Aggregation

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Notes on Aggregation

We have assumed a representative household.

How restrictive is this assumption?

If households are not identical, do they "aggregate" into a representative household?

Recall the Perpetual Youth model:
there was a representative household, but the Euler equation was different from that of an individual.

Example with Heterogeneity

Example with Heterogeneity

- Consider a Cass-Koopmans model with two types of households, $i = 1, 2$.
- Demographics:
  - The population of each type is constant (N^i).
- Preferences are identical: $\int_0^\infty e^{-\rho t} \frac{c^{1-\sigma} - 1}{1-\sigma} \, dt$.
- Endowments:
  - Each household starts with capital k_0^i.
  - Each has one unit of type i time at any moment.

Example with Heterogeneity

- Technology:

$$
\begin{aligned}
Y_t &= K_t^{\theta}\left[(L_t^1)^{1-\theta} + (L_t^2)^{1-\theta}\right] \\
&= \dot{K}_t + \delta K_t + C_t.
\end{aligned}
$$

- Note: Each household supplies a different type of labor.

Household

- The household problem is entirely standard.
- Solution is k_t^i and c_t^i which satisfy Euler equation

$$g\left(c_t^i\right) = (r - \rho) / \sigma \tag{1}$$

and budget constraint:

$$\dot{k}^i = rk^i + w^i - c^i \tag{2}$$

- Boundary conditions: k_0^i given and TVC.

Firm

- Factor prices equal marginal products.
- $q = F_k$ and $w^i = F_{L^i}$.

Equilibrium

A CE consists of functions of time $c^i, k^i, w^i, r, q, K, L^i$ that satisfy

- 2x2 household conditions
- 3 firm first order conditions
- Factor market clearing: $K = \sum k^i L^i$ and $L^i = N^i$
- Goods market clearing: $F(K, L^1, L^2) - \delta K = \dot{K} + \sum L^i c^i$
- Identity: $r = q - \delta$

Representative Household

- We now show that the entire economy behaves as if a representative household chose consumption.
- From lifetime budget constraint:
  present value of consumption = present value of income + initial assets

  $$c_0^i \Pi_0 = k_0^i + PV_0\left(w^i\right)$$

  where

  $$\Pi_0 = \int_0^\infty \exp\left(\int_0^t \left[g\left(c_\tau\right) - r_\tau\right] d\tau\right)$$

Representative Household

- Aggregate consumption

$$C_0 = \sum_i L_i c_0^i = \sum_i L_i \left(k_0^i + PV_0\left(w^i\right)\right) / \Pi_0 \tag{3}$$

$$= K_0 / \Pi_0 + PV_0 \left(\sum_i w^i L_i\right) / \Pi_0 \tag{4}$$

- The level is what a household who owns all capital and labor would choose.

Representative Household

The growth rate of aggregate consumption obeys the individual Euler equation:

$$g(C_t) = \frac{\sum_i L_i \dot{c}_t^i}{\sum_i L_i c_t^i} = \sum_i \frac{L_i c_t^i}{\sum_i L_i c_t^i} g\left(c_t^i\right) = g\left(c_t^i\right) \tag{5}$$

Why is this true?

Because the marginal propensity to consume out of capital / labor income is the same for all households.

This would fail if utility were not iso-elastic.

Then $g\left(c_t^i\right) = (r_t - \rho)/\sigma\left(c_t^i\right)$ is not independent of the level of c_t^i

Steady State

The same results are easier to see in steady state.

A steady state is: the same objects (but as scalars): $c^i, k^i, w^i, r, q, K, L^i$.

These satisfy, in **sequential** order:

- Labor inputs are exogenous.
- $F_K = \rho + \delta$ determines K.
- $r = \rho$.
- $w^i = (1-\theta)(K/L^i)^\theta$ determines w^i.

Steady State

We then have an additional 3 equations:

1. capital market clearing:

$$K = \sum k^i L^i \tag{6}$$

2. household budget constraints with $\dot{k}^i = 0$:

$$c^i = \rho k^i + w^i \tag{7}$$

The 3 equations are supposed to determine 4 variables: c^i, k^i.

Steady State

- The steady state is not unique.
- Any k^i that sum to K are a steady state.
- For any k^i pair we pick, the budget constraints tell us the corresponding steady state consumption levels.

Why is the steady state not unique?

- Both households have the same marginal propensity to consume: ρ.
- Redistribute a bit of k^1 to k^2. Aggregate C is unchanged. All markets clear.
- Effectively, the households behave as if they were one - a **representative household**.
- This is **good**: when it works, we don't have to explicitly model heterogeneous households.

The Representative Household

The representative household

How hard is it to get a representative household?

One perspective:

> *Any aggregate demand curve is consistent with optimal behavior by a set of households.*

Theorem

(Debreu-Mantel-Sonnenschein) Let $\varepsilon > 0$ be a scalar and $N < \infty$ be a positive integer. Consider a set of prices $P_\varepsilon = \left\{ p \in \mathbb{R}_+^N : p_j/p_{j'} \geq \varepsilon \forall j, j' \right\}$ and any continuous function $x : P_\varepsilon \to \mathbb{R}_+^N$ that satisfies Walras' Law and is homogeneous of degree 0. Then there exists an exchange economy with N commodities and $H < \infty$ households, where the aggregate demand is given by $x(p)$ over the set P_ε.

Why is aggregation so hard?

- The problem is income effects.
- Changing prices effectively redistributes income across households.
- If the income elasticities of various goods are very different, demand curves could be upward sloping over some intervals.
- But there is hope if income effects are not too strong.

Gorman aggregation

Theorem

(Gorman aggregation) Consider an economy with a finite number N of commodities and a set H of households. Suppose that the preferences of household $i \in H$ can be represented by an indirect utility function of the form

$$v^i\left(p, y^i\right) = a^i(p) + b(p)\, y^i$$

then these preferences can represented by those of a representative household with indirect utility

$$v(p, y) = \int a^i(p)\, di + b(p)\, y$$

where y is aggregate income.

Gorman aggregation

- Key feature of Gorman preferences:
  - All households have the same constant propensity to consume out of income.
- This is why redistributing income does not change consumption.
- Then aggregate income is sufficient to figure out demand.

CES Preferences

- The growth model has CES preferences:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

- CES preferences are consistent with balanced growth.
- This is because the marginal propensity to consume is constant on the balanced growth path.
- This is why redistribution does not change aggregate consumption.

Implications

Exact aggregation is rare.

How worried should we be?

One faction of economists views representative agent models as toy models.

Another faction is more pragmatic:

- start with a simple model
- check whether heterogeneity makes a quantitatively significant difference

Reading

- Acemoglu, "Introduction to modern economic growth," ch. 5.