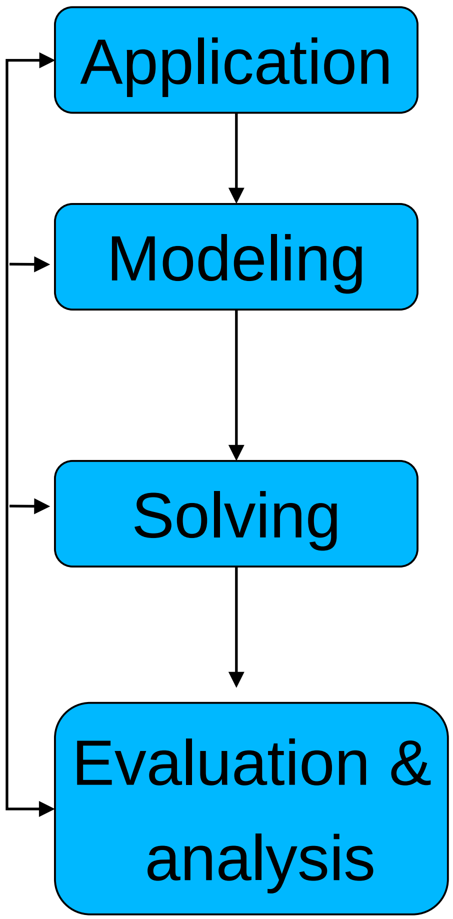
General methodology in Operations Research

OR is an interdisciplinary branch of **applied mathematics and formal science that uses advanced analytical methods** such as:

- mathematical modeling,
- statistical analysis and
- mathematical optimization

to arrive at optimal or near-optimal solutions to **complex decision-making problems**.



Flows, routing, scheduling, assignment, etc.

Mixed integer programming.

Heuristics, Metaheuristics, Approximations, Exact resolution.

Tests, Benchmarking, Simulations.

Freight Transportation and Service Network Design

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Outlines

- Introduction to transportation problems and service network design

- Use cases and MILP modeling

Stakes of the transportation problems (1/2)

Economic (costs), **ecologic** (nuisances, pollutions) and **societal** (opening up areas).

Freight transportation

- Essential link between production sites and clients, upstream (supply) and downstream (distribution) of a supply chain.
- Globalization → production located in low labor cost countries + worldwide clients.
- Freight transportation in cities represents 50 % of consumed diesel [Libération 16 mars 2006, Wuppertal et Certu].

  “The strawberry yogurt is greedy in oil”: traveled 9115km. “…Must be forwarded across Europe 70kg of goods per capita per day …”.
- Transportation costs represents 15% to 30% of the sale price of a product.

Stakes of the transportation problems (2/2)

- → **Rationalize transport policies and systems**
  - → **Strategic level:** Physical network design (long-term decision and heavy investments).
  - → ***<u>Tactical level</u>*:** Service network design (mid-term decision)
  - → **Operational level**: Resource assignement and management (short-terme decision depending on the service time)

Visions and service types

- **3 visions/actors**
  - **Regulation authority** (laws, general policy, heavy investments)
  - ***Carriers*** (route over a physical network, stops, equipments, frequences and schedules)
  - **Clients** (point to point, service offer, frequences and schedule, rate)
- **2 service types**
  - **customized**/« taxi » → fleet management, no service network planning is needed.
  - ***with consolidation***: gathering/dispatching, regular services

