Appendix

1. Proofs and Derivations

1.1 Balanced Growth Rates

Along a balanced growth path, the following relationships must hold between the various growth rates. From feasibility,

$$g(Y) = g(S) = g(k) = g(c) = (r - \rho)/\sigma .$$

A constant learning rate requires that θ be constant; therefore $g(A) = g(H)$. Efficiency units of equipment then grow at rate $g(K) = g(k) + g(A)$.

From the firm's first-order conditions r^S must be constant and $g(w) = g(Y)$. Moreover $g(\pi) = -g(A)$. The production function implies $g(Y) = \alpha_S\, g(S) + \alpha_E\, g(K)$, so that

$$g(A) = g(Y)(1 - \alpha_S - \alpha_E)/\alpha_E .$$

Constant (across technologies) are the age profiles of depreciation (δ_v, Δ_v).

1.2 Proof of Proposition 1

The positive slope of FOC-T is obvious. The slope of FOC-A can be found by implicitly differentiating the first-order condition for *A*. Write it as $F(\theta, T; \pi) = 0$. Then

$$\frac{\partial F}{\partial \theta} = \int_0^T e^{-(r+\gamma)s} \zeta_s\, f''(\zeta_s) e^{\gamma s} / \theta^2\, ds < 0$$

$$\frac{\partial F}{\partial T} = e^{-(r+\gamma)T} \left(f(\zeta_T) - \zeta_T\, f'(\zeta_T) \right) = e^{-(r+\gamma)T} ,$$

where $\zeta_s = e^{\gamma s}/\theta$ and the second equality follows from the assumption that capital attains full productivity at the end of its lifetime $(f'(\zeta_T) = 0)$. The slope of FOC-A is therefore positive and diminishing in f''.

The condition for concavity of the investor's problem ensures that f'' is sufficiently negative so that the intersection of FOC-A and FOC-T is unique. Reducing π shifts FOC-T left, but leaves FOC-A unchanged. The intersection therefore shifts down and to the left ($A \to B$ in Figure 1). ■

2. Cross-Country Data

The approach for establishing the relationship between equipment prices (or investment) and growth closely follows Jones (1995). Data on equipment prices, defined as the relative price of equipment to consumption goods, are taken from his paper. Equipment and non-equipment investment figures are taken from DeLong and Summers (1993). The remaining control variables are based on Levine and Renelt (1992), Barro (1991), or the Penn World Tables 5.6. Variable definitions are provided in Table 3.

I estimate a standard cross-country growth regression of the form

$$g_Y = a + b\,x + c\,\mathbf{X} + \varepsilon,$$

where g_Y is the average growth rate of real per capita GDP from 1960 to 1985, $\mathbf{X}$ is a matrix of control variables, ε is a random error term, and x denotes either the relative price of equipment or the share of equipment investment in GDP. The control variables are the log of 1960 per capita GDP, primary and secondary enrollment rates, the share of government consumption in GDP, the number of revolutions and coups, and dummy variables for socialist government, Latin America and Sub-Saharan Africa.

Figures 4 and 5 plot the component of the growth rate that is orthogonal to the control variables against equipment prices or investment, respectively. The data points representing the model's predictions are adjusted so that the residual growth rate for the baseline steady state equals that of the U.S. in the data. Figure 6 shows that raw data for equipment investment and prices together with the (unadjusted) model predictions.

