# Chapter 1

Part 3

## Fundamental and Elements of Logic

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# Why Are We Studying Logic?

## Some of the reasons:

- Logic is the foundation for computer operation
- Logical conditions are common in programs

  **Example**:

  **Selection**: `if (score <= max) { ... }`

  **Iteration:** `while (i<limit && list[i]!=sentinel) ...`
- All manner of structures in computing have properties that need to be proven (and proofs that need to be understood).

  **Example**: Trees, Graphs, Recursive Algorithms, . . .
- Programs can be proven correct.
- Computational linguistics must represent and reason about human language, and language represents thought (and thus also logic).

# PROPOSITION

A **statement** or a **proposition**, is a declarative sentence that is **either TRUE or FALSE, but not both.**

**Example:**

- 4 is less than 3.
- 7 is an even integer.
- Washington, DC, is the capital of United State.

# Example

i) Why do we study mathematics?
ii) Study logic.
iii) What is your name?
iv) Quiet, please.

**The above sentences are not propositions. Why ?**

(i) & (iii) : is question, not a statement.
(ii) & (iv) : is a command.

# Example

i) The temperature on the surface of the planet Venus is 800 F.

ii) The sun will come out tomorrow.

**Propositions? Why?**

- Is a statement since it is either true or false, but not both.
- However, we do not know at this time to determine whether it is true or false.

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# CONJUNCTIONS

- Compound propositions formed in English with the word "**and**"
- Formed in logic with the caret symbol **("$\wedge$")**
- **True** only when **both participating propositions are true**.

**TRUTH TABLE:** This tables aid in the evaluation of **compound propositions**.

| p | q | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

**True (T), False (F)**

# Example

p : 2 is an even integer
q : 3 is an odd number

**propositions**

p ∧ q

**symbols**

: 2 is an even integer and 3 is an odd number

**statements**

---

p : today is Monday
q : it is hot

p ∧ q: today is Monday and it is hot

# Example

## Proposition

p : 2 divides 4

q : 2 divides 6

## Symbol & Statement

$p \wedge q$: 2 divides 4 and 2 divides 6.

or,

$p \wedge q$: 2 divides both 4 and 6.

# Example

## Proposition

p : 5 is an integer

q : 5 is not an odd integer

## Symbol & Statement

$p \wedge q$: 5 is an integer and 5 is not an odd integer.

or,

$p \wedge q$: 5 is an integer but 5 is not an odd integer.

# DISJUNCTION

- Compound propositions formed in English with the word “**or**”,
- Formed in logic with the caret symbol (“ V ”)
- **True** when **one or both** participating propositions are true.

The truth table for $p \vee q$

| p | q | p Vq |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

# Example

***p***: 2 is an integer
***q***: 3 is greater than 5

***p*** ∨ ***q*** : 2 is an integer or 3 is greater than 5

***p*** : 1+1=3
***q*** *:* A decade is 10 years

***p*** ∨ ***q*** **:** 1+1=3 or a decade is 10 years

# Example

***p*** : 3 is an even integer
***q*** : 3 is an odd integer

$\mathbf{p} \vee \mathbf{q}$

3 is an even integer or 3 is an odd integer

or

3 is an even integer or an odd integer

# NEGATION

Negating a proposition simply flips its value. Symbols representing negation include:

$$\neg x\ ,\ \overline{x}\ ,\ {\sim}x,\ x' \quad \text{(NOT)}$$

Let *p* be a proposition.
The negation of *p, written* ¬ *p*
is the statement obtained by negating
statement *p.*

The truth table of ¬ *p*

| ***p*** | **¬*p*** |
|---|---|
| T | F |
| F | T |

# Example

$p$ : 2 is positive

$\neg p$ : 2 is not positive.

$p$ : 4 is less than 3

$\neg p$ : 4 is not less than 3.

# CONDITIONAL PROPOSITIONS

Let $\boldsymbol{p}$ *and* $\boldsymbol{q}$ be propositions.

"if $\boldsymbol{p}$, *then* $\boldsymbol{q}$"

is a statement called a **conditional proposition**, written as

$$p \rightarrow q$$

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# CONDITIONAL PROPOSITIONS

The truth table of $p \to q$

*=> Cause and effect relationship*

FALSE if $p$ = True and $q$ =false

| p | q | p→ q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

TRUE if both true or $p$=false for any value of $q$

# Example

***p*** : today is Sunday

***q*** : I will go for a walk

***p* → *q* :** If today is Sunday, then I will go for a walk.

***p*** : I get a bonus

***q*** : I will buy a new car

***p*** → ***q***: If I get a bonus, then I will buy a new car

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# Example

***p*** : x/2 is an integer.

***q*** : x is an even integer.

p → q : if x/2 is an integer, then x is an even integer.

