1 Model 2 (Borrowing Constraints)

2 Elements

2.1 Changes relative to model 1

Transfers are fixed

- function of parental income, tuition, ability (signal)

Students start out as HSD

- graduation is probabilistic, function of ability signal

For graduates, college can last 4 or 5 years

- to prevent overborrowing and to fix hours of high ability students

2.2 We need

important for efficiency: heterogeneity in returns to college

- first order reason: graduation rates depend on ability

borrowing constraints

- therefore consumption and hours in college

uncertain ability

- gets us imperfect ability sorting at entry
- important for efficiency, but not for fitting data (until we have earnings that depend on ability)
- having ability and learning about it will only matter with endogenous dropouts (which we will eventually need for comparative statics)

preference shocks at entry

- given ability uncertainty, they only serve a numerical purpose (make sure someone enters college for all param values)

I write the model for one cohort. There are no interactions between cohorts at this point

2.3 Notation

x: ability
a: age
j: type
J: number of types
k: assets
l: work time in college
N_a: size of ability grid
τ: college cost
A_s: time spent in college
t: time
Y: lifetime earnings
z: parental transfer for college
η: pref shock at entry
$\pi(x)$: prob of graduating (not dropping out after year 2)

3 Demographics

model period is 1 year

- not needed but simplifies notation and mapping into data

agents enter the model at model age 1 (age 17) as HS students
they live for A periods

4 Endowments

Each agent is endowed with a type $j \in \{1, ..., J\}$
All agents of type j share the same values for

- parental income y_p
- ability signal m
- college cost τ (currently not varying with j)
- college wage w_{coll}
- parental transfer z (not conditional on college entry)
- "free" consumption in college and leisure in college: $\bar{c}(j), \bar{l}(j)$
  - purpose: reduce marginal utility of consumption and leisure of high ability students in college, so they do not work to little, consume too much

We may not want all of those to vary by type, but we can allow for this.
The agents also has an ability x that is correlated with m

- Ability is not observed until the start of work
- The agent knows $\Pr(x|j)$ – a model primitive

5 Preferences

$$\mathbb{E}\sum_{a=1}^{A_s}\beta^{a-1}u\left(c_a+\bar{c}(j), 1+\bar{l}(j)-l_a\right)+\mathbb{E}\sum_{a=A_s+1}^{A}\beta^{a-1}u_w(c_a) \tag{1}$$

In addition, there are preference shocks for college.

6 Sequence of events

Household is born at age $a = 1$.
He observes j.
He graduates high school with probability $\pi_{HSG}(m)$. If he fails to graduate, he works as HSD with value $V_{HSD}(j)$.
If he graduates:

- he observes a preference shock.
- Household decides whether to enter college (value V_1) or work as HSG with value $V_{HSG}(j)$.

If the child enters college, he is committed to staying in college for 2 years. He chooses consumption, hours, saving subject to a borrowing constraint.

At the end of year 2, he drops out with probability $1 - \pi(x)$; he then must work as a college dropout. Value $V_{CD}(k, j)$.

Otherwise, he must stay in college for 2 more years, again choosing consumption, hours, and saving subject to a borrowing constraint.

Beliefs about ability are updated based on the fact that a favorable continuation shock was drawn. Beliefs are now $\Pr(x|j, CG)$.

At the end of 4 years in college, graduation is certain. But the timing is not. With probability π_4, he graduates in 4 years. Otherwise, he graduates in 5 years.

Once a household starts working, all uncertainty is resolved. He maximizes lifetime utility subject to a lifetime budget constraint.

7 Household solution

7.1 Probability of HS graduation

Depends on m (to avoid belief updating).

$\pi_{HSG}(m)$

7.2 College entry decision

- observe j
- observe pref shocks
- decide college entry

7.2.1 Equations

$$\max\{\underbrace{V_{HS}(j) - \gamma\eta_w}_{\text{work as HSG}}, \underbrace{V_1(j) - \gamma\eta_c}_{\text{enter college}}\} \tag{2}$$

- test: no test; only used through entry probability below

Probability of entry is then given by

$$\Pr(college|j) = \frac{\exp(V_1/\gamma)}{\exp(V_1/\gamma) + \exp(V_{HS}/\gamma)} \tag{3}$$

Test: no test; directly implemented in `college_entry`

7.2.2 Notation

η are type I extreme value shocks.

