

Bequests, Inter Vivos Transfers, and Wealth Distribution¹

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This paper constructs a heterogeneous agent overlapping generations model with bequests and inter vivos transfers. In the model, households in the same family line behave strategically to determine their consumption, working hours, gifts, and savings. Calibrating the model to the U.S. economy, the paper measures time preference and parental altruism consistent with the economy's capital-output ratio and the size of intergenerational transfers. The model with intergenerational transfers better explains, although not fully, the wealth distribution of the United States. The paper also analyzes the effects of government policy changes on wealth accumulation, distribution, and social welfare. *Journal of Economic Literature* Classification Numbers: D31, D64, D91, H31. © 2002 Elsevier Science (USA)

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1. INTRODUCTION

Macroeconomic analyses usually rely on either an infinite horizon model or an overlapping generations (OLG) model. Those analyses implicitly assume a household either perfectly altruistic toward its descendants or completely selfish. But, when economists evaluate fiscal policies that involve income redistribution among generations, the policy implication depends critically on the extent to which the Ricardian equivalence proposition holds.

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In Nishiyama (2000), I developed a heterogeneous agent overlapping generations model with altruistic and accidental bequests. The model assumes both imperfect altruism and lifetime uncertainty, and it captures the strategic behavior between a parent household and its adult child households. However, several panel data sets show a significant number of inter vivos transfers between a parent household and its child households, which the previous model does not consider (e.g., Gale and Scholz, 1994). This simplification is justified if the capital market is perfect. But, if borrowing constraints exist, it is also beneficial for an altruistic parent household to make transfers before it ceases to exist.

How will the economy change if we relax the model to allow parent households to make inter vivos transfers when the borrowing constraint of their child households is binding? More specifically, this paper answers the following questions: Can the model equipped with inter vivos transfers, as well as altruistic and accidental bequests, replicate the wealth distribution of the United States? If not, to what extent will the introduction of bequests and inter vivos transfers improve the model in explaining the skewed wealth distribution? The paper also analyzes how fiscal policy changes, such as estate tax repeal or a federal income tax cut, will affect national wealth, inequality, and social welfare.

To answer these questions, this study extends a heterogeneous agent overlapping generations model by adding one-sided altruism, lifetime uncertainty, and borrowing constraints, and it measures the degrees of time preference and parental altruism through the calibration of the model to the U.S. economy.² One of the main features of this extended model is that it involves both a dynasty economy (with one-sided altruism and borrowing constraint) and a pure life-cycle economy as two opposite cases. It is likely that the economy has imperfectly altruistic households, and it can be shown as an economy located between those two extremes.

The main parameters—time preference and parental altruism—are obtained simultaneously through the calibration of the model so that the steady-state equilibrium is consistent with the key statistics observed in the United States: capital-output ratio and the relative size of intergenerational transfers. For the steady-state economy to be consistent, a parent household would have to consider the utility of each adult child household 37 to 49% less than it considers its own utility.³

² The model was initially constructed as a two-sided altruism model in which child households could also make some gifts to their parents. However, the degree of child altruism implied by the model turns out to be negligible because the size of financial transfers from children to parents is very small.

³ The implied discount rate (the degree of altruism) depends on the coefficient of relative risk aversion as well as the wealth distribution in the steady-state equilibrium.

The calibrated model replicates the distributions of earnings, income, and wealth of the United States fairly well. The Gini coefficients of the distributions of the baseline economy are close to those in the data. Also, an improvement exists in replicating the wealth distribution. By introducing altruistic and accidental bequests to the model, the share of wealth held by the top 1% of households rises from 15.1 to 17.1%. By adding inter vivos transfers to the model, the share of wealth held by the top 1% of households rises further to 19.4% although the share is still lower than the 30.5% in the data.⁴

Under a plausible set of parameters and assumptions, the model predicts that eliminating estate tax would increase national wealth by between 1.6 and 2.8% in the long run, and that cutting the income tax rate by 10% would increase national wealth by between 2.3 and 3.7%.⁵ Both of those policy changes would only modestly intensify the inequality of wealth. However, introducing a perfect annuity market to the economy would reduce national wealth by between 6.0 and 9.1% and would intensify the wealth disparity significantly.

The rest of the paper is laid out as follows: Section 2 discusses previous literature about bequests and inter vivos transfers; Section 3 describes the economy and the extended model; Section 4 shows the calibration of the model, the obtained main parameters, and the characteristics of the steady-state economy; Section 5 implements policy experiments to examine the effects of intergenerational transfers on wealth accumulation, inequality, and social welfare; and Section 6 concludes the paper. Appendixes explain the algorithm of computing equilibria, the optimal annuity holdings, and welfare measures.

2. PREVIOUS LITERATURE

This paper constructs an altruistic model of bequests and inter vivos transfers based on the strategic behavior between a parent household and its adult child households. Clearly, this is not the first attempt to construct a heterogeneous agent overlapping generations model with altruistic bequests and inter vivos transfers.⁶

⁴ The share in the data is calculated from the sample of married households of ages 30–89 in the 1998 Survey of Consumer Finances. The calibration assumes that the coefficient of relative risk aversion is equal to 2.0.

⁵ The numbers depend on whether the economy is assumed to be closed or small and open, and how the government finances those tax cuts.

⁶ The earlier literature on an overlapping generations model with bequests and gifts (but without earnings ability shock) includes Buiter (1979), Carmichael (1982), and Burbidge (1983). Lord and Rangazas (1991) constructed a general equilibrium model with altruistic bequests and human capital investment.

Laitner (1992) constructed a two-sided perfect altruism model with liquidity constraints in which households receive lifetime earnings shocks and make nonnegative intergenerational transfers. Fuster *et al.* (2002) added differential lifetime uncertainty⁷ to Laitner's model to evaluate the welfare effects of unfunded Social Security. De Nardi (2001) introduced a "warm glow" bequest motive, skill inheritance, and lifetime uncertainty to her heterogeneous agent OLG model. Similar to the present paper, she evaluated the effect of bequests on wealth inequality. Laitner (2001) calibrated a one-sided imperfect altruism model with lifetime earnings shock and liquidity constraints to the U.S. economy to analyze the effects of bequests and inter vivos transfers on wealth accumulation and inequality.⁸

One of the main differences of the present model from the models above is the strategic interaction between a parent household and its child households. In this paper, the households in the same family line play a simultaneous game, and the economy is shown as the Nash equilibrium.

Intergenerational transfer motives. Many empirical papers have examined to what extent bequests are intentional or accidental, and if they are intentional whether bequests and inter vivos transfers are altruistic, selfish, or strategic. Hurd (1987) compared the wealth change of old households with children with that of households without children and concluded that bequests are mostly accidental. Wilhelm (1996) showed that parents tend to leave equal bequests to each of their children even if the children's earnings differ materially, and he concluded that bequests are not altruistic.

In contrast, Menchik and David (1983) demonstrated that elderly households do not dissave and showed that bequests are intentional. Bernheim (1991) used the same data set as Hurd, the Longitudinal Retirement Household Survey, and concluded that bequests are intentional. More recently, Kopczuk and Slemrod (2000) showed a strong negative relationship between the aggregate reported estates and the level of estate taxation, using estate tax return data from 1916 to 1996.

Altruistic hypotheses. The present study considers both intentional transfers (bequests and inter vivos transfers) and accidental bequests due to lifetime uncertainty. Regarding the question of whether the intentional transfers are altruistic, selfish, or strategic, this paper assumes that intentional

⁷ In the model, high income individuals have larger lifetime expectancy than low income individuals.

⁸ Laitner (2001) obtained the coefficient of relative risk aversion as well as the degrees of time preference and parental altruism through the calibration of the model.

transfers are motivated by altruism, although a part of bequests and gifts may be selfish or strategic.⁹

One of the main criticisms of the altruistic bequest model is that bequests are in many cases divided equally by parents even if the earnings of their children differ significantly. According to Wilhelm (1996), 76.6% of parents divide their estates almost equally (within $\pm 2\%$). Another criticism of altruistic hypotheses is that inter vivos transfers from parents to children do not completely compensate for the income changes of parents and children. Altonji *et al.* (1997) showed that parents increase transfers by only 13 cents when their income increases by one dollar and that of their children decreases by one dollar.

But, in the presence of asymmetric information about children's working ability and efforts, it may be optimal for parents to offer the partial insurance on children's income shocks to avoid moral hazard.¹⁰ In fact, empirical analyses by Altonji *et al.* (1997) and Wilhelm (1996) demonstrated that inter vivos transfers and bequests, respectively, are decreasing in the recipient's income and implied that intergenerational transfers are at least partially motivated by altruism.

Implicit insurance contracts. Intergenerational transfers may be motivated by income shock and lifetime uncertainty of households in the absence of perfect insurance and annuity markets. Even if parents and children are not benevolent toward each other, it is beneficial for them to make a risk-sharing contract if they can avoid enforceability problems and adverse selection. But, if parents and children are not altruistic at all, the sum of insurance payments should be close to that of insurance benefits. According to the Survey of Consumer Finances (SCF), inter vivos transfers from parents to children are about 4 to 10 times larger than those from children to parents; if we consider other transfers—such as bequests, trusts, and life insurance—that difference becomes much larger.

Those net transfers from parents to children are not explained solely by the risk-sharing motive. Also, for the implicit annuity contract between parents and children, it is enough to distinguish accidental bequests from other intentional transfers. This is because the price of the annuity that parents render is, on average, not very different from the amount of accidental bequests.

Strategic bequest motive. Parents may want to keep their wealth in a bequeathable form even in the presence of perfect annuity markets, to

⁹ Selfish bequests are sometimes called "joy-of-giving" bequests or "bequests-as-consumption." Also, "warm glow" bequests belong to that category. Strategic bequests are sometimes called "gift-exchange" bequests. Inter vivos transfers may be due to risk-sharing arrangements between households.

¹⁰ For example, see Nishiyama and Smetters (2002).

attract their children's attention (e.g., Bernheim *et al.* 1985). In that case, parents' bequests and other transfers to their children are the payments for their children's services, such as telephone calls and visits. But, as mentioned before, Wilhelm (1996) found that a majority of bequests are divided equally. Also, Behrman and Rosenzweig (1998) showed that the relationship between the amount of bequests and the number of visits among children is not significant and they rejected the framework in which parents use threats of disinheritance to elicit more visits from their children.

3. MODEL

This section describes a four-period heterogeneous agent overlapping generations model with bequests and inter vivos transfers. This model considers both parental altruism and lifetime uncertainty, and households in this economy play a noncooperative simultaneous game to make an optimal decision about their consumption, working hours, gifts (parents only), and savings.

3.1. Economy

The model is based on a standard growth economy that consists of a large number of households, a perfectly competitive firm, and a government. Each household is assumed to act as a single person. In the calibration of the model, a household is assumed to be a married couple that makes decisions jointly. Also, there is assumed to be no strategic interaction between siblings.¹¹

The life cycle of a household. Figure 1 shows the life cycle of a household. In each period, new households are born without wealth. The lifespan of each household is either three or four periods. One period in this model corresponds to 15 years starting from the actual age of 30. So, age 1 corresponds to 30–44 years old, age 2 corresponds to 45–59 years old, age 3 corresponds to 60–74 years old, and age 4 corresponds to 75–89 years old.¹² A household dies either at the end of age 3 or at the end of age 4. The mortality rate is known, but for each household its own lifespan is uncertain. When a household reaches age 3, its child households of age 1 are “born,” and the former becomes a parent household.

¹¹ It can be easily shown that a parent household leaves equal bequests to all children as long as the children's working abilities are the same. When a simultaneous game is assumed, the number of bequests will not change even if the children cannot collude.

¹² Because of this setting, households in this model are assumed to retire at the beginning of age 60, contrary to the fact that most people retire at either age 62 or age 65 in the United States.