# BICONDITIONAL

Let ***p*** *and* ***q*** *be propositions.*

"***p*** *if and only if* ***q***"

is a statement called a **biconditional proposition**, written as

$$p \leftrightarrow q$$

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# BICONDITIONAL

The truth table of $p \leftrightarrow q$:

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

# Example

***p*** *:* my program will compile
***q*** *:* it has no syntax error.

***p*** $\leftrightarrow$ ***q*** : My program will compile if and only if it has no syntax error.

***p*** : x is divisible by 3
***q*** : x is divisible by 9

***p*** $\leftrightarrow$ ***q*** : x is divisible by 3 if and only if x is divisible by 9.

# LOGICAL EQUIVALENCE

- The compound propositions ***Q*** and ***R*** are made up of the propositions $p_1, \ldots, p_n$.

- ***Q*** and ***R*** are logically equivalent and write,

  $Q \equiv R$

  provided that given any truth values of $p_1, \ldots, p_n$, either ***Q*** and ***R*** are **both true** or ***Q*** and ***R*** are **both false**.

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# Example

$\boldsymbol{Q} = p \rightarrow q$ $\boldsymbol{R} = \neg q \rightarrow \neg p$

Show that, $\boldsymbol{Q} \equiv \boldsymbol{R}$

The truth table shows that, $Q \equiv R$

| $p$ | $q$ | $p \rightarrow q$ | $\neg q \rightarrow \neg p$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | F |
| F | T | T | T |
| F | F | T | T |

# Example

Show that, $\neg (p \rightarrow q) \equiv p \wedge \neg q$

The truth table shows that, $\neg (p \rightarrow q) \equiv p \wedge \neg q$

| $p$ | $q$ | $\neg(p \rightarrow q)$ | $p \wedge \neg q$ |
|---|---|---|---|
| T | T | F | F |
| T | F | T | T |
| F | T | F | F |
| F | F | F | F |

# PRECEDENCE OF LOGICAL CONNECTIVES

| | Precedence of logical connectives is as follows: | | |
|---|---|---|---|
| not | ¬ | ↑ | Highest |
| and | $\wedge$ | | |
| or | $\vee$ | | |
| If…then | $\rightarrow$ | | |
| If and only if | $\leftrightarrow$ | | Lowest |

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# Example

Construct the truth table for, $\mathbf{A} = \neg(p \vee q) \rightarrow (q \wedge p)$

## Solution

| $p$ | $q$ | $(p \vee q)$ | $\neg(p \vee q)$ | $(q \wedge p)$ | A |
|---|---|---|---|---|---|
| T | T | T | F | T | T |
| T | F | T | F | F | T |
| F | T | T | F | F | T |
| F | F | F | T | F | F |

# LOGIC & SET THEORY

Logic and set theory go very well togather. The previous definitions can be made very succinct:

$$x \notin A \text{ if and only if } \neg(x \in A)$$

$$A \subseteq B \text{ if and only if } (x \in A \rightarrow x \in B) \text{ is True}$$

$$x \in (A \cap B) \text{ if and only if } (x \in A \wedge x \in B)$$

$$x \in (A \cup B) \text{ if and only if } (x \in A \vee x \in B)$$

$$x \in A - B \text{ if and only if } (x \in A \wedge x \notin B)$$

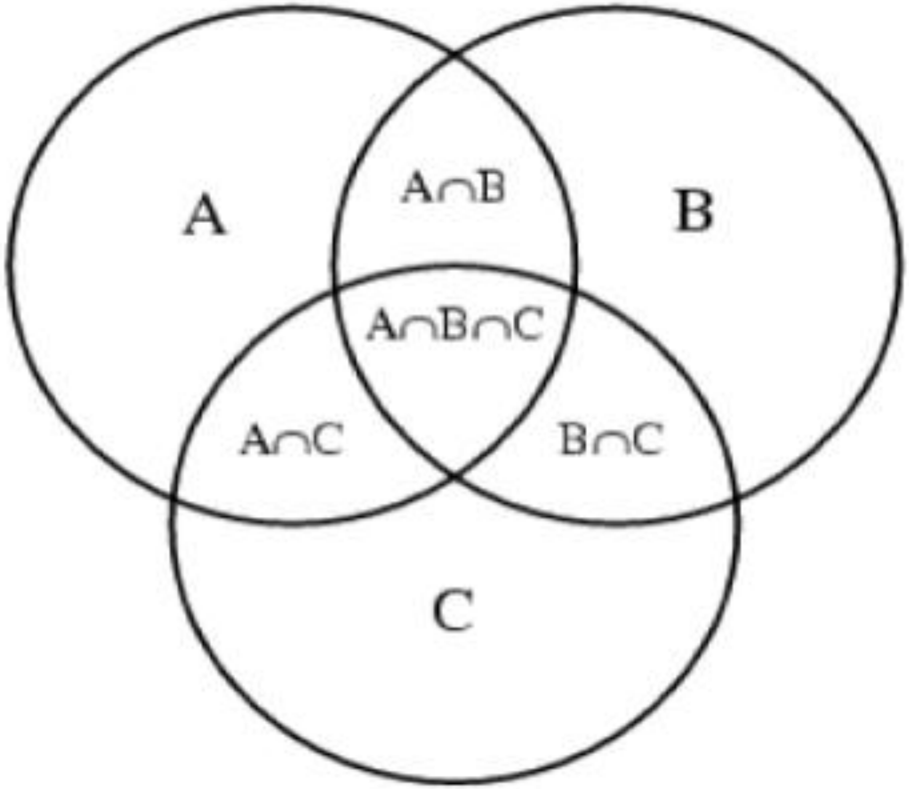
$$x \in A \Delta B \text{ if and only if } (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)$$