7.2.3 Algorithm

Computational note:

- if all / none go to college with nonnegligible probability, bound entry probs away from 0 and 1.

7.3 Years 1-2 in college:

Household who has decided to enter college. Household is committed to remain in college for 2 years.

- state: j
- everyone has the same inital assets (given j)
- choose c_a, l_a (constant across first 2 years in college)

7.3.1 Equations

period length = $\Delta a = 2$
budget constraint in college:

$$k_{a+\Delta a} = R^{\Delta a} k_a + (1 + R[\Delta a - 1])(w_{coll} l_a + z - c_a - \tau) \tag{4}$$

or

$$c = (R^{\Delta a} k - k') / (1 + R[\Delta a - 1]) + w_{coll} l + z_j - \tau_j \tag{5}$$

borrowing limit: $k_{a+\Delta a} \geq k_{min,a+\Delta a}$, $l \in [0,1]$.
Bellman

$$V_1(j) = \max_{k',l} (1 + \beta[\Delta a - 1]) u\left(c + \bar{c}_j, 1 + \bar{l}_j - l\right) + \beta^{\Delta a} V_m(k', j) \tag{6}$$

Test: `test_bc1.test_saved12` .

- tests that RHS is maximized and that value is correct.

7.3.2 Algorithm:

This is the same problem as years 3-4, except for the continuation value.
But I don't have a representation for the marginal value of k'.
Easier to max the RHS over c, where l is chosen from static condition

7.4 End of periods 1-2 in college

Draw graduation shock: with probability $1 - \pi(x)$ drop out

- or choose to drop out (we don't have that yet)

x is NOT known to agent, but he updated beliefs about it in light of graduation outcome
If agent drops out, he starts work at age 3 and receives remaining parental transfers until age 5.
Value:

$$V_m(k, j) = \sum_x \Pr(x|j) \left[(1 - \pi[x]) V_w \left(k + \sum_{a=3}^{5} R^{3-a} z_j, x, CD, 3 \right) + \pi[x] V_3(k, j) \right] \tag{7}$$

7.4.1 Algorithm

Compute a matrix by $[k, j]$:

$$X(k, j) = \sum_x \Pr(x|j)\, \pi[x]\, V_3(k, j) \tag{8}$$

Loop over j

- Compute present value of transfers
- Loop over ability
  - compute the 1st term

`coll_value_m`
Test: none. directly implemented equations.

7.5 Years 3-4 in college

Student has drawn a successful graduation outcome at the end of period 2. He is committed to staying in college for 2 more years
graduation at end of periods 4 or 5 is certain
Budget constraint is the same as in years 1-2
Bellman

$$V_3(k, j) = \max_{k', l} (1 + \beta)\, u\left(c + \bar{c}_j, 1 + \bar{l}_j - l\right) + \beta^2 V_5(k', j) \tag{9}$$

subject to $k' \geq k_{min,5}$, $l \geq 0$
Test: `test_bc1.college`

- `coll_pd3` tests that budget constraint and static condition hold

FOC:
k': $(1 + \beta)\, u_c/2 \geq \beta^2 \mathbb{E}_a \partial V_5(k', j) / \partial k'$ with equality if bc does not bind (`hh_eedev_coll3`) (not used)
l: $u_c w_{coll} \leq u_l$ with equality if $l > 0$ (tested in `hh_static_bc1`)
Assuming that utility is additively separable

7.5.1 Algorithm:

1. Try $k' = k_{min}$
   (a) find c, l that satisfy budget constraint, static condition
   (b) if Euler deviation > 0: done
2. Interior solution
   (a) Search over $c \in [c_{floor}, c_{max}]$. At c_{max} we hit corner.
   (b) Get l from static condition. Bound below from 0.
   (c) Get k' from budget constraint
   (d) find 0 of Euler dev
   (e) Closed form solution for $V_{k'}$

7.6 Value at start of period 5

Before learning whether student graduates today or tomorrow.
Assume: π_4 is simply a number.
If graduate today: receive the last transfer from the parent in addition to k.

$$V_5(k,j) = \pi_4 V_{CG}(k,j,5) + [1-\pi_4] V_5^c(k,j) \tag{10}$$

7.7 Value of studying in period 5

$$V_5^c(k,j) = \max_{c,l} u(c,l) + \beta V_{CG}(k,j,6) \tag{11}$$

subject to the same budget constraint as always. But now 1 period decision rather than 2
Test:

- uses same code as other periods in college

7.7.1 Beliefs after learning graduation outcome

$\Pr(x|j,CG) = \Pr(x,j,grad) / \Pr(j,grad)$
$\Pr(j,grad) = \Pr(grad|j)\Pr(j)$
$\Pr(j|grad) = \sum_x \Pr(x|j)\Pr(grad|x)$
$\Pr(x,j,grad) = \Pr(j)\Pr(x|j)\Pr(grad|x)$
Test: `pr_a_jgrad_test.m` by simulation

7.8 Value of working as CG

Can start at age 5 or 6. Beliefs are the same.

$$V_{CG}(k,j,a) = \sum_x \Pr(x|j,CG) V_w(k+Z,x,CG,a) \tag{12}$$

If $a=6$: $Z=0$.
If $a=5$: $Z=z_j$.
Beliefs: $\Pr(x|j,CG)$, takes into account that student did not drop out at end of year 2.
`value_work`.
Test:

- no test. Equations are directly implemented

7.9 Value of working as HSG / HSD

$$V_s(j) = \sum_x \Pr(x|j) V_w(k,x,s,1) + \bar{\eta}_c \tag{13}$$

$\bar{\eta}_c$ is the mean preference for working as HSG. Its model purpose is to match college entry rates.
$k = \sum_{a=1}^{5} R^{1-a} z_j$: present value of parental transfers
Test: none. Equation is directly implemented. `value_hsg`.

7.10 Value of Work

state: k, x, s, a
Except for CG: $a = A_s + 1$
For CG: $a = 5$ or 6.
receive present value of lifetime earnings $Y(x, s, a)$
choose consumption the usual way (perfect credit markets)

$$V_w(k, x, s, a) = \sum_{\hat{a}=a}^{A} \beta^{\hat{a}-a} u_w(c_{\hat{a}}) \tag{14}$$

subject to

$$Rk + Y(x, s, a) = \sum_{\hat{a}=a}^{A} R^{a-\hat{a}} c_a g_c^{\hat{a}-a} \tag{15}$$

Euler: $c^{-\sigma} = \beta R (c')^{-\sigma}$. Therefore $g_c = c'/c = (\beta R)^{1/\sigma}$.
Closed form solution. Uses general purpose code (`UtilCrraLH`).
Test: `test_bc1.work`

7.11 IQ

$IQ = m + \varepsilon_{IQ}$ (up to scale)
$\varepsilon_{IQ} \sim N(0, \sigma_{IQ})$
Divide into percentile groups (e.g. quartiles).
All I need to know is Pr(IQ group | j)
Test: `test_bc1.pr_xgroup_by_type` (simulation)

8 Computing Aggregates

Need aggregates by [IQ, school, type]. Type determines parental income etc.
These can be computed from aggregates by [s, a, j]
Mass(s,a,j) has closed form solution.
Mass(s,IQ,j) = sum over a (mass(s,a,j) * pr(IQ|a))

8.1 Stats by type

Prob(grad | entry, j) = $\sum_x \Pr(x|j)\, \pi(j)$

8.2 Stats by ability

$\Pr(HSD, x) = \sum_j \Pr(j) \Pr(x|j) [1 - \pi_{HSG}(j)]$
For $s \in \{HSD, HSG\}$: (because HS graduation depends on type, not ability)
$\Pr(s, x) = \sum_j \Pr(s, j) \Pr(x|j)$
.
$\Pr(college, x)$: works the same way b/c college entry depends on type not ability

8.3 Debt stats

Counting all transfers as paid out in period 1 understates debt (before the end of college).

Correct budget constraint:

$\hat{k}_{d+1} = R\hat{k}_d + 2\left(w_{coll} l_d - c_d - p + z\right)$

Iterate on this, starting from $\hat{k}_1 = 0$, to find correct debt stats.

This makes surprisingly little difference (`aggr_show`).

9 Calibration

9.1 Functional Forms

9.1.1 Preferences:

$$u\left(c,l\right) = \left[\omega_c \frac{c^{1-\varphi_c}}{1-\varphi_c} - 1\right] + \omega_l \frac{\left(1-l\right)^{1-\varphi_l}}{1-\varphi_l} \tag{16}$$

$$u_w\left(c\right) = \omega_w \frac{c^{1-\varphi_c}}{1-\varphi_c} \tag{17}$$

Normalize $\omega_c = 1$.