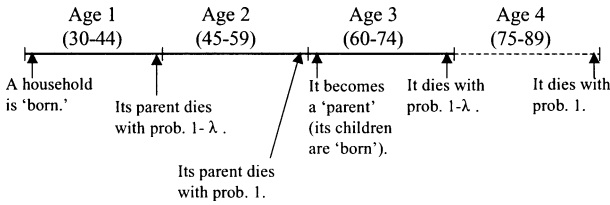


FIG. 1. The life cycle of a household.

Labor income and capital income. When a household is age 1 or 2, its working ability (labor productivity) at each age is stochastically determined. It receives labor income according to the market wage rate, its working hours, and its working ability. A household of age 3 or 4 is assumed to be retired. Though a household can work at home to produce a limited amount of consumption goods and services, its working ability is assumed to be low and deterministic. There is only one kind of asset a household can hold. It receives capital income according to its wealth level and the market interest rate. There is assumed to be a borrowing constraint, and the wealth of each household must be nonnegative.

Taxes and social security benefits. A household pays federal income tax according to its total income (the sum of labor income and capital income). A household that inherits wealth from its parent also pays federal and state estate taxes. In addition, a household of age 1 or 2 pays payroll tax for Social Security and Medicare based on its labor income. A household of age 3 or 4 receives Social Security benefits. The Social Security system is assumed to be one of the defined benefit types. Every household of age 3 or 4 is eligible for Social Security benefits and, for simplicity, the size of the benefit is assumed to be the same for all households.

Dynasty and altruism. Since each household lives either three or four periods, at any period of time there are two types of dynasties—the dynasties with both a parent household and its child households (Type I), and the dynasties with age 2 households only (Type II).¹³ Figure 2 shows the two types of dynasties in this economy. Every parent household cares about its child households and is assumed to be equally altruistic. Since a parent also knows its children are altruistic toward its grandchildren, it actually cares about all its descendants indirectly. Thus, a parent household chooses end-of-period wealth, which will be bequeathed to its child households if it dies. The wealth choice is made to maximize the weighted sum of its own utility and its children's utility.

¹³ The names, "Type I" and "Type II," were borrowed from Fuster *et al.* (2002).

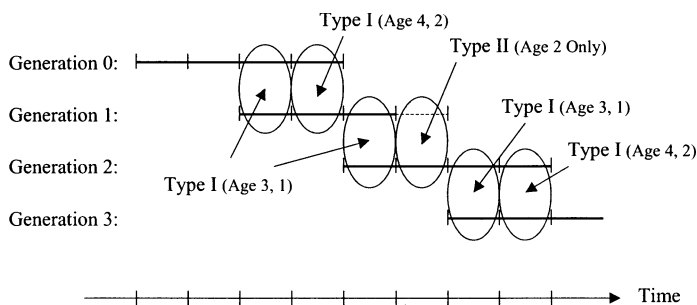


FIG. 2. Two types of dynasties.

Strategy of a parent and a child. Beginning-of-period wealth of a parent household and its child households, the working ability of the child households, and the mortality of the parent household (at the end of age 3) are known to each other. A parent and its children choose, simultaneously, their own optimal consumption, working hours, gifts (parents only), and end-of-period wealth.¹⁴ Since a parent household is altruistic toward its child households, and the children know that their parents are altruistic, the decisions of a parent and children are dependent on each other. For example, if a parent knew its children's wealth was going to be higher in the next period, it would reduce the amount of bequest since the marginal value of the bequest would be smaller. If a child knew the bequest of its parent was going to be higher, it would reduce its own savings and consume more. Also, the model exhibits the "Samaritan's dilemma" similar to that in Bruce and Waldman (1990); i.e., when intergenerational transfers are operative, the child households tend to save less in the first period to induce their parent household to make higher transfers in the second period.¹⁵

3.2. The Households' Problem

3.2.1. The Preference of a Household

Consider a household that lives either three or four periods and call it a Generation 0 household. If this household is selfish, the household's problem is shown as

$$\max_{\{c_i^0, h_i^0\}_{i=1}^4} u^0 = E \left[\sum_{i=1}^3 \beta^{i-1} u(c_i^0, h_i^0) + \lambda \beta^3 u(c_4^0, h_4^0) \right]$$

¹⁴ The numerical results will be different if we assume a two-stage game instead of a simultaneous game. Given the degrees of time preference and parental altruism, two-stage games will make national wealth smaller because of the strategic dissaving of the households who make their decisions first.

¹⁵ Models of strategic interaction are also considered in Laitner (1997).

subject to some budget constraint, where u^0 is the lifetime utility of Generation 0; $u(\cdot, \cdot)$ is an instantaneous utility function; c_i^0 and h_i^0 are consumption and working hours, respectively, at age i ; β is a time preference factor; and λ is the survival rate at the end of age 3. This household chooses its optimal consumption and working hours to maximize its lifetime utility u^0 .

Suppose that Generation 0 considers not only the utility from its own consumption and leisure (say, $h_{\max} - h_i^0$), but also the utility of Generation 1, that Generation 1 considers the utility of Generation 2 as well, and so on. Then the total utility of Generation 0 (including the utility from its descendants) U^0 is shown as

$$\begin{aligned} U^0 &= \max_{\{c_i^0, h_i^0\}_{i=1}^4} E[u^0 + \varphi U^1] \\ &= \max_{\{c_i^0, h_i^0\}_{i=1}^4} E\left[u^0 + \varphi \left\{ \max_{\{c_i^1, h_i^1\}_{i=1}^4} u^1 + \varphi \left\{ \max_{\{c_i^2, h_i^2\}_{i=1}^4} u^2 + \cdots \right\} \right\}\right], \end{aligned}$$

where φ denotes the discount factor by the parent household on the utility of its child households. This paper assumes that $\varphi = \beta^2 \eta n < 1$, where n is the number of child households per parent household and η is the degree of parental altruism. Since the consumption and leisure of the child households occur two periods later, the utility is discounted by β^2 . The parent cares about its children proportionally to the number n of its child households. The degree of parental altruism η shows how much the parent household cares about each of its adult child households relative to how much it cares about itself in the same period.

This paper decomposes φ into the degree of time preference β and the degree of parental altruism η (per child) or ηn (in total) through the calibration using the aggregate statistics of national wealth and intergenerational transfers.

3.2.2. The State of a Dynasty

Since the utility maximization problem of a household is nested as shown above, in the following sections, the preference of households is described by value functions.

For Type I dynasties, the state of each dynasty is shown by the ages of parent and child households $\{(3, 1), (4, 2)\}$, the beginning-of-period wealth of the parent $a_p \in A = [0, a_{\max}]$ and that of its children $a_k \in A$, and the labor productivity (which determines hourly wage) of the children $e_k \in E = [e_{\min}, e_{\max}]$. In the calibration, $e_{k,i}$ is a member of $\{e_i^1, e_i^2, \dots, e_i^{N_e}\}$ for age $i = 1$ or 2 , and it follows a Markov process. For Type II dynasties, the state of each dynasty is simply shown by the age $\{2\}$, the beginning-of-period wealth $a \in A$, and the working ability $e \in E$.

For notational simplicity, let \mathbf{s}_I and \mathbf{s}_{II} denote the states of a Type I dynasty and a Type II dynasty, respectively, where

$$\mathbf{s}_I = (a_p, a_k, e_k) \quad \text{and} \quad \mathbf{s}_{II} = (a, e).$$

Then the value function of a Type I household of age i is denoted as $v_{I,i}(\mathbf{s}_I)$, and the value function of a Type II household of age 2 is denoted as $v_{II,2}(\mathbf{s}_{II})$.

3.2.3. Type I Households

An age 3 parent and its age 1 child households. Let c_p , h_p , g_p , and a'_p denote the parent household's consumption, working hours, inter vivos transfers to its child households, and end-of-period wealth level (normalized by the steady-state economic growth), respectively. Similarly, let c_k , h_k , and a'_k denote each of its child household's consumption, working hours, and end-of-period wealth level, respectively. Also, let Φ_k denote the parent household's conjecture about its child households' decision, let n denote the number of child households, let λ denote the survival rate at the end of age 3, let r denote the rate of return on the capital, let w denote the wage rate per efficient unit of labor, let μ denote the growth rate of the economy, let $\tau_F(\cdot)$ be a federal income tax function, let $\tau_S(\cdot)$ be a payroll tax function for Social Security and Medicare, let $\tau_E(\cdot)$ be a federal and state estate tax function, let tr_{SS} denote Social Security benefits, and let tr_{LS} denote lump-sum transfers.

The value function of an age 3 parent household is shown as

$$v_{I,3}(\mathbf{s}_I; \Phi_k) = \max_{c_p, h_p, g_p, a'_p} \{u(c_p, h_p) + \eta nu(c_k, h_k) + \beta E[\lambda v_{I,4}(\mathbf{s}'_I) + (1 - \lambda)\eta nv_{II,2}(\mathbf{s}'_{II}) | e_k]\} \quad (1)$$

subject to

$$a'_p = \frac{1}{1 + \mu} \{we_p h_p + (1 + r)a_p + tr_{SS} + tr_{LS} - \tau_F(ra_p) - c_p - g_p\} \geq 0, \quad (2)$$

where \mathbf{s}_I is the state of this dynasty,

$$\mathbf{s}_I = (a_p, 0, e_k),$$

Φ_k is the parent's conjecture of its child households' decision,

$$\Phi_k = (c_k, h_k, a'_k),$$

and the law of motion of the state of this dynasty is

$$\begin{aligned} \mathbf{s}'_I &= (a'_p, a'_k, e'_k), \\ \mathbf{s}'_{II} &= (a'_k + (a'_p - \tau_E(a'_p))/n, e'). \end{aligned} \quad (3)$$

The parent household chooses its optimal consumption c_p , working hours (housework only) h_p , inter vivos transfers g_p , and end-of-period wealth level a'_p , taking the decision of its child households Φ_k as given. It discounts the utility of each of n child households by η . At the end of age 3, the parent household dies with probability $1 - \lambda$. The value of this household at the beginning of the next period is the weighted average of its own future value $v_{I,4}$ (when this household is alive) and its n children's future value $nv_{II,2}$ discounted by η . The term $E[\cdot | e_k]$ denotes a conditional expectation given that the current working ability of an age 1 child household is e_k ; i.e.,

$$E[v_{I,4}(s'_I) | e_k] = \int_E v_{I,4}(s'_I) \pi_{1,2}(e'_k | e_k) de'_k,$$

where $\pi_{1,2}(e'_k | e_k)$ is a conditional probability of the working ability being e'_k in the next period. Equation (2) is a budget constraint of this parent household. When the parent household dies, its end-of-period wealth a'_p is split equally and bequeathed to each of n child households.

Let Φ_p denote the child household's conjecture about its parent household's decision. The value function of an age 1 child household is shown as

$$v_{I,1}(s_I; \Phi_p) = \max_{c_k, h_k, a'_k} \{u(c_k, h_k) + \beta E[\lambda v_{I,2}(s'_I) + (1 - \lambda)v_{II,2}(s'_{II}) | e_k]\} \quad (4)$$

subject to

$$a'_k = \frac{1}{1 + \mu} \{we_k h_k + (1 + r)a_k - \tau_F(we_k h_k + ra_k) - \tau_S(we_k h_k) + tr_{LS} - c_k + g_p/n\} \geq 0, \quad (5)$$

where Φ_p is the child's conjecture of its parent household's decision,

$$\Phi_p = (c_p, h_p, g_p, a'_p),$$

and the law of motion of the state is (3).

The child household chooses its optimal consumption c_k , working hours h_k , and end-of-period wealth level a'_k , taking the decision of its parent household Φ_p as given. The value of the child household at the beginning of the next period is the weighted average of its own future value when its parent is alive, $v_{I,2}$, and its future value when its parent is deceased, $v_{II,2}$. Equation (5) is a budget constraint of this child household.

