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Introduction

Question

 Can real estate prices help us to learn about monetary policy regimes?

Approach

- Extend a partial equilibrium macro-finance model to price real estate assets
- Jointly estimate model of macroeconomic dynamics, the yield curve, & real estate prices in a ML framework

Value

- Identifying monetary policy regime switches has never been more difficult
- Post-Great Recession interest in the real estate sector
- Possible implications for other asset classes

What is a Monetary Policy Regime?

Taylor Rule

$$r_t = m_r(s_t^m) + \alpha(s_t^m)\pi_t + \beta(s_t^m)g_t + \sigma\epsilon_t$$
 (1)

 r_t :: target federal funds

Motivation

 π_t :: inflation measure

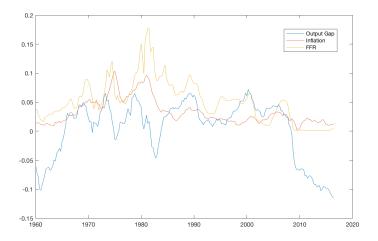
 g_t :: perceived output gap

 $s_t^m \in \{0,1\}$:: the monetary policy regime

 $\epsilon_t \sim \textit{Gaussian}$

- An aggressive monetary policy regime responds strongly to inflation ($\alpha > 1$)
- A passive monetary policy regime does not respond strongly to inflation (α < 1)

What Regime are We in Today?



Thought Experiment

Gordon Growth Model

$$\frac{D}{P} = r - g_d \tag{2}$$

D .. dividend

P:: price of asset

r :: cost of borrowing

 g_d :: growth rate of dividends

Assume

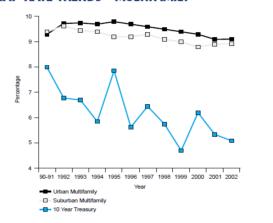
- $\rho(g_d,\pi) := corr(g_d,\pi) \approx 1$
- $\rho(r,\pi)$ is dependent on the monetary policy regime
 - If $s_t^m = 1$, $\rho(r, \pi) \approx 1 \Rightarrow \rho(\frac{D}{R}, \pi) \approx 0$
 - If $s_t^m = 0$, $\rho(r,\pi) \approx 0 \Rightarrow \rho(\frac{D}{R},\pi) < 0$

Deflated Rent Index

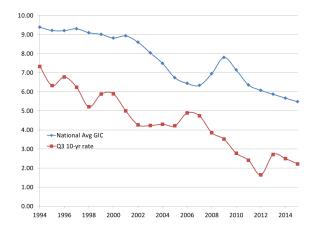


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TABLE 30 CAP RATE TRENDS—MULTIFAMILY



Cap Rate and Discount Rate: 1994-2014



Model Check List

In order to empirically evaluate the proposed mechanism, we must build a model of...

- Macroeconomic dynamics
 - Taylor rule
 - Forward-looking
 - Markov-switching
- 2 Term structure of interest rates
 - Affine term-structure model
- Real estate prices
 - Macro factors
 - Local factors

Bikbov & Chernov (2013, Journal of Econometrics)

- Argues "monetary policy regimes may not be estimated precisely if ones uses information from the short interest rate only"
- Long-term yields are dependent on current and expected future short-term yields

$$r_t^n = \left[\prod_{i=0}^{n-1} (1 + \mathbb{E}_t^{\mathbb{Q}}[r_{t+i}]) \right]^{\frac{1}{n}} + r p_t^n + \epsilon_t$$
 (3)

- (3) ⇒ long-term yields contain market expectations of current, and future monetary policy regimes
- Finds the econometrician's ex post regime probability through time resembles more of a binary process when including higher term yields than when not

Three Binary Regime Variables

- s_t^m :: Monetary policy rule regime
- s_t^d :: Discretionary monetary policy regime
- s_t^e :: Real volatility regime

Transition Dynamics

• Each regime follows Markov process w/ transition matrix $\Pi^{(k)}$ for $k \in \{m, d, e\}$

$$\Pi_{(i,j)}^{(k)} := \Pr(s_{t+1}^k = j | s_t^k = i) \tag{4}$$

 Equivalent to having single compound-regime variable, $S_t \in \{1, \dots, 8\}$

Macroeconomic Dynamics

Structural Framework

$$g_{t} = m_{g} + (1 - \mu_{g})g_{t-1} + \mu_{g}\mathbb{E}_{t}g_{t+1} - \phi(r_{t} - \mathbb{E}_{t}\pi_{t+1})$$

$$+ \sigma_{g}(s_{t}^{e})\epsilon_{t}^{g}$$

$$\pi_{t} = m_{\pi} + (1 - \mu_{\pi})\pi_{t-1} + \mu_{\pi}\mathbb{E}_{t}\pi_{t+1} + \delta g_{t} + \sigma_{\pi}(s_{t}^{e})\epsilon_{t}^{\pi}$$

$$r_{t} = m_{r}(s_{t}^{m}) + (1 - \rho(s_{t}^{m}))[\alpha(s_{t}^{m})\mathbb{E}_{t}\pi_{t+1} + \beta(s_{t}^{m})g_{t}]$$

$$+ \rho(s_{t}^{m})r_{t-1} + \sigma_{r}(s_{t}^{d})\epsilon_{t}^{r}$$

Forward-Looking Rational Expectations Solution

$$x_t = \mu(S_t) + \Phi(S_t)x_{t-1} + \Sigma(S_t)\epsilon_t$$

$$s.t. \quad x_t = [g_t, \pi_t, r_t]'$$
(5)

