Journal of Public Economics 87 (2003) 489–513
JOURNAL OF PUBLIC ECONOMICS
www.elsevier.com/locate/econbase

# Intergenerational transfers, production and income distribution

Itzhak Zilcha*

*The Eitan Berglas School of Economics Tel-Aviv University, Tel Aviv, Israel*

Received 18 November 1999; received in revised form 20 August 2000; accepted 9 September 2000

**Abstract**

We consider an overlapping-generations economy where the aggregative production process uses physical capital and human capital. Intergenerational transfers from parents to their children are motivated by 'altruism'. We distinguish between two types of transfers: Investment of parents in the education of their offspring, and capital transfer (the 'gift-bequest motive'). We show that the *intensity* of each type of altruism, and hence the *composition* of the two types of transfers, affect significantly the equilibrium output and the intragenerational income distributions. Comparing competitive equilibria, from the same initial distributions of capital transfers and human capital, we derive: (a) The economy with the higher 'education-inclined' altruism attains *lower* output and *more equal* intragenerational income distributions, if either the production function's elasticity of substitution is not 'large' or the altruism is 'weak' (b) For more 'bequest-inclined' altruism the effects are, usually, reversed (c) Under public provision of education, higher (bequest-inclined) altruism results in *lower* output and *less equal* distribution of income in all dates.


*JEL classification:* E13, D64, D91

## 1. Introduction

Intergenerational transfers, in their various forms, are among the significant factors affecting the aggregate human capital, physical capital stock and the

*Tel.: +972-3-640-9715; fax: +972-3-640-9908.
*E-mail address:* izil@ccsg.tau.ac.il (I. Zilcha).

0047-2727/00/$ – see front matter 
doi:10.1016/S0047-2727(00)00142-0

inequality in the distribution of income. In recent years economists studied extensively various types of endogenous growth models where human capital is assumed to be the engine for growth. The issues we consider here are related to the following two presumptions which are supported by empirical evidence:

1. Intergenerational transfers in the form of human capital and physical capital affect significantly intragenerational income distribution and capital accumulation.
2. These transfers are motivated mainly by some sort of 'altruism' between parents and children.

Should we distinguish between countries (or societies) where parental transfers to children consists mainly of investments in education and countries where parental transfers are biased towards physical capital? How does the 'mixture' of these two types of intergenerational transfers affect income inequality and output? The coexistence of these two types of transfers has received little attention in the literature (see, for example, Becker and Tomes, 1986; Lord and Rangazas, 1991. An extensive survey is brought in Laitner, 1997). Evidence regarding significant differences in investments in education between countries has been documented in recent years (among the OECD countries see: Education at Glance: OECD Indicators, 1997 for other countries see: World Development Indicators, 2000). It is our main aim to demonstrate the economic importance of the 'composition' of these two different types of intergenerational transfers. Differences between societies (and, hence, economies) emanate from cultural and traditional gaps as well as the nature of the 'altruism' of parents towards their children. We assume that,

(a) The investment of parents in education of their child emanates from their preference for a 'more educated' offspring, and also to affect this way her future earning capability (we call it 'education-inclined altruism').

(b) All sorts of physical capital transfers are motivated by the 'joy of giving' and/or the 'gift-bequest motive' (we call it 'bequest-inclined altruism').

The utility derived by parents from each kind of transfer is assumed to exist but the *intensity* (of the preferences) towards each transfer may differ between economies. Thus, in comparing economies, which differ only in the intensity of each type of altruism, we shall study the following questions:

- When parental transfers to children have larger bias towards education does it imply more equal intragenerational income distributions? higher output?
- Do the answers to the above questions depend on the education provision regime? Namely, is the private provision of education case different significantly from the public provision case?

In an OLG model we compare competitive equilibria of two economies, starting from some initial distributions of human capital and intergenerational transfers, which differ only in the 'composition' of transfers (due to different utility functions). Let $\sigma$ be the aggregate production function's elasticity of substitution. We obtain that under *private* provision of education, if either $\sigma$ is not 'much larger than 1', or the preference for human capital of the offspring is 'strong' then: A more (*education-inclined*) altruistic economy attains *lower* output, but *more equal* intragenerational income distribution. However, when we consider higher *bequest-inclined* altruism the *direction* reverses in some circumstances: For example, when $\sigma$ assumes 'low' values more altruism imply *higher* output and *less equal* income distribution in each date. Thus, in comparing income distributions in various countries we should also pay attention to the mode in which transfers from parents to children are carried out: Is it more in the form of enhancing human capital formation or is it mostly transfers of some sorts of capital.

Consider the case where education is provided by the government, financed by taxing the wage-income, and the provision is equal to all 'young' individuals (i.e., independent of ability and wealth of parents). It is shown that, regardless of the above-mentioned parameters, higher *bequest-inclined* altruism results in (a) *higher* output, and (b) *less equal* income distributions, in all dates. Thus, under public provision of education more altruism results in a significantly different economic consequences compared to the private provision case.

Our analysis shows that intergenerational transfers, and *the way* they are implemented, have consequences to the evolution of the economy. In the last two decades we have witnessed some disagreement between economists with regard to the quantitative importance and motivation of intergenerational transfers in enhancing savings and capital accumulation. Let us bring few, out of many, papers with evidence related to intergenerational transfers and their magnitudes. An extensive and insightful survey can be found in Laitner (1997). Considering all sorts of 'gift-bequest' types of transfers Kotlikoff and Summers (1981) find that, in the US economy, such transfers are responsible for more than three quarters of aggregate capital accumulation (see, also, Tomes, 1981; Kotlikoff, 1988; Gokhale et al., 1999). Others claim that it is only 20–30% (see, for example, Modigliani, 1988; Gale and Scholz, 1994). Bernheim (1991) presents empirical evidence supporting the view that private savings are strongly influenced by the desire for bequest. Bernheim's evidence is based on the presumption that bequest motives significantly alter the individual's attitude towards annuities markets. Weil (1994) shows that intergenerational transfers explain the substantial gap in the elderly saving rate between household (micro) data and the aggregate (macro) data. Gale and Scholz (1994) claim that the life-cycle model cannot be used to estimate properly the intergenerational transfers since most transfers are *inter vivos* which, in this case, are from living parents to adult children living in separate households; such transfers are estimated at 20% of aggregate wealth in the US, while total 'bequest transfers' are 31% of the net worth. Cox and Raines (1985) also show that inter-vivos gifts are three times as large as inheritance. On the other hand, some other studies find weak empirical evidence for intentional bequest transfers (see, e.g., Hurd, 1987; Altonji et al., 1992, 1997; Laitner and Juster, 1996). Thus, even if altruism is widespread, for many households binding non-negativity constraints may reduce the transfers to insignificant amounts (see Laitner, 1997 for more discussion).

The implications of human capital evolution to intragenerational income distribution have been examined widely (see, for example, Loury, 1981; Glomm and Ravikumar, 1992; Eckstein and Zilcha, 1994; Benabou, 1996; Galor and Tsiddon, 1997; Orazem and Tesfatsion, 1997). Empirical evidence shows that parental investments in education (as well as other family background) affect significantly the earnings distribution (see, e.g., Becker and Tomes, 1986; Galor and Zeira, 1993); Gokhale et al. (1999) consider evidence related to the transmission of inequality via bequests. The manner in which education is provided and its impact on income inequality and its efficiency has received attention (see, e.g., Hanushek, 1986; Glomm and Ravikumar, 1992; Fernandez and Rogerson, 1998). The role of the public sector, which is a major factor in most countries, in financing education and in redistribution of income has been studied widely in recent years.

The paper is organized as follows. In Section 2 we present an OLG model with altruistic individuals. Section 3 compares the aggregate output along competitive equilibrium paths, starting from the same initial conditions, in two economies which have different degrees of altruism. Section 4 compares the intragenerational distribution of incomes, under private provision of education, in two economies which differ only in the degree of altruism. In Section 5 we study this economy under *public provision* of education. Section 6 contains a discussion and the proofs are relegated to Section 7.

## 2. The model

### 2.1. Preferences and technology

Consider an overlapping generations economy with no population growth. Each individual has two economically active periods – a working period followed by a retirement period (let us say that the 'childhood' period is not explicitly considered). At the end of the first period, every individual gives birth to one offspring. Denote by $G_t$ the set of individuals born at the outset of period $t$ and refer to these individuals as generation $t$. The economy starts at $t = 0$, where $G_{-1}$ live their retirement period; their only source of income is their savings and they do not make economic decisions at date 0. Denote by $\Omega$ the set of families in each generation; $\Omega$ is time independent since we assume no population growth. Although our analysis can be carried out for any finite $\Omega$, to simplify the notations we assume a continuum of individuals (or families) in each generation, hence we take $\Omega = [0,1]$. Each individual in $G_t$ is characterized by its family 'name' $\omega \in [0,1]$. Denote by $\mu$ the Lebesque measure on [0,1].