    ++ economies of scale for carriers and clients

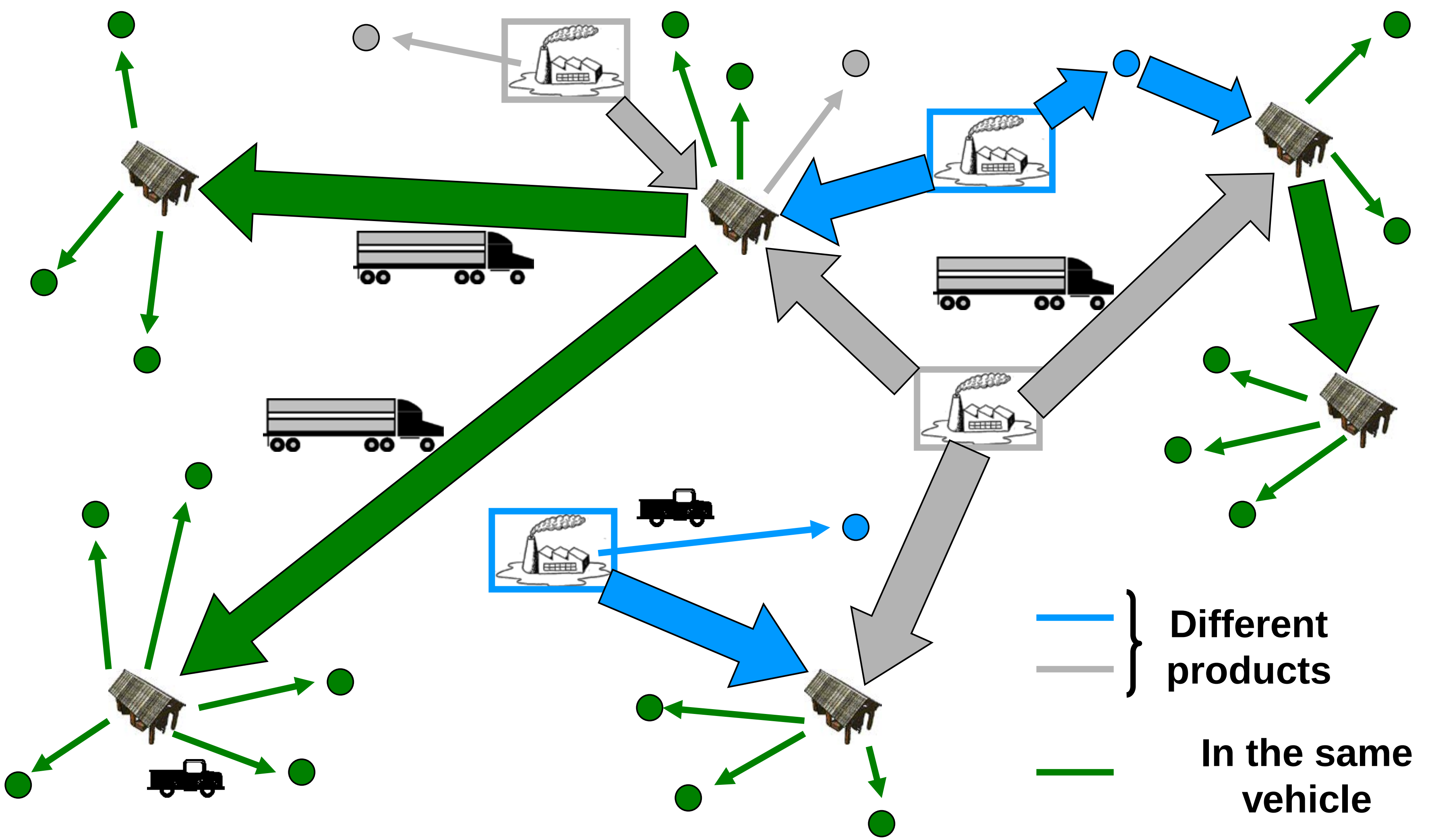
    -- less flexible and reliable for clients, extra operations for carriers at the hubs

    → Necessity of planning to optimize extra operations and to offer good quality of service.

