CONFIDENTIAL

UNIVERSITI TEKNOLOGI MALAYSIA

TEST

SEMESTER I 2022/2023

SUBJECT CODE	**: SCSI1013**
SUBJECT NAME	**: DISCRETE STRUCTURE**
TIME	**: 2 HOURS 30 MINUTES**
DATE	**: 19 DECEMBER 2022**

INSTRUCTIONS TO THE STUDENTS:

Answer all questions in the answer booklet.

NAME	
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SECTION	
LECTURER	

(This question paper consists of **9 (NINE)** pages including this pages)

QUESTION 1 **10 MARKS**

a) If $A = \{\ p, q, q, s\}, B = \{m, n, q, s, t\} and\ C = \{m, n, p, q, s\}$. then verify the associative property of the union below. (2 marks)

$$A \ \cup (B \ \cup C) = (A \ \cup B) \cup C$$

b) By referring to the properties of set operations, show that:

$$(X \cup Y) - Z = (X - Z) \cup (Y - Z)$$ (2 marks)

c) Given $A = \{\ 2, 3, 4, 5\}$ and $B = \{4, 16, 23\}, a \ \in A, b \ \in B$, find the set of ordered pairs such that $a^2 < b$. (2 marks)

d) There are 35 students in the art class and 57 students in the science class. Find the number of students who are either in art class or in science class.

i. When two classes meet at different hours and 12 students are enrolled in both activities. (2 marks)

ii. When two classes meet at the same hour. (2 marks)

QUESTION 2 **20 MARKS**

a) Consider the statement:

"**If you try hard and you have a talent then you will get rich**"

i. Translate the statement into logic symbols. Use p, q and r to represent the propositions. Clearly state which statement is p, q and r. (2 marks)

ii. Suppose you found out that the statement was a lie even you try hard. What can you conclude? (2 marks)

iii. If you are rich but you do not try hard or have talent. Was the statement true or false? Support your conclusion. (2 marks)

b) Use truth table to check if the compound propositions A and B are logically equivalent.

$$A = \neg(p \vee (q \wedge (\mathrm{r} \rightarrow p)))$$

$$B = \neg p \wedge (q \rightarrow r)$$ (5 marks)

c) Let $P(x, y)$: $x + 2y = 1$, where the domain of discourse is the set of all integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\neg\forall x P(x)$ (2 marks)

ii. $\exists x P(x)$ (2 marks)

d) Proof that if x is an odd integer and y is an even integer then $x^2 - 2y$ is an odd integer using direct proofing (5 marks)

QUESTION 3 **20 MARKS**

a) Given the digraph of relation R as in Figure 1.

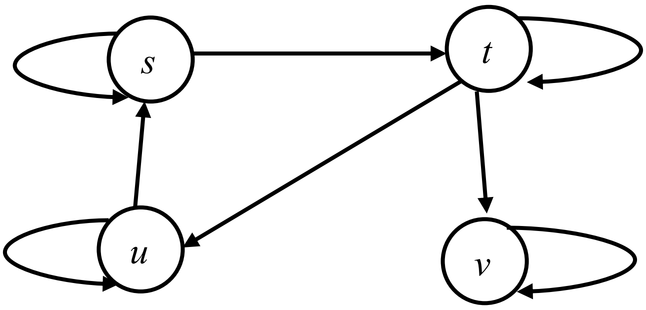

Figure 1

i. List the elements of R as a set of ordered pairs. (2 marks)

ii. What is matrix representation of R? (2 marks)

iii. Is the relation of R antisymmetric, asymmetric or/and partial order relation? Give justification for each of your answers. (5 marks)

b) Determine which of the relations are functions from $A = \{1, 2, 3\}$ to $B = \{4, 6, 8, 10\}$. Give justification for each of your answers.

i. $R_1 = \{(1, 4), (2, 8), (3, 8)\}$ (1 mark)

ii. $R_2 = \{(1, 4), (1, 8), (2, 6), (3,10)\}$ (1 mark)

iii. $R_3 = \{(1, 8), (3, 4)\}$ (1 mark)

c) Let $X=\{-3, 0, 3\}$ and $Y=\{-9, 0, 9\}$. For each $x \in X$, define functions $v: X \rightarrow Y$ and $w: X \rightarrow Y$ by:

$$v(x) = x^2$$

$$w(x) = 3x$$

Determine if v and w are one-to-one, onto Y, and/or bijection. (4 marks)

d) Let f dan g be functions from the positive integers to the positive integers defined by the equations,

$$f(x) = 2x + 3, \qquad g(x) = x \div 5$$

i. Find the inverse of $f(x)$ and $g(x)$. (2 marks)

ii. Find the compositions $fogof$ (2 marks)

