# Overlapping Generations Model: Dynamic Efficiency and Social Security

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# Issues

- The OLG model can have **inefficient equilibria**.
- We solve the problem of a ficticious **social planner**
  - This yields a Pareto optimal allocation by construction.
- We learn from this:

1. Solving the planning problem may be an easy way of characterizing CE (if it is optimal).
2. Comparing it with the CE points to sources of inefficiency.

# The Social Planner's Problem

# Planner's problem

- Imagine an omnipotent social planner.
- She can assign actions to all agents (consumption, hours worked, ...).
- She maximizes some average of individual utilities.
- She **only faces resource constraints**.

# Welfare function

- The planner's objective function is assumed to be a weighted average of individual utilities:

$$\omega_0 \beta u(c_1^o) + \sum_{t=1}^{\infty} \omega_t [u(c_t^y) + \beta u(c_{t+1}^o)]$$

- By varying the weights $(\omega_t)$ we can obtain all **Pareto optimal** allocations.
  - It makes sense even if comparing utilities across agents does not.
- To ensure that the objective function is finite, conditions need to be imposed on the weights such that $\sum_t \omega_t < \infty$.

# Planner's problem

The planner only faces feasibility constraints.

In this model:

$$K_{t+1} + N_t c_t^y + N_{t-1} c_t^o = F(K_t, N_t) + (1-\delta) K_t \tag{1}$$

Or, in per capita young terms $(k_t = K_t / N_t)$:

$$c_t^y + c_t^o/(1+n) + (1+n)k_{t+1} = (1-\delta)k_t + f(k_t)$$

# Planner's Lagrangian

$$\begin{aligned}
\Gamma &= \omega_0 \beta u(c_1^o) + \textstyle\sum_{t=1}^{\infty} \omega_t [u(c_t^y) + \beta u(c_{t+1}^o)] \\
&\quad + \textstyle\sum_{t=1}^{\infty} \lambda_t \left[ \begin{array}{c} (1-\delta)k_t + f(k_t) \\ -c_t^y - c_t^o/(1+n) - (1+n)k_{t+1} \end{array} \right]
\end{aligned}$$

Planner's FOCs:

$$\begin{aligned}
\omega_t u'(c_t^y) &= \lambda_t \\
\omega_{t-1} \beta u'(c_t^o) &= \lambda_t/(1+n) \\
\lambda_t [1 - \delta + f'(k_t)] &= \lambda_{t-1}(1+n)
\end{aligned}$$

# Planner's problem

Static optimality:

$$\omega_t u'(c_t^y) = \omega_{t-1}(1+n)\beta u'(c_t^o)$$

Euler equation:

$$\omega_t u'(c_t^y)[1-\delta+f'(k_t)] = \omega_{t-1} u'(c_{t-1}^y)(1+n)$$

Using the static condition, the Euler equation becomes

$$u'(c_t^y) = \beta u'(c_{t+1}^o)[1-\delta+f'(k_{t+1})] \qquad (2)$$

which looks like the Euler equation of the household.

This is not surprising: the planner should respect the individual FOCs unless there are externalities.

# Planner's Solution

Sequences $\left\{c_t^y, c_t^o, k_{t+1}\right\}_{t=1}^{\infty}$ that satisfy:

- Static and Euler equation.
- Feasibility.
- A transversality condition or $k_{t+1} \geq 0$.
  - We talk about those later.

# Interpretation of the Euler equation

- A feasible perturbation does not change welfare.
- In $t-1$:
  - $c_{t-1}^{y} \downarrow$ by $(1+n)$
  - $k_t \uparrow$ by 1 (per capita of the date $t$ young)
- In $t$:
  - output $\uparrow$ by $f'(k_t)$ (per capita $t$ young)
  - raise $c_t^y$ by $1-\delta+f'(k_t)$ or
  - raise $c_t^o$ by $(1+n)(1-\delta+f'(k_t))$
- From $t+1$ onwards: nothing changes
  - especially not $k_{t+1}$

# Planner's Steady State

For a steady state to exist, weights must be of the form

$$\omega_t = \omega^t, \quad \omega < 1$$

Otherwise the ratios $\omega_{t+1}/\omega_t$ in the FOCs are not constant.

Then the Euler equation becomes

$$\omega \left(1 - \delta + f'(k_{MGR})\right) = (1 + n)$$

This is the **Modified Golden Rule**. ($\omega = 1$ is the Golden Rule).

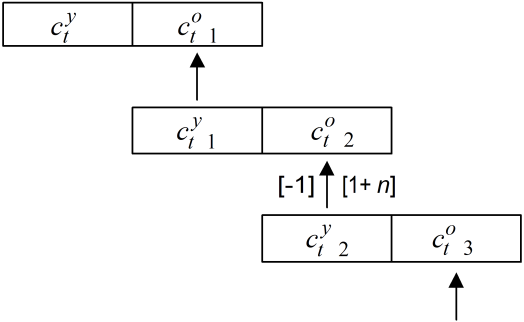
Because $\omega < 1$: $k_{MGR} < k_{GR}$ and the MGR is dynamically efficient.

# How does the planner avoid dynamic inefficiency?

If the planner desires lots of old age consumption, he can implement a "transfer scheme" of the following kind:

Take a unit of consumption from each young and give $(1+n)$ units to each old at the same date.