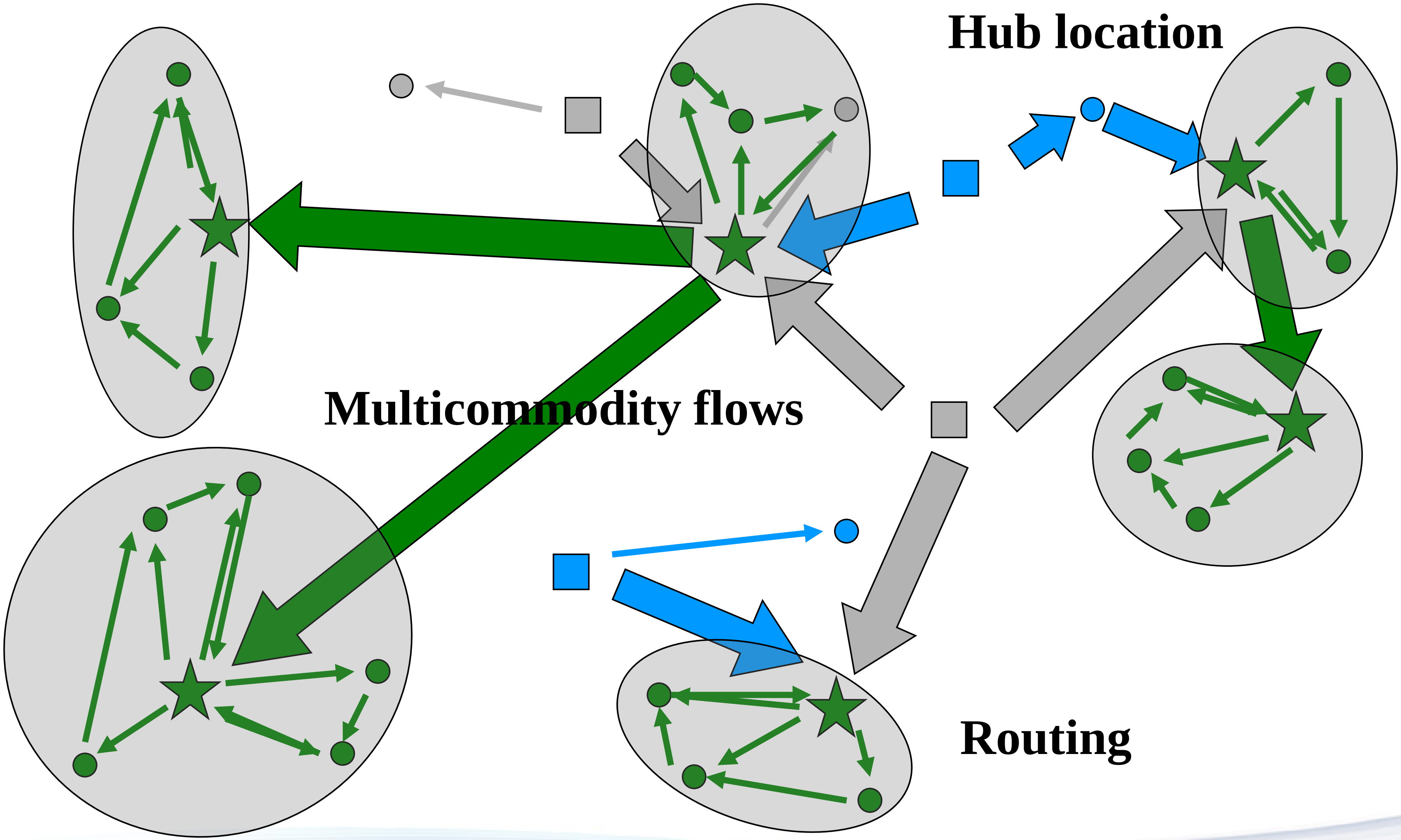
Demands, decisions and objectives

- **Demands**
  - Product, quantity, priority level
  - Origin, destination
  - Quality of service, hub operations
- **Decisions**: choice of routes and services
  - Routing of demands (goods or passengers) flows (routes, volumes, etc)
  - Selection of services (fixe & variable costs, capacities, vehicle types, hub operations, quality of service, etc.)
  - Interactions between these decisions
- **Objectives**: optimize costs and quality of service

Freight transport: a general description [Fournier 2008]

