1 Model 1 (Borrowing Constraints)

2 Elements

2.1 We need

important for efficiency: heterogeneity in returns to college

- first order reason: graduation rates depend on ability

borrowing constraints

- therefore consumption and hours in college

parental income affect transfers

- either a "transfer function" a la Keane/Wolpin (1997)
- or model parental transfers (probably more attractive)
- I currently have the parent as a "stub." We could make this full-blown OLG (but this seems overkill)

uncertain ability

- gets us imperfect ability sorting at entry
- important for efficiency, but not for fitting data (until we have earnings that depend on ability)
- having ability and learning about it will only matter with endogenous dropouts (which we will eventually need for comparative statics)

preference shocks at entry

- given ability uncertainty, they only serve a numerical purpose (make sure someone enters college for all param values)

I write the model for one cohort. There are no interactions between cohorts at this point

2.2 Notation

a: ability
d: age
j: type
J: number of types

k: assets
l: work time in college
N_a: size of ability grid
p: college cost
T_s: time spent in college
t: time
y: lifetime earnings
y_p: parental earnings
z: parental transfer for college

2.2.1 Greek

η: pref shock at entry
$\pi(a)$: prob of graduating

3 Demographics

model period is 1 year

- not needed but simplifies notation and mapping into data

agents enter the model at model age 1 (age 18) as HS grads
they live for T periods

4 Endowments

Each agent is endowed with a type $j \in \{1, ..., J\}$
All agents of type j share the same values for

- parental income y_p
- ability signal m
- college cost p
- college wage w_c
- "free" consumption in college and leisure in college: $\bar{c}(j), \bar{l}(j)$
  - purpose: reduce marginal utility of consumption and leisure of high ability students in college, so they do not work to little, consume too much

We may not want all of those to vary by type, but we can allow for this.

The agents also has an ability a that is correlated with m

- Ability is not observed until year 2 in college or start of work
- The agent knows $\Pr(a|j)$
- Note: currently, a is really not needed. But we keep it for later.

5 Preferences

The family makes a joint decision that maximizes

$$u_p(c_p) + \mathbb{E}\sum_{d=1}^{T_s}\beta^{d-1}u\left(c_d + \bar{c}(j), 1 + \bar{l}(j) - l_d\right) + \mathbb{E}\sum_{d=T_s+1}^{T}\beta^{d-1}u_w(c_d) \qquad (1)$$

The family likes consumption by parents and children; plus leisure in college

In addition, there are preference shocks for college.

6 Sequence of events

Household is born at age $d = 1$ and observes j. he also observes a preference shock.

Household decides whether to enter college (value V_1) or work as HSG with value V_w.

Conditional on college entry, household decides to split parental income between parental consumption and transfer to the child.

If the child enters college, he is committed to staying in college for 2 years. He chooses consumption, hours, saving subject to a borrowing constraint.

At the end of year 2, the student learns a. He drops out with probability $1 - \pi(a)$; he then must work as a college dropout. Otherwise, he must stay in college for 2 more years, again choosing consumption, hours, and saving subject to a borrowing constraint. At the end of 4 years in college, graduation is certain.

Once a household starts working, all uncertainty is resolved. He maximizes lifetime utility subject to a lifetime budget constraint.

7 Household solution

7.1 College entry decision

Year of birth:

- observe j
- observe pref shocks (so z can vary between college entry and non-entry)
- decide college entry
- decide c_p, z.
- budget constraint: The parent makes T_c equal transfers (one each year). For tractability, we put them into k_1.

Note: If z is paid out in each period,

- it becomes an additional state variable.
- this is expensive b/c the hh problem then needs to solved completely for each candidate z.
- in computing aggregates, we can choose how to map the combination of z and k_1 into data.