Note: we want the household to value leisure rather than dislike work. Otherwise the household never chooses $l = 0$.

9.1.2 Endowments:

Joint Normal: z, m, τ, y_p

I implement this by drawing independent standard Normal random variables for each endowment. Then correlate them using weights called $\alpha_{x.y}$. This is a detail. In effect, we are getting a joint Normal distribution.

- correlation parameters: $\alpha_{m,\tau}$ etc.
- marginal distributions: μ_τ, σ_τ etc.
- $m \sim N\left(0,1\right)$

$x \sim N\left(0,1\right)$

- correlation with m governed by $\alpha_{x,m}$.

Free college consumption / leisure:

- $\bar{c}\left(j\right) = \left(m_j - m_{\min}\right) \times \beta_c / \left(m_{\max} - m_{\min}\right)$
- In words: $\bar{c}$ is linear in m such that the lowest type gets 0 and the highest type gets β_c
- Analogous for $\bar{l}$.

With small J (e.g. 80), the realized means and std devs can be quite far from the targets. To avoid that, I scale all random vars to be N(0,1).

9.1.3 High school graduation

$$\pi_{HSG}\left(m\right) = \pi^0_{HSG} + \pi^1_{HSG} m \tag{18}$$

truncated in [0, 1]

`prob_hsg.m`.

9.1.4 College

Graduation probability is logistic:

$$\pi(x;t) = \pi_0 + \frac{\pi_1 - \pi_0}{\left[1 + \pi_a / \exp\left(\pi_b \left[x - x_0\right]\right)\right]^{1/\pi_c}}$$

Set $\pi_c = 1$, $x_0 = 0$. Calibrate the rest.

`pr_grad_a.m`

9.1.5 Work

CPS: median earnings by [age, school, year]. Base year prices.

For each cohort: compute present value using fixed interest rate (still in base year prices): $\bar{Y}(s,c)$

Detrend the present value, using detrending factor for work start age.

CPS measures average log lifetime earnings by (s,c)

$Y(x,s,c) = \phi_s (a - \bar{a}) + \hat{e}(s,c) + \bar{Y}(s,c)$

ϕ_s from one of our previous papers

$\hat{e}(s,c)$ is calibrated to match $\bar{Y}(s,c)$

skill premium for given a: $\Delta\phi_s (a - \bar{a})$ + common stuff

- set $\bar{a}$ arbitrarily so that $a - \bar{a} > 0$.

Calibration needs to ensure that lifetime earnings are rising in schooling (at least for most agents).

For now impose:

1. Not discounting to age 1, going to school longer raises lifetime earnings for all abilities (a bit strong b/c schooling shortens work lives). That requires $a - \bar{a} \geq 0$ and $\hat{e} + \bar{Y}$ rising in s.
2. HSD and HSG start working at age 1. CD start at age 3. CG start at ages 5 or 6. Starting at age 6 reduces lifetime earnings by factor $1/R$.

9.2 Fixed Parameters

1 shows fixed model parameters.

Mostly standard choices.

Directly from data

- borrowing limits $k_{min,d,c}$ (finaid.org)

φ_l: we don't really have any evidence on this. Set arbitrary for now.

π_4: matches average duration of college

- set to 0.5 to get duration of 4.5 years (HS&B)

9.3 Calibrated parameters for the NLSY79 or 97 cohort

2 shows calibrated parameters. The values are of course still invented.

9.4 Calibration targets for NLSY79 or 97 cohort

Complete summary in this table (showing model fit):

Table 1: Fixed Model Parameters

Parameter	Description	Value
Demographics		
A	Lifespan	58
A_s	School durations	0, 0, 2, 4
Endowments		
	Marginal distribution of τ	0.0
Preferences		
β	Discount factor	0.98
φ_c	Curvature of utility	2.00
ω_c	Weight on u(c)	1.00
φ_l	Curvature of utility	2.00
γ	Preference shock at college entry	0.20
Work		
	Returns to ability	0.155, 0.194
Other		
π_4	Prob of graduating in 4 years	0.50
R	Interest rate	1.04

