Contracts: Private Information

Prof. Lutz Hendricks

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Asymmetric Information

- We study a 2nd contracting friction: private information.
- Payoffs must be based on agents' **reports** of their information.
- We are looking for **incentive compatible** contracts in which agents report the truth.
- Applications:
  - Labor contracts: Employer cannot observe effort vs. luck. (Additional moral hazard.)
  - Investment contracts: Investor can hide income.

Environment

- The same as in the money lender model.
- Both sides commit to a contract.
- Promised utility is v^0 (exogenous).
- Lender cannot observe y or c.

Preferences

- As before, consumers' preferences are

$$E\sum_{t=1}^{\infty}\beta^{t}u(c_t)$$

- Consumption must be $\geq a$.
- $u'(c) \to \infty$ as $c \to a$.
- $u'(c) \to 0$ as $c \to \infty$.
    - interior solution
- u is bounded above (we will see why later).

Lender's problem

$$P(v) = \max_{b_s, w_s} \sum_{s=1}^{S} \Pi_s \left[-b_s + \beta P(w_s)\right]$$

subject to constraints (below):

1. Promise keeping
2. Incentive compatility

Notation:

- v: Promised utility by contract. As before.
- b_s: Payment to agent who reports $\bar{y}_s$.

Cannot specify consumption b/c y is not known.

Constraints: Promise Keeping

Agent value depends on

1. state $\bar{y}_s$
2. **reported** state $\bar{y}_k$

Value of agent with $\bar{y}_s$ who reports $\bar{y}_k$:

$$V_{s,k} = u(\bar{y}_s + b_k) + \beta w_k \tag{1}$$

Promise keeping:

An agent who tells the truth must get the promised value v.

$$v = \sum_{s=1}^{S} \Pi_s V_{s,s} \tag{2}$$

Constraints: Incentive Compatibility

Telling the truth is better than any lie:

$$C_{s,k} = V_{s,s} - V_{s,k} \geq 0 \quad \forall s,k \tag{3}$$

For technical reasons, we also need to make payoffs and values bounded:

$$\begin{aligned} b_s &\geq a - \bar{y}_s \\ w_s &\leq v_{\max} = \sup \frac{u(c)}{1-\beta} \end{aligned}$$

Properties of P(v)

We expect (intuitively) firm profits to be concave in promised value.

Low v:

- Household has low current utility, high $u'(c)$.
- It is cheap to raise v: $P'(v)$ should be small.

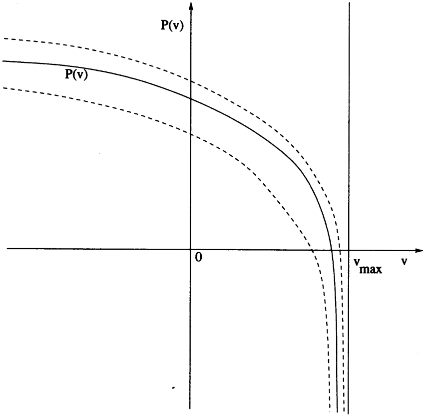
High v:

- Household has high $u'(c)$.
- $P'(v)$ should be large.

Suggests that $P''(v) < 0$ and $P \to -\infty$ as $v \to v_{\max}$ (and $c \to \infty$).

We assume this from now on.

Properties of P(v)



Ljungqvist and Sargent (2004)

Properties of the Optimal Contract

Some properties can be derived just from the constraints:

1. An agent who reports lower y gets punished via lower transfers b_s and lower future payoffs w_s.
2. Agents always want to report income that is lower than the truth
   "Downward" incentive compatibility constraints always bind.
   "Upward" constraints never bind.
3. Coinsurance: when y is high, household and firm split the benefits

Punishment for Low Income

- Result: Reporting lower y results in higher transfer b_s and lower future payoff w_s.
- Intuition:
  - low b and low w: household reports too high w
  - high b and high w: household reports too low w
  - low b and high w: no insurance value

Details

Downward constraints always bind

- Result: For the optimal contract, the downward constraints bind $(C_{s,s-1})$, the upward constraints don't $(C_{s,s+1})$.
- Agents would like to report lower than the true income.
- Proof idea:
  - Can raise profits by shrinking the w_s gaps until all downward constraints bind.
  - If expected w_s remains unchanged, the household is happier (risk aversion).
  - So the firm can raise profits by offering a less attractive contract.