Let \mathbf{d}_p and \mathbf{d}_k be the set of decisions of a parent household and each of its child households, respectively; i.e.,

$$\mathbf{d}_p = (c_p, h_p, g_p, a'_p), \quad \mathbf{d}_k = (c_k, h_k, a'_k).$$

Solving equations,

$$\mathbf{R}_3(\mathbf{d}_k; \mathbf{s}_I) = \mathbf{d}_p, \quad \mathbf{R}_1(\mathbf{d}_p; \mathbf{s}_I) = \mathbf{d}_k,$$

where $\mathbf{R}_3(\mathbf{d}_k; \mathbf{s}_I)$ and $\mathbf{R}_1(\mathbf{d}_p; \mathbf{s}_I)$ are the best response functions of a parent and its children, respectively, Nash equilibrium decision rules are obtained as

$$\mathbf{d}_{I,i}(\mathbf{s}_I) = (c_{I,i}(\mathbf{s}_I), h_{I,i}(\mathbf{s}_I), g_{I,i}(\mathbf{s}_I), a'_{I,i}(\mathbf{s}_I)), \quad g_{I,1}(\mathbf{s}_I) = 0,$$

for $\mathbf{s}_I \in A^2 \times E$ and $i \in \{3, 1\}$.

An age 4 parent and its age 2 child households. An age 4 parent household is assumed to die at the end of this period, and its child households become parent households at the beginning of the next period. So, the value function of an age 4 parent household is shown as

$$v_{I,4}(\mathbf{s}_I; \Phi_k) = \max_{c_p, h_p, g_p, a'_p} \{u(c_p, h_p) + \eta nu(c_k, h_k) + \beta \eta n E[v_{I,3}(\mathbf{s}'_I) | e_k]\} \quad (6)$$

subject to (2), where the law of motion of the state is

$$\mathbf{s}'_I = (a'_k + (a'_p - \tau_E(a'_p))/n, 0, e'_k). \quad (7)$$

The parent household considers its children's value at the beginning of the next period, $nv_{I,3}$, discounted by η . Similarly, the value function of an age 2 child household is shown as

$$v_{I,2}(\mathbf{s}_I; \Phi_p) = \max_{c_k, h_k, a'_k} \{u(c_k, h_k) + \beta E[v_{I,3}(\mathbf{s}'_I) | e_k]\} \quad (8)$$

subject to (5), where the law of motion of the state is (7).

The decision rule of an age i household is obtained as

$$\mathbf{d}_{I,i}(\mathbf{s}_I) = (c_{I,i}(\mathbf{s}_I), h_{I,i}(\mathbf{s}_I), g_{I,i}(\mathbf{s}_I), a'_{I,i}(\mathbf{s}_I)), \quad g_{I,2}(\mathbf{s}_I) = 0,$$

for $\mathbf{s}_I \in A^2 \times E$ and $i \in \{4, 2\}$.

3.2.4. Type II Households

The value function of an age 2 household without its parent household is simply

$$v_{II,2}(\mathbf{s}_{II}) = \max_{c, h, a'} \{u(c, h) + \beta E[v_{I,3}(\mathbf{s}'_I) | e]\} \quad (9)$$

subject to

$$a' = \frac{1}{1 + \mu} \{weh + (1 + r)a - \tau_F(weh + ra) - \tau_S(weh) + tr_{LS} - c\} \geq 0, \quad (10)$$

where the law of motion of the state is

$$\mathbf{s}'_I = (a', 0, e'_k).$$

The household chooses its optimal consumption c , working hours h , and end-of-period wealth level a' . The household's decision rule is obtained as

$$\mathbf{d}_{II,2}(\mathbf{s}_{II}) = (c_{II,2}(\mathbf{s}_{II}), h_{II,2}(\mathbf{s}_{II}), a'_{II,2}(\mathbf{s}_{II}))$$

for $\mathbf{s}_{II} \in A \times E$.

3.3. The Measure of Households

Let $x_{I,i}(\mathbf{s}_I)$ denote the measure of Type I households of age $i \in \{1, 2\}$, and let $x_{II,2}(\mathbf{s}_{II})$ denote the measure of Type II households of age 2.¹⁶ Also, let $X_{I,i}(\mathbf{s}_I)$ and $X_{II,2}(\mathbf{s}_{II})$ be the corresponding cumulative measures. The population of age 1 child households is normalized to be unity; i.e.,

$$\int_{A^2 \times B} dX_{I,1}(\mathbf{s}_I) = 1.$$

Let $\mathbf{1}_{[a'=y]}$ be an indicator function that returns 1 if $a' = y$ and 0 if $a' \neq y$. When the steady-state population growth rate is ν , the law of motion of the measure of households—normalized by the population growth—is

$$x'_{I,2}(\mathbf{s}'_I) = \frac{\lambda}{1+\nu} \int_{A^2 \times E} \mathbf{1}_{[a'_p=a'_{I,3}(\mathbf{s}_I)]} \mathbf{1}_{[a'_k=a'_{I,1}(\mathbf{s}_I)]} \pi_{1,2}(e'_k | e_k) dX_{I,1}(\mathbf{s}_I), \quad (11)$$

$$x'_{II,2}(\mathbf{s}'_{II}) = \frac{1-\lambda}{1+\nu} \times \int_{A^2 \times E} \mathbf{1}_{[a'=a'_{I,1}(\mathbf{s}_I)+(a'_{I,3}(\mathbf{s}_I)-\tau_E(a'_{I,3}(\mathbf{s}_I)))/n]} \pi_{1,2}(e' | e_k) dX_{I,1}(\mathbf{s}_I), \quad (12)$$

and

$$\begin{aligned} x'_{I,1}(\mathbf{s}'_I) &= (1+\nu) \\ &\times \left\{ \int_{A^2 \times E} \mathbf{1}_{[a'_p=a'_{I,2}(\mathbf{s}_I)+(a'_{I,4}(\mathbf{s}_I)-\tau_E(a'_{I,4}(\mathbf{s}_I)))/n]} \pi_{2,1}(e'_k | e_k) dX_{I,2}(\mathbf{s}_I) \right. \\ &\quad \left. + \int_{A \times E} \mathbf{1}_{[a'_p=a'_{II,2}(\mathbf{s}_{II})]} \pi_{2,1}(e'_k | e) dX_{II,2}(\mathbf{s}_{II}) \right\}. \end{aligned} \quad (13)$$

The steady-state condition is

$$\begin{aligned} x'_{I,i}(\mathbf{s}_I) &= x_{I,i}(\mathbf{s}_I) \quad \text{for } i \in \{1, 2\}, \\ x'_{II,2}(\mathbf{s}_{II}) &= x_{II,2}(\mathbf{s}_{II}), \end{aligned} \quad (14)$$

for all $\mathbf{s}_I \in A^2 \times E$ and $\mathbf{s}_{II} \in A \times E$.

¹⁶ For Type I households, since the number of child households per parent household is fixed to n , we do not need the measure $x_{I,i}(\mathbf{s}_I)$ for $i = \{3, 4\}$.

3.4. The Firm's Problem

National wealth W is the sum of total private wealth and government's net wealth W_g , and total labor supply L is measured in efficiency units.

$$W = \sum_{i=1}^2 \int_{A^2 \times E} (a_p/n + a_k) dX_{I,i}(s_I) + \int_{A \times E} a dX_{II,2}(s_{II}) + W_g, \quad (15)$$

$$L = \sum_{i=1}^2 \int_{A^2 \times E} (e_p h_{I,i+2}(s_I)/n + e_k h_{I,i}(s_I)) dX_{I,i}(s_I) + \int_{A \times E} e h_{II,2}(s_{II}) dX_{II,2}(s_{II}). \quad (16)$$

There is only one perfectly competitive firm in this economy. In a closed economy, the capital stock is equal to national wealth, i.e.,

$$K = W, \quad (17)$$

and the gross national product Y is determined by a constant-returns-to-scale production function,

$$Y = F(K, AL),$$

where A is the measure of labor productivity. The profit-maximizing condition of the firm is

$$r + \delta = F_K(K, AL), \quad (18)$$

$$w(1 + \tau'_S) = F_L(K, AL), \quad (19)$$

where δ is the depreciation rate of capital and τ'_S is the marginal payroll (Social Security) tax rate.

In a small open economy, the gross national product Y_S is defined as

$$Y_S = (r + \delta)W + w(1 + \tau'_S)L,$$

where r and w are international factor prices. The capital stock used in domestic production K and the net foreign wealth B are determined by (18), (19), and

$$B = W - K.$$

3.5. The Government's Policy Rule

Government tax revenue consists of federal income tax T_F , payroll tax for Social Security T_S , and federal and state estate taxes T_E . These revenues are calculated as follows:

$$T_F = \sum_{i=1}^2 \int_{A^2 \times E} \{ \tau_F(ra_p)/n + \tau_F(we_k h_{I,i}(\mathbf{s}_I) + ra_k) \} dX_{I,i}(\mathbf{s}_I) \\ + \int_{A \times E} \tau_F(weh_{II,2}(\mathbf{s}_{II}) + ra) dX_{II,2}(\mathbf{s}_{II}), \quad (20)$$

$$T_S = \sum_{i=1}^2 \int_{A^2 \times E} \tau_S(we_k h_{I,i}(\mathbf{s}_I)) dX_{I,i}(\mathbf{s}_I) \\ + \int_{A \times E} \tau_S(weh_{II,2}(\mathbf{s}_{II})) dX_{II,2}(\mathbf{s}_{II}), \quad (21)$$

$$T'_E = (1 - \lambda) \int_{A^2 \times E} \tau_E(a'_{I,3}(\mathbf{s}_I))/n dX_{I,1}(\mathbf{s}_I) \\ + \int_{A^2 \times E} \tau_E(a'_{I,4}(\mathbf{s}_I))/n dX_{I,2}(\mathbf{s}_I). \quad (22)$$

Total tax revenue is the sum of these three tax revenues and payroll tax from employers; i.e.,

$$T = T_F + 2T_S + T_E.$$

Social Security benefits per elderly household are determined by

$$tr_{SS} = 2T_S/N_{OLD},$$

where N_{OLD} is the population of households of age 3 or 4.¹⁷ The law of motion of the government wealth (normalized by productivity growth μ and population growth ν) is

$$W'_g = \frac{1}{1 + \mu + \nu} \{ (1 + r)W_g + T - C_g - tr_{SS}N_{OLD} - tr_{LS}N \}, \quad (23)$$

where C_g is government consumption and N is total population.

¹⁷ In the policy experiments in Section 5, tr_{SS} is assumed to be exogenous.

3.6. Recursive Competitive Equilibrium

The definition of a steady-state recursive competitive equilibrium (which is also a Markov perfect equilibrium) of this model is as follows:

DEFINITION (STEADY-STATE RECURSIVE COMPETITIVE EQUILIBRIUM). Let \mathbf{s}_I and \mathbf{s}_{II} be the state of a Type I dynasty and that of a Type II dynasty, respectively, where

$$\mathbf{s}_I = (a_p, a_k, e_k), \quad \mathbf{s}_{II} = (a, e).$$

Given the time-invariant government policy rules,

$$\Psi = \{\tau_F(\cdot), \tau_S(\cdot), \tau_E(\cdot), tr_{SS}, tr_{LS}, C_g, W_g\};$$

factor prices, r and w ; the value functions of households,

$$\{v_{I,i}(\mathbf{s}_I)\}_{i=1}^4 \quad \text{and} \quad \{v_{II,2}(\mathbf{s}_{II})\};$$

the decision rules of households,

$$\{c_{I,i}(\mathbf{s}_I), h_{I,i}(\mathbf{s}_I), g_{I,i}(\mathbf{s}_I), a'_{I,i}(\mathbf{s}_I)\}_{i=1}^4 \quad \text{and} \quad \{c_{II,2}(\mathbf{s}_{II}), h_{II,2}(\mathbf{s}_{II}), a'_{II,2}(\mathbf{s}_{II})\};$$

and the measures of dynasties,

$$\{x_{I,i}(\mathbf{s}_I)\}_{i=1}^2 \quad \text{and} \quad \{x_{II,2}(\mathbf{s}_{II})\}$$

are in a steady-state recursive competitive equilibrium if, in every period, a household solves the utility maximization problem, (1)–(10), taking its counterpart's (either its parent's or child's) decision as given; the firm solves the profit maximization problem, and the capital and labor markets clear; i.e., (15)–(19) hold; the government policy rules satisfy (20)–(23); the goods market clears; and the measures of dynasties are constant; i.e., (11)–(14) hold.