3. Figures

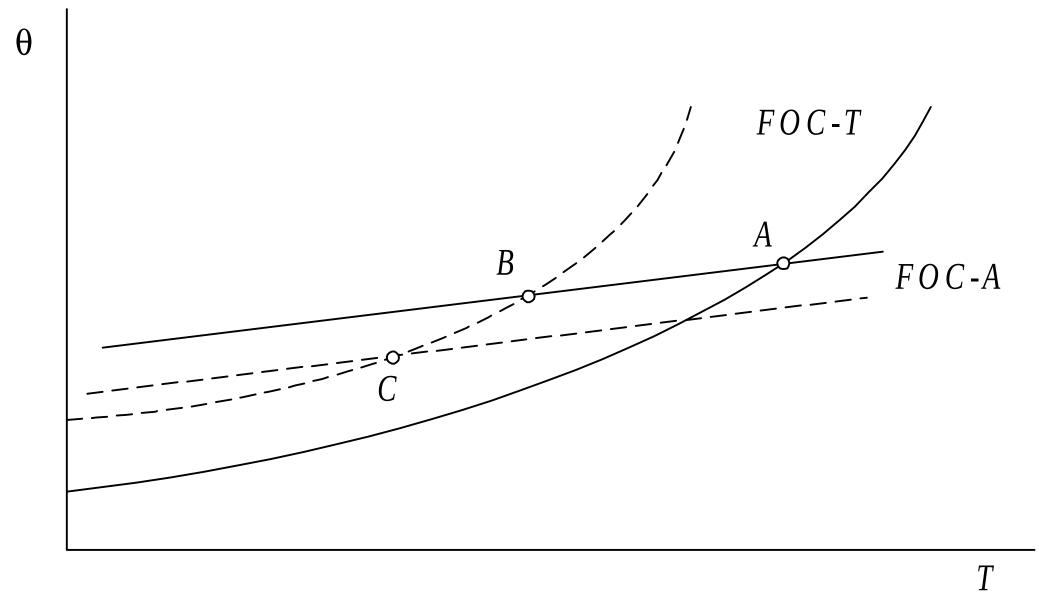


Figure 1. The Investor's Problem

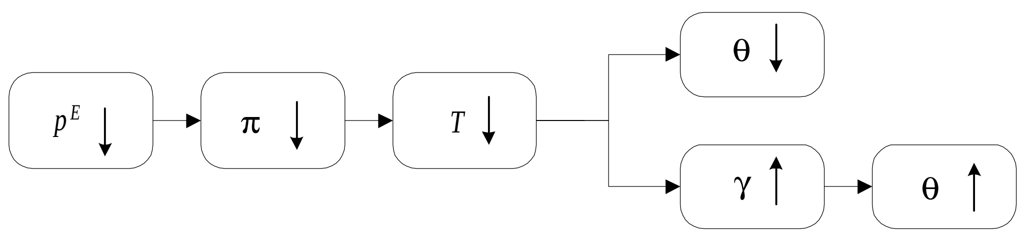


Figure 2. The effect of lower equipment prices

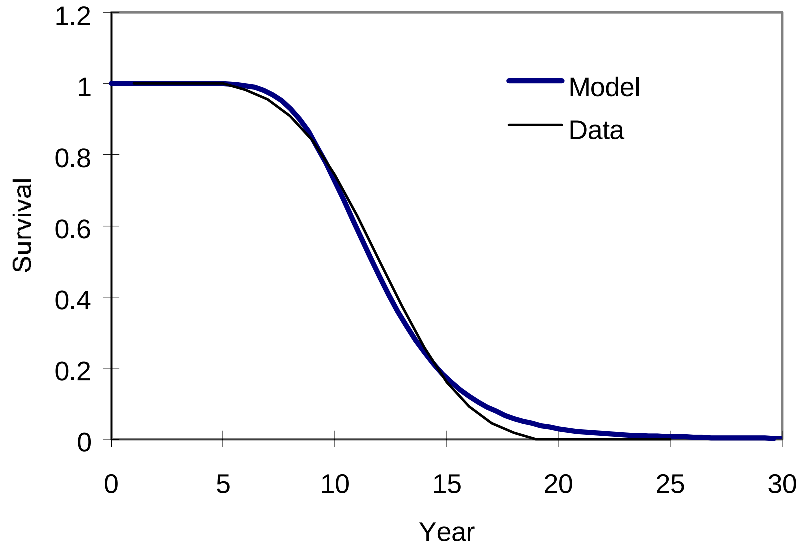


Figure 3. Age-survival profiles for equipment

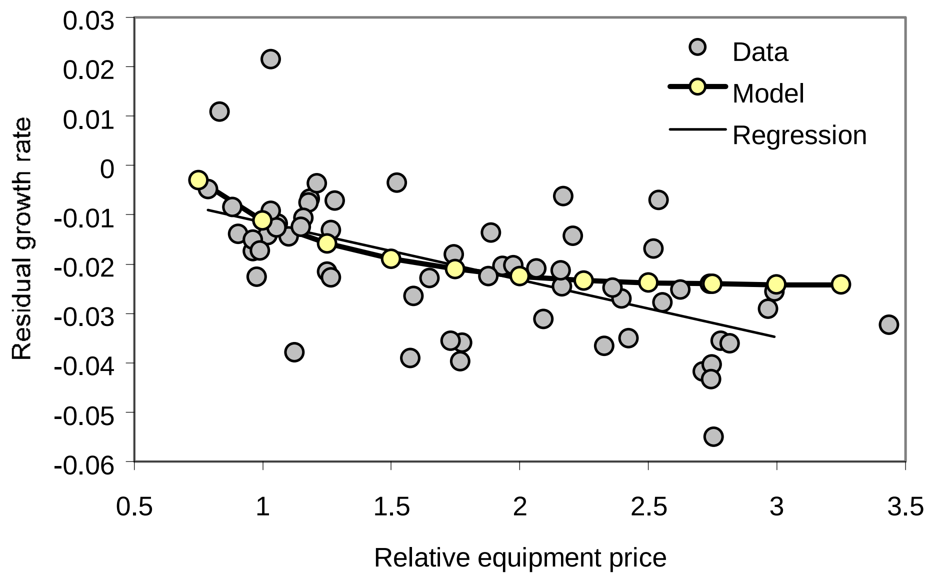


Figure 4. Growth rates and equipment prices (1960-85)

