Example: Heterogeneous Households

Econ720

Prof. Lutz Hendricks

September 29, 2014

Model

Demographics:

- There are $j = 1, ..., J$ types of households.
- The mass of type j households is μ_j.

Preferences:

- $\max \sum_{t=0}^{\infty} \beta^t u_j(c_{jt})$.
- u_j is increasing and strictly concave and obeys Inada conditions.

Model

Technology: $F(K_t, L_t) + (1 - \delta) L_t = C_t + K_{t+1}$

Endowments:

- Each household is endowed with one unit of labor in each period.
- At $t = 0$ household j is endowed with k_{j0} units of capital and with $b_{j0} = 0$ units of one period bonds.

Market arrangements are standard.

Household Problem

- Nothing new here, except everything is indexed by j.
- Define wealth as $a_{jt} = k_{jt} + b_{jt}$.
- Impose no-arbitrage: $R = q + 1 - \delta$
- Bellman equation:

$$V_j(a) = \max u_j(w + Ra - a') + \beta V_j(a') \tag{1}$$

- Euler Equation:

$$u_j'(c) = \beta R' u_j'(c') \tag{2}$$

- Solution (sequence language): $\{c_{jt}, a_{jt}\}$ that solve the Euler equation and budget constraint.
- Boundary conditions: a_{j0} given and TVC $\lim_{t\to\infty} \beta^t u'(c_{jt}) a_{jt} = 0$.

Competitive Equilibrium

A CE consists of sequences $\{c_{jt}, k_{jt}, b_{jt}, k_t, n_t, R_t, q_t\}$

which satisfy:

- 2 household conditions
- 2 firm first-order conditions (standard)
- Market clearing:

$$\begin{aligned}
k_t &= \sum \mu_{jt} k_{jt} \\
n_t &= \sum \mu_{jt} \\
F(k_t, n_t) + (1-\delta) k_t &= \sum \mu_{jt} c_{jt} + k_{t+1} \\
\sum b_{jt} &= 0
\end{aligned}$$

We need to distinguish k_{jt} from k_t in the equilibrium definition.

Steady State

- Similar to CE without time subscripts.
- Euler equation becomes:

$$\beta R = 1$$

- Interesting: we can find R without knowing preferences or wealth distribution.

Are there steady states with persistent inequality?

- Let's solve for steady state c_j as a function of prices and endowments (k_{j0}, b_{j0}).
- With constant prices, the household's present value budget constraint implies

$$k_{j0} + b_{j0} = \frac{c_j - w}{R - 1} \tag{3}$$

- Endowing households with any k_{j0}'s that sum to the steady state k yields a steady state with persistent inequality.
- It would be harder to show that persistent inequality follows from *any* initial asset distribution which features capital inequality.

Redistribution

How does the steady state allocation change when a unit of capital is taken from household j and given to household j'?

- There is a steady state with the same aggregate levels of c and k for any initial distribution of assets.
- The new steady state differs from the previous one only in that type 2 households consume more and type 1 households consume less.
- Total consumption remains constant: $\mu_1 \, dc_1 + \mu_2 \, dc_2 = 0$.

Lump-sum Taxes

Impose a lump-sum tax τ on type j households. The revenues are given to type j' households.

How does the steady state change?

- The Euler equation still pins down the same capital stock (and R).
- The new present value budget constraint is

$$k_{j0} + b_{j0} = \frac{c_j - w - \tau_j}{R - 1} \tag{4}$$

  where $\tau_1 = \tau$ and $\tau_2 = -\tau \mu_1 / \mu_2$.
- Households change consumption without affecting the aggregate.

Lump-sum Taxes

What if revenues are thrown into the ocean instead?

- Still no change in the Euler equations and steady state capital stocks.
- Households must cut consumption by the tax amount.
- Nothing else changes.

Differences in β

- Now imagine households differ in their β's, but not in their u functions.
- For simplicity, assume that $u(c) = c^{1-\sigma}/(1-\sigma)$.
- What would the asset distribution look like in the limit as $t \to \infty$?

- Consumption growth in steady state is given by $1 + g(c_j) = (\beta_j R)^{1/\sigma}$.
- The most patient households have the highest consumption growth rate.
- They must save a higher fraction of lifetime income than all other types.
- In the limit, prices will be constant.
- The most patient household must have zero consumption growth (otherwise feasibility would be violated eventually).
- All other households must have negative consumption growth: $c_{j,t} \to 0$.

A famous result: The most patient household ends up holding all wealth.