In describing consumers' behavior economists consider well-being of an entity called 'household'. This family unit may include mature children. Thus maximizing the household's lifetime well-being may include besides the parent own consumption her contribution to her children's future consumption, human capital etc., Thus, using Laitner's (1997, page 193) words 'altruism means that parents objective function includes a term reflecting empathy toward the household of its grown children'. Specifically, in our 'one-sided altruism' case, in addition to its own lifetime consumption the parent cares about consumption and human capital of her (mature) child.

In this economy we have one-sided transfers: parents derive utility from the human capital level of their child as well as from (physical) capital transfers, or the 'joy of giving'. This motivates the transfers to their offspring in the form of human and nonhuman capital. The parent's, say $\omega \in G_t$, investment in their offspring's education in conjunction with her (random) ability, determine her level of human capital $h_{t+1}(\omega)$. The intergenerational transfer of physical capital $b_t(\omega)$ and her human capital$h_{t+1}(\omega)$, together with the relevant interest rate and wages determine the offspring's total income $y_{t+1}(\omega)$. We assume that the human capital of the child is important to the parent beyond its contribution to her future wage income. Thus, it is represented in the utility function as a separate type of 'joy of giving'. In many societies investment in human capital is rewarded with social status (which certainly rewards parents as well). For instance, scientists and professors often obtain reward in the form of social esteem in addition to the fungible form (see Hodge et al., 1966; Treiman, 1977). The distinction between these two types of transfers (human capital vs. physical capital) is the basis for our choice of utility function where each 'type' of transfer appears as a separate argument in the utility function. A special case, frequently used in the literature, is the case where the parent's utility depends only the 'joint contribution' of both transfers to the offspring's future income, i.e., the parent derives utility from $y_{t+1}(\omega)$.

Preferences of each individual in $G_t$ are represented by a Cobb-Douglas utility function:

$$U = c_{1t}^{\alpha_1} c_{2t}^{a_2} [\hat{b}_t]^{\alpha_3} [h_{t+1}]^{\alpha_4} \tag{1}$$

where $\alpha_i$ are known parameters, $\alpha_i > 0$ and bounded from above for $i = 1, 2, 3, 4$; $c_{1t}$ and $c_{2t}$ denote, respectively, consumption in the first and second periods of the individual's life; $\hat{b}_t$ denotes the capital transferred to the child at the end of date $t$ (or beginning of date $t+1$), while $h_{t+1}$ denotes her human capital level.

Our utility function includes a taste for transfers to one's offspring. This is supported by Becker and Tomes (1986), who claim that 'Parents influence the economic welfare of their children primarily by influencing their potential earnings'. We shall use the term 'altruism', even though it is somehow a different notion (altruism, usually, takes into account how the transfers affect the recipient's well-being). This will be further discussed in the next section. Our choice of utility implies that individuals do not optimize over an infinite horizon, which is the case where parent's utility function contains the utility of the offspring, and hence the utilities of *all future generations*. Such linkage between generations has been criticized (see, e.g., Bernheim and Bagwell, 1988) due to its extensive neutrality implications. However, assuming that parent's utility function depend directly on the utility function of her offspring will vary the results obtained in this work. We shall (mis)use the notion '*altruism*' when we refer to utility functions given by (1).

Let $e_t(\omega)$ be the investment of $\omega \in G_t$ in educating her child. We assume that the human capital $h_{t+1}(\omega)$ of the offspring is determined by the investment in education $e_t(\omega)$ and the random *innate ability*, denoted by $\tilde{\theta}_{t+1}(\omega)$. Moreover, for some constant $\gamma$, $0<\gamma \leq 1$, the level of human capital is given by the process:

$$h_{t+1}(\omega) = A[e_t(\omega)]^{\gamma}\tilde{\theta}_{t+1}(\omega)\ , \quad t = 0,1,2,\ldots \tag{2}$$

Even though some evidence supporting the dependence of a child's human capital on his parent's human capital exists, we shall not assume such a direct relationship. In general, such an assumption will change the dynamics of the equilibrium path *significantly*. However, in this framework, $h_{t+1}$ is tied indirectly to $h_t$ through the parent's income (which depends on $h_t$) and affects the optimal choice of $e_t$. The theoretical literature related to intergenerational links has various approaches to modelling the evolution of human capital (see, e.g., Lucas, 1988; Jovanovic and Nyarko, 1995; Benabou, 1996; Galor and Tsiddon, 1997). A more general process of the human capital evolution was presented by Orazem and Tesfatsion (1997). They assume that, although parents control all physical resources invested in their child's human capital, the effectiveness of this investment depends on the child's choice of how much effort to exert in school. Our analysis can be generalized to include additional factors in the production process of human capital, given by (2), but the results (and the complexity of proofs) will differ significantly. Without loss of generality let us take $A=1$.

The random variables $\tilde{\theta}_t(\omega)$, $t = 1,2,3,\ldots;$ $\omega \in \Omega$ are identically distributed with support $[a, b]$, $0<a<b<\infty$, independent over time and across dynasties. We denote the mean of (each) $\tilde{\theta}_t(\omega)$ by $\tilde{\theta}$. It is not difficult to relax the independence assumption and to allow some correlation between $\tilde{\theta}_t(\omega)$ and $\tilde{\theta}_{t+1}(\omega)$ for each $\omega$; however, we do not pursue this case here. Each individual, in his 'working' period, supplies inelastically 1 unit of labor. We do not consider the possibility that an individual can increase investment in her own education. Usually, imperfect capital markets prevent children from investing in their own education (some empirical evidence can be found in Zeldes, 1989).

The dependence of human capital upon some random innate ability has an important consequences to the heterogeneity in this economy. Earnings heterogeneity, coupled with the inequality in intergenerational transfers, result in intragenerational dispersion in incomes along the equilibrium path. In a steady state, however, income dispersion will result solely from the random ability process; thus each dynasty would 'asymptotically' face the same *expected income*. In other words, since abilities are assumed to be random variables within the same dynasty and across dynasties, heterogeneity in the long run persists in a limited sense: income dispersion results from the random ability process only. However, since we consider the *whole* equilibrium path, initial distributions of capital transfers and human capital are additional factors which contribute to heterogeneity.

Production in this economy is carried out by competitive firms that use labor and capital to produce a single commodity. The commodity serves for consumption and investment in production as well as in human capital. For expository convenience, and to avoid having to deal with details that are not essential for the main issue, we choose to disregard investment in human capital as a separate production processes. Consequently, our formulation abstracts from possible effects of changes in the relative price of education on the evolution of the economy. The aggregate stock of capital in period t is $K_t$, and it is determined by the aggregate savings in the preceding period. The production function $F(K_t, L_t)$ is assumed to exhibit constant returns-to-scale, where $L_t$ denotes the aggregate 'effective labor' or, since labor is supplied inelastically, the stock of human capital. We assume that $F_{KK} < 0$, $F_{LL} < 0$, and $F_{KL} > 0$. We denote by $\sigma$ the elasticity of substitution of the aggregate production function.

### 2.2. Equilibrium

In each period the economy features three markets: two factor markets (labor and capital), and one commodity market. To define a competitive equilibrium, we begin by considering the state of the economy at the outset of period $t$. Each family $\omega \in \Omega$ consists of two members, the 'old' member belonging to $G_{t-1}$ and the 'young' member belong to $G_t$. Suppose that the distribution of the bequests received by individuals in generation $G_t$ is given by the function $b_{t-1}(\omega)$. Note that in our economy inheritances are received at the beginning of the 'youth' period and gifts/bequests are made at the end of the 'youth' period. The human capital in $G_t$ is denoted by $h_t(\omega)$ while the initial distributions, $b_{-1}(\omega)$ and $h_0(\omega)$, are given at the outset of date 0. Clearly, both functions $b_{t-1}(\omega)$ and $h_t(\omega)$ depend upon the history up to date $t$.

We assume that parents are aware of the native abilities of their offsprings at the outset of their 'youth' period, i.e., $\omega \in G_t$ observes the realization of $\tilde{\theta}_{t+1}(\omega)$ prior to making her decision about the investment in the offspring's education. Thus, parents are not making decisions under uncertainty when they choose their saving and transfers. Consider now replacing this assumption with the assumption that the uncertainty about ability of children has not been resolved *before* parents make this decision about education, i.e., they maximize *expected* lifetime utility. In this case the optimal decision about saving and transfers will not be a function of the realization of $\tilde{\theta}_{t+1}(\omega)$. However, under our choice of the random processes, the utility function and the process governing the production of human capital, this will have no affect on the *aggregate* savings and transfers in each generation; thus such an assumption will bear no significant effect on our subsequent analysis and results. In particular, such modification will not change meaningfully the equilibrium.