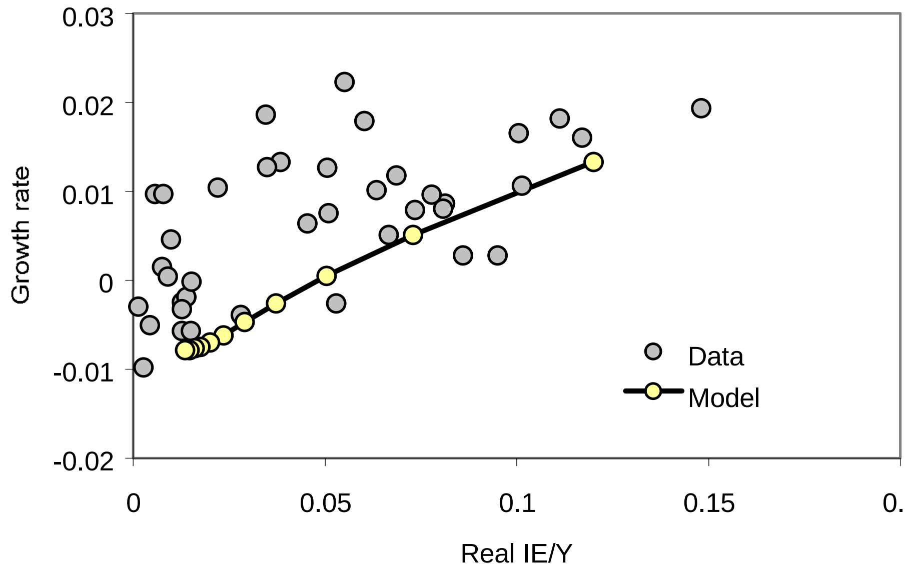


Figure 5. Equipment investment and growth, 1960-85

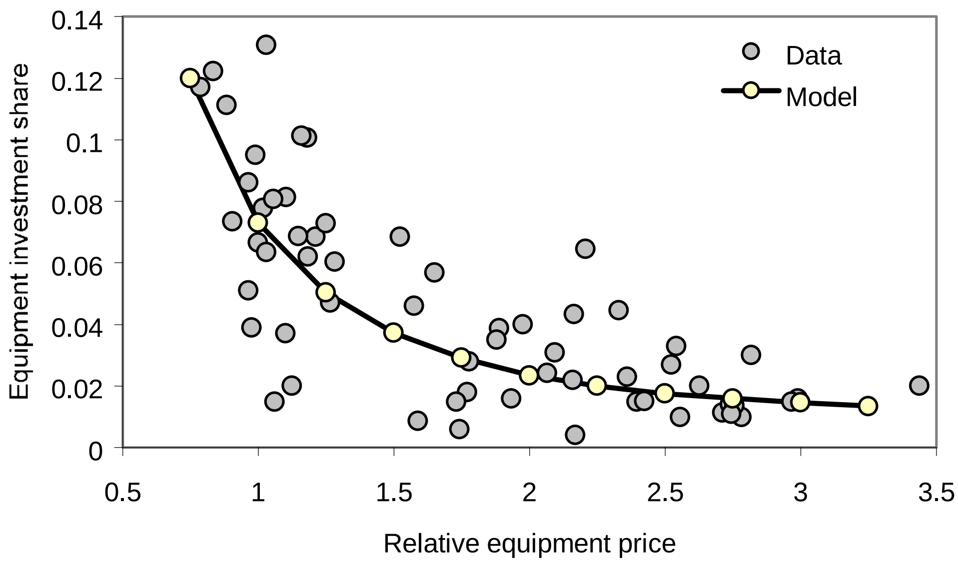