7.1.1 Equations

Budget constraint:
$y_p = c_p + z$

- no test, direct in solution code

$k_1 = T_c z$

- test: test_bc1.college

Choice of z:

$$V_{HS}\left(z_w,j\right) = \max_{z_w \geq 0} T_c u_p\left(y_p - z_w\right) + \mathbb{E}_a\left\{V_w\left(T_c z_w, HS, a\right)|j\right\} \tag{2}$$

$$V_{entry}\left(z_c, j\right) = \max_{z_c \geq 0} T_c u_p\left(y_p - z_c\right) + V_1\left(T_c z_c, j\right) \tag{3}$$

- test: no test; directly implemented as objective function in `college_entry`
- for numerical purposes: add $\omega_z \ln z$ to parental utility (so that transfers do not get stuck at 0).

Entry decision

$$\max\left\{V_{HS}\left(z_w, j\right) + \bar{\eta}_c - \gamma\eta_w, V_{entry}\left(z_c, j\right) - \gamma\eta_c\right\} \tag{4}$$

- test: no test; only used through entry probability below

where
$$\mathbb{E}_a V_w | j = \sum_a \Pr(a|j) \, V_w \left(T_c z, HS, a\right) \tag{5}$$

- Test: Equation directly implemented in `value_work`. No test

Probability of entry is then given by
$$\Pr(college|j) = \frac{\exp\left(V_{entry}/\gamma\right)}{\exp\left(V_{entry}/\gamma\right) + \exp\left(\left[\mathbb{E}V_{HS} + \bar{\eta}\right]/\gamma\right)} \tag{6}$$

- test: no test; directly implemented in `hh_solve`

7.1.2 Notation

η are type I extreme value shocks.

$\bar{\eta}_c$ is the mean preference for working as HSG. Its model purpose is to match college entry rates.

The T_c factor in front of parental utility just simplifies the intuition. When borrowing constraints do not bind, we expect $u_p' = u_c$ (consumption smoothing)

7.1.3 Algorithm

1. Find optimal z for the college option
   (a) make a continuous approximation of V_1
   (b) max over interval $T_c z \in [0, \min\{k_{max}, y_p - cFloor\}]$
2. Find optimal z for the non-college option
3. Pick the max.

Computational note:

- if all / none go to college with nonnegligible probability, bound entry probs away from 0 and 1.

7.2 Years 1-2 in college:

Household who has decided to enter college. Household is committed to remain in college for 2 years.

- state: $k_1 = z$, j
- choose c_d, l_d (constant across first 2 years in college)

7.2.1 Equations

budget constraint in college: $k_{d+1} = Rk_d + 2\left(w_{coll}l_d - c_d - p\right)$
borrowing limit: $k_{d+1} \geq k_{min,d+1}$, $l \in [0,1]$.
Bellman

$$V_1\left(k,j\right) = \max_{k',l}\left(1+\beta\right)u\left(Rk/2 + w_{coll}l - p - k'/2, 1-l\right) + \beta^2 V_m\left(k',j\right) \quad (7)$$

Test: `test_bc1.test_saved12` .

7.2.2 Algorithm:

This is the same problem as years 3-4, except for the continuation value.
But I don't have a representation for the marginal value of k'.
Easier to max the RHS over c, where l is chosen from static condition

7.3 End of periods 1-2 in college

Draw graduation shock: with probability $1-\pi\left(a\right)$ drop out

- or choose to drop out (we don't have that yet)
- a is NOT known to agent, but he updated beliefs about it in light of graduation outcome

Value:

$$V_m\left(k,j\right) = \sum_a \Pr\left(a|j\right)\left[\left(1-\pi\left[a\right]\right)V_w\left(k,a,CD\right) + \pi\left[a\right]V_3\left(k,j\right)\right] \quad (8)$$