Since each generation contains a continuum of individuals there is no uncertainty about future (effective) aggregate labor supply. This fact also implies that future wages and interest rates *are not* random variables. Consider now the optimization problem of individual $\omega \in G_t$, $t=0,1,2,\ldots$ where the bequest transfer, $b_{t-1}(\omega)$, the stock of human capital, $h_t(\omega)$, the effective wage rate, $w_t$, the interest rate $r_t$ for date $t$, the interest rate $r_{t+1}$, for date $t+1$, and the realization of $\theta_{t+1}(\omega)$ are given. The timing of receiving gift-bequest transfers is unimportant due to the perfect capital markets. However, the timing we have chosen is consistent with the starting point of our economy at date 0, where the 'young' individuals receive the transfers at the outset of this period. Each $\omega \in G_t$ chooses the levels of saving, $s_t(\omega)$, bequest transfer, $b_t(\omega)$, and investment in his offspring's education, $e_t(\omega)$, so as to maximize:

$$\text{Max } c_{1t}(\omega)^{\alpha_1} c_{2t}(\omega)^{\alpha_2} [(b_t(\omega)(1+r_{t+1})]^{\alpha_3} [h_{t+1}(\omega)]^{\alpha_4} \tag{3}$$

subject to constraints:

$$c_{1t}(\omega) = (1+r_t)b_{t-1}(\omega) + w_t h_t(\omega) - s_t(\omega) - b_t(\omega) - e_t(\omega) \geq 0 \tag{4}$$

$$c_{2t}(\omega) = (1+r_{t+1})s_t(\omega) \tag{5}$$

$$h_{t+1}(\omega) = A[e_t(\omega)]^{\gamma} \tilde{\theta}_{t+1}(\omega) \tag{6}$$

Let the aggregate capital stock $K_0$, the initial capital transfers $b_{-1}(\omega)$, and the human capital levels $h_0(\omega)$, for all $\omega \in G_0$, are given at date 0. A *competitive equilibrium* from these initial conditions is a sequence of functions $[c_{1t}(\omega), c_{2t}(\omega), s_t(\omega), b_t(\omega), e_t(\omega)]_{t=0}^{\infty}$ and a sequence of prices $[w_t, r_t]_{t=0}^{\infty}$ such that for all t, $t=0, 1, 2, \ldots$ (a) Given the above prices $[w_t, r_t]_{t=0}^{\infty}$, for all $\omega \in G_t$, $[c_{1t}(\omega), c_{2t}(\omega), s_t(\omega), b_t(\omega), e_t(\omega)]$ is the optimum for the lifetime utility maximization given by (3)–(6).

Given the initial human capital distribution $h_0(\omega)$, the optimal consumption–saving–transfer decisions given by (a), the aggregate labor supply in date t $L_t$ is $\int_{\Omega} h_t(\omega)\, d\mu(\omega)$; the aggregate supply of capital at date $t+1$, $K_{t+1}$, is given by:

$$K_{t+1} = \int_{\Omega} [s_t(\omega) + b_t(\omega)]\, \mathrm{d}\mu(\omega) \tag{7}$$

Then, each production factor's price is determined by the value of its marginal product, namely,

(*b*) $w_t = F_L(K_t, L_t)$ for all $t$.

(*c*) $1 + r_t = F_K(K_t, L_t)$ for all $t$.

The dynamics of the equilibrium follows, basically, from conditions (7)(b), (c). The initial $K_0$, $L_0$ determine $r_0$ and $w_0$, and hence, since the initial human capital distribution $h_0(\omega)$ is given, the income distribution for $G_0$ , $y_0(\omega)$, is known. Due to our assumption that parents know the ability of their offspring prior to their decision about investment in her education, each $\omega$ in $G_0$ with $y_0(\omega)$, observing interest rate $r_1$, determines $s_0(\omega)$, $e_0(\omega)$ and $b_0(\omega)$ by maximizing the lifetime utility as in (3)–(6). In addition, $K_1$ (given by $s_0(\omega)$ and $b_0(\omega)$) will give rise, by the equilibrium condition (c), to $r_1$. Also, since at the outset of period 0 the random $\tilde{\theta}_1(\omega)$ realizes, given $e_0(\omega)$, the distribution of $h_1(\omega)$ is known and thus the aggregate labor $L_1$ as well. Now, $w_1$ is given by (b) since $K_1$ and $L_1$ are known. This process can be continued, in a similar way, for all $t \geq 1$ to determine the equilibrium path: At the outset of period t, $b_{t-1}(\omega)$, $h_t(\omega)$ and $K_t$ are given, hence $L_t$ is determined. Each individual of dynasty $\omega$ in $G_t$ knows $y_t(\omega)$ and the realization of his child's ability $\theta_{t+1}(\omega)$. Thus, for a given $r_{t+1}$ the values of $s_t(\omega)$, $b_t(\omega)$ and $e_t(\omega)$ are determined optimally. Hence, $K_{t+1}$ is given as a function of $r_{t+1}$,is determined by the equilibrium condition (c).

Condition (a) asserts that consumers are price takers. Conditions (b) and (c) are the equilibrium conditions in the labor and capital markets, respectively. The specification of the demand functions is based on the assumption that firms are price takers in the factor markets. Eq. (7) describes the dynamic adjustment of the aggregate physical capital stock in the economy assuming full depreciation of capital. Aggregate capital stock is determined by aggregate savings and capital transfers.

In such OLG models with intergenerational transfers motivated by 'altruistic' parents, there exists inherent externality due to underinvestment in education. Parents undervalue the benefits from investment in human capital ignoring its impact upon wages and output in the future dates (see Eckstein and Zilcha, 1994 for more elaborate analysis). Thus, competitive equilibria are not Pareto optimal (hence, a central planner can Pareto improve utilities). Since in each generation dynasties differ in their levels of wealth and human capital, this will be the case also when parent's utility function depends on the child's utility directly.

Assuming strict concavity of the utility functions, necessary and sufficient conditions for optimum are: for $t=0,1,2,\ldots$.

$$\frac{c_{1t}}{c_{2t}}=\frac{\alpha_1}{\alpha_2(1+r_{t+1})} \tag{8}$$

$$\frac{c_{1t}}{e_t}=\frac{\alpha_1}{\gamma\alpha_4} \tag{9}$$

$$\frac{e_t}{b_t}=\frac{\gamma\alpha_4}{\alpha_3} \tag{10}$$

Now, from (8) and (9) we also obtain that

$$\frac{b_t}{s_t}=\frac{\alpha_3}{\alpha_2}. \tag{11}$$

Given the realization of $\tilde{\theta}(\omega)$, let $\alpha^*=\alpha_1+\alpha_2+\alpha_3+\gamma\alpha_4$, then the lifetime income of $\omega$, $y_t(\omega)$, is

$$\begin{aligned} y_t(\omega) &= (1+r_t)b_{t-1}(\omega)+w_t[e_{t-1}(\omega)]^{\gamma}\theta_t(\omega) \\ &= (1+r_t)\left\{b_{t-1}(\omega)+\frac{w_t}{1+r_t}\left[\frac{\gamma\alpha_4}{\alpha^*}y_{t-1}(\omega)\right]^{\gamma}\theta_t(\omega)\right\} \end{aligned} \tag{12}$$

## 3. Altruism and production

Our main purpose in this section is to study the impact of 'higher altruism' on the equilibrium output. Intergenerational transfers are driven by two main motivations: (a) Improving the offspring's earning capability (and perhaps, other non-income effects which may enhance parent's utility, such as social status) via education; (b) Due to the 'joy of giving' and/or the 'gift-bequest motive', parents transfer directly income to their offspring. Thus in (b) we include *inter vivos* transfers as well as inheritance transfers. We shall bring now two different criteria comparing 'similar' economies (which differ only in preferences to transfers) with respect to their 'altruistic behavior'. Consider two economies with the same structure, as the above one, which differ only in the preferences of the individuals (thus, each individual has a utility function given by (1)). Consider individuals from both economies who face the same economic circumstances (namely, the same income $y(\omega)$ and with an offspring who has ability $\theta(\omega)$). If individuals from the first economy *invest more in education* than those from the second one, we say that the first economy is **more education-inclined altruistic** than the second one. Similarly, other things are equal, when facing the same circumstances individuals from the first economy transfer more (nonhuman) capital to their child than those from the second one, we say that the first economy is **more bequest-inclined altruistic** than the second economy. Basically, these definitions compare just the preferences of individuals in the two economies, where the utility functions are given by (1). Hence, taking $\alpha_1$ and $\alpha_2$ to be fixed, we are comparing the ratios $\alpha_4/\alpha_3$. Consequently, facing the same economic circumstances, parents in $G_t$ with higher education-inclined altruism choose larger $e_t(\omega)$, while more bequest-inclined altruism implies a choice of a higher $b_t(\omega)$.

