Asset Pricing

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Econ720

September 9, 2015

Pricing Assets: An Example

The infinite horizon model can be used to price long-lived assets.

- This is more interesting in stochastic economies.
- It then yields the famous β measure of risk and the CAPM.

Environment

Demographics:

- a representative, infinitely lived household
- mass 1

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$

Endowments:

- in each period: N units of labor time
- at $t = 0$: L units of land

Environment

Technologies: $F(N_t, L_t; A_t) = c_t$

- constant returns to (N, L)
- the productivity sequence $\{A_t\}$ is given.

Markets:

- goods (numeraire)
- land rental: r_t
- labor: w_t
- land purchase: p_t

Firm's problem

The firm's problem is standard:

$$\max F\left(N_t, L_t; A_t\right) - w_t N_t - r_t L_t \tag{1}$$

FOCs:

$$r = F_L \tag{2}$$
$$w = F_N \tag{3}$$

Solution: N_t, L_t that satisfy the 2 FOCs.

Household

Budget constraint:

Bellman equation:

First-order conditions:

Household

The Euler equation is standard

$$u'(c) = \beta\, u'(c') \frac{r' + p'}{p}$$

Solution: $\{c_t, l_t\}$ that solve the Euler equation and budget constraint.

Equilibrium

A competitive equilibrium is a set of sequences

that satisfy:

- household: Euler equation and budget constraint;
- firm: 2 FOCs:
- market clearing for land:
- market clearing for goods:

The price of land

- We find a difference equation for p_t.
- Substitute the goods market clearing condition and the first-order condition for r into the Euler equation to obtain

$$u'(F(N,L)) = \beta\, u'(F(N',L'))\frac{F_L(N',L')+p'}{p} \tag{4}$$

- Note that this difference equation only contains p and exogenous variables.

The price of land

- We solve the difference equation for p_t by **forward iteration**.

$$
\begin{aligned}
p_t &= \beta \frac{u'(c_{t+1})}{u'(c_t)} \left\{ F_L(t+1) + \beta \frac{u'(c_{t+2})}{u'(c_{t+1})} \left[F_L(t+2) + p_{t+2} \right] \right\} \\
&= \sum_{j=0}^{\infty} F_L(t+j+1) \frac{\beta^{j+1} u'\left(F(N,L,A_{t+j+1})\right)}{u'\left(F(N,L,A_t)\right)} \qquad (5)
\end{aligned}
$$

- In deriving (5) I made use of the fact that $c_t = F(N,L,A_t)$ and that
$$\frac{u'(c_{t+1})}{u'(c_t)} \frac{u'(c_{t+2})}{u'(c_{t+1})} \dots \frac{u'(c_{t+1+T})}{u'(c_{t+T})} = \frac{u'(c_{t+1+T})}{u'(c_t)}$$

The price of land

- The asset price equals the discounted present value of "divdends."

$$p_t = \sum_{j=0}^{\infty} F(t+j+1)\ MRS(t, t+j+1) \tag{6}$$

- The discount factor is the Marginal Rate of Substitution

$$MRS(t, t+j) = \frac{\beta^j u'(t+j)}{u'(t)} \tag{7}$$

- This is a fairly general result.

The price of land: Intuition

- Start from the equilibrium price.
- Add ε to the date $t+j$ payoff.
- The household gains $\beta^j u'(t+j)\varepsilon$.
- The household's willingness to pay for this: $u'(t)\varepsilon$.
- The derivative of the price:

$$\partial p_t / \partial F(t+j) = \frac{\beta^j u'(t+j)}{u'(t)} \tag{8}$$

The price of land: stationary economy

- We calculate the price of land for a stationary economy, in which $A_t = A^S$.

$$p_t^S = F_L(N, L, A^S)\,\frac{\beta}{1-\beta}$$

The price of land: fluctuating economy

- We calculate p_t for an economy which is subject to deterministic fluctuations.
- In even periods $A_t = A^H$
- In odd periods $A_t = A^L \leq A^H$.
- Simplifying assumptions:
  - for given factor inputs, the marginal product of land is independent of A_t.
  - $2F^S = F^H + F^L$, where $F^j = F(N, L; A^j)$.

The price of land: fluctuating economy

- The trick is to break the sum in (5) into two parts, one for even and one for odd periods:

$$
\begin{aligned}
p_t^R &= \beta F_L \left\{ \sum_{j=0}^{\infty} \beta^{2j} \frac{u'(c_{t+1+2j})}{u'(c_t)} + \beta \sum_{j=0}^{\infty} \beta^{2j} \frac{u'(c_{t+2+2j})}{u'(c_t)} \right\} \\
&= \beta F_L \left\{ \frac{u'(c_{t+1})}{u'(c_t)} \frac{1}{1-\beta^2} + \frac{u'(c_{t+2})}{u'(c_t)} \frac{\beta}{1-\beta^2} \right\}
\end{aligned}
$$

- Denote the marginal rate of substition between odd and even periods by

$$\alpha = u'(F^H)/u'(F^L) < 1$$

The price of land: fluctuating economy

- If t is even, then

$$p_t^{even} = (1/\alpha + \beta) F_L \frac{\beta}{1-\beta^2}$$

- If t is odd, then

$$p_t^{odd} = (\alpha + \beta) F_L \frac{\beta}{1-\beta^2}$$

The price of land: fluctuating economy

- Since $1-\beta^2 = (1+\beta)(1-\beta)$,

$$\begin{aligned} \frac{p_t^{even}}{p_t^S} &= \frac{1/\alpha+\beta}{1+\beta} > 1 \\ \frac{p_t^{odd}}{p_t^S} &= \frac{\alpha+\beta}{1+\beta} < 1 \end{aligned}$$

- In even periods, the "risky asset" is always worth more than the "safe asset." In odd periods, the reverse is true.
- The word "risky asset" is misleading; it actually provides insurance against low consumption states.

The price of land: Intuition

- What is the intuition?
- Consider an even period.
  - Times are good, so that saving is easy.
  - And the return tomorrow is worth a lot because times will be bad.
  - Hence, the demand for land is high and so is the price.
- In odd periods, saving is painful and the return won't be worth much tomorrow. So the price is low.

Summary

- The standard growth model is also the standard framework for pricing assets.
- The price of an asset equals the present value of "dividends."
- The discount factors are the Marginal Rates of Substitution.
- This survives in stochastic environments. Just add $E[.]$.