Details

Coinsurance

- When a higher y_s is drawn, $u(.) + \beta w_s$ and firm profits both rise.
- Contrast with the frictionless case where the risk neutral firm fully insures the risk averse household.
- **Household utility** rises because the downward constraint binds: $C_{s,s-1} = 0$:

$$\begin{aligned} u(\bar{y}_s + b_s) + \beta w_s - u(\bar{y}_s + b_{s-1}) - \beta w_{s-1} &= 0 \\ &\Longrightarrow \\ u(\bar{y}_s + b_s) + \beta w_s - u(\bar{y}_{s-1} + b_{s-1}) - \beta w_{s-1} &> 0 \end{aligned}$$

Coinsurance

Firm profits:

$$-b_s + \beta P(w_s) \geq -b_{s-1} + \beta P(w_{s-1}) \tag{4}$$

Proof:

- Suppose (4) does not hold.
- Then change the contract to: $(b_s, w_s) \to (b_{s-1}, w_{s-1})$.
- Profits rise.
- Household utility is unchanged b/c the downward constraint $C_{s,s-1}$ binds.

Contract Design Problem

$$
\begin{aligned}
P(v) \quad = \quad & \max_{b_s, w_s} \sum_{s=1}^{S} \Pi_s \left[-b_s + \beta P(w_s)\right] \\
& + \lambda \left[\sum_{s=1}^{S} \Pi_s \left[u(\bar{y}_s + b_s) + \beta w_s\right] - v \right] \\
& + \sum_{s=2}^{S} \mu_s \left[u(\bar{y}_s + b_s) + \beta w_s - u(\bar{y}_s + b_{s-1}) - \beta w_{s-1}\right]
\end{aligned}
$$

We need promise keeping and downward incentive compatibility constraints.

First order conditions

$$
\begin{aligned}
b_s &: \quad -\Pi_s + \lambda \Pi_s u'(\bar{y}_s + b_s) + \mu_s u'(\bar{y}_s + b_s) - \mu_{s+1} u'(\bar{y}_{s+1} + b_s) = 0 \\
w_s &: \quad \Pi_s \beta P'(w_s) + \lambda \Pi_s \beta + \mu_s \beta - \mu_{s+1} \beta = 0
\end{aligned}
$$

where $\mu_1 = \mu_{S+1} = 0$ (there are no such terms in the FOCs).

In words:

1. Raise b_s: direct cost is 1 with probability Π_s
2. Raise w_s: direct cost is the marginal profit
3. In both cases:
   3.1 it contributes to promise keeping (λ)
   3.2 it relaxes the downward constraint in state s, but worsens that in $s+1$

Simplify FOCs

$$
\begin{aligned}
\Pi_s \left[1 - \lambda u'(\bar{y}_s + b_s)\right] &= \mu_s u'(\bar{y}_s + b_s) - \mu_{s+1} u'(\bar{y}_{s+1} + b_s) \\
\Pi_s \left[P'(w_s) + \lambda\right] &= \mu_{s+1} - \mu_s
\end{aligned}
$$

Sum the FOCs for w_s:

$$
\begin{aligned}
\sum \Pi_s P'(w_s) + \lambda &= \sum \mu_{s+1} - \mu_s \\
&= \mu_{S+1} - \mu_1 \\
&= 0
\end{aligned}
$$

First order conditions

$$P'(w_s) = P'(v) + \frac{\mu_{s+1} - \mu_s}{\Pi_s} \tag{5}$$

If truth-telling constraints were non-binding: $\mu_s = \mu_{s+1} = 0$.

Then the full info optimality condition returns:

$$P'(w_s) = P'(v) = -\lambda = 1/u'(\bar{y}_s + b_s) \tag{6}$$

On average, this still holds: $\sum \Pi_s P'(w_s) = P'(v)$.

But now there is an additional cost to raising w_s: it increases the incentive to lie in state $s+1$.

- μ_{s+1} is that cost.

But higher w_s also reduces the incentive to lie in state s.

- This saves the planner μ_s.