Test: inside `coll_value_m`

7.4 Years 3-4 in college

Student has drawn a successful graduation outcome at the end of period 2. He is committed to staying in college for 2 more years, after which he starts working as CG.
graduation at end of period 2 is certain
Budget constraint is the same as in years 1-2
Beliefs: $\Pr\left(a|j,grad\right)$
Bellman

$$V_3(k,j) = \max_{k',l} (1+\beta)\, u(Rk/2 + w_{coll} l - p_j - k'/2, 1-l) + \beta^2 \sum_a \Pr(a|j, grad)\, V_w(k', a, CG) \tag{9}$$

subject to $k' \geq k_{min,5}$, $l \geq 0$

Test: `test_bc1.college`

FOC:

k': $(1+\beta)\, u_c/2 \geq \beta^2 \mathbb{E}_a \partial V_w(k', a, CG)/\partial k'$ with equality if bc does not bind (`hh_eedev_coll3`) (not used)

l: $u_c w_{coll} \leq u_l$ with equality if $l > 0$ (tested in `hh_static_bc1`)

Assuming that utility is additively separable

Note:

- it could help with debt stats if the household does not know whether or not he will graduate when choosing consumption

7.4.1 Algorithm:

1. Try $k' = k_{min}$
   (a) find c, l that satisfy budget constraint, static condition
   (b) if Euler deviation > 0: done
2. Interior solution
   (a) Search over $c \in [c_{floor}, c_{max}]$. At c_{max} we hit corner.
   (b) Get l from static condition. Bound below from 0.
   (c) Get k' from budget constraint
   (d) find 0 of Euler dev
   (e) Closed form solution for $V_{k'}$

7.5 Beliefs after learning graduation outcome

$\Pr(a|j, grad) = \Pr(a, j, grad) / \Pr(j, grad)$

$\Pr(j, grad) = \Pr(grad|j) \Pr(j)$

$\Pr(j|grad) = \sum_a \Pr(a|j) \Pr(grad|a)$

$\Pr(a, j, grad) = \Pr(j) \Pr(a|j) \Pr(grad|a)$

Test: `pr_a_jgrad_test.m` by simulation

7.6 Work

state: k_d, s, a

receive present value of lifetime earnings $Y(s,a)$

choose consumption the usual way (perfect credit markets)

$$V_w(k,a,s) = \sum_{d=1}^{T-T_s} \beta^{t-1} u_w(c_d) \tag{10}$$

Euler: $c^{-\sigma} = \beta R (c')^{-\sigma}$. Therefore $c'/c = (\beta R)^{1/\sigma}$.

Present value of lifetime income: $Rk_d + Y(s,a)$

Present value of consumption $= \sum_{d(s)}^{T} R^{d(s)-d} c_{d(s)} g_c^{d(s)-d}$

Budget constraint equates the present values.

Test: `test_bc1.work`

7.7 IQ

$IQ = m + \varepsilon_{IQ}$ (up to scale)

$\varepsilon_{IQ} \sim N(0, \sigma_{IQ})$

Divide into percentile groups (e.g. quartiles).

All I need to know is Pr(IQ group | j)

Test: `test_bc1.pr_xgroup_by_type` (simulation)

8 Computing Aggregates

Need aggregates by [IQ, school, type]. Type determines parental income etc.

These can be computed from aggregates by [s, a, j]

Mass(s,a,j) has closed form solution.

Mass(s,IQ,j) = sum over a (mass(s,a,j) * pr(IQ|a))

8.1 Debt stats

Counting all transfers as paid out in period 1 understates debt (before the end of college).

Correct budget constraint:

$\hat{k}_{d+1} = R\hat{k}_d + 2(w_{coll} l_d - c_d - p + z)$

Iterate on this, starting from $\hat{k}_1 = 0$, to find correct debt stats.

This makes surprisingly little difference (`aggr_show`).

9 Calibration

9.1 Functional Forms

9.1.1 Preferences:

$$u(c,l) = \left[\omega_c \frac{c^{1-\varphi_c}}{1-\varphi_c} - 1\right] + \omega_l \frac{(1-l)^{1-\varphi_l}}{1-\varphi_l} \tag{11}$$

$$u_p(c_p) = \omega_p \left[\frac{c_p^{1-\varphi_p}}{1-\varphi_p} - 1\right] \tag{12}$$

$$u_w(c) = \omega_w \frac{c^{1-\varphi_c}}{1-\varphi_c} \tag{13}$$

Normalize $\omega_c = 1$.