This type of comparison between countries is rather theoretical. In practice, it is very hard to compare countries with respect to their 'altruistic' behavior towards their younger generation. Some extensive data regarding the various types of investments in education in the developed and the developing countries exists (see, for example: Education At Glance: OECD Indicators, 1997; World Development Indicators, 2000). Data regarding intergenerational transfers of capital can be found only for few countries (some related discussions can be found, e.g., in Gokhale et al. (1999) as well as the other studies mentioned in the introduction). In some countries (e.g., India, China) the 'main' intergenerational transfers are land and property, while in others (e.g., the US, UK) the 'main' type of intergenerational transfers is education. Even though we do not have data to conduct properly such comparisons, we consider our theoretical exercise as a first step in this direction.

Let us assume in the sequel, without loss of generality, that $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$. Considering economies from the same initial distributions of capital and human capital, let us compare competitive equilibria period by period. We prove.

**Theorem 1.** *Consider economies from the same initial distributions of capital transfers and human capital. Higher education-inclined altruism results in*:

(a) *Lower output in all dates if* $\sigma \leq 1 + \alpha_4/1 - \gamma\alpha_4$.
(b) *Higher output in all dates if either* $\sigma$ *is 'large', or* $\gamma\alpha_4$ *is 'small'*.

Thus, in comparing economies, from the same initial conditions, which differ only in their altruism (namely, in our case, in $\alpha_4/\alpha_3$, given that $\alpha_1$ and $\alpha_2$ are fixed throughout this discussion), more education-inclined altruism means stronger preference for investment in human capital of the offspring. The above proposition indicates that the effect of more altruism on output depends not only on parameters of the utility, but also on the aggregate production function and the human capital production process. When either the elasticity of substitution $\sigma$ is 'not large' or altruism is significant, hence $\gamma\alpha_4$ is close to 1, thus the inequality in (a) holds, more education-inclined altruism results in lower capital–labor ratio and, hence, lower price ratio of labor to capital. As a result, since for each dynasty investment in education increases while physical capital transfer declines, *lifetime income* declines due to lower wages and lower capital transfer. To some extent, the result in (a) is not obvious since diverting more resources to education generates more human capital which is a production factor; but this comes at the expense of accumulation of physical capital, hence declines under these conditions. This explains the role played by $\sigma$ in determining the total effect of these variations in the production factors. When altruism is sufficiently ‘weak’, or if $\sigma$ is ‘large’, more altruism implies higher capital–labor ratio, hence higher wages and lower rates of interest. Hence, in equilibrium, lifetime incomes increase in this case and the economy attains higher output.

Now, comparing two economies which differ only at the level of bequest-inclined altruism we obtain

**Theorem 2.** *Consider two economies from the same initial distributions of transfers and human capital. The economy with the higher bequest-inclined altruism attains*:

(a) *lower output in all dates if* $\sigma \geq \frac{\alpha_3}{\gamma}\left[\frac{1}{\alpha_2+\alpha_3}+\gamma-1\right]$,
(b) *higher output in all dates if* $\sigma \leq \alpha_3$.

Note that for some $\gamma^* < 1$, whenever $\gamma \in (\gamma^*,1]$ the condition in part (a) holds for all $\sigma \geq 1$. This demonstrates the contrast in the effects of these two types of altruism on the equilibrium outputs. Assuming that the condition in (a) holds, we find, along the equilibrium path, that stronger preference for gift-bequest transfers results in lower investment in education, and hence, increases the aggregate labor supply while capital labor ratios decline. Consequently, this rise in altruism results in lower wage rates and higher interest rates which reinforces the attractiveness of the gift-bequest transfers vs. the investment in human capital. Given that the elasticity of substitution $\sigma$ is ‘not small’ (as in case (a)), these changes yield lower output.

The above two Theorems demonstrate the gap in economic implications that each type of altruism entails. We conclude that **the way** in which transfers are implemented has a significant impact on economic activity. For example, the effect of higher bequest-inclined altruism on factor price ratios in equilibrium, assuming that $\sigma$ is large, is opposed to that attained by higher education-inclined altruism. Similar phenomena exists when we consider the effect of intergenerational transfers (under private provision of education) on income distributions in the next section.

## 4. Intergenerational transfers and income distribution: private provision of education

Before presenting the effect of altruism on intragenerational distribution of incomes in equilibrium let us define income inequality measurement. Let $X$ and $Y$ be two random variables with values in a bounded interval in $\Re$ and let $m_x$ and $m_y$ denote their respective means. Define $\hat{X} = X/m_x$ and $\hat{Y} = Y/m_y$. Denote by $F_x$ and

$F_y$ the cumulative distribution functions of $\hat{X}$ and $\hat{Y}$, respectively. Let $[a, b]$ be the smallest interval containing the supports of $\hat{X}$ and $\hat{Y}$.

**Definition.** $F_x$ *is more equal* than $F_y$ if, for all $t \in [a, b]$, $\int_a^t [F_x(s) - F_y(s)]\, \mathrm{d}s \leq 0$.

Thus, $F_x$ is more equal than $F_y$ if $F_x$ **dominates** in the second-degree stochastic dominance $F_y$ (denoted also as $\hat{X} >_2 \hat{Y}$). This definition, due to Atkinson (1970), is equivalent to the requirement that the Lorenz curve corresponding to $X$ is everywhere above that of $Y$. We say that $X$ is *more equal* than $Y$ if the c.d.f.'s of $\hat{X}$ and $\hat{Y}$ satisfy: $F_x$ is more equal than $F_y$. Henceforth, the relation '$X$ is more equal than $Y$' is denoted $X \gg Y$. We say that $X$ is equivalent to $Y$, denoted by $X \approx Y$, if $X$ is more equal than $Y$ and $Y$ is more equal than $X$.

We present now, under private provision of education, the effect of higher education-inclined altruism on the income distribution in each generation. Comparing two economies which differ only at the importance attached to the human capital of the offspring we obtain:

**Theorem 3.** *Consider two economies starting from the same initial distributions of capital transfers and human capital. Higher education-inclined altruism results in*:

(a) *More equal intragenerational income distribution, in each date, if* $\sigma \leq 1 + \alpha_4/1 - \gamma\alpha_4$.
(b) *Less equal intragenerational income distribution, in each date, if* $\gamma\alpha_4$ *is 'small', or if* $\sigma$ *is 'large'*.

Inequality in income distribution in each generation has two sources: (1) unequal intergenerational transfers of physical capital, and (2) unequal human capital levels. Stronger preference for human capital of offsprings results in more investment in education, at the expense of physical capital transfers, and thus, larger aggregate labor supply and *lower* capital–labor ratio. This implies lower wages, which *weaken* the contribution of the first component to inequality, while higher interest rates *reinforce* the contribution of the second component to inequality. When the utility and production function parameters satisfy the conditions in (a) (see Eq. (12)) the *total effect* will be more equality in the intragenerational income distributions. When $\sigma$ is 'large enough' the rise in the capital–labor ratio (due to higher $\alpha_4/\alpha_3$) results in higher price ratio (wages to interest rates) which reverses the total effect of the above two opposing effects to income distribution inequality. Hence in this case we obtain less equality in each date. Intuitively, attaching more importance to education, under private provision, should lessen the inequality in *human capital distribution*, but this does not necessarily imply more equal income distribution due to its impact on the intergenerational distribution of transfers. However, when weight to education in the utility function is significant (hence $\gamma\alpha_4$ is not 'small') we obtain, for all $\sigma$ which are not extremely large, that attaching more importance to education enhances equality in the distributions of income.

Let us show, once again, that higher *bequest-inclined* altruism implies different results:

**Theorem 4.** *Consider two economies starting from the same initial distributions of capital transfers and human capital. Higher bequest-inclined altruism results, in all dates, in*:

(a) *More equality in the intragenerational distributions of income if* $\sigma \geq \frac{\alpha_3}{\gamma}\left[\frac{1}{\alpha_2 + \alpha_3} + \gamma - 1\right]$.
(b) *Less equality in the distributions of income if* $\sigma \leq \alpha_3$.

More bequest-inclined altruism induces less investment in education and higher physical capital transfers (hence, higher aggregate stock). In this case we obtain higher capital–labor ratio and, hence, higher (labor to capital) price ratio. Increasing wages reinforce the contribution to inequality of the component stemming from unequal distribution of human capital. Moreover, when $\sigma$ is 'not small', this will be outweighed by the decline in the contribution to inequality of the component stemming from unequal transfers (due to the decline in interest rates), which results in more equality.