Envelope Condition

$$P'(v) = -\lambda$$

Therefore:

$$P'(v) = -\lambda = \sum \Pi_s P'(w_s) \tag{7}$$

Marginal profits are a **martingale**:

$$P'(v) = EP'(v') \tag{8}$$

Spreading continuation values

One can show:

- $w_S > v$: When the household draws the best income, he is rewarded.
- $w_1 < v$: When the household draws the worst income, he is punished.

Sketch of proof:

- If $w_S < v$: it violates the martingale property.
- Then the household would be punished in all states (since w is increasing in s).
- If $w_S = v$: the martingale property would require $w_s = v$ for all s.
- This would violate incentive compatibility (no punishment for reporting bad incomes).

Poverty

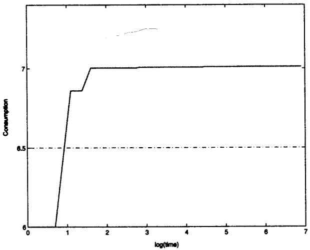
Result: $v \to -\infty$ **almost surely**.

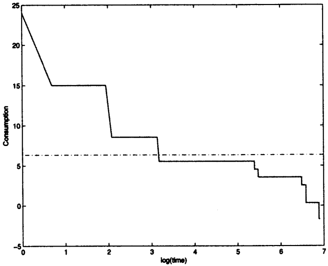
- $P'(v)$ is a non-positive Martingale.
- Theorem: A non-positive Martingale converges almost surely.
- Therefore, v converges.
- But v cannot converge to a strictly positive value.
- If it did, incentive compatibility would require strictly positive fluctuations in w_s.
- Then $P'(v)$ would not converge.

Summary

- The type of contract depends on the friction.
- When the friction is commitment:
  - Raise the rewards over time to prevent agents from walking off.
- When the friction is asymmetric information:
  - Make the payoff an increasing function of the reported income.
  - For low reports: punish the worker (to induce truth-telling)
  - Payoffs drift down over time.

Summary: Typical consumption profiles





Ljungqvist and Sargent (2004)

Private Storage

Private storage

- Modify the model so that agents can store goods.
- But agents cannot borrow (the planner can).
- The gross return is the same for planner and agent (R).
- Main result:
  1. The optimal contract provides no risk sharing across households.
  2. The optimal allocation is the same as in an economy where each household can borrow / lend at rate R.

Model

- The world lasts for T periods.
- Agents observe histories of incomes: $h_t = \{y_1, ..., y_t\}$.
- Agents report $\hat{y}_t(h_t)$ (that may not be truthful) and make storage decisions $\hat{k}_t(h_t)$ (without report).
- Agents receive transfers $b_t\left(\hat{h}_t\right)$.
- Budget constraint:

$$
\begin{aligned}
c(h_t) + \hat{k}(h_t) &= y(h_t) + R\hat{k}_{t-1}(h_{t-1}) + b_t\left(\hat{h}_t[h_t]\right) && (9) \\
\hat{k}(h_t) &\geq 0 && (10)
\end{aligned}
$$

- $\hat{h}$ is the reported history ending in $\hat{y}_t(h_t)$.

Household

- Preferences:

$$\Gamma\left(\hat{y},\hat{k};b\right)=\max\sum_{t=1}^{T}\beta^{t-1}\sum_{h_t}\pi(h_t)\ u(c(h_t)) \tag{11}$$

- Strategies: $\hat{k}(h_t),\hat{y}(h_t)$.
- Take as given transfer rule b.

Planner

- Budget constraint:

$$K_t + \sum_{h_t} \pi(h_t)\, b_t\left(\hat{h}_t[h_t]\right) = RK_{t-1} \tag{12}$$

- $K_T \geq 0$.
- Incentive compatibility: For any history, lifetime utility must be higher under truth-telling than under any lying strategy:

$$\Gamma\left(\hat{k}, \hat{y}; b\right) \geq \Gamma\left(\tilde{k}, \tilde{y}; b\right)$$

for any alternative strategy $\left(\tilde{k}, \tilde{y}\right)$.

Planner's problem:

Choose b to max $\Gamma\left(\hat{k}, \hat{y}; b\right)$

subject to:

1. Incentive compatibility: $\Gamma\left(\hat{k}, \hat{y}; b\right) \geq \Gamma\left(\tilde{k}, \tilde{y}; b\right)$.
2. Budget constraint.