Note: we want the household to value leisure rather than dislike work. Otherwise the household never chooses $l = 0$.

9.1.2 Endowments:

Joint Normal: z, m, p

I implement this by drawing independent standard Normal random variables for each endowment. Then correlate them using weights called $\alpha_{x.y}$. This is a detail. In effect, we are getting a joint Normal distribution.

- correlation parameters: $\alpha_{m,p}$ etc.
- marginal distributions: μ_p, σ_p etc.
- $m \sim N(0,1)$

$a \sim N(0,1)$

- correlation with m governed by $\alpha_{a,m}$.

Free college consumption / leisure:

- $\bar{c}(j) = (m_j - m_{\min}) \times \beta_c / (m_{\max} - m_{\min})$
- In words: $\bar{c}$ is linear in m such that the lowest type gets 0 and the highest type gets β_c
- Analogous for $\bar{l}$.

With small J (e.g. 80), the realized means and std devs can be quite far from the targets. To avoid that, I scale all random vars to be N(0,1).

Table 1: Fixed Model Parameters

Table 2: Calibrated Model Parameters

9.1.3 College

Graduation probability is logistic:

$$\pi(a;t) = \pi_0 + \frac{\pi_1 - \pi_0}{\left[1 + \pi_a / \exp\left(\pi_b\left[a - a_0\right]\right)\right]^{1/\pi_c}}$$

Set $\pi_c = 1$, $a_0 = 0$. Calibrate the rest.

9.1.4 Work

CPS measures average lifetime earnings by (s, c): $\bar{Y}(s, c)$
$Y(s, a, c) = \phi_s(a - \bar{a}) + \hat{e}(s, c) + \bar{Y}(s, c)$
ϕ_s from one of our previous papers
$\hat{e}(s, c)$ is calibrated to match $\bar{Y}(s, c)$
skill premium for given a: $\Delta\phi_s(a - \bar{a})$ + common stuff

- set $\bar{a}$ arbitrarily so that $a - \bar{a} > 0$.

9.2 Fixed Parameters

1 shows fixed model parameters.
Mostly standard choices.
Directly from data

- age wage profiles $\bar{e}(d; s, c)$ (CPS)
- borrowing limits $k_{min,d,c}$ (finaid.org)

φ_l: we don't really have any evidence on this. Set arbitrary for now.

9.3 Calibrated parameters for the NLSY79 or 97 cohort

2 shows calibrated parameters. The values are of course still invented.

9.4 Calibration targets for NLSY79 or 97 cohort

9.4.1 Endowments

Directly from data:

- y_p quartile means (NLSY) -> μ_y, σ_y.

College costs:

- p_t: tuition + fees + room/board (?) - scholarships - grants + supplies
- p: match mean from time series source (Chris?)
- p std dev:
  - take ratio of std dev / mean from HS&B
  - assume constant over time
- assume that std/mean stays constant over time
- also try with room and board; and including private

Correlations:

- y,m: match mean log y_p by IQ quartile (NLSY)
- p,m: match mean p by IQ quartile (HSB)
- y,p: match mean p by y_p quartile (HSB?, not constructed +++)
- a,m: match fraction CD, CG by IQ quartile (NLSY)
- σ_{IQ}: fix for now at what we estimate from other papers (either Todd and Lutz's or Oksana and Lutz's)

9.4.2 Preferences

γ: size of pref shocks at college entry; match fraction CD, CG by IQ quartile
ω_l: weight on leisure; match hours worked in college (NLSY)

- mean by IQ, y_p (NLSY)
- unconditional mean hours: set to 20/week, unless we have direct data (NLSY only)

ω_c: match average **debt** (at end of college, grads and dropouts) (NLSY)

- earlier cohorts: average student debt (Trends in Student Aid)

ω_p : match mean transfer by y_p, IQ quartile (could also be used to match curvature of u_p) (NELS)

9.4.3 College

$\pi(a)$ parameters: match fraction CD, CG by IQ quartile

9.4.4 Other

w_{coll}: match college earnings and hours (NLSY)
College earnings:

- NLSY: mean by IQ, yp

10 Time series calibration

Detrending:

- all dollar figures are detrended
- for each cohort: construct constant composition mean earnings, ages 30-50
- scale so that mean earnings are the same as for reference cohort
- `cal_targets.m`

We want to replicate

- college entry pattern by IQ, fam income
- college graduation pattern by IQ, fam income
- average debt
- financing shares
- hours worked in college

10.1 Time varying parameters

10.1.1 Directly from data

$k_{min,d,c}$:

- directly from statutory borrowing limits

w_{coll}

- match earnings in college (source?)
- for now: keep constant (in stationary model)

$\pi(a)$

- could have one parameter time varying to match graduation rate (not done)

10.1.2 Calibrated

$\hat{e}(s)$: match lifetime earnings by s
μ_p: match mean college cost
$\bar{\eta}_c$: preference for work as HSG

- match college entry rate overall

$\alpha_{a,m}$: signal precision:

- match college entry rates, graduation rates by IQ

ω_p: weight on parental utility

- match financing shares

10.1.3 Currently fixed

μ_y: mean log parental income: treat as fixed (stationary model)

10.1.4 Group sizes vary with cohort

E.g. family income is not in quartiles.
If close to quartiles: interpolate (e.g. Project Talent)

10.2 Time series calibration targets

All targets that are available.
Not available:

- financing: transfers, earnings

10.2.1 How college is financed

share of total spending from earnings and loans
Data (`cal_targets`)

- for cohorts born before 1950: Hollis (1957)
- total = 100 - scholarships - veterans
- family = family + 0.5 * other
- earnings = earnings + 0.5 * other

Model

- Total spending $= 2(c+p)$
- Earnings: $2w_{coll}l$
- Debt: k' (if < 0)
- Family: the rest.

11 Interesting counterfactuals

1. Vary one observable input at a time
   (a) college premium (really: earnings profiles)
   BUT: must also vary $\hat{e}$. That requires to first calibrate each cohort.
   (b) borrowing limits
   (c) college costs (need to recalibrate that one parameter to match 1 target)
2. Replicate all data for a cohort
   (a) recalibrate $\bar{\eta}$, signal precision, ω_p, $\hat{e}$
   (b) also calibrate μ_p (because it directly matches a data moment)
3. Hold all parameters fixed, except those taken directly from data (e.g. college costs, earnings profiles and $\hat{e}$, borrowing limits)

12 Notes

12.1 Organization

Steps:

1. Calibrate all params for reference cohort (NLSY79)
2. Calibrate a subset of parameters for every other cohort (experiments)
3. Counterfactuals: Set selected parameters to values for other cohorts (e.g. borrowing limits or present value of earnings by ability / schooling).

12.2 Parental transfers

Key: how responsive to changes in tuition, borrowing ...?

Altruism cannot work

Transfers are a tiny fraction of parental income.

They will (at least for rich parents) almost fully offset changes in financial conditions of the child (e.g. tuition). At least for the NLSY cohort (few borrowing limits).

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With altruism, transfers are too steep in parental income.

Transfers approximately equal MU(c) across generations.

Double parental income => double parental and child consumption.

Child hours fall (static condition)

The entire increase in consumption must come out of transfers.

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We need to fundamentally rethink transfers.

We should really use better data to pin this down (NELS). It's important for our paper.

12.3 Time series scaling

Everything, including preference shocks, must scale properly as we take the model back in time.

How to scale pref shocks is not clear.

Sidestep this problem by detrending the model.

1. Compute average earnings for the baseline cohort
   (a) weights: for now simple; age range 30-50; school weights = fraction by s from base cohort
2. For each cohort: compute a scale factor that makes average earnings the same as base cohort
3. Multiply all dollar variables by that scale factor

Result display: divide all dollar figures by the scale factor (inconvenient)

12.4 Corners

For calibration, should prevent a situation where nobody /everybody enters college for some parameter values.

13 eof