Problem Set 6: Innovation and Growth

Econ720. Fall 2016. Prof. Lutz Hendricks

1 Stochastic patent duration

[Due to Matt Doyle] Consider a version of the "Expanding Variety of Goods" model in which innovators' monopoly power diminishes over time. Otherwise the model is standard.

Demographics: There is a single representative household.

Endowments: The household is endowed with L units of labor, which can only be used for work.

Preferences:

$$U = \int_0^\infty \frac{c^{1-\theta} - 1}{1-\theta} \cdot e^{-\rho t} dt. \tag{1}$$

Technology:

- Final goods are produced from labor and intermediate inputs according to

$$Y = AL^{1-\alpha} \cdot \sum_{j=1}^{N} (X_j)^\alpha, \tag{2}$$

where $0 < \alpha < 1$, Y is output, L is labor input, X_j is the input of the $j'th$ type of the intermediate good, and N is the number of varieties.

- It takes one unit of final goods to produce one unit of intermediates.
- It costs η units of the final good to create a new type of intermediate good.

Market arrangements:

- The final goods sector is perfectly competitive.
- Intermediate goods producers hold monopolies.
- There is free entry for innovators.
- Households own all firms in the economy.

Patents: Upon innovation, the innovator receives a patent. If intermediate good j is currently monopolized, it becomes competitive in the next instant dT with probability $p \cdot dT$, where $p \geq 0$. Thus, if good j is invented at time t, the probability of it still being monopolized at some future date $v \geq t$ is $e^{-p \cdot (v-t)}$.

Notation: Denote by N^c, the number of intermediate goods produced competitively and by N the total number of intermediate goods.

Answer the following questions:

1. State the household problem and its solution.
2. Solve the problem of the final goods producer.
3. Solve the problem of the intermediate input producer.
4. State the free entry condition for innovation.
5. Define an equilibrium.
6. Derive the quantity of X_j produced when the j'th producer is a monopolist. D
7. Derive the quantity of X_j produced when the j'th intermediate good is produced competitively.
8. Using free entry and the definition of profits, show that:

   $$r = (L/\eta) \cdot A^{1/(1-\alpha)} \cdot \frac{1-\alpha}{\alpha} \cdot \alpha^{2/(1-\alpha)} - p \tag{3}$$

   Note that a higher p (shorter patents) reduces growth in this model. This is, of course, not a general result.
9. Solve for a balanced growth values of $\dot{c}/c, N^c/N$, and Y/N. Hint: Use the following approximation: $\dot{N}^c = p \cdot (N - N^c)$.

2 Answer: Stochastic patent duration

1. The household's budget constraint is:
$$\dot{a} = ra + w - c \tag{4}$$
Maximizing (1) subject to (4) is a standard problem. The (PV) Hamiltonian is:
$$H = e^{-\rho t}\frac{c^{1-\theta} - 1}{1-\theta} + \lambda\{w + ra - c\} \tag{5}$$
Obtaining and manipulating first order conditions gives:
$$\dot{c}/c = (1/\theta)(r - \rho) \tag{6}$$

2. Final goods firms maximize profits:
$$Y - wL - \sum_{j=0}^{N} P_j X_j \tag{7}$$
subject to (2). The first order conditions yield the demand function for each intermediate good:
$$X_j = L(A\alpha/P_j)^{1/(1-\alpha)} \tag{8}$$

3. A monopolistic intermediate goods producer maximizes profits:
$$P_j X_j - X_j \tag{9}$$
subject to (8). The first order conditions imply
$$P_j = 1/\alpha. \tag{10}$$

4. Zero profits for entrepreneurs mean that the costs (η units of output) equal the expected benefits (expected value of future monopoly profits, appropriately discounted). This is identical to the condition in the standard model, except that future profits have to be discounted more heavily because of the possibility that the intermediate good producer will lose his or her monopoly before date v.
$$\eta = \int_t^{\infty} \pi_t^m \cdot e^{-[p+\bar{r}(t,v)]\cdot(v-t)} dt. \tag{11}$$

5. Equilibrium: $Y, c, X_j^m, X_j^c, L, w, r, p_j$
   - Household's maximize utility taking prices as given
   - Final goods firms maximize profits, taking prices as given
   - Competitive intermediate goods firms maximize profits, taking prices as given
   - Monopolistic intermediate goods firms choose prices to maximize profits, taking the demand function as given
   - Entrepreneurs maximize profits, taking prices as given (free entry).
   - Markets clear

6. Substituting the optimal price back into (8) gives the output produced by a monopolist:
$$X_j^m = LA^{1/(1-\alpha)}\alpha^{2/(1-\alpha)} \tag{12}$$

7. Under perfect competition, price equals marginal cost, which is equal to 1 in this case. Substituting $Pj = 1$ into 8 gives the output produced in a competitive sector:
$$X_j = L(A\alpha)^{1/(1-\alpha)} \tag{13}$$

8. Free entry implies:

$$r(t) = \pi^m/\eta - p \tag{14}$$

Substitution of the monopolists optimal price (10 and output (12) into the profit function (9) gives:

$$\pi^m = \frac{1-\alpha}{\alpha} \cdot LA^{1/(1-\alpha)}\alpha^{2/(1-\alpha)} \tag{15}$$

Simple substitution of the monopoly profits derived above into the interest rate equation gives (3).

9. On a BGP, N, N^c, C and Y all grow at the same rate. Using 3 and 6, we get an expression for a constant growth rate of consumption:

$$\dot{c}/c = (1/\theta)((L/\eta) \cdot A^{1/(1-\alpha)} \cdot \frac{1-\alpha}{\alpha} \cdot \alpha^{2/(1-\alpha)} - p - \rho) \equiv \gamma^* \tag{16}$$

In a steady state, N, N^c, C and Y all grow at rate γ^*.

Since each monopolized good becomes competitive with probability p per unit time, the change in N^c over time can be approximated using $\dot{N}^c = p \cdot (N - N^c)$. In the steady state, this implies:

$$N^c/N = \frac{p}{\gamma^* + p} \tag{17}$$

Using the final goods firm's production functions and the quantities of intermediate goods produced (derived in part i) we get an expression for aggregate output:

$$Y = A^{1/(1-\alpha)}\alpha^{2/(1-\alpha)}LN \cdot [1 + N^c/N \cdot (\alpha^{-\alpha/(1-\alpha)} - 1)] \tag{18}$$

If we substitute for N^c/N, we can determine a formula for output that applies on the steady state path.

$$Y = A^{1/(1-\alpha)}\alpha^{2/(1-\alpha)}LN \cdot [1 + \frac{p}{\gamma^* + p} \cdot (\alpha^{-\alpha/(1-\alpha)} - 1)] \tag{19}$$

Note that Y does indeed grow at the same rate as N.