## 5. Public provision of education

Consider now the case where education is provided by the government with equal opportunities to all. Basically, this is the case prevalent in most countries today (with some variations). We do not consider here the 'mixed regime' case, namely, when both public and private provisions co-exist, although our analysis can be generalized to this framework (this type of 'mixed' provision has been studied by Eckstein and Zilcha, 1994). Such a generalization requires much more complex analysis, especially for the income distribution section, weaker results and does not add much to our understanding of this comparative dynamics exercise (see the discussion in Section 6). Thus, we prefer to consider only the private provision and public provision cases in this work. In line with reality we assume 'no discrimination', i.e., the investment in education is the same for each young person, regardless of his/her ability. In our framework education costs are covered by taxing wage incomes at a fixed rate (in each date), while parents do not complement the investment in education provided by the government. There is no tax on gift-bequest transfers, but, as shown later, such additional tax will not alter our results. Our subsequent analysis can be easily extended to the case where the tax rate $\tau$ is time-dependent. Denote by $\tau$ the tax rate on wage incomes, and by $\bar{e}_t$ the public investment in educating each young person in generation $t$. Given $b_{t-1}(\omega)$ and $h_t(\omega)$, the maximization problem of individual $\omega$ in $G_t$ (choosing $s_t$ and $b_t$ only) is the following:

$$\text{Max } c_{1t}(\omega)^{\alpha_1} c_{2t}(\omega)^{\alpha_2} [(b_t(\omega)(1+r_{t+1})]^{\alpha_3} [h_{t+1}(\omega)]^{\alpha_4} \tag{13}$$

subject to the constraints:

$$c_{1t}(\omega) = (1+r_t)b_{t-1}(\omega) + (1-\tau)w_t h_t(\omega) - s_t(\omega) - b_t(\omega) \geq 0. \tag{14}$$

$$c_{2t}(\omega) = (1+r_{t+1})s_t(\omega) \tag{15}$$

$$h_{t+1}(\omega) = A[\bar{e}_t]^{\gamma} \tilde{\theta}_{t+1}(\omega) \tag{16}$$

The equilibrium conditions will now include the requirement that revenues from taxing wage incomes cover the total cost of education, i.e.,

$$\tau \int w_t h_t(\omega)\, \mathrm{d}\mu(\omega) = \int \bar{e}_t\, \mathrm{d}\mu(\omega)$$

which implies that,

$$\tau w_t L_t = \bar{e}_t \quad \text{for } t = 0,1,2,\ldots. \tag{17}$$

On the other hand the aggregate labor supply can be expressed as follows:

$$L_t = \int h_t(\omega)\, \mathrm{d}\mu(\omega) = \int [\bar{e}_{t-1}]^{\gamma} \tilde{\theta}_t(\omega)\, \mathrm{d}\mu(\omega) = [\bar{e}_{t-1}]^{\gamma} \bar{\theta}. \tag{18}$$

The income in this case can be written as:

$$\begin{aligned} y_t(\omega) &= (1+r_t)b_{t-1}(\omega) + (1-\tau)w_t h_t \\ &= (1+r_t)\left\{ b_{t-1}(\omega) + \frac{w_t}{1+r_t}(1-\tau)[\bar{e}_{t-1}]^{\gamma} \tilde{\theta}_t(\omega) \right\} \end{aligned} \tag{19}$$

Let us examine now the effects of higher altruism on output and intragenerational income distribution when education is provided equally by the public sector. It is not hard to verify that when education is provided by the government (and financed by tax on wage income) then the level of *education-inclined altruism has no effect* on the equilibrium output and the intragenerational income distribution.

Now, let us consider the effect of bequest-inclined altruism on aggregate output. Regardless of the size of $\sigma$ we obtain:

**Theorem 5.** *Assume a public provision of education. Higher bequest-inclined altruism results in higher aggregate output in all dates.*

This result stands in contrast to that obtained for the *private* education case. When the level of education investment is determined exogenously and taxes are used solely to provide equal education to all individuals, higher bequest-inclined altruism results in higher transfers. Since aggregate effective labor supply, given the initial distribution of human capital, evolution is based on the publicly provided education we obtain that the more altruistic economy attains higher aggregate output regardless of the size of $\sigma$. This economy contains subsidization of education to the low-income families: the level of education investment (and hence, the tax rate) is determined by the government (perhaps by some political process), and it is provided equally to all the young individuals. Now, higher bequest-inclined altruism yields higher capital transfers to the young generation without lowering the investment in education. Since this expenditure is financed by a fixed tax rate on wages, it implies higher $\overline{e_t}$ and higher stock of human capital as well. On the other hand, this tax lowers the aggregate stock of capital (via savings and transfers), which is outweighed by the increase in aggregate labor, resulting in higher output.

Let us show now that under public provision of education the result of Theorem 4(a) reverses; namely, in this case, regardless of the parameters' magnitudes, higher bequest-inclined altruism results in *less* equal intragenerational income distributions.

**Theorem 6.** *Assume that education is publicly provided. Then, higher bequest-inclined altruism results in less equal intragenerational income distributions.*

This is surprising since the provision of equal education to all young individuals (regardless of their ability), while financing it by a fixed tax rate on wage earnings (thus subsidizing the education of the low income families), should reduce inequality in the income dispersion in comparison to the private provision case. This result is compatible with the observations made by Davies (1986). Under public provision of education income inequality between households results from the random ability (which causes unequal labor earnings) and inequality in the transfers between generations. Since education is provided equally regardless of ability and parental wealth one source of inequality has been removed (talents are assumed to be i.i.d. random variables). However, in the more altruistic economy the inequality in the gift-bequest transfers distribution is higher, hence this source of income inequality is reinforced. Moreover, the higher intergenerational transfers will reverse the result attained in the privately provided education case (when $\sigma$ is 'not small'); namely, that more altruism yields less equality. To understand this reversal in the change of inequality (of income distribution), let us note the following: At each date t higher bequest-inclined altruism will increase the physical capital accumulation while the human capital distribution remains unchanged; therefore, the factors price ratio $w_t/1+r_t$ increases. Observing Eq.

(19), this change in price ratio raises the inequality in the distribution of incomes $y_t(\omega)$ (this follows from Lemma 2 in the Appendix).

**Remark 7.** *We have assumed so far that the government taxes wage incomes* (*only*) *to finance the cost of education. It is possible to generalize the results of Theorem* 5 *and Theorem* 6 *to the case where the same tax applies to gift-bequest transfers as well*: *In this case the income* $y_t(\omega)$ *is given by*:

$$y_t(\omega) = (1-\tau)[(1+r_t)b_{t-1}(\omega) + w_t h_t(\omega)]$$

*while condition* (17) *should be replaced by*:

$$\tau[(1+r_t)B_{t-1} + w_t L_t] = \bar{e}_t \quad t = 0,1,\ldots.$$

We shall not prove this claim since the adjustment of the proofs of the two theorems to this case is straightforward.

## 6. Concluding remarks

The attempts to explain the persistent differences in economic development between countries, using new models of growth, were concentrated on the technological aspects and the endogenous human capital accumulation. We claim here that intergenerational transfers, motivated by 'altruism' within families, including values and cultural background, and their composition, play an important role. Put it differently, the interactions between generations, which are affected greatly by altruistic considerations, have significant impact on the production activities and inequality in income distributions. However, as was shown in this work, it is important to distinguish between the intensities in which the two types of transfers are carried out: Whether the emphasis is on children's human capital formation or, perhaps, on direct transfers of wealth from parents to children in various ways. Whether the transfers from parents to their children are mostly some sort of physical capital, or predominantly investments in education and knowledge, have different economic implications. Such comparison between countries (or societies) may explain part of the persistent differences in growth rates and/or income distributions.

In this paper we assume a Cobb-Douglas type of utility function in order to derive 'sharp' results. Some restriction of the homothetic utility family is essential when we need to aggregate, in a manageable way, over dynasties' savings, transfers and human capital. Most of our results are robust to *some* generalization of the Cobb-Douglas preferences. However, we were unable to obtain our results assuming, for example, any homothetic utility function. Since any generalization of the utility function entails more complex proofs we did not proceed this way.

In the Becker and Tomes type of altruism, even though transmission of capital from parents to children is based on utility maximization of parents (concerned about their child's welfare), the composition of transfers is driven by the relative returns on human and physical capital, which depends upon family wealth. In our case, although we take each country's preferences to be 'given', it is the intragenerational income distributions that are affected by the relative rates of return. A modest generalization of our preferences will yield 'more' dependence of the composition of transfers on the relative returns.

Surprisingly, in our economy, when education is provided equally by the government the impact of higher altruism (the relevant one to this case) is reversed, for most relevant values of the parameters, compared to the private provision case. More altruism reinforces the role played by the initial income dispersion, caused by historical reasons, and hence results in less equality. It seems to us that the 'mixed' provision case, i.e., the case where a certain level of education is provided by the government while parents can increase the investment in education, should be examined as well. We claim that in such a case the results obtained in Section 5 will change.

