

Growth Through Product Creation

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Issues

- ▶ We study a GE model of growth driven by innovation.
- ▶ Innovation takes the form of inventing new goods.
- ▶ Alternative: Quality ladders.

A Model of Product Innovation

- ▶ A representative household supplies capital and labor to firms
- ▶ Final goods firms use labor and intermediate inputs
- ▶ Intermediate inputs are produced from final goods
- ▶ Final goods can be used to create new varieties of intermediate inputs
- ▶ Innovators receive permanent monopolies

Note: Now that models get more complicated, it really pays off to be pedantic about details.

Demographics and Preferences

Demographics:

- ▶ A representative household.

Preferences:

$$\int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\theta} - 1}{1-\theta} dt \quad (1)$$

- ▶ C : the final good

Technology: Final Goods

Resource constraint:

$$C_t + X_t + Z_t = Y_t$$

Final goods Y are used for

- ▶ Z : R&D investment.
- ▶ X : Inputs into the production of intermediates x .
- ▶ C : consumption

Technology: Final Goods

Production of **final goods** from intermediates and labor:

$$Y_t = (1 - \beta)^{-1} \left[\int_0^{N_t} x(v, t)^{1-\beta} dv \right] L^\beta \quad (2)$$

This is of the Dixit-Stiglitz form:

Write $\left[\int x^{1-\beta} dv \right]^{\frac{1-\beta}{1-\beta}}$ to see that this is a CES aggregator of x .

Technology: Intermediate Inputs

Each unit of x requires ψ units of Y :

$$X = \psi \int_0^{N_t} x(v, t) dv \quad (3)$$

Intermediate inputs fully depreciate in use.

Technology: Innovation

Investing the final good yields a flow of new patents:

$$\dot{N} = \eta Z_t \quad (4)$$

Think of this as the aggregate (deterministic) outcome of the (stochastic) innovation efforts of many firms.

Market arrangements

- ▶ Final goods and labor markets are competitive.
- ▶ Intermediates are sold by **monopolists** (the innovators).
 - ▶ Monopolies are permanent.
 - ▶ What the monopolists do with their profits is not clear.
- ▶ Free entry into innovation
 - ▶ ensures zero present value of profits
- ▶ The household owns the innovating firms.
- ▶ Asset markets are complicated
 - ▶ there is often no need to spell out the details

Notes

Production is cyclical:

- ▶ today's Y is used to make X which makes Y
- ▶ the alternative: durable X (more complicated)
- ▶ implication: the efficient allocation maximizes $Y - X = C + Z$

The only long-lived object is a patent

- ▶ this keeps the model simple

Assuming that intermediates are made from final goods fixes marginal costs (and prices)

Solving Each Agent's Problem

Final goods producers

- ▶ Maximize period profits by choosing L and $x(v, t)$.
- ▶ Normalize the price Y to 1.
- ▶ Profits

$$Y_t - w_t L_t - \int_0^{N_t} p^x(v, t) x(v, t) dv \quad (5)$$

where

$$Y_t = (1 - \beta)^{-1} \left[\int_0^{N_t} x(v, t)^{1-\beta} dv \right] L_t^\beta \quad (6)$$

Final goods producers

FOCs:

- ▶ $\partial Y / \partial x(v) = L^\beta x(v)^{-\beta} = p^x(v)$
- ▶ $\partial Y / \partial L = \beta Y / L = w$

Demand function (cf. the Dixit Stiglitz discussion):

$$x(v, t) = L p^x(v, t)^{-1/\beta} \quad (7)$$

Solution to the firm's problem: $L_t, x(v, t)$ that satisfy the "2" first-order conditions.

Intermediate input producers

Problem after inventing a variety.

x is produced at constant marginal cost ψ .

Maximize present value of profits

$$V(v, t) = \int_t^{\infty} e^{-rs} \pi(v, s) ds \quad (8)$$

Instantaneous profits are

$$\pi(v, t) = (p^x(v, t) - \psi) x(v, t) \quad (9)$$

where $x(v, t) = Lp^x(v, t)^{-1/\beta}$

This is a sequence of static problems

Intermediate input producers

- ▶ First order condition (standard monopoly pricing formula):

$$p^x = \psi / (1 - \beta) \quad (10)$$

- ▶ Profits are

$$\pi(v, t) = \psi \frac{\beta}{1 - \beta} x(v, t) \quad (11)$$

- ▶ Solution: A constant p^x .

