UNIVERSITI TEKNOLOGI MALAYSIA

TEST 1

SEMESTER I - 2017/2018

SUBJECT CODE	**:**	**SCSI1013**
SUBJECT	**:**	**DISCRETE STRUCTURE**
TIME	**:**	**2 HOURS (8.00 P.M - 10.00 P.M)**
DATE	**:**	**23rd OCTOBER 2017**

INSTRUCTIONS TO THE STUDENTS:

Please answer all questions in the answer booklet.

QUESTION 1 **[10 MARKS]**

a) Sketch Venn diagrams that show the universal set, **U**, the sets A and B, and a single element x in each of the following cases:

(i) $x \in A$; $x \notin B$; $B \subset A$ (2 marks)

(ii) $x \in A$; $x \in B$; A is not a subset of B; B is not a subset of A (2 marks)

b) Prove the following identity, stating carefully which of the set laws you are using at each stage of the proof.

$$B \cup (\varnothing \cap A) = B$$ (4 marks)

c) In a certain programming language, all variable names have to be 3 characters long. The first character must be a letter from 'a' to 'z'; the others can be letters or digits from 0 to 9. If L = {a, b, c, ... , z}, D = {0, 1, 2, ..., 9}, and V = {permissible variable names}, use a Cartesian product to complete:

$V = \{pqr \mid (p, q, r) \in \ldots\ldots \}$

(2 marks)

QUESTION 2 **[20 MARKS]**

a) Suppose that the variable x represents student, $F(x)$ means *"x is freshman,"* and $M(x)$ means *"x is a math major"*. Determine the symbolic statement equivalent to the following statement.

i) No math major is a freshman. (1 mark)

ii) Some freshmen are math major. (1 mark)

b) Let $P(x, y): 2x + y = 1$, where the domain of discourse is the set of all integers. What are the truth values of the following:

i) $\forall x \exists y P(x, y)$ (2 marks)

ii) $\forall x \forall y P(x, y)$ (2 marks)

c) Evaluate the following logical equivalent using truth table.

$$(\neg p \rightarrow (q \vee r)) \equiv (q \rightarrow (p \vee r))$$ (5 marks)

e) Prove the following compound proposition using logical law.

$$(P \vee \neg Q) \vee (R \wedge (Q \vee \neg Q)) \equiv R \vee (P \vee \neg Q)$$

(4 marks)

f) Proof directly that,

"If m is an even integer and n is an odd integer then $m+n$ is an odd"

(5 marks)

QUESTION 3 **[25 MARKS]**

a) $A = \{(1,3),(2,4),(-4,-8),(3,9),(1,5),(3,6)\}$. For all $(a,b),(c,d) \in A$. Determine R, a relation defined on A as

$$(a,b)R(c,d) \Leftrightarrow ad = bc$$ (5 marks)

b) Determine whether the following relation is reflexive, symmetric, antisymmetric and/or transitive. (Note: Use diagram to assist your justification and you can define an appropriate set if necessary).

i) R is relation on the set of people where $(a,b) \in R$, if a is taller than b.

(4 marks)

ii) R is a relation on the set of all web pages where $(a,b) \in R$, if everyone who has visited web page a also visited web page b.

(3 marks)

iii) R is a relation on the set of people in the Facebook friend list, where $(a,b) \in R$, if a is a friend of b.

(3 marks)

c) Determine whether the relation presented by the following matrices is equivalence relation.

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

(10 marks)

QUESTION 4 **[14 MARKS]**

a) Which of the following relations are functions from A to B. Write their domain and range. If it is not a function, give a reason.

i) $R = \{(1,-2),(3,7),(4,-6),(8,1)\},\ A = \{1,3,4,8\},\ B = \{-2,7,-6,1,2\}$

(2 marks)

ii) $R = \{(1,0),(1,-1),(2,3),(4,1)\},\ A = \{1,2,4\},\ B = \{0,-1,3,1,0\}$

(2 marks)

i) $R = \{(1,-1),(2,-2),(3,-3),(4,-4),(5,-5)\},\ A = \{0,1,2,3,4,5\},\ B = \{-1,-2,-3,-4,-5\}$

(2 marks)

b) Prove that

$f : X \to Y$ defined by $f(x) = 3y - 7$ is a one-to-one function.

(4 marks)

c) If $f(x) = \sqrt{x+1}$ and $g(x) = x^2 + 2$

i) Calculate $f \circ g$ (2 marks)

ii) Calculate $g \circ f$ (2 marks)

QUESTION 5 **[16 MARKS]**

a) Determine s_5 if $s_0, s_1, s_2, \ldots.$ is a sequence satisfying the given recurrence relation and initial conditions.

$s_n = 2s_{n-1} + s_{n-2} - s_{n-3}$ for $n \geq 3,\ s_0 = 2,\ s_1 = -1,\ s_2 = 4$

(3 marks)

b) A consumer purchased items costing RM280 with a department store credit card that charges 1.5% interest per month compounded monthly. Write a recurrence relation and initial condition for b_n, the balance of the consumer's account after n months if no further charges occur and the minimum monthly payment of RM25 is made.

(3 marks)

c) Write a recursive algorithm to find the n term of sequence defined by $a_1 = 2,\ a_2 = 3,$ and $a_n = a_{n-1}a_{n-2}$ for $n \geq 3$. Then, use the algorithm to trace a_7.

(10 marks)

Theorem for Logic

Indempotent Laws	$p \wedge p \equiv p$ $p \vee p \equiv p$
Double negation laws	$\neg\neg p \equiv p$
Commutative law	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Absorption Laws	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$
De Morgan's Laws	$\neg (p \wedge q) \equiv (\neg p) \vee (\neg q)$ $\neg (p \vee q) \equiv (\neg p) \wedge (\neg q)$