Private storage restricts allocations

- Result: Any allocation that can be implemented with private storage can also be implemented when $k = 0$.
- Intuition:
  - Private storage makes it harder to manipulate continuation values through a contract (self-insurance).
  - This makes incentive problems more severe.

Characterizing the optimal contract

- The constraints are complicated.
  - Need to consider lifetime utility for any feasible reporting strategy.
- The only method: guess and verify.
- Find a problem with a smaller set of constraints.
- Show that the optimal allocation is incentive compatible and feasible with the larger set of constraints.

Characterizing the optimal contract

In this model, the optimal contract solves

$$\max \sum_{t=1}^{T} \beta^{t-1} \sum_{h_t} \pi(h_t)\; u(c_t(h_t)) \tag{13}$$

subject to

$$\sum_{t=1}^{T} R^{1-t}\; [y_t(h_T) - c_t(h_t(h_T))] \geq 0 \;\; \forall h_t \tag{14}$$

In words: The allocation the household could achieve through self-insurance with the borrowing constraint $k_T \geq 0$.

Proof: Cole and Kocherlakota (2001)

The trick: Only consider lying strategies where the household reports y_{s-1} instead of y_s.

Characterizing the optimal contract

- The planner only relaxes the individual's borrowing constraints: $k_T \geq 0$ instead of $k_t \geq 0$.
- The planner cannot achieve insurance across agents.

Details

Punishment For Low Income I

- The result follows directly from incentive compatibility and concave utility.
- Downward constraint:

$$V_{s,s} - V_{s,s-1} = u(\bar{y}_s + b_s) + \beta w_s - u(\bar{y}_s + b_{s-1}) - \beta w_{s-1} \geq 0$$

- Upward constraint:

$$V_{s-1,s-1} - V_{s-1,s} = u(\bar{y}_{s-1} + b_{s-1}) + \beta w_{s-1} - u(\bar{y}_{s-1} + b_s) - \beta w_s \geq 0$$

- Add the two:

$$u(\bar{y}_s + b_s) - u(\bar{y}_{s-1} + b_s) \geq u(\bar{y}_s + b_{s-1}) - u(\bar{y}_{s-1} + b_{s-1}) \quad (15)$$

Punishment For Low Income II

- A given increment in $\bar{y}_{s-1}$ to $\bar{y}_s$ implies a larger utility increment for b_s than for b_{s-1}.
- Therefore:

$$b_{s-1} \geq b_s \tag{16}$$

- If reporting a higher state reduces transfers, $C_{s,s-1}$ requires that it has a higher future payoff:

$$w_{s-1} \leq w_s \tag{17}$$

Local constraints are enough

- A bit of algebra shows: If $C_{s,s-1}$ and $C_{s,s+1}$ hold, then all $C_{s,k}$ hold.

Downward constraints always bind

Proof (by contradiction)

- Suppose that some downward constraint does not bind: $C_{s,s-1} > 0$:

$$u(\bar{y}_s + b_s) + \beta w_s - u(\bar{y}_s + b_{s-1}) - \beta w_{s-1} > 0$$

- We construct an alternative contract that yields higher profits.
- Since $b_s \leq b_{s-1}$: $u(\bar{y}_s + b_s) - u(\bar{y}_s + b_{s-1}) \leq 0$.
- Therefore $w_s > w_{s-1}$ (strictly).
- Reduce w_2 until $C_{2,1} = 0$.
- Then reduce w_3 until $C_{3,2} = 0$. Etc.
- Add a constant to all w_s to keep promised value unchanged.
- The new contract satisfies all constraints (check that upward constraints don't bind).

Proof (by contradiction)

- $EP(v) = \sum \Pi_s P(w_s)$.
- Ew_s is unchanged.
- $w_s - w_{s-1}$ has been reduced.
- The new contract is a mean-reducing spread of the old one.
- Since $P(v)$ is strictly concave, $EP(v)$ has increased.

- Ljungqvist and Sargent (2004), ch. 19.

References I

Cole, H. L. and N. R. Kocherlakota (2001): "Efficient allocations with hidden income and hidden storage," *Review of Economic Studies*, 523–542.

Ljungqvist, L. and T. J. Sargent (2004): *Recursive macroeconomic theory*, 2nd ed.