4. CALIBRATION

The two main parameters—the degree of time preference β and the degree of parental altruism η —are determined simultaneously so that the steady-state equilibrium of the model replicates the U.S. economy in terms of two key statistics: the capital-output ratio and the relative size of inter-generational transfers. The functional forms and other parameters are chosen to be consistent with macroeconomic and cross-section data in the United States.

As is explained below, a Cobb–Douglas utility function with constant relative risk aversion and a Cobb–Douglas production function are used for the calibration. Table I summarizes the choice of these parameters. The following sections describe the choice of functional forms and parameter values, and the choice of target variables and values.

TABLE I
Parameters

Share parameter for consumption	α	0.76
Coefficient of relative risk aversion	γ	2.0
Capital share of output	θ	0.32
Depreciation rate of capital stock	δ	0.046 ^a
Long-term real growth rate	μ	0.018 ^a
Population growth rate	ν	0.010 ^a
Survival rate at the end of age 3	λ	0.546

^aAnnual rate.

4.1. Households

Utility function. The model uses the following Cobb–Douglas utility function with constant relative risk aversion;

$$u(c_i, h_i) = \frac{\{c_i^\alpha (h_i^{\max} - h_i)^{1-\alpha}\}^{1-\gamma} - 1}{1 - \gamma}.$$

Here, γ is the coefficient of relative risk aversion, and it is set to be 2.0 in the main calibration. Later, I also show the results when the coefficient is changed to 1.0 or 4.0. The maximum working hours h_i^{\max} depend on the age of the household and are explained below.

Working hours. The model uses the statistics from the Panel Study of Income Dynamics (PSID) 1993 Family Data. The annual working hours in the model are the sum of the working hours of a husband and a wife (“Head” and “Wife” in PSID), including housework.

$$\begin{aligned} \text{Total working hours} = & \text{Head’s market work hours} \\ & + \text{Head’s housework hours} \\ & + \text{Wife’s market work hours} \\ & + \text{Wife’s housework hours.} \end{aligned}$$

The average working hours of married households of ages between 30 and 59 are 4810 h. Suppose that the 90th percentile (6430 h) is regarded as the maximum working hours h_i^{\max} ($i = 1$ or 2).¹⁸ The parameter α is chosen to be 0.76 so that average working hours of age 1 and age 2 households become 4810 h in the steady state.

¹⁸ The maximum working hours in the PSID data are actually 11,400 h in 1992. But, the 90th percentile is used instead because the average of the maximum in 15 years must be significantly smaller.

Market work and housework. According to the PSID data, for married couples between ages 30 and 59, about 68% of total working hours are declared as market work and the remaining hours are declared as housework. This ratio is used to compute the taxable earnings of each household in the model. Also, 1 h of housework is assumed to produce the same value of consumption goods or services as the hourly market wage of this household.

For simplicity, there is no retirement decision in this model, and all of the age 3 and age 4 households are retired. The maximum working hours of age 1 and age 2 are assumed to be 6430 h, in which 68% are assumed to be market working hours. Subtracting this amount, the maximum working hours of age 3 and age 4, h_i^{\max} ($i = 3$ or 4), are set to be 2058 h.

Working ability. The working ability in this calibration corresponds to the hourly wage (labor income per hour) of each household in the 1998 Survey of Consumer Finances (SCF). The average hourly wage of a married couple (family members #1 and #2 in SCF) is calculated by

$$\begin{aligned} & \text{Hourly wage (annual average)} \\ &= \frac{\text{Regular salary and Additional salary (\#1 + \#2)}}{\text{Working hours (\#1 + \#2)}}. \end{aligned}$$

The first two columns of Table II show the eight discrete levels of working abilities based on the 1998 SCF.

Markov transition matrix. Since one period in this model corresponds to 15 years, to compute a Markov transition matrix of working ability from the data, we need to have 30 years of longitudinal hourly wage data. To keep the sample size larger, this paper constructs a Markov transition matrix from age 1 (30–44 years old) to age 2 (45–59 years old), Γ^{12} , based on the five-year-average wages in 1974–1978 and those in 1989–1993 in the PSID individual data;

$$\Gamma^{12} = \begin{pmatrix} 0.5851 & 0.2699 & 0.1149 & 0.0187 & 0.0114 & 0.0000 & 0.0000 & 0.0000 \\ 0.2543 & 0.3057 & 0.2456 & 0.1368 & 0.0201 & 0.0375 & 0.0000 & 0.0000 \\ 0.0987 & 0.1840 & 0.3782 & 0.2858 & 0.0419 & 0.0000 & 0.0114 & 0.0000 \\ 0.0505 & 0.1480 & 0.1753 & 0.3887 & 0.1509 & 0.0505 & 0.0361 & 0.0000 \\ 0.0066 & 0.0146 & 0.0752 & 0.2016 & 0.3676 & 0.2276 & 0.1068 & 0.0000 \\ 0.0000 & 0.2407 & 0.1430 & 0.0968 & 0.2229 & 0.1928 & 0.0570 & 0.0468 \\ 0.0399 & 0.1249 & 0.0631 & 0.1657 & 0.1816 & 0.0000 & 0.3653 & 0.0595 \\ 0.0000 & 0.0000 & 0.0000 & 0.2367 & 0.0000 & 0.0000 & 0.2356 & 0.5277 \end{pmatrix},$$

where $\Gamma^{12}(i, j) = \pi(e_2 = e_2^j \mid e_1 = e_1^i)$. The correlation of hourly wages of age 1 and age 2 in the steady state is 0.62. The Markov transition matrix

TABLE II
Working Abilities of a Household (in U.S. Dollars per Hour)

		Annual average		Period average	
	Percentile	Age 1 (30–44)	Age 2 (45–59)	Age 1 (30–44)	Age 2 (45–59)
e^1	0–20th	6.98	7.03	8.27	8.50
e^2	20–40th	11.24	11.90	12.46	13.34
e^3	40–60th	15.06	16.20	15.63	16.97
e^4	60–80th	20.57	22.81	20.79	23.20
e^5	80–90th	27.79	31.36	29.37	33.62
e^6	90–95th	37.44	43.39	35.53	42.01
e^7	95–99th	62.85	79.09	54.66	68.62
e^8	99–100th	208.70	285.69	169.25	230.24

Source: The author’s calculation from the 1998 SCF. Period average (15-year average) is estimated from annual numbers and a Markov transition matrix.

from age 2 parents to age 1 children is simply assumed to be $\Gamma^{21} = (\Gamma^{12})^{3/2}$ so that the correlation of hourly wages in the steady state is 0.49.¹⁹

Period-average working ability. The period-average working ability of a married couple is calculated as the average of current working ability and the expected working ability after 15 years. Let $\hat{e}_i = [\hat{e}_i^1, \hat{e}_i^2, \dots, \hat{e}_i^8]'$ be the vector of annual-average working ability of age i . Then, the vector of period-average working ability is

$$e_i = [e_i^1, e_i^2, \dots, e_i^8]' = (\Gamma^{12}\hat{e}_i + \hat{e}_i)/2.$$

The last two columns of Table II show the period-average working abilities used in the calibration.

Population growth and mortality. The population growth rate ν is assumed to be annually 1.0 and 16.1% per period (15 years). Since new households are born to the dynasty every 30 years, the number of child households per parent household n is 1.348 (=1.01³⁰). In the United States, the life expectancy of a 60-year-old male is 80.68 and that of a 60-year-old female is 85.71.²⁰ Taking the simple average of male and female, the life expectancy becomes 83.20 years. The survival rate at the end of age 3 (75 years old) λ is set at 0.546 so that the life expectancy in this model is also 83.20.

¹⁹ Solon (1992) showed that the intergenerational income correlation in the United States is at least 0.4 and possibly around 0.5. Zimmerman (1992) also estimated the intergenerational income correlation to be on the order of 0.4 or higher. The correlation is partly due to the parental investment in the children’s schooling.

²⁰ Source: <http://www.retireweb.com/death.html> (retrieved on September 1, 2001), which is based on the Group Annuity Mortality Table.

4.2. The Firm

Production function. The model uses the Cobb–Douglas production function,

$$F(K_t, A_t L_t) = K_t^\theta (A_t L_t)^{1-\theta},$$

where

$$A_t = e^{\mu t} A, \quad L_t = e^{\nu t} L,$$

μ is the growth rate of labor productivity, and ν is the population growth rate. To compute GNP, the calibration uses the sum of market work hours in efficiency units as total labor supply L . The capital share of output θ is chosen by

$$\theta = 1 - \frac{\text{Compensation of employees} + (1 - \theta) \times \text{Proprietors' income}}{\text{National income} + \text{Consumption of fixed capital}}.$$

The average of θ in 1996–1998 is 0.32. The annual growth rate μ is assumed to be 1.8%. The annual population growth rate ν is assumed to be 1.0%. The labor productivity A is chosen to be 0.983 so that the wage per unit of efficient labor is normalized to be unity.

Fixed capital and private wealth. The fixed capital K in the model is defined by the following formula:

$$\begin{aligned} \text{Fixed capital} &= \text{Fixed reproducible Tangible wealth} \\ &\quad - \text{Durable goods owned by consumers.} \end{aligned}$$

These data are taken from the Survey of Current Business (1997). Between 1990 and 1996, fixed capital accounted for 89.7% of fixed reproducible tangible wealth, and durable goods owned by consumers accounted for the remainder.

To connect the total private wealth with the fixed capital, it is assumed that all of the private capital is owned by households and that part of the government-owned fixed capital is effectively owned by households in the form of government bonds.

$$\begin{aligned} \text{Fixed capital} &= \text{Private wealth} + \text{Government-owned fixed capital} \\ &\quad - \text{Government bonds owned by private sector} \\ &\quad - \text{Durable goods owned by consumers.} \end{aligned}$$

Based on the data in 1990–1996, an approximate relationship,

$$\text{Fixed capital} = 0.956 \times \text{Private wealth},$$

is used in the calibration.

The depreciation rate of fixed capital. The depreciation rate of fixed capital δ is chosen by

$$\delta = \frac{\text{Total gross investment}}{\text{Fixed capital}} - \mu - \nu.$$

In the period 1997–2000, gross private domestic investment accounted, on average, for 17.5% of the GDP, and gross government investment (federal and state) accounted for 3.2% of the GDP. When the capital-output ratio is 2.8 (see Section 4.4), the ratio of gross investment to fixed capital is 7.4%. Subtracting the productivity and population growth rates, the annual depreciation rate is assumed to be 4.6%.

4.3. Taxes and Transfers

Federal income taxes. For federal income tax, the model uses the tax function estimated by Gouveia and Strauss (1994),

$$\text{Federal income tax} = \phi_0(y - (y^{-\phi_1} + \phi_2)^{-1/\phi_1}),$$

where y is the total income of a household after the standard deduction and exemption for four people. In 1997, the sum of this deduction and exemption was \$17,900. When the parameters are $\phi_0 = 0.41$, $\phi_1 = 0.85$, and $\phi_2 = 0.015$, this function replicates the statutory income tax schedule. But, because of itemized deductions, the effective tax rate of high-income households is regarded as much lower.²¹ Since in 1997 the ratio of total private income tax to nominal GDP was 0.09, ϕ_0 is assumed to be 0.262 so that income tax revenue is 9.0% of GDP in the steady-state equilibrium.