Bond Prices & Yield Curve

The conditional price of an n-period face value bond

$$B_t^n(x_t, S_t) = \mathbb{E}^{\mathbb{Q}}[\exp\{-\sum_{i=0}^{n-1} r_{t+i}\} | x_t, S_t]$$
 (6)

Let $\delta := (0,0,1)'$, then $r_t = \delta' x_t$ and Equation (7) is transformed:

$$B_t^n(x,i) = \mathbb{E}^{\mathbb{Q}}[\exp\{-\xi_{t,n}\}|x_t = x, S_t = i]$$
 (7)

$$\xi_{t,n} := \delta' \sum_{i=0}^{n-1} x_{t+i} \tag{8}$$

Finally, via the cumulant generating function the term structure is derived

$$r_t^n(x,i) = -\frac{1}{n}\log(B_t^n(x,i)) = -\frac{1}{n}\sum_{k=1}^{\infty}\frac{(-1)^k}{k!}\mu_n^{(k)}(x,i)$$
 (9)

Takeaway

Model Features

- Markov-Switching,
- Forward-Looking Rational Expectations,
- Macro-Finance Model of Macroeconomic Dynamics & the Yield Curve.

Takeaway

- Econometrically equivalent to a MS-VAR(1), where the coefficient matrix is subject to non-linear constraints
- Lots of parameters, not so many covariates

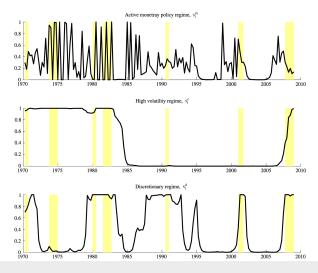
Maximum Likelihood

- Draw 1,000,000 quasi-random points from constrained parameter space
- Evaluate each draw
- Keep highest 10% of initial draw
- Perform local optimization using each remaining draw of initial parameter guess

Highly Persistent Data

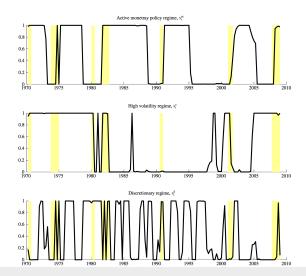
Parametric bootstrap over 1,000 simulated data paths

Results: No Term Structure



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Results: Term Structure Included



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Real Estate Model

Cap Rate

 Real estate is similar to equity in the sense that they are both assets with stochastic cash flows potentially into perpetuity

$$\hat{Q}_{i,t} = \eta_{i,t} (1 + \mathbb{E}_t^{\mathbb{Q}}[\hat{Q}_{i,t+1}])$$
 (10)

$$\eta_{i,t} = \exp\{\lambda_i' x_t\} \tag{11}$$

•
$$(12)$$
- $(13) \Rightarrow$

$$\hat{Q}_{i,t} = \eta_{i,t} + \mathbb{E}^{\mathbb{Q}}[\eta_{i,t}\eta_{i,t+1}] + \mathbb{E}^{\mathbb{Q}}[\eta_{i,t}\eta_{i,t+1}\eta_{i,t+2}] + \dots
= \exp\{\lambda'_{i}x_{t}\} + \mathbb{E}^{\mathbb{Q}}[\exp\{\lambda'_{i}(x_{t} + x_{t+1})\}]
+ \mathbb{E}^{\mathbb{Q}}[\exp\{\lambda'_{i}(x_{t} + x_{t+1} + x_{t+2})\}] + \dots$$
(12)

Simulation Methodology

Conditional Monte-Carlo

- Conditioning on each initial regime and some state vector, simulate 1,000 time series under the risk-neutral dynamics using the parameter estimates from Bikbov & Chernov (2013, JOE)
- Along each path, calculate realizations of each component of Equation (14), and the treasury yields
- For each initial state, the average across paths at a given time is an estimate of its expectation under the risk-neutral dynamics conditional on the initial regime and state vector
- Finally, calculate model approximates and compare

Extension & Simulation

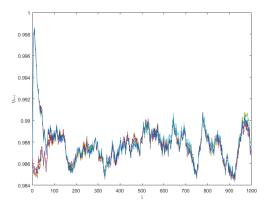
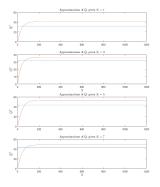


Figure: Simulated $\mathbb{E}^{\mathbb{Q}}[\eta_{t+i}|S_t=k,x_t=x]$



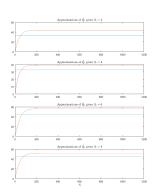


Figure: Simulated $\hat{Q}(S_t, x_t = x)$ (Blue) & Model Implied \hat{Q}^N (Red) Using First N Terms

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Current Challenges

Computational

- Likelihood function is costly to evaluate
- Slow convergence
- Feasibility of bootstrap
 - Metropolis-Hastings

Model Problems

- Approximation of cap. rate worse than yields
 - Approximate continuous state model in discrete state-space

Conclusion

- It is theoretically feasible that real estate prices contain information regarding current & future expected monetary policy regimes
- The idea has potential value to both policy makers and real estate investors who are interested in the feedback loop between monetary policy and the real estate cycle
- Properly estimating such a mechanism requires a model framework that justifies its complexity & computational challenges
- Model is capable of producing significant variation based on the monetary policy regime alone