Household

- ▶ The household holds shares of all intermediate input firms.
- ▶ Each firm produces a stream of profits.
- ▶ New firms issue new shares.
- ▶ But: the details don't matter to the household.
- ▶ There simply is an asset with rate of return r .
- ▶ Euler equation is standard:

$$g(C) = \frac{r - \rho}{\theta} \quad (12)$$

- ▶ Invoke Walras' law - so you never have to write down the budget constraint!

Equilibrium

- ▶ Objects: $C_t, X_t, Z_t, x(v, t), V(v, t), N_t$ and prices $p^x(v, t), r(t), w(t)$.
- ▶ Conditions:
 - ▶ "Everybody maximizes." (see above)
 - ▶ Markets clear.
 1. Goods: resource constraint.
 2. Shares: omitted b/c I did not write out the household budget constraint.
 3. Intermediates: implicit in notation.
 - ▶ Innovation effort satisfies a **free entry** condition: present value of profits equals 0.

Symmetric Equilibrium

We assume (and then show) that all varieties v share the same x , V and p^x .

Intuition:

- ▶ p^x : monopoly pricing with a constant elasticity
- ▶ x : varieties enter final goods production symmetrically
- ▶ V : the age of a variety does not matter
(no stock of x to build; permanent patents)

Equilibrium: Characterization

The growth rate follows from the Euler equation:

$$g(C) = (r - \rho)/\theta.$$

- ▶ We need to solve for r .

Simplifications:

- ▶ Normalize marginal cost $\psi = 1 - \beta$
 - ▶ so that $p^x = 1$.
 - ▶ Why can I do that?
- ▶ Focus on balanced growth paths.

Equilibrium: Characterization

Free entry will determine the interest rate

Spend 1 to obtain η new patents, each valued (initially) at $V(v, t)$

$$\eta V(v, t) = 1 \quad (13)$$

- ▶ Then V is constant over time.
- ▶ This assumes that innovation takes place.

With balanced growth and constant profits (to be shown):

$$V = \pi/r \quad (14)$$

Profits

With a fixed markup, profits are a multiple of revenues:

$$\pi(t) = \psi \frac{\beta}{1-\beta} x(t) \quad (15)$$

$$= \beta x(t) \quad (16)$$

Demand for intermediates:

$$\begin{aligned} x(t) &= L p^x(t)^{-1/\beta} \\ &= L \end{aligned}$$

Profits: $\pi = \beta L$.

Free Entry

- ▶ Free entry:

$$\eta V = \eta \beta L / r = 1 \quad (17)$$

- ▶ This is the closed form solution for r .
- ▶ Balanced **growth** rate then follows from the Euler equation.

$$g(C) = \frac{\eta \beta L - \rho}{\theta} \quad (18)$$

Equilibrium: Characterization

Production function for final goods with $x = L$:

$$Y = \frac{N_t L}{1 - \beta} \quad (19)$$

Wage (from firm's FOC):

$$w_t = \beta \frac{Y_t}{L_t} = \frac{\beta}{1 - \beta} N_t \quad (20)$$

Total expenditure on intermediates:

$$X_t = \psi N_t x_t = (1 - \beta) N_t L \quad (21)$$

Summary of Equilibrium

Prices and quantities of intermediate inputs are constant.

- ▶ the model is rigged to deliver this
- ▶ for tractability

Growth comes from rising N

No Transition Dynamics

The equilibrium looks like an AK model with production function

$$\begin{aligned} Y_t &= \frac{L}{1-\beta} N_t \\ \dot{N}_t &= \eta s_z Y_t \end{aligned}$$

Intuition:

- ▶ Period profits π are constant at βL .
- ▶ At any moment we need $\eta V = 1$.
- ▶ V is the present value of (constant) profits.
- ▶ Constant V is only possible with constant r .
- ▶ Intuition: There is a reduced form AK structure.

Scale Effects

$$g(C) = \frac{\eta\beta L - \rho}{\theta}$$

Larger economies (L) grow faster.