Estate taxes. For federal estate tax, the model uses the similar function as federal income tax; i.e.,

$$\text{Estate tax per capita} = \psi_0(b - (b^{-\psi_1} + \psi_2)^{-1/\psi_1}).$$

In this function, b is the number of bequests per capita minus the exemption of \$600,000 (before 1998). The parameters $\psi_0 = 0.6$, $\psi_1 = 0.32$, and $\psi_2 = 0.046$ are chosen so that the function replicates the federal estate tax schedule (before state death tax credits). It is simply assumed that each household consists of a married couple, and that a husband and a wife leave bequests to their children separately. In total, a married household can leave at most \$1.2 million without filing federal estate tax returns. The estate tax that each household pays is twice the number in the above equation. State estate tax is assumed to be equal to the state death tax credit.

²¹ See Gouveia and Strauss (1994) for effective federal tax rates.

The calibration does not consider the additional estate tax or inheritance tax levied in some states.

Each person can make transfers up to \$10,000 per recipient every year without filing federal estate and gift tax returns. So, a household with two parents and two children can make a transfer up to \$40,000 without paying a tax. The transfers above this amount are accumulated, and an estate and gift tax is levied after the exemption of \$600,000. In this calibration, however, a gift tax is not considered because of the difficulty of the computation.²²

Social security. The tax rate for old-age, survivors, disability, and insurance (OASDI) is 6.2% and the tax rate for Medicare (HI) is 1.45%. In 1997, employee compensation above \$68,400 was not taxable for OASDI. The same amount of taxes is also paid by employers. So, the firm's profit-maximization problem becomes

$$w \times (1 + \text{Marginal payroll tax rate}) = AF_L(K, AL),$$

where the marginal payroll tax rate is 0.0765.

Social Security benefits are determined by the average indexed monthly earnings and the corresponding replacement rates. In this calibration, for simplicity, the annual benefits per married couple are assumed to be the same for all eligible households.

4.4. Target Variables

Capital-output ratio. The capital stock used here is measured by "fixed reproducible tangible wealth" minus "durable goods owned by consumers." These data are taken from the Survey of Current Business (1997). For the output data, nominal gross domestic product is used. The average capital-output ratio for the period 1990–1996 is approximately 2.8.

The relative sizes of intergenerational transfers. Aggregate inheritance and trust received from parents or grandparents, a five-year average in 1993–1997 calculated from the 1998 SCF, are 0.32% of the total private wealth ("net worth" in SCF). Adjusted taxable estates in taxable returns, in federal estate tax returns for 1997, account for 0.30% of total private wealth. Comparing these two numbers, the inheritance data in SCF are likely to be significantly underreported. But, the federal estate tax returns (taxable returns) covered only 1.5% of decedents in 1993. So, the calibration based on taxable estates or estate tax revenue will be inaccurate unless the model replicates the wealth distribution of elderly households precisely. The calibration in this paper thus uses the annual flow of bequests, 1.00% of net worth, estimated by Gale and Scholz (1994). (See Table III.)

²² The model needs to have the taxable inter vivos transfers made by each household as an additional state variable.

TABLE III
The Annual Flows of Intergenerational Transfers

	Annual flow as a percentage of total private wealth	
	1986	1997
Inheritance and trust received from: parents or grandparents ^a		0.32
Adjusted taxable estates in taxable returns ^b		0.30
Estimated bequests ^c	1.00	
Support given to: ^d		
Children or grandchildren	0.32	0.15
Parents or grandparents	0.03	0.04

^a The author's calculation from the 1998 Survey of Consumer Finances (SCF). A five-year average in 1993–1997.

^b Source: SOI Bulletin (1999).

^c The data come from Table 4 in Gale and Scholz (1994). The number is calculated from the sum of bequests and estimated trusts.

^d The 1986 data come from Table 4 in Gale and Scholz (1994). Source: 1986 SCF. The 1997 data are the author's calculation from the 1998 SCF.

In the 1998 SCF, financial support given to children and grandchildren accounts for 0.15% of total private wealth. Also, financial support given to parents and grandparents accounts for 0.04% of total private wealth. If we look at the 1986 SCF, summarized by Gale and Scholz (1994), support given to children and grandchildren are 0.32% of total private wealth. One of the possible explanations of this difference is that the numbers in the 1986 survey include gifts as well as financial support. This calibration uses the relative sizes of inter vivos transfers in the 1986 SCF. The size of inter vivos gifts to parents and grandparents is very small if we only consider financial transfers in the data. So, in this paper, the model assumes one-sided altruism.

In the calibration of the model, the target variable for the size of intergenerational transfers is the sum of bequests and inter vivos transfers, and the target value is set to 1.32 as a percentage of total private wealth.

4.5. The Distributions in the Steady-State Economy

Table IV compares the distributions of earnings, income, and wealth in the model economy with those in the U.S. data from the Survey of Consumer Finances. The first panel of the table shows the distributions of married households of ages 30–89 calculated from the 1998 SCF. The second panel shows the distributions in the steady-state economy. The model replicates the distributions fairly well in terms of Gini coefficients, although the

TABLE IV
The Distributions of Earnings, Income, and Wealth

	Gini	Percentages of earnings, income, and wealth of top					
	coefficient	1%	5%	10%	20%	40%	60%
U.S. data (married households, the 1998 survey of consumer finances) ^a							
Earnings	0.60	13.6	29.2	40.8	58.2	82.5	96.8
Income	0.52	17.2	31.9	41.8	55.7	75.5	88.7
Wealth	0.75	30.5	53.9	64.9	77.2	90.4	96.9
Economy with altruistic and accidental bequests, and inter vivos transfers							
Earnings	0.60 (0.31) ^b	11.8	27.3	39.7	57.7	82.7	98.3
Income	0.47	11.3	25.1	35.8	51.4	72.6	87.5
Wealth	0.74	19.4	41.2	55.7	74.4	93.3	99.6
Economy with altruistic and accidental bequests ^c							
Earnings	0.60 (0.31)	11.6	27.1	39.5	57.5	82.6	98.2
Income	0.46	10.6	24.2	34.9	50.5	71.8	87.0
Wealth	0.73	17.1	38.6	53.5	72.8	92.9	99.5
Economy with accidental bequests only							
Earnings	0.60 (0.31)	11.6	27.0	39.5	57.5	82.6	98.2
Income	0.46	10.6	24.1	34.8	50.4	71.8	86.9
Wealth	0.71	15.9	36.5	51.4	70.8	91.7	99.0
Economy without any intergenerational transfers							
Earnings	0.60 (0.31)	11.5	27.0	39.4	57.4	82.5	98.2
Income	0.46	10.5	23.9	34.6	50.0	71.3	86.4
Wealth	0.67	15.1	34.5	48.7	67.5	88.4	97.0
Economy with altruistic and accidental bequests, and inter vivos transfers ($\gamma = 4.0$)							
Earnings	0.60 (0.31)	11.8	27.3	39.7	57.7	82.7	98.3
Income	0.48	11.4	25.4	36.2	51.8	72.9	87.8
Wealth	0.74	20.4	42.3	56.7	75.0	93.5	99.5
Economy with altruistic and accidental bequests, and inter vivos transfers (with the adjustment of hourly wages of the top 1 percent of households)							
Earnings	0.63 (0.35)	17.5	31.9	43.6	60.5	83.9	98.4
Income	0.53	17.9	31.4	41.3	55.8	75.4	89.2
Wealth	0.79	30.7	51.0	63.7	80.0	95.3	99.8

^a The author's calculation based on the data of married households of ages 30–89 and widowed households of ages 60–89.

^b The Gini coefficients of earnings in the parentheses are those of lifetime earnings in the model.

^c The relative size of taxable bequests is assumed to be 1.00% of total private wealth.

shares of wealth held by the top 1, 5, and 10% of households are smaller than those in the data.²³

²³ We cannot exactly compare the Gini coefficients of earnings and income in the model with those in the U.S. data because the former are calculated from 15-year average numbers. The earnings Gini coefficient in the model is higher than it is supposed to be because all

Table IV also shows how the distribution of wealth has been intensified by considering intergenerational transfers. The share of wealth held by the top 1% is only 15.1% in the model without any intergenerational transfers (the fifth panel), the share becomes 15.9% if the model considers accidental bequests (the fourth panel), and it becomes 17.1% if the model considers both altruistic and accidental bequests (the third panel). The share of wealth held by the top 1% of households becomes 19.4% in the economy with inter vivos transfers.

These results imply that both bequests and inter vivos transfers can help explain the distribution of wealth in the United States, although there are likely to be other causes of the skewed distribution, such as stochastic discount rates (Krusell and Smith, 1998), entrepreneurship (Quadrini, 2000), or risky assets. The sixth panel of Table IV shows the distributions in the economy in which households are more risk averse, assuming γ equals 4.0. The share of wealth held by the top 1% of households rises to 20.4% but it is still lower than the share in the U.S. data.

Table V compares the inequality and wealth distribution in the steady-state economy of the present model with those of other models in the literature. The present model explains the share of wealth held by the top 1% of households better than the Huggett (1996) model, which considers earnings shock and lifetime uncertainty only. The share of wealth held by the top 1% of households in the present model is also larger than that of De Nardi (2001), which considered parent's "warm glow" bequest motive and productivity inheritance.²⁴ Castañeda *et al.* (2000) replicated the top 20% of wealth distribution almost perfectly, partly because their model was calibrated to replicate the wealth distribution.

4.6. Obtained Main Parameters

The number of bequests a parent household leaves depends on its wealth level. To measure the degree of altruistic bequest motive, the distribution of wealth produced in the model must be similar to that in the U.S. economy.

One of the possible explanations of the discrepancy between the model and the data is that the salary data in SCF do not include capital gains, and the income data in SCF do not include unrealized capital gains. To

households of ages 60–89 are retired in the model but they are not in the data. The Gini coefficient of lifetime earnings (shown in the parentheses) in the model economy is 0.31. This number is somewhat lower than 0.38 estimated by Knowles (1999). This is partly because the present model does not include single households and does not distinguish one-earner couples and two-earner couples.

²⁴ A "warm glow" bequest motive can likely explain the skewness of wealth distribution more than an altruistic bequest motive. In an altruistic bequest model, a parent household would reduce the number of transfers if it expected its child households were going to be wealthier.

TABLE V
The Comparison with Other Models in the Literature

	Gini coefficient		Percentage of wealth held by top				
	Earnings	Wealth	1%	5%	10%	20%	40%
U.S. data (all households) ^a	0.60	0.80	33.9	57.2	68.7	81.5	93.9
U.S. data (married households) ^b	0.60	0.75	30.5	53.9	64.9	77.2	90.4
Nishiyama (2002)	0.60 (0.31)	0.74	19.4	41.2	55.7	74.4	93.3
Castañeda <i>et al.</i> (2000) ^c	0.60	0.79	29.5	44.6	61.4	81.0	96.1
De Nardi (2001) ^d	0.43	0.73	15	38	n.a.	75	94
Huggett (1996) ^e	0.42	0.76	11.8	35.6	n.a.	75.5	n.a.

^a The author's calculation using data from the 1998 Survey of Consumer Finances.

^b See the first footnote of Table IV.

^c The data come from Table IV in Castañeda *et al.* (2000).

^d The data come from Table VI in De Nardi (2001) with parent's bequest motive and productivity inheritance assumed. The earnings Gini coefficient is that of the households whose head is ages between 25 and 60.