Table 2: Calibrated Model Parameters

Parameter	Description	Value
Endowments		
	Marginal distribution of τ	1.8
	Marginal distribution of z	1.64, 0.28
	IQ noise	1.57
Preferences		
ω_l	Weight on leisure	0.11
ω_w	Weight on u(c) at work	9.76
$\bar{\eta}$	Preference for HS	23.66
$\bar{c}_{max}$	Max free consumption	9.8
$\bar{l}_{max}$	Max free leisure	0.16
Work		
$\bar{Y}_s$	Log skill prices	6.16, 6.50, 6.53, 6.73
Other		
w_c	College wage	28.8
	High school graduation	0.93, 0.11

Table 3: Model Fit

Description	Model	Data	Dev
Fraction by schooling	0.08, 0.37, 0.29, 0.25	0.08, 0.38, 0.30, 0.24	0.02
Fraction entering college by IQ quartile	0.30, 0.52, 0.67, 0.84	0.31, 0.47, 0.66, 0.86	0.19
Fraction graduating by IQ quartile	0.12, 0.22, 0.31, 0.43	0.04, 0.13, 0.27, 0.55	3.16
Fraction entering college by y quartile	0.37, 0.44, 0.58, 0.83	0.37, 0.49, 0.61, 0.85	0.22
Fraction graduating by y quartile	0.15, 0.19, 0.28, 0.41	0.10, 0.14, 0.25, 0.52	1.89
Fraction entering college by qy quartile			0.31
Fraction graduating college by qy quartile			0.95
Regression entry on iq, yp	0.62, 0.44	0.58, 0.39	0.70
Fraction graduating high school by y quartile	0.88, 0.89, 0.92, 0.97	0.81, 0.91, 0.96, 0.99	0.42
Fraction graduating high school by IQ quartile	0.86, 0.91, 0.94, 0.97	0.76, 0.94, 0.99, 1.00	0.83
Joint distribution of q,y			2.27
Lifetime earnings CD	6.85	6.85	0.00
Lifetime earnings premiums	-0.55, -0.20, 0.00, 0.37	-0.53, -0.19, 0.00, 0.37	0.02
Mean of college cost	1.31	1.95	0.21
Mean hours in college	0.24	0.24	0.00
Mean hours in college by IQ	0.28, 0.26, 0.24, 0.21	0.24, 0.26, 0.24, 0.23	0.14
Mean hours in college by y	0.29, 0.26, 0.23, 0.22	0.26, 0.24, 0.25, 0.22	0.17
Mean earnings in college	4.94	5.03	0.02
Mean earnings in college by IQ	5.72, 5.39, 5.04, 4.34	4.92, 5.90, 5.49, 4.54	0.58
Mean earnings in college by y	5.90, 5.28, 4.77, 4.44	5.26, 5.28, 5.46, 4.61	0.47
Fraction with debt by IQ	0.43, 0.51, 0.59, 0.72	0.53, 0.56, 0.60, 0.61	0.23
Fraction with debt by yp	0.77, 0.66, 0.57, 0.56	0.68, 0.66, 0.65, 0.54	0.14
Mean college debt	2.02, 2.96, 4.05, 6.18	2.63, 5.03, 3.69, 4.54	0.95
Mean college debt	4.57, 5.61, 4.88, 3.68	4.81, 4.21, 4.95, 3.85	0.26
Mean transfer	4.01	4.96	0.45
Mean transfer by y quartile	2.57, 3.25, 3.99, 5.09	2.13, 3.25, 4.81, 6.97	0.56
Mean transfer by IQ quartile	3.95, 3.99, 4.02, 4.04	2.05, 3.33, 4.21, 4.87	0.61

9.4.1 Schooling

fraction by schooling overall
by IQ, and yp quartile

- fraction HSG+
- fraction entering college and graduating (conditional on HSG)

β_{IQ}, β_F.
one could just target the entire matrix of mass by (school, IQ, yp), but this seems more transparent

9.4.2 Lifetime earnings

Mean log lifetime earnings by schooling

9.4.3 College earnings and hours

- mean by IQ, y_p (NLSY)
- unconditional mean hours: set to 20/week, unless we have direct data (NLSY only)

9.4.4 College costs

p_t: tuition + fees + room/board (?) - scholarships - grants + supplies
p mean: match mean from time series source (Chris?)
p std dev:

- take ratio of std dev / mean from HS&B
- assume constant over time
- currently set to 0.

also try with room and board; and excluding private

9.4.5 College debt

by IQ, yp: mean debt, fraction in debt (college graduates, end of college) (NLSY)

9.4.6 Parental income

Target joint distribution of y_p, IQ (quartiles).