Figure 6. Equipment investment and equipment prices

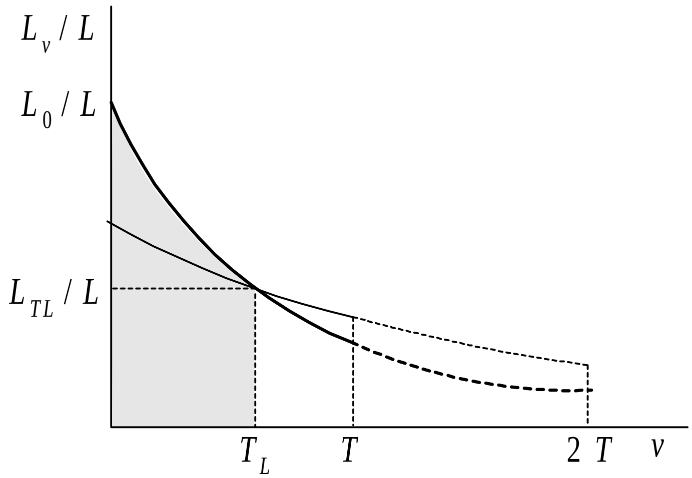
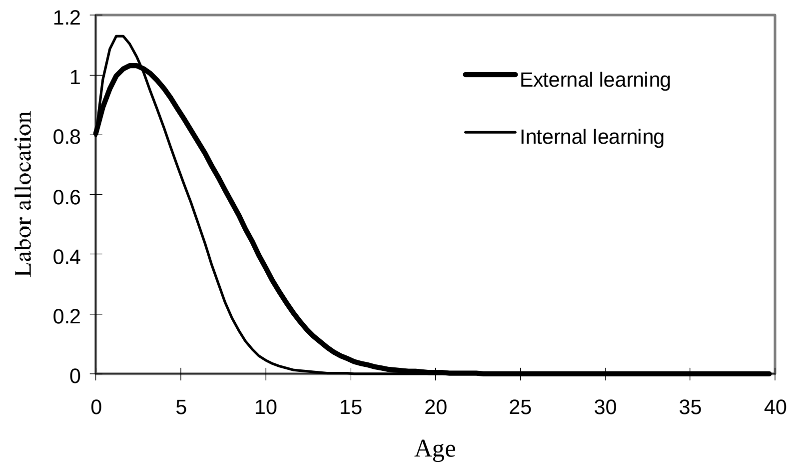
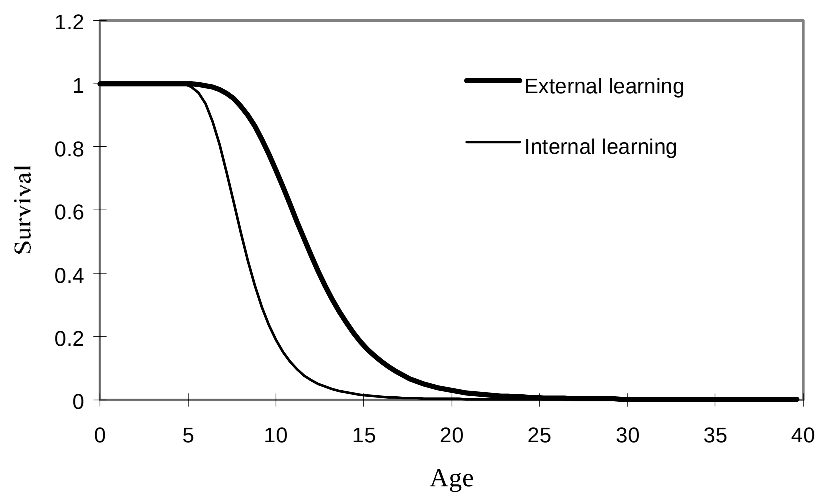