When parents can add some investment $e_t(\omega)$ to the government's provision to education $\bar{e}_t$, in (16) we insert $\bar{e}_t + e_t(\omega)$. Since the results differ significantly in the case where $\tau$ is small (hence $\bar{e}_t$ is very small and $e_t^*(\omega) > 0$ for almost all) to the case of large $\tau$ (hence $\bar{e}_t$ is large and hence $e_t^*(\omega) = 0$ for almost all) the analysis in this regime will be inconclusive. Namely, the impact of more altruism will depend on the size of the public investment $\bar{e}_t$ in education as well as the *initial distributions* of wealth and human capital.

We conjecture that if labor supply is elastic *and* each young individual can increase the investment made by her parent, the economic implications of the variations in the two types of altruism will coincide.

## 7. Proofs

**Proof of Theorem 1.** Consider Eqs. (8)–(11) derived from the optimality conditions. Given income $y_t(\omega)$ (see (12)) we can write:

$$b_t(\omega) = \frac{\alpha_3}{\alpha^*} y_t(\omega); \quad e_t(\omega) = \frac{\alpha_4}{\alpha^*} y_t(\omega) \tag{20}$$

At date $t = 0$, $K_0$, $b_{-1}(\omega)$ and $h_0(\omega)$ are given. Denote the equilibrium with individuals having utility functions as in (2) by $\{(c_{1t}, c_{2t}, s_t, b_t, e_t), (w_t, r_t)\}$. We shall conduct our experiment as follows: in varying the taste either for human capital or for gift-bequest transfers we shall vary $\alpha_4/\alpha_3$ accordingly, while keeping $\alpha_1$ and $\alpha_2$ fixed throughout our proofs. The equilibrium with the more altruistic individuals, i.e., with $\alpha_4' > \alpha_4$(while $\alpha_1$, $\alpha_2$, $\alpha_3$ remain the same) is denoted by $\{(c_{1t}', c_{2t}', s_t', b_t', e_t'), (w_t', r_t')\}$. Let us write $\alpha_4' = (1 + m)\alpha_4$, where $m > 0$.

Since the distribution of savings and human capital $h_0(\omega)$ are given at date 0 $L'_0 = L_0$ and also $K'_0 = K_0$. Thus we also have $w'_0 = F_L(K'_0, L'_0) = F_L(K_0, L_0) = w_0$ and similarly $r'_0 = r_0$. The change in $\alpha_4$ does not affect the level of incomes in period 0, hence $y_0(\omega) = y'_0(\omega)$.

**Part (a).** Under the assumptions in this part we prove that

**Claim 1.** For all $t$, $t = 1,2,\dots$ $y'_t(\omega) < y_t(\omega)$ for all $\omega$.

Assume that $y'_{t-1}(\omega) \leq y_{t-1}(\omega)$ for all $\omega$. Using (20), we have:

$$L'_t = \int [e'_{t-1}(\omega)]^\gamma \theta_t(\omega)\,\mathrm{d}\mu = \bar{\theta}\int \left[\frac{\alpha_4(1+m)}{\alpha^* + m\alpha_4\gamma}\right]^\gamma [y'_{t-1}(\omega)]^\gamma\,\mathrm{d}\mu$$

$$K'_t = \int [s'_{t-1}(\omega) + b'_{t-1}(\omega)]\,\mathrm{d}\mu = \frac{\alpha_2+\alpha_3}{\alpha^* + m\alpha_4\gamma}\int y'_{t-1}(\omega)\,\mathrm{d}\mu.$$

Hence

$$\frac{K'_t}{L'_t} = \frac{\alpha_2+\alpha_3}{\bar{\theta}[(1+m)\alpha_4\gamma]^\gamma[\alpha^* + m\alpha_4\gamma]^{1-\gamma}}\,\frac{\int y'_{t-1}}{\int [y'_{t-1}]^\gamma}, \quad t = 1,2,3,\dots \tag{21}$$

Without loss of generality let us take $\alpha^* = 1$. Considering the expressions in Eq. (12), let us write for $t \geq 1$: Under our assumptions, for some constant $\hat{\mu}$ we have $F_L(K, L)/F_K(K, L) = \hat{\mu}(K/L)^{1/\sigma}$. Since $0 < \gamma \leq 1$ it can be verified that:

**Claim 2.** For any $t$ if, $y'_t(\omega) < y_t(\omega)$ for all $\omega$ then, $\int y'_t/\int[y'_t]^\gamma \leq \int y_t/\int[y_t]^\gamma$.

Assume now that for $t \geq 1$ we have $y'_{t-1}(\omega) < y_{t-1}(\omega)$ for all $\omega$, then:

$$\begin{aligned}
\frac{w'_t}{1+r'_t}\cdot\left[\frac{\gamma\alpha_4(1+m)}{1+\gamma\alpha_4 m}\right]^\gamma &= \hat{\mu}\left[\frac{\gamma\alpha_4(1+m)}{1+\gamma\alpha_4 m}\right]^\gamma \\
&\quad\times\left(\frac{\alpha_2+\alpha_3}{\bar{\theta}[(1+m)\alpha_4\gamma]^\gamma[\alpha^* + m\alpha_4\gamma]^{1-\gamma}}\,\frac{\int y'_{t-1}}{\int [y'_{t-1}]^\gamma}\right)^{1/\sigma} \\
&\leq [\gamma\alpha_4]^\gamma\hat{\mu}\left(\frac{K_t}{L_t}\right)^{1/\sigma} \leq [\gamma\alpha_4]^\gamma\frac{w_t}{1+r_t}
\end{aligned} \tag{22}$$

where the last two inequalities were obtained due to Claim 2 and the assumptions in part (a) which imply that:

$$(1+m)^{\gamma-(\gamma/\sigma)} \leq (1+m\gamma\alpha_4)^{\gamma+(1-\gamma/\sigma)} \tag{23}$$

Denote by,

$$Z_t(\omega) = b_{t-1}(\omega) + \frac{w_t}{1+r_t}\left[\frac{\gamma\alpha_4}{\alpha^*} y_{t-1}(\omega)\right]^{\gamma} \theta_t(\omega)$$

Then, by the induction step, we have $b_{t-1}(\omega) \geq b'_{t-1}(\omega)$, hence from the above result we obtain that: $Z'_t(\omega) < Z_t(\omega)$ for all $\omega$.

Since $y'_{t-1}(\omega) \leq y_{t-1}(\omega)$ for all $\omega$, we derive that $K'_t < K_t$ and $L'_t < L_t$ (due to $h'_t(\omega) < h(\omega)$ for all $\omega$). Thus $Q'_t = F(K'_t, L'_t) < F(K_t, L_t) = Q_t$. Now let us show that this implies that $\int y'_t < \int y_t$.

From the above analysis we see that for all $t$: $K'_t/L'_t \leq K_t/L_t$, which implies that $w'_t \leq w_t$ for all t. Therefore, $w'_t L'_t \leq w_t L_t$ for all $t$. Now let us use the relations attained in Section 2 to derive that:

$$\int y_t = \frac{\alpha_3}{\alpha_2+\alpha_3} Q_t + \frac{\alpha_2}{\alpha_2+\alpha_3} w_t L_t$$

Which shows that $\int y'_t < \int y_t$. Thus $(1+r'_t) \int Z'_t(\omega) < (1+r_t) \int Z_t(\omega)$. Since $Z'_t(\omega) < Z_t(\omega)$ for all $\omega$ we obtain that $y'_t(\omega) < y_t(\omega)$ for all $\omega$. But this implies that $b'_t(\omega) < b_t(\omega)$ for all $\omega$ and so on, to continue the proof by induction.