- matched by endowment correlation matrix

Scale no longer matters.

9.4.7 Parental transfers

means by IQ, yp

9.5 Intuition on how parameters are identified

9.5.1 Correlations:

- p,m and (p,y): would like to match mean p by IQ quartile and by yp quartile (HSB)
  - but not clear how to scale that consistently with the mean series
  - possible: rescale so that mean equals time series
- a,m: match fraction CD, CG by IQ quartile (NLSY)
- σ_{IQ}: fix for now at what we estimate from other papers (either Todd and Lutz's or Oksana and Lutz's)

9.5.2 Preferences

γ: size of pref shocks at college entry; match fraction CD, CG by IQ quartile

- currently fixed at 0.2; model wants something very small (0.05) which implies that college entry rates are at corners for most types

ω_l: weight on leisure; match hours worked in college (NLSY)
ω_c: match average **debt** (at end of college, grads and dropouts) (NLSY)

- earlier cohorts: average student debt (Trends in Student Aid)

9.5.3 College

$\pi(a)$ parameters: match fraction CD, CG by IQ quartile

9.5.4 Other

w_{coll}: match college earnings and hours (NLSY)
College earnings:

- NLSY: mean by IQ, yp

9.5.5 Detrending

We have preference shocks and the preference for working as HSG. So detrending affects choices.
Disposable income per capita has doubled since 1960. Median (or mean log) CPS wages are roughly constant.
If we detrend by CPI, present value of lifetime earnings falls by half across cohorts we use.
Probably better to not detrend at all.

10 Time series calibration

Detrending is important because it scales preference shocks (such as pref for HS work, but that is actually time varying; but also $\bar{c}$).
We want to replicate

- college entry pattern by IQ, fam income
- college graduation pattern by IQ, fam income
- average debt
- financing shares
- hours worked in college

10.1 Time varying parameters

10.1.1 Directly from data

$k_{min,d,c}$:

- directly from statutory borrowing limits

w_{coll}

- match earnings in college (source?)
- for now: keep constant (in stationary model)

$\pi(a)$

- π_0 matches college graduation rate

π_4: matches average duration of college (source?)

- currently set to 0.5 to get duration of 4.5 years (HS&B)

10.1.2 Calibrated

$\hat{e}(s)$: match lifetime earnings by s
μ_p: match mean college cost
π^0_{HSG}: match high school graduation rate
$\bar{\eta}_c$: preference for work as HSG

- match college entry rate overall

$\alpha_{a,m}$: signal precision:

- match college entry rates, graduation rates by IQ

anything to match hours? +++

10.1.3 Currently fixed

μ_y: mean log parental income: treat as fixed (stationary model)

10.1.4 Group sizes vary with cohort

E.g. family income is not in quartiles.
If close to quartiles: interpolate (e.g. Project Talent)

10.2 Time series calibration targets

All targets that are available.
Not available:

- financing: transfers, earnings

10.2.1 Hours worked in college

Currently set to 6.7 (arbitrary)

10.2.2 College earnings

No data.

10.2.3 Transfers

No data.

10.2.4 Financing shares

share of total spending from earnings and loans
Data (`cal_targets`)

- for cohorts born before 1950: Hollis (1957)
- total = 100 - scholarships - veterans
- family = family + 0.5 * other
- earnings = earnings + 0.5 * other

Model

- Total spending $= 2\,(c+p)$
- Earnings: $2w_{coll}l$
- Debt: k' (if < 0)
- Family: the rest.

11 Interesting counterfactuals

1. Vary one observable input at a time
   (a) college premium (really: earnings profiles)
   BUT: must also vary $\hat{e}$. That requires to first calibrate each cohort.
   (b) borrowing limits
   (c) college costs (need to recalibrate that one parameter to match 1 target)
2. Hold all parameters fixed, except those taken directly from data (e.g. college costs, earnings profiles and $\hat{e}$, borrowing limits)

   Also vary $\bar{\eta}$ so that model matches schooling. Otherwise the college premium produces a large change in schooling, which makes results hard to interpret.

12 Notes

12.1 Organization

Steps:

1. Calibrate all params for reference cohort (NLSY79)
2. Calibrate a subset of parameters for every other cohort (experiments)
3. Counterfactuals: Set selected parameters to values for other cohorts (e.g. borrowing limits or present value of earnings by ability / schooling).

12.2 Identification

Key: how responsive is entry of different students to changing college costs, debt limits, college premium?