^e The data come from Table III in Huggett (1996) with earnings shock and lifetime uncertainty assumed. The earnings Gini coefficient is that of the population ages between 20 and 65.

avoid complicating the model further, alternative calibrations are made by adjusting the hourly wage of the top 1% of working-age households.

The bottom panel in Table IV shows the distributions when the hourly wages of the top 1% of households are inflated by 70% so that the shares of wealth held by the top 1, 5, and 10% of households are roughly the same as those in the U.S. data.

The obtained main parameters are shown in Table VI. The parameter for the annual time preference β_A was obtained as follows.

$$\beta_A = \{\beta(1 + \mu)^{-\alpha(1-\gamma)}\}^{1/15}.$$

In the main calibration, the coefficient of relative risk aversion γ is assumed to be 2.0. The annual time preference parameter is 0.946 and the degree of parental altruism turns out to be 0.626. When the earnings distribution is adjusted so that the wealth distribution is similar to the U.S. data, the degree of altruism lowers slightly to 0.512. The last mentioned

TABLE VI
Obtained Main Parameters

Coefficient of relative risk aversion γ	Annual time preference β_A	Parental altruism η
2.0	0.946	0.626
2.0 (with the earnings adjustment)	0.940	0.512
1.0	0.947	0.792
4.0	0.937	0.388

number implies that a parent household, on average, cares about each of its adult child households about 50% less than it cares about itself. Table VI also shows the parameters when the coefficient of relative risk aversion γ is assumed to be 1.0 and 4.0. Both the time preference and the degree of altruism become lower as the coefficient of relative risk aversion increases.

5. POLICY EXPERIMENTS

In this section, first, federal and local estate taxes are eliminated from the baseline economy, and the effect of altruistic bequests on wealth accumulation, inequality, and social welfare is shown. Next, a perfect annuity market is introduced to the baseline economy, and the effect of eliminating accidental bequests due to lifetime uncertainty is shown. Finally, marginal and average income tax rates are reduced proportionally by 10% and the effect of the progressive income tax is shown.

All of those policy experiments would reduce government tax revenue. For each case, two different budget rules are assumed. First, government consumption is cut contemporaneously so that government net wealth level is unchanged from the baseline economy. Second, a lump-sum tax is introduced so that government wealth is unchanged. The model assumes both a closed economy and a small open economy. Due to limits of space, all of the following policy experiments use the baseline economy with the earnings adjustment.

5.1. Eliminating Federal and Local Estate Taxes

In the baseline economy, both federal and local estate taxes are considered. If those estate tax rates were reduced to zero, how much would national wealth, inequality, and social welfare change? This policy change would increase parents' savings aimed for altruistic bequests. It would also increase the after-tax receipts of altruistic and accidental bequests by children. At the same time, those children who expect taxable bequests would reduce their own life-cycle savings. Table VII illustrates the long-run effect of estate tax repeal.

When the government revenue decrease was compensated by the reduction in government consumption, national wealth would increase by 1.6% in a closed economy and by 2.6% in a small open economy. The effect on wealth is smaller in a closed economy because the interest rate would fall by 0.1 percentage points and the wage rate would rise by 0.5%. The ratio of net foreign wealth to baseline GDP would rise by 7.6 percentage points in a small open economy.²⁵

²⁵ In the baseline economy, net foreign wealth is set to zero, and the GNP is equal to the GDP.

TABLE VII
When Federal and Local Estate Taxes Are Eliminated from the Economy

	Adjusted by government consumption			Adjusted by lump-sum tax	
	Baseline economy	Closed economy	Small open economy	Closed economy	Small open economy
%Δ National wealth		1.6	2.6	1.7	2.8
%Δ Labor		0.0	-0.1	0.0	-0.1
%Δ GNP		0.5	0.8	0.5	0.8
%Δ Domestic capital		1.6	-0.1	1.7	-0.1
%Δ GDP		0.5	-0.1	0.5	-0.1
%Δ Private consumption		0.4	0.4	0.3	0.3
Δ Government consumption/GDP*		-0.2	-0.2	n.c.	n.c.
Δ Lump-sum tax/GDP*		n.c.	n.c.	0.2	0.2
Δ Bequests/GDP*		0.1	0.2	0.1	0.2
Δ Interest rate		-0.1	n.c.	-0.1	n.c.
%Δ Wage rate		0.5	n.c.	0.5	n.c.
Δ Net foreign wealth ratio		n.c.	7.6	n.c.	7.9
Welfare changes (%)					
Compensating wealth ^a		1.7	4.2	1.2	3.8
Equivalent wealth ^b		1.8	4.2	1.5	4.0
Gini coefficients					
Earnings	0.63	0.63	0.63	0.63	0.63
Income	0.53	0.53	0.53	0.53	0.53
(After tax and transfers)	0.46	0.47	0.46	0.46	0.46
Income & bequests	0.53	0.53	0.53	0.53	0.53
(After tax and transfers)	0.47	0.47	0.47	0.47	0.47
Wealth	0.79	0.80	0.79	0.80	0.79
Share of wealth (%)					
Top 1%	30.7	31.8	31.6	31.7	31.6
Top 5%	51.0	52.4	52.2	52.4	52.1
Top 10%	63.7	64.9	64.6	64.9	64.6
Top 20%	80.0	80.8	80.5	80.8	80.5
Top 40%	95.3	95.5	95.4	95.5	95.4
Top 60%	99.8	99.9	99.8	99.9	99.8

Note. %Δ and Δ denote the percent change and the change in percentage points, respectively, from the baseline economy; GDP* denotes GDP in the baseline economy. "n.c." denotes "no change" by definition.

^a (Total wealth/Compensating wealth - 1) × 100.

^b (Equivalent wealth/Baseline total wealth - 1) × 100.

Although estate tax repeal would reduce labor supply slightly in a small open economy, the GNP would increase by 0.5% in a closed economy and by 0.8% in a small open economy. For the government wealth to stay at the same level, the ratio of government consumption to baseline GDP would have to fall by 0.2 percentage points. Based on the wealth measures

(a compensating variation and an equivalent variation), households would be better off on average by 1.7 to 1.8% in a closed economy and by 4.2% in a small open economy.

Repeal of the estate tax would intensify inequality slightly. The wealth Gini coefficient would rise by 0.01 in a closed economy. The share of wealth held by the top 1% of households would rise by 1.1 percentage points in a closed economy and by 0.9 percentage points in a small open economy.

When the government revenue decrease was compensated by the lump-sum tax to all households, the effect on national wealth would be slightly larger.²⁶ National wealth would rise by 1.7% in a closed economy and by 2.8% in a small open economy. The effect of estate tax repeal on inequalities would be almost the same as in the case of the government consumption adjustment.

5.2. Introducing a Perfect Annuity Market

If a perfect annuity market was introduced to the baseline economy to eliminate accidental bequests, how much would national wealth, inequality, and social welfare change? This policy change would reduce the savings and bequests of elderly households, and the savings of child households. Table VIII shows the overall effect of introducing a perfect annuity market.

When the government revenue decrease was adjusted by government consumption, national wealth would decrease by 6.2% in a closed economy and by 9.1% in a small open economy. Since child households would not expect accidental bequests from their parents, labor supply would increase by 0.2% in a closed economy and by 0.3% in a small open economy. The GNP would decrease by 1.9 and 2.7% in a closed economy and a small open economy, respectively.

In a closed economy, the interest rate would rise by 0.5 percentage points and the wage rate would fall by 2.1%. In a small open economy, the ratio of net foreign wealth to baseline GDP would fall by 26.4 percentage points. For the government wealth to stay at the same level, the ratio of government consumption to baseline GDP would have to fall by 0.4 percentage points in both a closed economy and a small open economy.

Although private consumption would decrease and working hours would increase, households would be better off on average by 7.2 to 7.6% in a closed economy based on the wealth measures. This is because a household would be able to allocate its lifetime resources more efficiently with a perfect annuity market. In a small open economy, however, households

²⁶ In the economy with income uncertainty, a lump-sum tax will increase precautionary savings.

TABLE VIII
When a Perfect Annuity Market Is Added to the Economy

	Baseline economy	Adjusted by government consumption		Adjusted by lump-sum tax	
		Closed economy	Small open economy	Closed economy	Small open economy
%Δ National wealth		-6.2	-9.1	-6.0	-8.9
%Δ Labor		0.2	0.3	0.3	0.4
%Δ GNP		-1.9	-2.7	-1.8	-2.6
%Δ Domestic capital		-6.2	0.3	-6.0	0.4
%Δ GDP		-1.9	0.3	-1.8	0.4
%Δ Private consumption		-0.8	-0.5	-1.0	-0.8
Δ Government consumption/GDP*		-0.4	-0.4	n.c.	n.c.
Δ Lump-sum tax/GDP*		n.c.	n.c.	0.4	0.4
Δ Bequests/GDP*		-1.7	-1.8	-1.7	-1.8
Δ Interest rate		0.5	n.c.	0.5	n.c.
%Δ Wage rate		-2.1	n.c.	-2.0	n.c.
Δ Net foreign wealth ratio		n.c.	-26.4	n.c.	-26.0
Welfare changes (%)					
Compensating wealth ^a		7.6	-2.0	6.1	-2.7
Equivalent wealth ^b		7.2	-1.1	6.2	-2.0
Gini coefficients					
Earnings	0.63	0.62	0.62	0.62	0.62
Income	0.53	0.52	0.53	0.52	0.53
(After tax and transfers)	0.46	0.46	0.47	0.46	0.47
Income & bequests	0.53	0.52	0.53	0.52	0.53
(After tax and transfers)	0.47	0.47	0.47	0.47	0.47
Wealth	0.79	0.80	0.80	0.80	0.80
Share of wealth (%)					
Top 1%	30.7	32.6	33.2	32.6	33.1
Top 5%	51.0	53.0	53.6	52.9	53.6
Top 10%	63.7	65.6	66.5	65.6	66.4
Top 20%	80.0	80.7	81.3	80.6	81.2
Top 40%	95.3	95.6	95.9	95.6	95.9
Top 60%	99.8	99.8	99.9	99.8	99.9

Note. %Δ and Δ denote the percent change and the change in percentage points, respectively, from the baseline economy; GDP* denotes GDP in the baseline economy. “n.c.” denotes “no change” by definition.

^a (Total wealth/compensating wealth - 1) × 100.

^b (Equivalent wealth/Baseline total wealth - 1) × 100.

would be worse off by 1.1 to 2.0% because the decline of private wealth would be larger.

Introducing a perfect annuity market would increase inequality, too. The wealth Gini coefficient would rise by 0.01 in both a closed economy and a small open economy. The share of wealth held by the top 1% of households

would rise by 1.9 percentage points in a closed economy and by 2.5 percentage points in a small open economy.

Whether accidental bequests increase or decrease wealth inequality depends on the baseline economy. The policy experiment shown here implies that accidental bequests would *decrease* wealth inequality, because the introduction of a perfect annuity market would increase inequality. In the absence of altruistic bequests, however, accidental bequests would rather *increase* wealth inequality. See the fourth and fifth panels in Table IV.

When the government revenue decrease was compensated by lump-sum tax, the reduction of national wealth would be slightly smaller. National wealth would decrease by 6.0% in a closed economy and by 8.9% in a small open economy. The effect of a perfect annuity market on inequality is almost the same as the case of the government consumption adjustment.

5.3. Cutting Federal Income Tax Rates by 10%

If the marginal income tax rates were cut proportionally by 10%, how much would national wealth, inequality, and social welfare change? Table IX shows the effect of this marginal rate cut.

When the government revenue decrease was adjusted by government consumption, national wealth would increase by 2.3% in a closed economy and by 3.4% in a small open economy. The income tax cut would increase labor supply by 0.3% in both a closed economy and a small open economy. The GNP would increase by 1.0% in a closed economy and by 1.3% in a small open economy.

