CONFIDENTIAL

UNIVERSITI TEKNOLOGI MALAYSIA

TEST 1

SEMESTER I 2016/2017

SUBJECT CODE	**: SCSI1013**
SUBJECT NAME	**: DISCRETE STRUCTURE**
YEAR/COURSE	**: 1SCSJ/SCSR/SCSV/SCSB/SCSD**
TIME	**: 2 HOURS (8.00 PM – 10.00 PM)**
DATE	**: 13 OKTOBER 2016**
VENUE	**: BK1-6, N28**

INSTRUCTIONS TO THE STUDENTS:

Answer all questions in the answer booklet.

NAME	
IC NO	
SECTION	
LECTURER	

(This question paper consist of 5 pages including this pages)

Question 1 **[10 marks]**

a) Let $A = \{a, \{a\}, b, \{a, b\}\}$. State whether the following statement is TRUE or FALSE.

i. $\{a\} \subseteq A$ (1 mark)

ii. $\{a, b\} \in A$ (1 mark)

b) Given set $B = \{x \mid x \in Z, x^2 + 6 \leq 10\}$, where Z is the set of integers. List the elements of set B. (1 mark)

c) A group of 35 students attended English class for one semester. In order to sit for final exam, they need to have at least 80% attendance. 10 students have 100% attendance. 15 students have $\geq$90% attendance. 30 students have $\geq$75% attendance. 5 students have lower than 75% attendance and 8 are not allowed to sit for final exam.

i. How many students are not allowed to sit for final exam with attendance < 80% but $\geq$ 75%? (1 mark)

ii. Find the number of students with attendance <90% but $\geq$ 80%? (1 mark)

d) The following Venn Diagram in Figure 1 shows set A, B and C. Find $(A \cap B \cap C) \times (B - C)$. (2 marks)

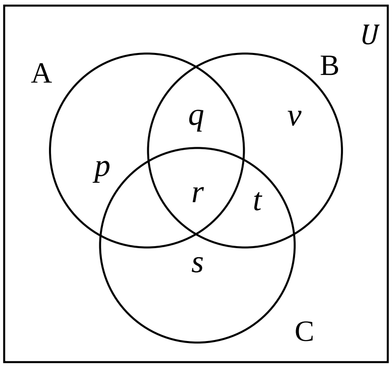

Figure 1

e) Let A and B be sets. Use properties of sets to simplify $(A \cup B') \cap (B \cap C)'$.
(3 marks)

Question 2 **[25 marks]**

a) Base on the propositions $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$, rewrite the given statements using logic symbols.

$\boldsymbol{p}$: Power button is on
$\boldsymbol{q}$: Computer shuts down
$\boldsymbol{r}$: Computer displays start screen

i. Computer displays start screen only with the condition that the power button is on. (1 mark)

ii. If the power button is on then the computer does not shuts down or displays start screen. (2 marks)

iii. Once the power button is off, it is either the computer shuts down or does not displays start screen. (2 marks)

b) State the truth value for the given compound propositions below provided the truth values for $\boldsymbol{p}$ and $\boldsymbol{r}$ is **FALSE** and $\boldsymbol{q}$ is **TRUE**.

i. $(p \rightarrow \neg q) \wedge \neg(r \vee q)$ (2 marks)
ii. $(\neg p \wedge \neg q) \rightarrow (p \vee \neg r)$ (2 marks)
iii. $\neg(\neg p \leftrightarrow \neg q) \wedge r$ (2 marks)

c) Let $P(x, y) = (x * y)^2 \geq 1$. Given the domain of discourse for x and y is set of integer, **Z**. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\exists x \exists y P(x, y)$ (2 marks)
ii. $\forall x \forall y P(x, y)$ (2 marks)

d) Use truth table to check if the compound propositions A and B are logically equivalent.

$$A = (\neg p \vee q) \rightarrow r$$
$$B = \neg q \leftrightarrow (\neg p \vee r)$$

(6 marks)

d) Simplify the following compound propositions using Laws of Logic.

$$(a \vee b) \wedge \neg (\neg a \wedge b)$$

(3 marks)

Question 3 **[25 marks]**

a) Let R be the relation from $X = \{1, 2, 3, 4\}$ to $Y = \{1, 3, 5, 7\}$ defined by xRy if and only if $x \in X$, $y \in Y$ and $x + 3y \leq 12$.

i. List the elements of the set R. (3 marks)
ii. Find the domain of R. (1 mark)
iii. Find the range of R. (1 mark)

b) Let $A = \{a, b, c\}$ and $R: A \longrightarrow A$. Draw the digraph representing the following properties:

i. Symmetric and reflexive but not transitive (5 marks)
ii. Reflexive but not anti-symmetric. (3 marks)
iii. Write a matrix representing an irreflexive relation R on a set A. (3 Marks)

c) Let $B = \{d, e, f, g\}$ and R be the relation on B that has the matrix M_R

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

i. List in-degrees and out-degrees of all vertices. (4 marks)
ii. Determine whether the relation R on the set B represented by M_R is an equivalence relation. Explain. (5 marks)

Question 4 **[10 marks]**

Here are two functions f:{1, 2, 3} → {10, 11, 12, 13} and g:{10, 11, 12, 13}→ {4, 5, 6} whose rules are given in Table 1.

Table 1

x	1	2	3
$f(x)$	11	13	10

x	10	11	12	13
$g(x)$	4	5	4	6

a) What is $f(1)$? (1 mark)

b) What is $g(11)$? (1 mark)

c) Draw arrow diagrams for f and g. (3 marks)

d) Which of these compositions can be defined: $g \circ f$, $g \circ g$, $f \circ g$, or $f \circ f$? (1 mark)

e) For any of the compositions above that are defined, give the domain and codomain, and draw the arrow diagram. (4 marks)

Question 5 **[10 marks]**

a) Let, f: $\boldsymbol{R} \to \boldsymbol{R}$ be the function with rule $f(x) = 5x{-}7$

i. Show that f is one-to-one. (2 marks)
ii. Find the inverse of f. (2 marks)

(note: $\boldsymbol{R}$ is the set of real numbers)

b) Let $X=\{-1, 0, 1\}$ and $Y = \{-2, 0, 2\}$. For each $x \in X$, define functions f: $X \to Y$ and g: $X \to Y$ by:

$$f(x) = x^2 - x$$
$$g(x) = 2x$$

Determine if f and g are one-to-one, onto Y, and/or bijection. (6 marks)

Question 6 **[10 marks]**

A robot can take steps of 1, 2 or 3 meters. As examples (Table 2):

Table 2

Distance	Sequence of Steps	Number of Ways to Walk
1	1	1
2	1, 1 or 2	2
3	1, 1, 1 or 1, 2 or 2,1 or 3	4
4	1, 1, 1, 1 or 1, 1, 2 or 1, 2, 1 or 2, 1, 1 or 2, 2, or 1, 3 or 3,1	7

Let $walk(n)$ denote the number of ways the robot can walk n meters. We have observed that,

$walk(1) = 1$, $walk(2) = 2$, $walk(3) = 4$.

Now suppose that $n>3$. Since the walk must begin with either a 1-meter, 2-meter or a 3-meter step, all of the ways to walk n meters are accounted for. We obtain the formula,

$$walk(n) = walk(n-1) + walk(n-2) + walk(n-3)$$

For example,

$$walk(4) = walk(3) + walk(2) + walk(1) = 4+2+1 = 7$$

a) Write the recursive algorithm to calculate the number of ways the robot can walk n meters. (6 marks)

b) Find $walk(6)$. (4 marks)

*** end of questions ***