There is no need to save more than the GR.




Of course, there aren't really any transfers in the planner's world.

# Social Security

# Social Security

A transfer scheme akin to Social Security can replicate the Planner's allocation and avoid dynamic inefficiency.

Social Security consists of

- a **payroll tax** on workers;
- a **transfer** payment to the retired.

# Two flavors of Social Security

- **Fully funded**:
  - For each worker, the government invests the tax payments.
  - This is equivalent to a forced saving plan.
- **Pay-as-you-go**:
  - Current transfers are paid from current tax revenues.

# Household with Social Security

The household maximizes

$$u\left(c_t^y\right) + \beta u\left(c_{t+1}^o\right)$$

subject to the present value budget constraint

$$w_t - \tau_t^y - \frac{\tau_{t+1}^o}{1+r_{t+1}} = c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} \tag{3}$$

Lump-sum taxes do not change the Euler equation (prove this):

$$\beta(1+r_{t+1})u'([1+r_{t+1}]s_{t+1} - \tau_{t+1}^o) = u'(w_t - s_{t+1} - \tau_t^y)$$

# Household with Social Security

- The saving function remains the same

$$s_{t+1} = s\left(w_t - \tau_t^y, -\tau_{t+1}^o, r_{t+1}\right) \tag{4}$$

- For given prices, Social Security reduces saving for two reasons:
  - Higher income when old.
  - Lower income when young.

# Household with Social Security

- If a tax change does not alter the present value of taxes,

$$d\tau^y + \frac{d\tau^o}{1 + r_{t+1}} = 0$$

then the optimal consumption path does not change.

- Reason: present value budget constraint and first-order condition unchanged.
- This is the Permanent Income Hypothesis.

# Fully funded Social Security

- Young: pay tax $\tau_t^y$.
- Old pay: $\tau_{t+1}^o = -(1 + r_{t+1})\ \tau_t^y < 0$.
- Government supplies revenues as capital to firms.
- For the household:
    - Forced saving at rate of return $r$.
    - No change to the present value budget constraint.
- Therefore, if prices remain fixed:
    - No change to optimal consumption plan.
    - Private saving (of the young) drops by the Social Security tax amount.

# Fully Funded Social Security

- We prove that unchanged $(w_t, r_t)$ clear the markets with Social Security.
- Household:
  - By PIH: no change in consumption plan.
  - Household fully dissaves the tax: $\Delta s_{t+1} = -\tau_t^y$.
- Government saves: $s_{t+1}^G = N_t \tau_t^y$.
- Capital market clearing:

$$\Delta K_{t+1} = N_t \, \Delta s_{t+1} + s_{t+1}^G = 0 \tag{5}$$

- Fully funded SS is neutral.
  - Essentially, the government just relabels some private savings as public.

# Pay-as-you-go Social Security

- Assume population growth at rate $n$: $N_t = (1+n)N_{t-1}$.
- Tax collection from the current young: $N_t \ \tau_t^y$.
- Transfer payments to the current old: $-N_{t-1} \ \tau_t^o$.
- The budget balances in each period:

$$\tau_t^o = -\tau_t^y \ (1+n) \tag{6}$$

- From the household's perspective:
  - Forced saving with return $n$.
  - Saving drops by an amount different from $\tau_t^y$.

# Pay as you go Social Security

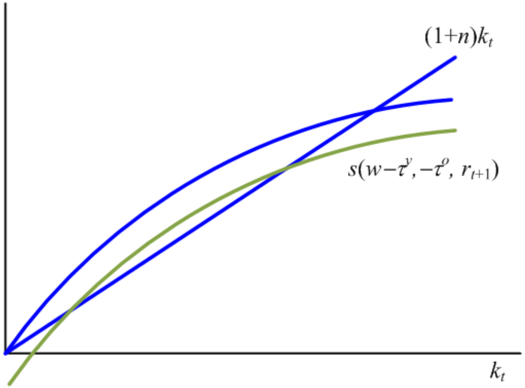
- We prove that unchanged $r_{t+1}$ imply excess demand for $K_{t+1}$.
- Household: $\Delta s_{t+1} < 0$.
- Government: Balanced budget.
- Capital market: $\Delta K_{t+1} = N_t \ \Delta s_{t+1} < 0$.

# Illustration

- Capital market clearing:

$$k_{t+1}(1+n) = s\left(w(k_t) - \tau_t^y,\ w(k_{t+1}) - \tau_{t+1}^o,\ r(k_{t+1})\right) \qquad (7)$$

- Assume that the saving function is well-behaved (e.g. log utility and Cobb-Douglas).



# Complications

- Since prices change, we cannot guarantee that Pay-as-you-go SS reduces steady state $k$.
- Totally differentiate the saving function:

$$[1+n-s_3 f''(k_{t+1})]dk_{t+1} = -s_1 d\tau^y - s_2 d\tau^o < 0$$

- A sufficient condition for $dk_{t+1} < 0$ is that $s_3 > 0$. Then the law of motion unambiguously shifts down.

# Dynamic efficiency

- If SS reduces the steady state capital stock, it can alleviate dynamic inefficiency.
- Note that the argument is not reversible:
  - in a dynamically efficient economy, “reverse social security” is not a Pareto improvement.
  - why not?

# Reading

- Acemoglu, ch. 9.