Population growth implies exploding income growth (!)

Mechanical reason:

- ▶ Innovation technology is linear in goods.
- ▶ Larger economy \rightarrow higher $Y \rightarrow$ higher $Z \rightarrow$ faster growth.

We will return to this later.

Pareto Efficient Allocation

Efficiency

Two distortions prevent efficiency of equilibrium:

1. Monopoly pricing.
2. Inefficient innovation due to aggregate demand externality.

Planner's Problem

Solve in two stages:

1. Given N , find optimal static allocation $x(v, t)$.
 - ▶ That is: maximize $Y - X$ which is available for consumption and investment.
 - ▶ An odd feature of the model: goods are produced from goods without delay.
2. Given the reduced form production function from #1, find optimal Z .

Static Allocation

Given N , choose $x(v, t)$ to maximize $Y - X$:

$$\max (1 - \beta)^{-1} L^\beta \int_0^{N_t} x(v, t)^{1-\beta} dv - \int_0^{N_t} \psi x(v, t) dv \quad (22)$$

First-order condition

$$L^\beta x^{-\beta} = \psi \quad (23)$$

with $\psi = 1 - \beta$:

$$x = (1 - \beta)^{-1/\beta} L \quad (24)$$

The planner's x is larger than the equilibrium x (Intuition?)

Static Allocation

Next: find $Y - X$.

$$X = \psi N x = (1 - \beta) N (1 - \beta)^{-1/\beta} L \quad (25)$$

Reduced form production function:

$$Y_t = (1 - \beta)^{-1} L^\beta N [(1 - \beta)^{1-1/\beta} L]^{1-\beta} \quad (26)$$

$$= (1 - \beta)^{-1/\beta} L N_t \quad (27)$$

Net output

$$\begin{aligned} Y - X &= (1 - \beta)^{-1/\beta} L N - (1 - \beta)^{1-1/\beta} L N \\ &= (1 - \beta)^{-1/\beta} \beta L N \end{aligned} \quad (28)$$

Planner: Dynamic Optimization

$$\max \int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\theta} - 1}{1-\theta} dt$$

subject to

$$\dot{N} = \eta Z$$

$$Y = (1-\beta)^{-1/\beta} \beta L N = C + Z$$

Or

$$\dot{N} = A N - \eta C \quad (29)$$

$$A = \eta (1-\beta)^{-1/\beta} \beta L \quad (30)$$

Hamiltonian

$$H = \frac{C^{1-\theta} - 1}{1-\theta} + \mu [AN - \eta C] \quad (31)$$

FOC

$$\partial H / \partial C = C^{-\theta} - \mu \eta = 0 \quad (32)$$

$$\partial H / \partial N = \rho \mu - \dot{\mu} = \mu A \quad (33)$$

Optimal growth

The same as in an AK model with

$$A = \eta (1 - \beta)^{-1/\beta} \beta L \quad (34)$$

we have

$$\dot{C}/C = \frac{A - \rho}{\theta} \quad (35)$$

Comparison with CE

- ▶ CE interest rate: $\eta\beta L$.
- ▶ Planner's "interest rate:" $(1 - \beta)^{-1/\beta} \eta\beta L$.
- ▶ The planner chooses faster growth.
- ▶ Intuition:
 - ▶ CE under-utilizes the fruits of innovation: x is too low.
 - ▶ This reduces the value of innovation.

Policy Implications

- ▶ One might be tempted to reduce monopoly power.
- ▶ A policy that encourages competition (e.g. less patent protection, forcing lower p^x) reduces the static price distortion.
- ▶ But it also reduces growth: innovation is less valuable.
- ▶ Similar result for shorter patents.
- ▶ Policy trades off static efficiency and incentives for innovation.

Reading

- ▶ Acemoglu (2009), ch. 13.
- ▶ Krusell (2014), ch. 9
- ▶ Romer (2011), ch. 3.1-3.4.
- ▶ Jones (2005)

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- Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.
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- Krusell, P. (2014): "Real Macroeconomic Theory," Unpublished.
- Romer, D. (2011): *Advanced macroeconomics*, McGraw-Hill/Irwin.