

Assignment 2

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Tuesdays from 1:15PM to 3PM in Extranef 126

1. **Normal distribution for stock returns (45 points).** Using the Python code developed in the first lecture, download daily stock returns over the period December 31st, 1999, to December 31st, 2021 from Yahoo finance for the following 5 companies: Apple, General Electrics, Goldman Sachs, Microsoft, and Procter & Gamble.
 - (a) For each stock compute the mean and variance of daily simple returns.
 - (b) Plot the empirical density function of stock returns (i.e., a histogram estimate of the underlying density function) and compare to (i) the normal distribution with the same mean and variance as the empirical distribution and (ii) the normal distribution with mean and variance of the ‘winsorized’ empirical distribution. To compute the ‘winsorized’ empirical distribution, simply keep only the daily returns with absolute value less than 4%.

(Effectively you should make one separate graph for each stock with the empirical histogram and two different normal densities, corresponding to the two calibrations of means and variances).

(Hint: If \mathbf{S} is a `panda DataFrame`, then the method `S.plot.bin` might be useful.)
 - (c) For each stock, compute the 95% and 99% Value-at-Risk and Conditional Expected Shortfall from the empirical distribution of returns, and compare to the Value-at-Risk and Conditional Expected Shortfall, that would obtain if the distributions were normal with corresponding means and variances.

(Hint: If \mathbf{S} is a `panda DataFrame`, then the method `S.quantile` might be useful. Also, you might want to import the function `scipy.stats.norm` which contains a lot of useful methods linked to the normal distribution.)
 - (d) Based on your computations, does the normal distribution seem to describe appropriately the distribution of daily stock returns?

- (e) Compare your results from the task (b) with empirical density function of 2-year and 10-year bond returns (Us bond from Yahoo finance). (If you can find)

Plot the empirical density function of two bonds (i.e., a histogram estimate of the underlying density function) and compare to a normal distribution with the same mean and variance as the empirical distribution (one separate graph for each bond with the empirical histogram and normal densities, corresponding to the two calibrations of means and variances). Effectively, repeat the same analysis as in (b) and compare results of bonds' analysis and stocks' analysis.)

2. Mean-Variance Frontier with 2 risky assets (45 points).

Download weekly stock returns for Apple and Procter & Gamble for the period 01/01/2010-31/12/2021 from Yahoo finance. Assume the risk-free rate over the period was 1.5%.

- (a) Estimate the expected return and standard deviation for each stock, as well as the correlation between both stocks.
- (b) Plot the mean-standard deviation frontier you can achieve by combining both risky assets.
- (c) Compute the portfolio weights of the global minimum variance portfolio, as well as its expected return, standard deviation, and Sharpe Ratio.
- (d) Plot the mean-standard deviation frontier if you can also invest in the risk-free asset (i.e., combining the two risky assets and the risk-free asset).
- (e) Suppose you run a fund with 10 million USD and you would like to increase your wealth by 50% in one year in expectation. What is the optimal portfolio (i.e, the optimal dollar investment in the three assets)?
- (f) What is your implied risk-aversion coefficient?
- (g) What is the volatility of your portfolio (expressed both in percentage and in dollar terms)? What is its Sharpe ratio?