	A	B	C	D	E	F	G	H	I	J	K	
I	(7)	6	4	(7)	5	(7)	3	2	0	1	4	
		1	4	7	5	7	4	5		8	11	
IUA	0	1	4	(7)	5	(7)	3	2	0	1	4	
		6	3		2	4	9	9		8	8	
IUAUD	0	1	3	0	2	(4)	3	2	0	1	(4)	4

$C_{11}^{3} \quad \binom{11}{3} = 165$

$C_{1000}^{50} = ?$

MILP

Data!!!

Data:

$[D_{ij}]_{i \in \mathcal{C}, j \in H}$

Set of the cities $= \mathcal{C} = \{A, B, \dots, K\} \Rightarrow |\mathcal{C}| = 11$

Set of potential hubs $= H = \mathcal{C}$

$k = 3$

Decision variables:

Y_j = City j has an opened hub $= \begin{cases} 0 & \text{not opened} \\ 1 & \text{opened} \end{cases}$ $j \in \mathcal{C}$

X_{ij} = City i is served by hub $j = \begin{cases} 0 & \text{not served} \\ 1 & \text{served} \end{cases}$ $i \in \mathcal{C}, j \in \mathcal{C}$

(D_{ij}, X_{ij}) $i \leftarrow j$

JER

Obj. function: "K-center"

$$\text{Min}\left(\max_{i\in\mathcal{C},\, j\in H} D_{ij}\cdot X_{ij}\right)$$

Not linear

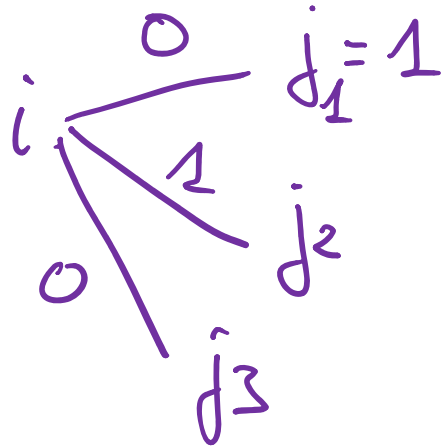
$$\text{Min}\quad \lambda$$

$$\text{s.t.}\quad D_{ij}\cdot X_{ij} \le \lambda,\ \forall i\in\mathcal{C},\ \forall j\in H \quad (|\mathcal{C}|\times|H|)$$

$$\sum_{j\in H} D_{ij}\, x_{ij} \le \lambda,\ \forall i\in\mathcal{C} \quad (|\mathcal{C}|)$$

Constraints :

$$\Rightarrow \sum_{j \in H} y_j = k \quad (1)$$



$$\sum_{j \in H} x_{ij} = 1, \quad \forall i \in \mathcal{C} \quad (|\mathcal{C}|)$$

if $x_{ij} = 1$ then $y_j = 1$

$$x_{ij} \leq y_j \quad \forall i \in \mathcal{C}, \forall j \in H \quad (|\mathcal{C}| \times |H|)$$

$$\sum_{i \in \mathcal{C}} x_{ij} \leq |\mathcal{C}| y_j \quad \forall j \in H \quad (|H|)$$

Obj function 2 : "K-median"

$$\text{Min} \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{C}} D_{ij} \cdot x_{ij}$$

Obj function 3 : Opening costs of the hubs h_j

$$\text{Min} \left(\sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{C}} D_{ij} \cdot x_{ij} + \sum_{j \in \mathcal{C}} h_j y_j \right)$$

"Uncapacitated Facility Location"