A federal income tax cut would improve social welfare. Based on wealth measures, households would be better off on average by 2.8 to 3.0% in a closed economy and by 5.6 to 6.0% in a small open economy.

The effect of the marginal tax rate cut on inequality is relatively small. The after-tax income Gini coefficient would rise by 0.01 in both a closed economy and a small open economy. The share of wealth held by the top 1% of households would rise 0.6 percentage points in a closed economy and by 0.3 percentage points in a small open economy.

If the progressive income tax were completely replaced with a flat-rate income tax, how much would inequality be intensified? The last two columns of Table IX reveal the overall effect of a progressive income tax on the economy.

The changes in national wealth, labor supply, and GNP would be much larger than those in the previous case. National wealth would increase by 9.6% in a closed economy and by 15.0% in a small open economy. The average income tax rate to keep the same government wealth level as that

TABLE IX

When Federal Income Tax Rates Are Cut by 10 Percent or the Progressive Tax Is Replaced with a Flat-Rate Tax

	Baseline econ.	A 10% federal income tax cut adjusted by				Replacing the progressive tax w/flat-rate tax	
		Government consumption		Lump-sum tax		Closed econ.	S.open econ.
		Closed econ.	S.open econ.	Closed econ.	S.open econ.		
%Δ National wealth		2.3	3.4	2.7	3.7	9.6	15.0
%Δ Labor		0.3	0.3	0.5	0.4	1.9	1.7
%Δ GNP		1.0	1.3	1.2	1.5	4.3	5.9
%Δ Domestic capital		2.3	0.3	2.7	0.4	9.6	1.7
%Δ GDP		1.0	0.3	1.2	0.4	4.3	1.7
%Δ Private consumption		1.5	1.5	1.0	0.9	4.1	4.3
Δ Government consumption/GDP*		-0.7	-0.7	n.c.	n.c.	n.c.	n.c.
Δ Lump-sum tax/GDP*		n.c.	n.c.	0.7	0.7	n.c.	n.c.
Δ Average income tax rate		n.c.	n.c.	n.c.	n.c.	-0.9	-1.0
Δ Bequests/GDP*		0.1	0.1	0.1	0.2	0.4	0.8
Δ Interest rate		-0.1	n.c.	-0.2	n.c.	-0.6	n.c.
%Δ Wage rate		0.6	n.c.	0.7	n.c.	2.4	n.c.
Δ Net foreign wealth ratio		n.c.	8.9	n.c.	9.1	n.c.	37.4
Welfare changes (%)							
Compensating wealth ^a		2.8	5.6	0.8	3.6	6.1	17.6
Equivalent wealth ^b		3.0	6.0	1.5	4.3	10.1	23.6
Gini coefficients							
Earnings	0.63	0.63	0.63	0.63	0.63	0.63	0.63
Income	0.53	0.53	0.53	0.53	0.52	0.55	0.54
(After tax and transfers)	0.46	0.47	0.47	0.47	0.46	0.51	0.51
Income & bequests	0.53	0.53	0.53	0.53	0.53	0.55	0.55
(After tax and transfers)	0.47	0.48	0.47	0.47	0.47	0.52	0.52
Wealth	0.79	0.79	0.79	0.79	0.79	0.83	0.82
Share of wealth (%)							
Top 1%	30.7	31.3	31.0	31.2	30.7	35.0	34.3
Top 5%	51.0	51.7	51.3	51.6	51.0	57.2	56.1
Top 10%	63.7	64.4	63.9	64.3	63.7	69.6	68.7
Top 20%	80.0	80.5	80.0	80.4	79.8	84.7	83.6
Top 40%	95.3	95.5	95.3	95.4	95.2	97.2	96.9
Top 60%	99.8	99.9	99.8	99.9	99.8	100.0	99.9

Note. %Δ and Δ denote the percent change and the change in percentage points, respectively, from the baseline economy; GDP* denotes GDP in the baseline economy. “n.c.” denotes “no change” by definition.

^a (Total wealth/Compensating wealth - 1) × 100.

^b (Equivalent wealth/Baseline total wealth - 1) × 100.

in the baseline economy would be lower by 0.9 percentage points in a closed economy and by 1.0 percentage point in a small open economy.

Replacing the progressive income tax with a flat-rate tax, inequality would increase significantly. The Gini coefficient of after-tax income would rise by 0.05 in both a closed economy and a small open economy. The wealth Gini coefficient would rise by 0.04 and 0.03 in a closed economy and a small open economy, respectively. The share of wealth held by the top 1% of households would rise by 4.3 percentage points in a closed economy and 3.6 percentage points in a small open economy.

6. CONCLUDING REMARKS

There are several hypotheses to explain why a household leaves bequests and why it makes inter vivos transfers to their offspring. Besides accidental bequests due to lifetime uncertainty, this paper assumes that altruism is the primary motive of intergenerational transfers. But, if other motives, such as “joy-of-giving” and “gift-exchange,” are stronger, the effect of policy changes will be different.

Still, the policy experiments in this paper demonstrate how the economy could be after the policy change—such as estate tax repeal or a federal income tax cut—if bequests and inter vivos transfers were motivated in large part by altruism and lifetime uncertainty. Comparing the distributions of earnings, income, and wealth in the model with those in the previous models, the paper shows how intergenerational transfers are likely to affect wealth accumulation and its distribution of households.

In this paper, the model assumes a noncooperative simultaneous game. If we assume a two-stage game instead, then the steady-state economy and the policy implication of the model might be different. Given the degree of time preference, national wealth will be smaller because of the stronger strategic dissaving. Given the degree of parental altruism, bequests will be smaller if the parent household decides the level of bequests first (and that decision is credible), and bequests will be larger if the child households make their decision first. I am planning to explore these other types of models in the near future.

According to the survey data, financial transfers from children to parents are very small. So, the model assumes one-sided altruism. But, there are other unmeasured transfers, such as informal caregiving. Arno *et al.* (1999) estimated the national economic value of informal caregiving as \$196 billion in 1997. If we also consider those nonfinancial transfers in a two-sided altruism model, the wealth inequality of the steady-state economy will be slightly stronger.

Other important topics, which were not considered in this paper, include the education investment—both time and money—by altruistic parents (e.g., Aiyagari *et al.*, 2000), the retirement decision of elderly households, and the working decision jointly determined by couples.

APPENDIXES

A. The Computation of Equilibria

In this model, the state of a dynasty is shown as $\mathbf{s}_I = (a_p, a_k, e_k) \in A^2 \times E$ for Type I dynasties and $\mathbf{s}_{II} = (a, e) \in A \times E$ for Type II dynasties, where $A = [0, a_{\max}]$ and $E = [e_{\min}, e_{\max}]$. To compute an equilibrium, the state space of a dynasty is discretized as $\hat{\mathbf{s}}_I \in \hat{A}^2 \times \hat{E}$ and $\hat{\mathbf{s}}_{II} \in \hat{A} \times \hat{E}$, where $\hat{A} = \{a^1, a^2, \dots, a^{N_a}\}$ and $\hat{E} = \{e^1, e^2, \dots, e^{N_e}\}$. In this paper, N_a is set at 31 and N_e is set at 8. The total number of discrete states for Type I dynasties is 7680 in each age; for Type II dynasties, the total number is 248. The asset space is discretized as $\hat{A} = \{i^3\}_{i=0}^{30}$.

For all these discrete points, compute

- (1) the optimal decision of households, $\{\mathbf{d}_{I,i}(\hat{\mathbf{s}}_I)\}_{i=1}^4$ and $\mathbf{d}_{II,2}(\hat{\mathbf{s}}_{II})$, where $\mathbf{d}_{I,i}(\hat{\mathbf{s}}_I) \in (0, c_{\max}] \times [0, h_{i\max}] \times [0, g_{\max}] \times A$ and $\mathbf{d}_{II,2}(\hat{\mathbf{s}}_{II}) \in (0, c_{\max}] \times [0, h_{i\max}] \times A$,
- (2) the marginal values, $\mathbf{v}'_{I,i}(\hat{\mathbf{s}}_I) = (\frac{\partial}{\partial a_p} v_{I,i}(\hat{\mathbf{s}}_I), \frac{\partial}{\partial a_k} v_{I,i}(\hat{\mathbf{s}}_I))$ and $\mathbf{v}'_{II,2}(\hat{\mathbf{s}}_{II}) = (\frac{\partial}{\partial a} v_{II,2}(\hat{\mathbf{s}}_{II}), \frac{\partial}{\partial e} v_{II,2}(\hat{\mathbf{s}}_{II}))$, given the government policy rule and factor prices.

To find the optimal end-of-period wealth, the model uses the Euler equation method and bilinear (for Type I households) or linear (for Type II households) interpolation of marginal value functions in the next period.²⁷

A.1. Steady-State Equilibria

The algorithm to compute a steady-state equilibrium is as follows: Let Ψ denote the time-invariant government policy rule $\Psi = \{\tau_F(\cdot), \tau_S(\cdot), \tau_E(\cdot), tr_{SS}, tr_{LS}, C_g, W_g\}$.

- (1) Set the initial values of factor prices (r^0, w^0) and the government policy variables $(tr_{SS}^0, tr_{LS}^0, C_g^0, W_g^0)$ if these are determined endogenously.²⁸

²⁷ Because of this bilinear interpolation of marginal value functions, any equilibrium in this model is shown as a mixed strategy equilibrium.

²⁸ If the capital-labor ratio is found, both r and w are calculated from the production function and the depreciation rate. So, the iteration uses the capital-labor ratio instead of r and w .

- (2) Find the decision rule of households given factor prices and the government policy variables, $\{\mathbf{d}_{I,i}^0(\hat{\mathbf{s}}_I)\}_{i=1}^4$ and $\mathbf{d}_{I,2}^0(\hat{\mathbf{s}}_{II})$, for all $\hat{\mathbf{s}}_I \in \hat{A}^2 \times \hat{E}$ and $\hat{\mathbf{s}}_{II} \in \hat{A} \times \hat{E}$.
- (3) Find the steady-state measure of dynasties, $\{x_{I,i}^0(\hat{\mathbf{s}}_I)\}_{i=1}^2$ and $x_{II,2}^0(\hat{\mathbf{s}}_{II})$, using the decision rule obtained in Step (2) and the Markov transition matrix of the working ability of households.
- (4) Compute the aggregate capital stock K , labor supply L , and government aggregate variables. Then, find factor prices (r^1, w^1) and the government policy variables $(tr_{SS}^1, tr_{LS}^1, C_g^1, W_g^1)$.
- (5) Compare $(r^1, w^1, tr_{SS}^1, tr_{LS}^1, C_g^1, W_g^1)$ with $(r^0, w^0, tr_{SS}^0, tr_{LS}^0, C_g^0, W_g^0)$. If the difference is sufficiently small then stop. Otherwise, replace $(r^0, w^0, tr_{SS}^0, tr_{LS}^0, C_g^0, W_g^0)$ with $(r^1, w^1, tr_{SS}^1, tr_{LS}^1, C_g^1, W_g^1)$ and return to Step (2).

A.2. The Decision Rule of Households

The algorithm to find the decision rule of Type I households is as follows. For simplicity, the explanation is abstracted from population growth, productivity growth, and lifetime uncertainty.