QUESTION 4 **10 MARKS**

As a lead computer scientist in a chemical industry plant, you are assigned to design and develop algorithms that simulate chemical reaction processes. Two chemicals *A* and *B* are combined to produce a third chemical *C*. The initial temperature F_0, of chemical *A*, is 3.0 Fahrenheit and the initial temperature F_1 for chemical *B* is 2.5. When chemicals *A* and *B* are combined to produce chemical *C*, the increment in each minute $t = 0, 1, 2, 3 \ldots$, which chemical *C* warms up to room temperature is a recurrence sequence, with F_0 and F_1 as initial conditions. For $t \geq 2$, this recurrence sequence is found by summing the previous element of the sequence (*t*-1), with one-fourth of the previous two elements of the sequence (*t*-2). From the above given information,

a) Find the recurrence relation of chemical *C* that models the warming to room temperature. (3 marks)

b) Using the recurrence relation obtained in (a), list down the sequence from $F_0, F_1, F_2, \ldots F_5$. (2 marks)

c) Write a recursive algorithm to model the warming of chemical *C*. Name your recursive function as `warmC()`. (5 marks)

QUESTION 5 **15 MARKS**

a) Suppose there are five people in a corporate army dinner, including the captain and two vice-captain that are sitting around a circular table

i. How many ways for these people to be seated around the table. (1 mark)

ii. If the captain and both vice-captains should be seated next to each other. (1 mark)

b) In a camping site, five people are staying in one camp with five allocated beds. The head of each camp must not sit next to their assistant to ensure safety of the camp in case of emergency. In that case, how many ways can you allocate the beds to all people in each camp? (4 marks)

c) Suppose there are 10 types of chocolate that are available in a chocolate store. How many ways for you to buy half a dozen chocolate:

i. If there are no restrictions. (1 mark)

ii. If there at least 4 hazelnut flavoured chocolate. (1 mark)

iii. If there are no two chocolates of the same type. (1 mark)

d) In a football match, there are thirteen players that show up to play for the next game.

i. How many ways are there to choose 11 players to take start the game? (1 mark)

ii. How many ways are there to assign 11 positions from the pool of 13 players? (1 mark)

iii. Three players from 13 that showed up are woman. How many ways to choose 11 players to start the game if at least one player must be a woman? (4 marks)

QUESTION 6 **5 MARKS**

a) A box contains three colour balls, which are red, yellow, and green. At least how many balls must be taken from the box to get two balls of the same colour? (1 mark)

b) Ten cheesecakes are shared among thirty students and two teachers. However, each cheesecake can be cut into eight pieces. Show that they can have at least three pieces of cheesecake. (2 marks)

c) From the integers in the set $\{2, 3, 4, 6, 7, 8\}$, what is the smallest number of integers that must be chosen so that at least one pair of them has a sum of 10? (2 marks)

Formulas:

Set Identities (Properties of Set)

- **Commutative laws**

  $A \cap B = B \cap A$

  $A \cup B = B \cup A$

- **Associative laws**

  $A \cap (B \cap C) = (A \cap B) \cap C$

  $A \cup (B \cup C) = (A \cup B) \cup C$

- **Distributive laws**

  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

- **Absorption laws**

  $A \cup (A \cap B) = A$

  $A \cap (A \cup B) = A$

- **Idempotent laws**

  $A \cap A = A$

  $A \cup A = A$

- **De Morgan's laws**

  $(A \cap B)' = A' \cup B'$

  $(A \cup B)' = A' \cap B'$

- **Complement laws**

  $A \cap A' = \varnothing$

  $A \cup A' = \mathrm{U}$

- **Double complement laws**

  $(A')' = A$

- **Complement of U and $\varnothing$**

  $\varnothing' = U$

  $U' = \varnothing$

- **Properties of universal set**

  $A \cup U = U$

$A \cap U = A$

- **Set difference laws**

  $A - B = A \cap B'$

- **Identity laws**

  $A \cup \varnothing = A$

  $A \cap U = A$

- **Properties of empty set**

  $A \cup \varnothing = A$

  $A \cap \varnothing = \varnothing$

Permutations (Repetition is allowed)

$$P_n \quad n^r$$

Permutations (Repetition is not allowed)

$$P(n, r) = \frac{n!}{(n-r)!}$$

Permutations (*n* objects, *k* different types)

$$P(n) = \frac{n!}{(n_1! n_2! \ldots n_k!)}$$

Combinations (without Repetition)

$$C(n, r) = {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Combinations (Repetition Allowed)

$$C(n+r-1, r) = \binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$