** (hubs → wholesalers). To achieve this, it is necessary to have hubs as close as possible to all the wholesalers. The idea is that if you have k logistics hubs, it would be interesting to have these hubs located at the **centres** of k groups of wholesalers.

In mathematics, this is the notion of centre of gravity or barycentre. A logistics hub is then located at the centre of gravity of a group of wholesalers. However, the barycentre technique is not always operational because the formulas could locate theoretically a hub in the middle of a corn field and far away from any road access. To overcome this drawback, we propose to locate a hub in the city of a wholesaler. We assume that each wholesaler's city can host a hub.

After several months of works at SLog doing interviews and reading the cost accounting documents provided by the company, you have finally managed to retrieve the following information:

- You know specifically the cities where are located the wholesalers of Slog and moreover, you have all the shortest paths between each pair of cities. In practice, you have a shortest distance matrix [Dij], where Dij is the shortest distance between city i and city j.
- You have also the following constraints:
    - A wholesaler is located at only one city and a city has only one wholesaler. We can so considered without confusion a city as a wholesaler.
    - SLog wants to setup only k hubs (k<<N), k and N are known.
    - Each hub is located in a city, and each hub can serve several other wholesalers/cities. But each wholesaler is served by only one hub.
    - The delivery cost to a wholesaler is equal (for the sake of simplicity but without lost of generality) the shortest path distance between a hub and this wholesaler.

Assume that we have the following network of wholesalers' cities identified from A to K with the existing distance between them:



On the previous network, you have the associated symmetrical shortest paths matrix as follows:

	A	B	C	D	E	F	G	H	I	J	K
A	0	1	4	7	5	7	4	5	7	8	11
B		0	3	6	4	6	3	4	6	7	10
C			0	3	1	3	6	6	4	5	7
D				0	2	4	9	9	7	8	8
E					0	2	7	7	5	6	6
F						0	9	9	7	7	4
G							0	1	3	4	7
H								0	2	3	6
I									0	1	4
J										0	3
K											0

Question: find “by hand” a feasible (respecting all the constraints expressed above) solution containing a hub on city I for this instance. Give the cost of the solution and its structure. You need to specify clearly your method.