We don't have anything that identifies these responses.

In the baseline cohort, costs are essentially irrelevant. Nobody is borrowing constrained and costs are a tiny fraction of the reward (positive or negative) from going to college.

As a result, there is nearly perfect sorting by m.

This would change in a model with dropout decisions. Then we could identify the distribution of surplus by matching dropout rates.

12.3 Modeling Dropout Decisions

At the end of year 2, each students receives an additional ability signal $\hat{m}$. We need that to get dropouts. Or we could have preference shocks, but that seems hokey.

From then on the state is $(k, \hat{m}, m, t)$.

If the signal is discrete, we can calculate $\Pr(x|\hat{m}, m)$. Very much like in the current model we update beliefs after receiving the dropout shock.

The dropout problem is then easy.

Value functions in years 3+ are not changed much. Just have the additional state that affects beliefs.

Aggregation: All we need is $\Pr(x|s, a)$, which we can compute from $\Pr(grad|m)$.

12.4 Endowments

12.4.1 An alternative way of drawing endowments.

Each agent draws (x, m, z, y_p, IQ) from a joint normal distribution.

The household observes all endowments except for ability.

We draw J realizations.

For each j, x is a Normal random variable.

it should be possible to compute its mean and std deviation, so we have the entire distribution.

This gives us the beliefs of the household.

Now we make an ability grid and compute $\Pr(x|j)$. Done.

Code: Define a lower diagonal weight matrix W. 5x5

compute the implied Cov matrix. Compute the implied scaled weight matrix such that $VV' = Cov$ using Cholesky.

Endowment draws: $y_{is} = \mu_s + \sum_k V_{sk}\varepsilon_{ik}$ with $\varepsilon \sim N(0,1)$

$\mathbb{E}y_{is}|rest = \mu_s + \sum_{k \neq s} V_{sk}\varepsilon_{ik}$

$Var(y_{is}|rest) = V_{ss}^2$

Test this by simulation.

Now we have the entire conditional distribution of each variable.

12.5 Years with small college premium

Model fails to match small college premium (or dropout premium is 0).

Reason:

- when college premium gets small, high ability agents stay out of college (which is painful)
- because of selection into graduation, the college premium cannot get too small
- for the same reason, when entry selection is weak, the CD premium goes negative

12.5.1 Modeling dropout

would weaken selection into graduation => CD premium can rise, CG premium can fall

12.5.2 Allow for shallower $\pi(x)$

tried that - effect was pretty small

12.5.3 Calibrate γ

tried that - effect pretty small

12.5.4 Psychic costs correlated with m

and time varying

Idea:

have a fudge factor that allows the model to "exactly" match entry by IQ for each cohort

then we could get positive selection with small college premium

Problem:

CD premium would likely remain low

12.6 Parental transfers

Key: how responsive to changes in tuition, borrowing ...?

Altruism cannot work

Transfers are a tiny fraction of parental income.

They will (at least for rich parents) almost fully offset changes in financial conditions of the child (e.g. tuition). At least for the NLSY cohort (few borrowing limits).

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With altruism, transfers are too steep in parental income.

Transfers approximately equal MU(c) across generations.

Double parental income => double parental and child consumption.

Child hours fall (static condition)

The entire increase in consumption must come out of transfers.

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We need to fundamentally rethink transfers.

We should really use better data to pin this down (NELS). It's important for our paper.

12.7 Time series scaling

Everything, including preference shocks, must scale properly as we take the model back in time.

How to scale pref shocks is not clear.

Sidestep this problem by detrending the model.

1. Compute average earnings for the baseline cohort
   (a) weights: for now simple; age range 30-50; school weights = fraction by s from base cohort
2. For each cohort: compute a scale factor that makes average earnings the same as base cohort
3. Multiply all dollar variables by that scale factor

Result display: divide all dollar figures by the scale factor (inconvenient)

12.8 Corners

For calibration, should prevent a situation where nobody /everybody enters college for some parameter values.

12.8.1 Debt corners

Can have parameter values where everyone maxes out on debt in year 4.
Then using only debt tg for grads gets stuck.
Using aggregate debt does not help all that much either. Can have params where (nearly) only grads borrow.
Solution:

- start from a guess where not everyone hits the borrowing limit
- penalty when more than a fixed fraction hits borrowing limit
- only when borrowing limit is positive

13 eof