- (1) Set the initial numbers of marginal values $\{\mathbf{v}_{I,i}^0(\hat{\mathbf{s}}_I)\}_{i=2}^4$.
- (2) For each $(\hat{\mathbf{s}}_I, i) \in \hat{A}^2 \times \hat{E} \times \{1, 2, 3, 4\}$ find the decision rule of all households, $\mathbf{d}_{I,i}(\hat{\mathbf{s}}_I) = \mathbf{d}_p$ or \mathbf{d}_k , taking the government policy rule $\Psi = \{\tau_F(\cdot), \tau_S(\cdot), \tau_E(\cdot), tr_{SS}^0, tr_{LS}^0, C_g^0, W_g^0\}$, factor prices (r^0, w^0) , and the marginal values as given.
 - (a) Set the initial values on the decision of the child household $\mathbf{d}_k^0 = (c_k^0, h_k^0, a_k^0)$.
 - (b) Given the decision of the child household \mathbf{d}_k^0 , find the optimal decision of its parent household $\mathbf{d}_p^0 = (c_p^0, h_p^0, g_p^0, a_p^0)$.
 - (i) Set the initial value of the parent's end-of-period wealth $a_p^0(\mathbf{d}_k^0)$ and the gift to its child household $g_p^0(a_p^0, \mathbf{d}_k^0)$.
 - (ii) Find the level of consumption and working hours, $c_p^0(g_p^0, a_p^0, \mathbf{d}_k^0)$ and $h_p^0(g_p^0, a_p^0, \mathbf{d}_k^0)$, using the marginal rate of substitution of c_p^0 for h_p^0 and after-tax marginal wage rate.
 - (iii) Compare the marginal utility of consumption of its own and of its child household. If $u_1(c_p^0, h_p^0) \geq \eta u_1(c_k^0, h_k^0)$ with equality holds when $g_p^0 > 0$, go to Step (iv). Otherwise, replace g_p^0 with g_p^1 that solves

$\varepsilon = \arg \min |\eta u(c_k^0 + \varepsilon, h_k^0) - u(c_p^0 - \varepsilon, h_p^0)|$ subject to $g_p^1 = g_p^0 + \varepsilon \geq 0$, and return to Step (ii).

(iv) Check the Euler equation of the parent household. If

$$\frac{\partial}{\partial c_p} u(c_p^0, h_p^0) \geq \begin{cases} \beta E \frac{\partial}{\partial a'_p} v_{I, i+1}(\hat{s}'_I) & (\text{if } i = 3) \\ \beta E \eta \frac{\partial}{\partial a'_p} v_{I, i-2}(\hat{s}'_I) & (\text{if } i = 4) \end{cases}$$

with equality holds when $a_p^0 > 0$, go to Step (c). Otherwise, replace a_p^0 with a_p^1 , where

$$a_p^1 = \begin{cases} \arg \min \left| \beta E \frac{\partial}{\partial a'_p} v_{I, i+1}(\hat{s}'_I) - \frac{\partial}{\partial c_p} u(c_p^0, h_p^0) \right| & (\text{if } i = 3) \\ \arg \min \left| \beta E \eta \frac{\partial}{\partial a'_p} v_{I, i-2}(\hat{s}'_I) - \frac{\partial}{\partial c_p} u(c_p^0, h_p^0) \right| & (\text{if } i = 4) \end{cases}$$

subject to $a_p^1 \geq 0$, and return to Step (ii).

- (c) Similarly, given the decision of the parent household \mathbf{d}_p^0 obtained in Step (b), find the optimal decision of its child household $\mathbf{d}_k^1 = (c_k^1, h_k^1, a_k^1)$.
 - (d) Compare the new decision of the child household, \mathbf{d}_k^1 , with the old one, \mathbf{d}_k^0 . If the difference is sufficiently small, then go to Step (e). Otherwise, replace \mathbf{d}_k^0 with \mathbf{d}_k^1 and return to Step (b).
 - (e) Compute the marginal values $(\mathbf{v}_{I,4}^1(\hat{s}_I), \mathbf{v}_{I,2}^1(\hat{s}_I))$ or $\mathbf{v}_{I,3}^1(\hat{s}_I)$ using $(\mathbf{d}_p^0, \mathbf{d}_k^0)$.
- (3) Compare the new marginal values $\{\mathbf{v}_{I,i}^1(\hat{s}_I)\}_{i=2}^4$ with $\{\mathbf{v}_{I,i}^0(\hat{s}_I)\}_{i=2}^4$. If the difference is sufficiently small, then stop. Otherwise, replace $\{\mathbf{v}_{I,i}^0(\hat{s}_I)\}_{i=2}^4$ with $\{\mathbf{v}_{I,i}^1(\hat{s}_I)\}_{i=2}^4$ and return to Step (2).

B. Optimal Annuity Holdings of a Household

In this model, death is uncertain at the end of age 3. In the presence of perfect annuity markets, age 3 households choose the optimal level of end-of-period annuity holdings, $qb'_{p,3}(s_I)$, where q is the price of annuity and $q = \lambda$. Clearly, it will not exceed end-of-period wealth level; i.e., $0 \leq qb'_{p,3}(s_I) \leq a'_{p,3}(s_I)$. If an age 3 household has qb'_p of its wealth in the form of an annuity at the end of this period and if it is alive in the next period, its wealth at the beginning of age 4 is $(a'_p - qb'_p) + b'_p = a'_p + (1 - \lambda)b'_p$.

If the household dies at the end of age 3, the wealth inherited by its child is simply $(a'_p - \lambda b'_p)/n$.

Let $\tilde{\mathbf{d}}_p$ and $\tilde{\mathbf{d}}_k$ be the set of decisions of a parent household and its child households, respectively; i.e., $\tilde{\mathbf{d}}_p = (c_p, h_p, g_p, a'_p, b'_p)$ and $\tilde{\mathbf{d}}_k = (c_k, h_k, a'_k)$. The best response functions of an age 3 parent and an age 1 child are written as

$$\tilde{\mathbf{R}}_3(\tilde{\mathbf{d}}_k; \mathbf{s}_I) = \arg \max_{c_p, h_p, a'_p, b'_p} \{u(c_p, h_p) + \eta nu(c_k, h_k) + \beta E[\lambda v_{I,4}(\mathbf{s}'_I) + (1 - \lambda)\eta nv_{II,2}(\mathbf{s}'_{II}) \mid e_k]\},$$

$$\tilde{\mathbf{R}}_1(\tilde{\mathbf{d}}_p; \mathbf{s}_I) = \arg \max_{c_k, h_k, a'_k} \{u(c_k, h_k) + \beta E[\lambda v_{I,2}(\mathbf{s}'_I) + (1 - \lambda)v_{II,2}(\mathbf{s}'_{II}) \mid e_k]\},$$

where the law of motion of the state is

$$\begin{aligned} \mathbf{s}'_I &= (a'_p + (1 - \lambda)b'_p, a'_k, e'_k), \\ \mathbf{s}'_{II} &= (a'_k + ((a'_p - \lambda b'_p) - \tau_E(a'_p - \lambda b'_p))/n, e'). \end{aligned}$$

Solving equations,

$$\tilde{\mathbf{R}}_3(\tilde{\mathbf{d}}_k; \mathbf{s}_I) = \tilde{\mathbf{d}}_p, \quad \tilde{\mathbf{R}}_1(\tilde{\mathbf{d}}_p; \mathbf{s}_I) = \tilde{\mathbf{d}}_k,$$

Nash equilibrium decisions rules are obtained as

$$\begin{aligned} \tilde{\mathbf{d}}_{I,3}(\mathbf{s}_I) &= (c_{I,3}(\mathbf{s}_I), h_{I,3}(\mathbf{s}_I), g_{I,3}(\mathbf{s}_I), a'_{I,3}(\mathbf{s}_I), b'_{I,3}(\mathbf{s}_I)), \\ \tilde{\mathbf{d}}_{I,1}(\mathbf{s}_I) &= (c_{I,1}(\mathbf{s}_I), h_{I,1}(\mathbf{s}_I), a'_{I,1}(\mathbf{s}_I)), \end{aligned}$$

for $\mathbf{s}_I \in A^2 \times E$.

C. The Computation of Welfare Measures

Since the labor supply of households is endogenous in this paper, the following welfare measures are constructed based on the wealth level of the households.

C.1. *The Compensating Variation*

Let $v_{I,i+2}^0(\mathbf{s}_I)$, $v_{I,i}^0(\mathbf{s}_I)$, and $v_{II,2}^0(\mathbf{s}_{II})$ be the value functions of a Type I parent household, a Type I child household, and a Type II household, respectively, in the baseline economy, where $i \in \{1, 2\}$. Let $X_{I,i}^0(\mathbf{s}_I)$ and $X_{II,2}^0(\mathbf{s}_{II})$ be the cumulative measures of Type I households and Type II households, respectively, in the baseline economy. Let $v_{I,i+2}^1(\mathbf{s}_I)$, $v_{I,i}^1(\mathbf{s}_I)$, and $v_{II,2}^1(\mathbf{s}_{II})$ be the corresponding value functions in the alternative economy. Suppose that the functions $a_{I,i+2}^C(\mathbf{s}_I)$, $a_{I,i}^C(\mathbf{s}_I)$, and $a_{II,2}^C(\mathbf{s}_{II})$ solve

$$\begin{aligned} v_{I,i+2}^1(a_{I,i+2}^C(\mathbf{s}_I), a_{I,i}^C(\mathbf{s}_I), e_k) &= v_{I,i+2}^0(\mathbf{s}_I), \\ v_{I,i}^1(a_{I,i+2}^C(\mathbf{s}_I), a_{I,i}^C(\mathbf{s}_I), e_k) &= v_{I,i}^0(\mathbf{s}_I), \\ v_{II,2}^1(a_{II,2}^C(\mathbf{s}_{II}), e) &= v_{II,2}^0(\mathbf{s}_{II}). \end{aligned}$$

Then the compensating wealth W^C is defined as

$$\begin{aligned} W^C &= \sum_{i=1}^2 \int_{A^2 \times E} (a_{I,i+2}^C(\mathbf{s}_I)/n + a_{I,i}^C(\mathbf{s}_I)) dX_{I,i}^0(\mathbf{s}_I) \\ &\quad + \int_{A \times E} a_{II,2}^C(\mathbf{s}_{II}) dX_{II,2}^0(\mathbf{s}_{II}). \end{aligned}$$

Let W^1 be the total private wealth in the alternative economy. The welfare change, in percent, measured by the compensating variation is defined as $(W^1/W^C - 1) \times 100$. The alternative economy is potentially Pareto preferred to the baseline economy if $W^1 > W^C$.

C.2. *The Equivalent Variation*

Let $X_{I,i}^1(\mathbf{s}_I)$ and $X_{II,2}^1(\mathbf{s}_{II})$ be the cumulative measures of Type I households and Type II households, respectively, in the alternative economy. Suppose that the functions $a_{I,i+2}^E(\mathbf{s}_I)$, $a_{I,i}^E(\mathbf{s}_I)$, and $a_{II,2}^E(\mathbf{s}_{II})$ solve

$$\begin{aligned} v_{I,i+2}^0(a_{I,i+2}^E(\mathbf{s}_I), a_{I,i}^E(\mathbf{s}_I), e_k) &= v_{I,i+2}^1(\mathbf{s}_I), \\ v_{I,i}^0(a_{I,i+2}^E(\mathbf{s}_I), a_{I,i}^E(\mathbf{s}_I), e_k) &= v_{I,i}^1(\mathbf{s}_I), \\ v_{II,2}^0(a_{II,2}^E(\mathbf{s}_{II}), e) &= v_{II,2}^1(\mathbf{s}_{II}). \end{aligned}$$

Then the equivalent wealth W^E is defined as

$$\begin{aligned} W^E &= \sum_{i=1}^2 \int_{A^2 \times E} (a_{I,i+2}^E(\mathbf{s}_I)/n + a_{I,i}^E(\mathbf{s}_I)) dX_{I,i}^1(\mathbf{s}_I) \\ &\quad + \int_{A \times E} a_{II,2}^E(\mathbf{s}_{II}) dX_{II,2}^1(\mathbf{s}_{II}). \end{aligned}$$

Let W^0 be the total private wealth in the baseline economy. The welfare change measured by the equivalent variation is defined as $(W^E/W^0 - 1) \times 100$. The alternative economy is potentially Pareto preferred to the baseline economy if $W^E > W^0$.

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