1 Definitions

$G = (V, E)$ is a unoriented graph. $\omega(G)$ is the clique number, $\Delta(G)$ the maximum degree of G. We will denote for $v \in V$ by $G - v$ the induced subgraph of G that contains all the elements of V but v.
Let $C := \{C_1, C_2, ..., C_k\}$ be a set of such that $\forall i \leq k, |C_i|$ is K_1, K_2 or K_3. The triangle number of G is the minimal size of C. We denote it by $\Omega(G)$.

2 Lemmas

Lemma 2.1. *For any graph G, $\Omega(G) \leq |A|$.*

Remark. *Proof is trivial; there is equality if $\omega(G) = 2$, so for forests, in particular.*

Lemma 2.2. *Let V_1, V_2 be a partition of V such that for $v_1 \in V_1$, $v_2 \in V_2$, there exist no path between v_1 and v_2. Then $\Omega(V) = \Omega(V_1) + \Omega(V_2)$.*

Remark. *Proof is probably less easy but should not be too complicated. Discussing this lemma allows us to only think about connected graphs.*

Theorem 2.3 (Triangle number of complete graphs)**.**

$$\Omega(K_n) = \binom{n-1}{2}\frac{n}{3}$$

Proof. It suffice to count how many triangles there are in $G = K_n$.
Consider $v \in V$. Notice that $d(v) = n - 1$. As $G = K_n$, when we choose any two edges that have an end in v, the other two ends are neighbours themselves. Thus, we have exactly $\binom{n-1}{2}$ triangles that contains v in K_n. By doing that for every vertex of G, and dividing by 3 as we counted each triangle 3 times, we obtain the result. □

The following lemmas are corollaries:

Lemma 2.4.

$$\Omega(G) \leq \binom{|V|-1}{2}\frac{|V|}{3}$$

Lemma 2.5.

$$\Omega(G) \geq \binom{\omega(G)-1}{2}\frac{\omega(G)}{3}$$