Figure 7. Service lives and the learning rate



(a) Labor allocation



(b) Age-survival profile

Figure 8. Age profiles with and without learning spillovers

4. Tables

Table 1. Parameters

$\mu_m = 0.111$, $\sigma_m = 0.033$,	Match equipment age-survival profile (Coen 1980)
$T = 40$	
$\lambda = 0.768$, $\varphi = 2$	Match amount and duration of learning (Bahk and Gort 1993)
$\alpha_L = 0.7$	Labor income share in GDP (Greenwood et al. 1997)
$\alpha_E = 0.143$	Match equipment investment shares in GDP (Greenwood et al. 1997)
$\delta^S = 0.056$	Greenwood et al. (1997)
$\rho = 0.075$ $\sigma = 2$	Match structures investment share in GDP
$\bar{h} = 1.64$ $\psi = 1/6$	Matches growth rate of per capita output of 0.02

Table 2. Sensitivity Analysis

	Δg_Y [%]	$\Delta T_{1/2}$ [years]	$\Delta(\theta - 1)$ [%]	$f(1/\theta)^{-1}$	$\frac{\Delta g_Y}{\Delta I_E / Y}$
Baseline	–1.13	20.0	–8.8	1.08	0.23
Longer learning $T_L = 10$	–1.02	17.6	–5.0	1.10	0.21
Higher curvature of f $\varphi = 4$	–1.05	18.1	–4.8	1.07	0.22
Longer halflife $\Delta T_{1/2} = 16$	–0.92	18.5	–7.0	1.09	0.20
Substitutability in h					
$\psi = 0.5$	–0.52	10.0	–0.5	1.10	0.12
$\psi = 1.0$	–0.23	7.4	+2.0	1.11	0.06
No learning spillovers	–0.19	6.7	–0.9	1.20	0.05

Notes: The table shows the comparative balanced growth effects of doubling p^E.

Table 3. Variable definitions

Variable	Definition	Source
GDP60	Real GDP per worker [1985 international prices]	PWT #19 (RGDPW)
GOV	Government Consumption Share of Gross Domestic Product	Barro (1991)
INV	Real investment share in GDP [current international prices]	PWT #11 (CI)
PI	Price level of investment	PWT #15 (PI)
PC	Price level of consumption	PWT #14 (PC)
PRI	Primary Enrollment Rate 1960	Barro (1991)
SEC	Secondary Enrollment Rate 1960	Barro (1991)
GR6089	Growth of Real per Capita GDP, 1960-89	Levine and Renelt (1992)
REVC	Number of Revolution and Coups per Year	Barro (1991)
LAAM	Latin America dummy	Levine and Renelt (1992)
AFRICA	Sub-Saharan Africa dummy	Levine and Renelt (1992)
SOC	Dummy for Socialist Economy	Barro (1991)