To prove part (b) we note that when either $\sigma$ is large or $\gamma\alpha_4$ is sufficiently small then, starting from certain $t$ with $y'_{t-1}(\omega) > y_{t-1}(\omega)$ the inequality in Claim 2 will be reversed. Also, the inequalities in (22) and in (23) are reversed as well. It is easy to verify that due to the reversed inequalities we obtain that $y'_t(\omega) > y_t(\omega)$ for all $\omega$. The rest of the proof follows as in part (a). □

**Proof of Theorem 2.** In this case increasing $\alpha_3$ to $\alpha'_3 = (1+m)\alpha_3$, keeping $\alpha_1, \alpha_2, \alpha_4$ unchanged, results in the following:

$$K'_t = \frac{\alpha_2 + \alpha_3(1+m)}{\alpha^* + m\alpha_3} \int y'_{t-1}(\omega)$$

$$L'_t = \bar{\theta}\left[\frac{\gamma\alpha_4}{\alpha^* + m\alpha_3}\right]^{\gamma} \int [y'_{t-1}(\omega)]^{\gamma}$$

Hence

$$\frac{K'_t}{L'_t} = \frac{\alpha_2 + \alpha_3(1+m)}{\bar{\theta}[\gamma\alpha_4]^{\gamma}[\alpha^* + m\alpha_3]^{1-\gamma}} \frac{\int y'_{t-1}(\omega)}{\int [y'_{t-1}(\omega)]^{\gamma}}, \qquad t = 1,2,\ldots \tag{24}$$

As in the previous proof we have $y_0(\omega) = y'_0(\omega)$ for all $\omega$. Assume now, using an induction step, that $y'_{t-1}(\omega) \leq y_{t-1}(\omega)$ for all $\omega$. Without loss of generality let $\alpha^* = 1$. Then, using Eq. (12) and the assumption in the theorem, we can compare the following expressions:

$$\frac{w_t'}{1+r_t'}\left[\frac{\gamma\alpha_4}{\alpha_3(1+m)}\right]^{\gamma} = \hat{\mu}\left[\frac{\gamma\alpha_4}{\alpha_3(1+m)}\right]^{\gamma}\left(\frac{K_t'}{L_t'}\right)^{1/\sigma}$$
$$\leq \hat{\mu}\left[\frac{\gamma\alpha_4}{\alpha_3(1+m)}\right]^{\gamma}\left[\frac{\alpha_2+\alpha_3(1+m)}{(\alpha_2+\alpha_3)[1+m\alpha_3]^{1-\gamma}}\right]^{1/\sigma}$$
$$\times\left(\frac{K_t}{L_t}\right)^{1/\sigma} = \frac{w_t}{1+r_t}\cdot\left[\frac{\gamma\alpha_4}{\alpha_3}\right]^{\gamma} \tag{25}$$

The inequality in (25) holds since under the assumption about $\sigma$ the following inequality holds:

$$(1+m)^{\sigma\gamma} \geq \frac{\alpha_2+\alpha_3(1+m)}{(\alpha_2+\alpha_3)[1+m\alpha_3]^{1-\gamma}}. \tag{26}$$

Since $y_{t-1}'(\omega) \leq y_{t-1}(\omega)$ for all $\omega$, we obtain also that $b_{t-1}'(\omega) \leq b_{t-1}(\omega)$ for all $\omega$. From (24) and (25) we conclude that $Z_t'(\omega) \leq Z_t(\omega)$ for all $\omega$. By the same argument we used in our earlier proof, we derive that $y_t'(\omega) < y_t(\omega)$ for all $\omega$. This implies, using (20), that $b_t'(\omega) < b_t(\omega)$ for all $\omega$. As before, this result demonstrates that the output is lower when the economy is more bequest-inclined altruistic. □

**Poof of Theorem 3.** Denote by $y_t'(\omega)$ the incomes when $\alpha_4$ increases to $\alpha_4' = (1+m)\alpha_4$. Define:

$$X_t = [\frac{\gamma\alpha_4}{\alpha^*}]^{\gamma}\frac{w_t}{1+r_t} = [\frac{\gamma\alpha_4}{\alpha^*}]^{\gamma}\hat{\mu}\left(\frac{K_t}{L_t}\right)^{1/\sigma}. \tag{27}$$

We conclude from the proof of Theorem 1 that:

(i) If $\sigma \leq 1+\alpha_4/1-\gamma\alpha_4$ then $X_t$ decreases as $\alpha_4$ increases.
(ii) When either $\sigma$ is 'large' or $\gamma\alpha_4$ is 'small', then $X_t$ increases as $\alpha_4$ increases.

We have shown in the proof of Theorem 1 that under the assumption in (a) we have $X_1' < X_1$. To proceed we need the following Lemma.

**Lemma 1.** *Let X and Y be random variables taking values in* $(-\infty, \infty)$. *Let Z be a positive random variable with compact support and assume that Z is independent of X and Y, then*:

(a) $X \approx Y$ *implies that* $XZ \approx YZ$
(b) $X \gg Y$ *implies that* $XZ \gg YZ$.

The proof of this lemma follows the same lines as in the proof of Lemma 1 in Karni and Zilcha (1995), hence it is omitted (see also Karni and Zilcha (1994)).

Rewriting Eq. (12):

$$\begin{aligned} y_t(\omega) &= (1+r_t)\left\{ b_{t-1}(\omega) + \frac{w_t}{1+r_t}\left[\frac{\gamma\alpha_4}{\alpha^*}\right]^{\gamma}[y_{t-1}(\omega)]^{\gamma}\theta_t(\omega)\right\} \\ &= (1+r_t)\{b_{t-1}(\omega) + X_t[y_{t-1}(\omega)]^{\gamma}\theta_t(\omega)\}. \end{aligned} \tag{28}$$

It is easy to verify that since $0<\gamma<1$ if $y_t(\omega) \gg y'_t(\omega)$ then $[y_t(\omega)]^{\gamma} \gg [y'_t(\omega)]^{\gamma}$. It follows from Lemma 1 in Karni and Zilcha (1995) that if $X_t > X'_t$ then:

$$1 + X'_t\theta_t(\omega) \gg 1 + X_t\theta_t(\omega). \tag{29}$$

Before proceeding with the proof let us state the following two Lemmas.

**Lemma 2.** *Consider two independent random variables $\tilde{X}$ and $\tilde{Y}$ with values in $[a, b]$, $0 < a < b < \infty$. Let $\bar{X} = E\tilde{X}$, $\bar{Y} = E\tilde{Y}$ and define for $\lambda > 0$, $Z(\lambda) = \tilde{X} + \lambda\tilde{Y}/\bar{X} + \lambda\bar{Y}$. Then, $\lambda > \hat{\lambda}$ implies that $Z(\hat{\lambda}) \gg Z(\lambda)$.*

**Lemma 3.** *Let the nonnegative random variables $\tilde{X}$, $\tilde{Y}$, $\hat{X}$, $\hat{Y}$ satisfy*:

(a) *$\tilde{X}$, $\tilde{Y}$ are independent and $\hat{X}$, $\hat{Y}$ are independent.*
(b) *$\tilde{X}$ dominates (in the second degree) $\hat{X}$, i.e., $\tilde{X} >_2 \hat{X}$. Also $\tilde{Y} >_2 \hat{Y}$. Then $Z = \tilde{X} + \tilde{Y} >_2 \hat{Z} = \hat{X} + \hat{Y}$.*

The proofs of Lemmas 2 and 3 are available from the author upon request.

Since $\theta_t(\omega)$ are i.i.d. across individuals, we reach the following conclusions: Assuming that σ satisfies the condition in (a). Since $y'_0(\omega) \approx y_0(\omega)$, then $b'_0(\omega) \approx b_0(\omega)$, while $X'_1 < X_1$ ($X'_1$ corresponds to a higher $\alpha_4$). By the Lemma 3 above we obtain that $y'_1 \gg y_1$. But for each $t$ $b_t(\omega) = \alpha_3/\alpha^* y_t(\omega)$ while $b'_t(\omega) = \dfrac{\alpha_3}{\alpha^* + m\alpha_4} y'_t(\omega)$ hence, $b_1 \gg b'_1$. Using the same argument for $t=2$, we derive that $y'_2 \gg y_2$ and so on.

Assume now that the assumption in (b) holds. In this case from the proof of Theorem 1 we have $X'_1 > X_1$ while $b'_0 \approx b_0$, hence $y_1 \gg y'_1$; Similarly, $b'_1 \gg b_1$. But $X'_t > X_t$ for any $t$, thus the inequality is 'reversed' in this case. Consequently $b'_{t-1}(\omega) \gg b_{t-1}(\omega)$ and $X'_t > X_t$ imply that $y_t \gg y'_t$ for all $t$. □

**Proof of Theorem 4.** As before let us define:

$$X_t = \frac{w_t}{1+r_t}\left[\frac{\gamma\alpha_4}{\alpha_3}\right]^{\gamma}. \tag{30}$$

From the proof of Theorem 2 it is easy to verify that:

$$\frac{\partial X_t}{\partial \alpha_3} < 0 \;\; if \;\; \sigma \geq \frac{\alpha_3}{\gamma}\left[\frac{1}{\alpha_2+\alpha_3} + \gamma - 1\right] \tag{31}$$

$$\frac{\partial X_t}{\partial \alpha_3} > 0 \quad if \quad \sigma \leq \alpha_3. \tag{32}$$

When the assumption in (a) holds, using Eq. (30) together with (32), since $b'_{-1}(\omega) = b_{-1}(\omega)$ for all $\omega$ we conclude that (again we apply Lemma 1 in Karni and Zilcha, 1995) $y'_0(\omega) \gg y_0(\omega)$, hence $b'_0(\omega) \gg b_0(\omega)$. Applying once again the same arguments we obtain that $y'_1(\omega) \gg y_1(\omega)$, hence $b'_1 \gg b_1$. This process can be continued, since $X'_t < X_t$, to obtain that $y'_t \gg y_t$ for $t=0,1,2,\ldots$. The proof of part (b) uses the same technique; however, since $X'_t > X_t$ we obtain now $y'_t \gg y_t$ for $t=0,1,2,\ldots$. □

**Proof of Theorem 5.** Let $\alpha'_3 = (1+m)\alpha_3$ for $m > 0$. Denote the equilibrium corresponding to $\alpha'_3$ with ‘′’. Since $y'_0(\omega) = y_0(\omega)$ by (20) we obtain that $b'_0(\omega) > b_0(\omega)$ and hence, $b'_0(\omega) + s'_0(\omega) > b_0(\omega) + s_0(\omega)$ for all $\omega$. Thus $K'_1 > K_1$. Since $w'_0 = w_0$ we obtain that $\bar{e}_0$ remains unchanged, and hence $L'_1 = L_1$. This implies that $K'_1/L'_1 > K_1/L$, hence, $w_1/1 + r_1 < w'_1/1 + r'_1$ which implies, by (19), that $y'_1(\omega) > y_1(\omega)$ for all $\omega$. As in our earlier analysis this yields that $Q_1 < Q'_1$.