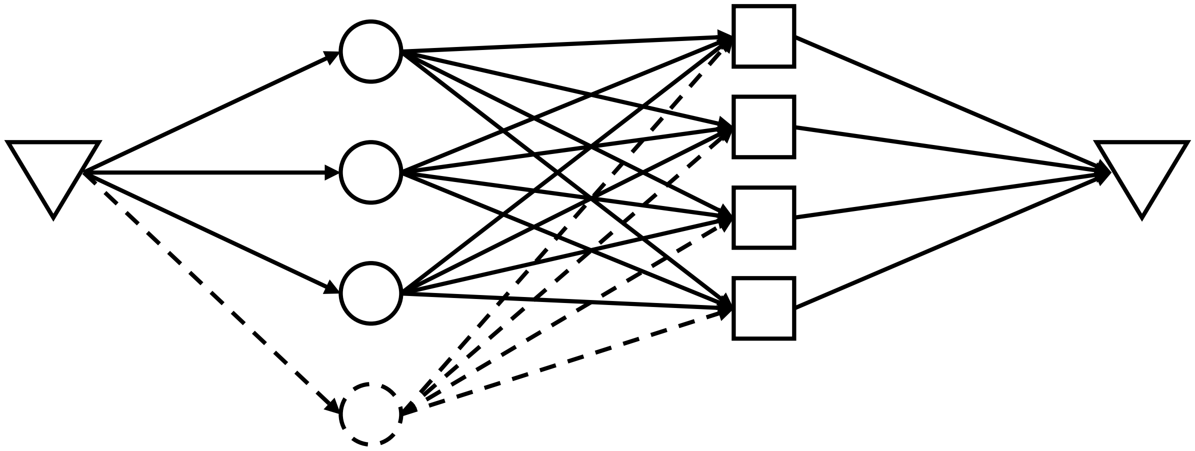


« Hub-and-Spoke » graph



A basic transportation problem

		Sale centres				
Transport cost per unit		1	2	3	4	Productions
Factories	1	10	3	7	5	10
	2	1	20	13	15	20
	3	8	4	16	11	30
	Demands	5	25	15	35	



ILP modelling (1/2)

- Network
  - Vertices=Factory *i* and sale centre *j*
  - Arcs=transport link

$$i, j \in T = U \cup Cv$$

$$s = (i, j) \in S; i, j \in T$$

- Data
  - Variable transport cost *i* → *j*
  - Production p_i and demands d_j

$$c_{ij}$$

$$p_i, d_j$$

- Decisions
  - Flow variables to transport x_{ij} units from factory *i* to sale centre *j*

$$x_{ij} \in \mathbb{N}$$

ILP modelling (2/2)

Minimize $$\sum_{i \in U} \sum_{j \in Cv} c_{ij} x_{ij}$$

s.t.

$$\forall i \in U, \sum_{j \in Cv} x_{ij} = p_i$$

- i factory

$$\forall j \in Cv, \sum_{i \in U} x_{ij} \leq d_j$$

- j sale centre

$$x_{ij} \in \mathbb{N}$$

- Entirety of flow variables

Service network design MILP modelling (1/2)

- Network
  - Vertices=terminals/hubs
  - Arcs=services

$$i, j \in T$$

$$s = (i, j) \in S; i, j \in T$$

- Data
  - Fixe cost and capacity to operate service s
  - Variable cost of service s for product p
  - Demands on product p

$$f_s, w_s$$

$$c_{sp}, p \in P$$

$$D_p$$

- Decisions
  - Binaries variables for services s
  - Flow variables for product p transported by service s

$$y_s \in \{0,1\}$$

$$x_{sp} \geq 0$$

Service network design MILP modelling (2/2)

Minimize $\sum_{s \in S} f_s y_s + \sum_{p \in P} \sum_{s \in S} c_{sp} x_{sp}$

s.t.

Product flow conservation

$$\forall p \in P, \sum_{s_{i-} \in S} x_{sp} - \sum_{s_{i+} \in S} x_{sp} = \begin{cases} -D_p & \text{if i is the origin} \\ 0 & \text{if i is a hub} \\ D_p & \text{if i is the destination} \end{cases}$$

$$\forall s \in S, \sum_{p \in P} x_{sp} \le w_s y_s$$

Coupling/capacity constraints

$$x, y \in \Phi$$

Other specific constraints

$$x_{sp} \ge 0 \qquad y_s \in \{0,1\}$$

Arc formulation of a multi-product capacitated problem, with fixed costs and transshipment, it is NP-hard.

Extensions

- **Take into account frequences**: variables y become integers

  → Increase the number of combination
- **Take into account schedules**: discretize time by adding a time indice

  → Increase the number of combination

  → Synchronization problem at the transsphipment
- **Take into account specific constraints on services like** vehicle repositionning

  → Second flow problem for the vehicles

→ ***Solving is very difficult !***

Use cases

- Basic Transportation (flow problem)
- Transportation with hub
- Transportation of multi-products (multicommodity flow problem)
- SE example