$$x \in A' \text{ if and only if } \neg(x \in A)$$

$$X \in P(A) \text{ if and only if } X \subseteq A$$

# Venn Diagrams

Venn Diagrams are used to depict the various unions, subsets, complements, intersections etc. of sets.



# Logic and Sets

## are closely related

| Tautology | Set Operation Identity |
|---|---|
| $p \vee q \leftrightarrow q \vee p$ | $A \cup B = B \cup A$ |
| $p \wedge q \leftrightarrow q \wedge p$ | $A \cap B = B \cap A$ |
| $p \vee (q \vee r) \leftrightarrow (p \vee q) \vee r$ | $A \cup (B \cup C) = (A \cup B) \cup C$ |
| $p \wedge (q \wedge r) \leftrightarrow (p \wedge q) \wedge r$ | $A \cap (B \cap C) = (A \cap B) \cap C$ |
| $p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$ | $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ |
| $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$ | $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ |
| $p \wedge \neg q \leftrightarrow p \wedge \neg (p \wedge q)$ | $A - B = A - (A \cap B)$ |
| $p \wedge \neg (q \vee r) \leftrightarrow (p \wedge \neg q) \wedge (p \wedge \neg r)$ | $A - (B \cap C) = (A - B) \cup (A - C)$ |
| $p \wedge \neg (q \wedge r) \leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r)$ | $A - (B \cup C) = (A - B) \cap (A - C)$ |
| $p \wedge (q \wedge \neg r) \leftrightarrow (p \wedge q) \wedge \neg (p \wedge \neg r)$ | $A \cap (B - C) = (A \cap B) - (A \cap C)$ |
| $p \vee (q \wedge \neg r) \leftrightarrow (p \vee q) \wedge \neg (r \wedge \neg p)$ | $A \cup (B - C) = (A \cup B) - (C - A)$ |
| $p \wedge \neg \vee (q \wedge \neg r) \leftrightarrow (p \wedge \neg q) \vee (p \wedge r)$ | $A - (B - C) = (A - B) \cup (A \cap C)$ |

The above identities serve as the basis for an "algebra of sets".

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# Logic and Sets
## are closely related

**Tautology**

$p \wedge p \leftrightarrow p$

$p \vee p \leftrightarrow p$

$p \wedge \neg(q \wedge \neg q) \leftrightarrow p$

$p \vee \neg(q \wedge \neg q) \leftrightarrow p$

**Set Operation Identity**

$A \cap A = A$

$A \cup A = A$

$A - \varnothing = A$

$A \cup \varnothing = A$

**Contradiction**

$p \wedge \neg p$

$p \wedge (q \wedge \neg q)$

$p \wedge \neg p$

**Set Operation Identity**

$A - A = \varnothing$

$A \cap \varnothing = \varnothing$

$A - A = \varnothing$

The above identities serve as the basis for an "algebra of sets".

# Theorem for Logic

Let ***p***, ***q*** and ***r*** be propositions.

## Idempotent laws:

$p \wedge p \equiv p$

$p \vee p \equiv p$

### Truth table

| $p$ | $p \wedge p$ | $p \vee p$ |
|---|---|---|
| T | T | T |
| F | F | F |

# Theorem for Logic

**Double negation law:**

$$\neg\neg p \equiv p$$

**Commutative laws:**

$$p \wedge q \equiv q \wedge p$$
$$p \vee q \equiv q \vee p$$

**Associative laws:**

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$
$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

# Theorem for Logic

**Distributive laws:**

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

| $p$ | $q$ | $r$ | $p \vee (q \wedge r)$ | $(p \vee q) \wedge (p \vee r)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | T | T |
| T | F | F | T | T |
| F | T | T | T | T |
| F | T | F | F | F |
| F | F | T | F | F |
| F | F | F | F | F |

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# Theorem for Logic

**Absorption laws:**

$$p \land (p \lor q) \equiv p$$
$$p \lor (p \land q) \equiv p$$

| $p$ | $q$ | $p \land (p \lor q)$ | $p \lor (p \land q)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | T | T |
| F | T | F | F |
| F | F | F | F |

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# Theorem for Logic

**De Morgan's laws:**

$$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$$
$$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$$

The truth table for $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$

| $p$ | $q$ | $\neg(p \vee q)$ | $\neg p \wedge \neg q$ |
|---|---|---|---|
| T | T | F | F |
| T | F | F | F |
| F | T | F | F |
| F | F | T | T |

# Thank You

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