Since in our framework: $w_t/1 + r_t < w'_t/1 + r'_t \Leftrightarrow K_t/L_t < K'_t/L'_t$, let us examine the change in factor price ratio as we increase $\alpha_3$. Assume, using induction step, that $y'_{t-1}(\omega) > y_{t-1}(\omega)$ for all $\omega$, hence $b'_{t-1}(\omega) > b_{t-1}(\omega)$ for all $\omega$ and $[\bar{e}'_{t-1}]^\gamma > [\bar{e}_{t-1}]^\gamma$. Using and the inequality in Claim 2, we obtain that $K_t/L_t < K'_t/L'_t$ since:

$$\frac{\mathrm{d}}{\mathrm{d}m}\left[\frac{\alpha_2 + (1+m)\alpha_3}{(1+m\alpha_3)^{1-\gamma}}\right] > 0$$

Therefore, by Eq. (28), we obtain (redefining $Z_t(\omega)$ for the public education case) that $Z'_t(\omega) > Z_t(\omega)$. As we did before this implies that $y'_t(\omega) > y_t(\omega)$ for all $\omega$, and hence $Q_t < Q'_t$ for all t, which completes the proof. □

**Proof of Theorem 6.** This is proved again by induction. Since $y'_0(\omega) = y_0(\omega)$ we have $b'_0(\omega) \approx b_0(\omega)$. Assume now that $y_{t-1} \gg y'_{t-1}$. This implies that $b_{t-1}(\omega) \gg b'_{t-1}(\omega)$. Define: $X_t = \frac{w_t}{1+r_t}(1-\tau)[\bar{e}_{t-1}]^\gamma$. Observe the expression,

$$y_t(\omega) = (1 + r_t)\{b_{t-1}(\omega) + X_t\bar{\theta}_t(\omega)\} \tag{33}$$

By the same arguments we had in the proof of Theorem 5 we derive that for the public provision case we have: $X'_t > X_t$ for all t. Using Lemma 2 and Lemma 3 for this case we obtain that $y_t(\omega) \gg y'_t(\omega)$ for all t, which completes the proof. □

## Acknowledgements

I have benefitted from discussions with Larry Ball, Jim Davies, Gerhard Glomm, Bruce Hamilton, Edi Karni, Omer Moav Jean-Marie Viaene, Yoram Weiss and Caspar de-Vries. I would like to thank my referees for their comments and suggestions which resulted in significant improvements.

## References

Altonji, J.G., Hayashi, L.J., Kotlikoff, L.J., 1992. Is the extended family altruistically linked? Direct testing using micro data. American Economic Review 82, 1177–1198.

Altonji, J.G., Hayashi, L.J., Kotlikoff, L.J., 1997. Parental altruism and inter vivos transfers: Theory and evidence. Journal of Political Economy 105, 1121–1166.

Atkinson, A., 1970. On the measurement of inequality. Journal of Economic Theory 2, 244–263.

Becker, G.S., Tomes, N., 1986. Human capital and the rise and fall of families. Journal of Labor Economics 4, S1–S38.

Benabou, R., 1996. Equity and efficiency in human capital investment: The local connection. Review of Economic Studies 63, 237–264.

Bernheim, D., 1991. How strong are bequest motives? Evidence based on estimates of the demand for life insurance and annuities. Journal of Political Economy 99, 899–956.

Bernheim, B., Bagwell, K., 1988. Is everything neutral? Journal of Political Economy 96, 308–338.

Cox, D., Raines, F., 1985. Interfamily transfers and income redistributions. In: David, M., Smeeding, T. (Eds.), Horizontal Equity, Uncertainty and Measures of Well-Being. University of Chicago Press, Chicago, IL.

Davies, J.B., 1986. Does redistribution reduce inequality? Journal of Labor Economics 4 (4), 538–557.

Eckstein, Z., Zilcha, I., 1994. The effects of compulsory schooling on growth, income distribution and welfare. Journal of Public Economics 54, 339–359.

Education at Glance: OECD Indicators, 1997. Center for Educational Research and Innovation: www.oecd.org

Gale, W.G., Scholz, J.K., 1994. Intergenerational transfers and accumulation of wealth. Journal of Economic Perspectives 8 (4), 145–160.

Galor, O., Zeira, J., 1993. Income distribution and macroeconomics. Review of Economics Studies 60, 35–52.

Galor, O., Tsiddon, D., 1997. The distribution of human capital and economic growth. Journal of Economic Growth 2, 93–124.

Glomm, G., Ravikumar, B., 1992. Public versus private investment in human capital: endogenous growth and income inequality. Journal of Political Economy 100, 818–834.

Gokhale, J., Kotlikoff, L.J., Sefton, J., Weale, M., 1999. Simulating the transmission of wealth inequality via bequests. NBER, Working Paper #7183.

Hanushek, E.A., 1986. The economics of schooling: production and efficiency in public schools. Journal of Economic Literature XXIV, 1141–1177.

Hodge, R.W., Siegel, P.M., Rossi, P.H., 1966. Occupational prestige in the United States: 1925–1963. In: Bendix, R., Lipset, S.M. (Eds.), Class, Status and Power: Social Stratification in Comparative Perspective. Free Press of Glencoe, New York.

Hurd, M.D., 1987. Saving of the elderly and desired bequests. American Economic Review 77, 298–312.

Jovanovic, B., Nyarko, Y., 1995. The transfer of human capital. Journal of Economic Dynamics and Control 19, 1033–1064.

Karni, E., Zilcha, I., 1994. Technological progress and income inequality: a model with human capital and bequests. In: Bergstrand et al. (Eds.), The Changing Distribution of Income in an Open US Economy. Elsevier Science, BV.

Karni, E., Zilcha, I., 1995. Technological progress and income inequality. Economic Theory 5, 277–294.

Kotlikoff, L.J., Summers, L.H., 1981. The role of intergenerational transfers in aggregate capital accumulation. Journal of Political Economy 89 (4), 706–732.

Kotlikoff, L.J., 1988. Intergenerational transfers and savings. Journal of Economic Perspectives 2 (2), 41–58.

Laitner, J., 1997. Intergenerational and interhousehold economic links. In: Rosenzweig, M.R., Stark, O. (Eds.), Handbook of Population and Family Economics. Elsevier, North Holland.

Laitner, J., Juster, F.T., 1996. New evidence on altruism: a study of TIAA-CREF retirees. American Econmic Review 86, 893–906.

Lord, W., Rangazas, P., 1991. Savings and wealth in models with altruistic bequests. American Economic Review 81, 289–296.

Loury, G., 1981. Intergenerational transfers and the distribution of earnings. Econometrica 49 (4), 843–867.

Modigliani, F., 1988. The role of intergenerational transfers and life cycle saving in the accumulation of wealth. Journal of Economic Perspectives 2 (2), 15–40.

Orazem, P., Tesfatsion, L., 1997. Macrodynamic implications of income-transfer policies for human capital investment and school effort. Journal of Economic Growth 2, 305–329.

Tomes, N., 1981. The family, inheritance and intergenerational transmission of inequality. Journal of Political Economy 89 (5), 928–958.

Treiman, D.J., 1977. Occupational Prestige in Comparative Perspective. Academic Press, New York.

Weil, D.N., 1994. The saving of the elderly in micro and macro data. Quarterly Journal of Economics 108, 55–81.

World Development Indicators, 2000. The World Bank.

Zeldes, S., 1989. Consumption and liquidity constraints: an empirical investigation. Journal of Political Economy 97, 305–346.