Secular stagnation and bubble boom-bust

By Toan Phan and Andrew Hanson

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We show that in the presence of nominal wage rigidities, a leveraged bubble boom-bust cycle may lead to persistent unemployment and depressed output. This is the "secular stagnation" equilibrium. The leveraged bubble bursting acts as a deleveraging shock, but whose size and therefore effect is endogenous. We find that periods of low real interest rates encourage bubble existence and growth, exacerbating exposure to the persistent unemployment equilibrium. Larger bubble bursts are associated with deeper potential recessions.

The boom and bust cycle of the housing bubble in the decade leading up to the 2007-2009 recession is widely regarded as a critical element of the antecedent financial weakness. When the bubble burst, housing net worth fell 9.5 percent on average in the US; an asset which accounts for a large proportion of the average household's leveraged assets. Simultaneously, aggregate employment fell by 5.3 percent from 2007-2009 and has been sluggish in correcting.[1]

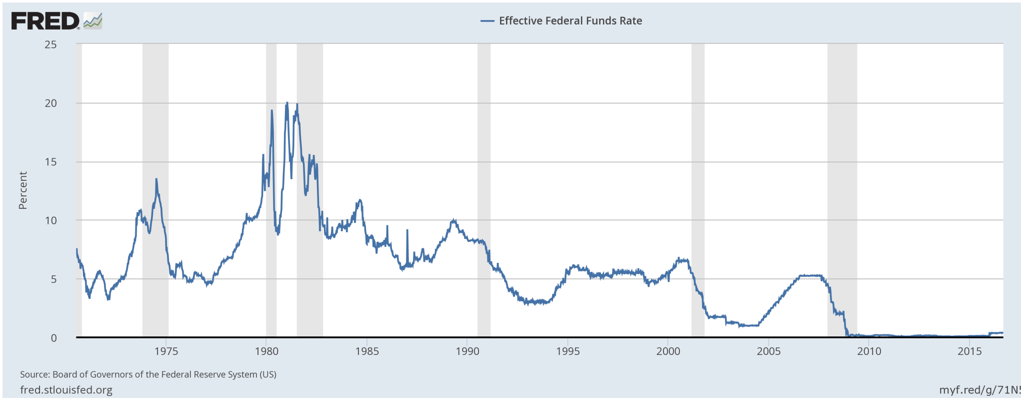
The economy has been a fertile ground for the emergence of bubble speculation. Low real interest rates reduce demand for savings in traditional vehicles in favor of high variance speculation. As seen in Figure 1, the economy has experienced a slow, secular decline in interest rates since the 1980s. The liquidity trap is also evident in the years following 2008, which creates the risk of a secular stagnation equilibrium in which the aggregate employment decline persists.

The final ingredient which can lead to secular stagnation is a period of deflation which coincides with the bubble bursting. In the presence of nominal wage rigidities, this artificially inflates real wages and therefore reduces labor demand by firms. As Figure 2 shows, not only did the US economy experience weak inflation in the years around the recession; in 2009 the economy did indeed experience a deflationary episode.

The combination of a bubble boom-bust cycle and a period of deflation carries great risk for persistently depressed aggregate demand and a secular reduction in unemployment. We formalize this risk in a theoretical framework using an overlapping generations model with nominal wage rigidities. The mapping between bubble asset size and subsequent depth of output loss is clear.

[1]Mian and Sufi (2014)

Figure 1. : The Secular Decline in US Interest Rates



I. Literature Review

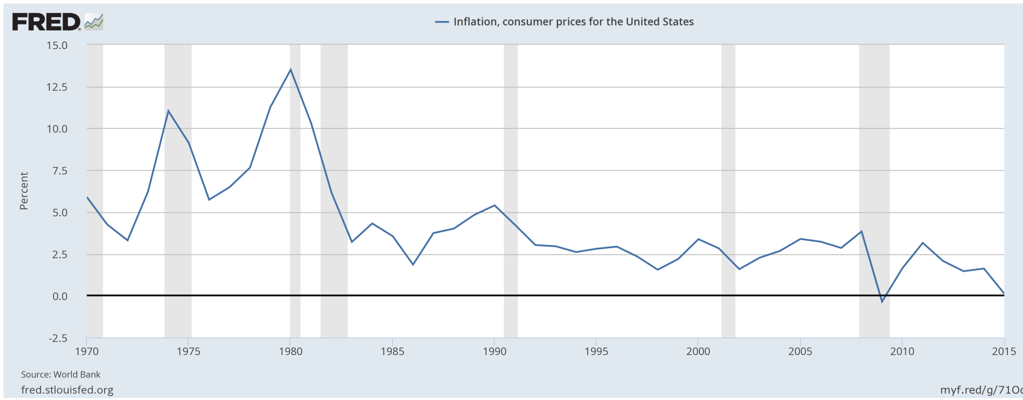
Our model attempts to join two similar fields of study: the existence of a "secular stagnation" equilibrium and its antecedents, and the rational bubble literature. The literature surrounding leveraged bubbles as in Bengui and Phan (2016) offers a vehicle through which bubble boom-bust may cause a secular stagnation equilibrium.

Conceptualization of the secular stagnation premise began with Alvin Hansen in 1938. Hansen sought to explain persistently poor employment recovery following the Great Depression, but was quickly overshadowed by the fiscal spending on the advent of the Second World War. The topic has resurfaced following the 2007-2009 recession, notably in commentary form (see Summers (2013), Taylor (2014), Krugman (2013), and Delong (2014) for example). The main cause of secular stagnation comes from an inefficient suppression of aggregate demand. Hansen argues that this was due to a decline in birth rates as well as a savings glut.

Our model of secular stagnation follows current literature which chooses to focus on the savings glut. As is the case in Mian and Sufi (2014), this glut arises due to a collapse in the balance sheet of households. Mian and Sufi (2014) argue that a collapse in household housing net worth should cause a stratified reduction in employment across non-tradable and tradable sectors due to the differing demand elasticities. Additionally, Mian and Sufi (2014) show that the reduction in employment in the non-tradable sector was not driven by industry-specific supply shocks, contemporaneous credit supply tightening, or other effects. This lends empirical weight to theoretical work on debt deleveraging in the presence of New Keynesian rigidities. For further empirical motivation, see Mian and Sufi (2011).

Much of the theoretical work surrounding the secular stagnation premise follows closely with Eggertsson and Krugman (2012). For these works two essential

Figure 2. : Deflationary Episode Following Housing Bubble Burst



ingredients are present: an unexpected debt deleveraging in the form of a tightening collateral constraint and monetary policy subject to the zero lower bound. As documented by Eggertsson and Krugman (2012), a large deleveraging shock - which acts as the reduction in household net worth - suppresses aggregate demand to such an extent that monetary policy is forced to the zero lower bound. Thus the central bank cannot prop up the real interest rate enough to discourage savings and fully counteract the aggregate demand suppression, which leads to involuntary unemployment. Critically, this result is demand-driven.

Eggertsson and Mehrotra (2014) and Schmitt-Grohé and Uribe (2015) add nominal rigidities to the Eggertsson and Krugman (2012) class of models to show that this involuntary unemployment can be a secular, not episodic, phenomenon. Our model closely follows that of Eggertsson and Mehrotra (2014) which uses nominal wage rigidity to show that a large enough debt deleveraging phenomenon can lead to a unique secular stagnation steady state. They also show that, under specific conditions, this steady state is stable even in the presence of full employment steady states. Schmitt-Grohé and Uribe (2015) develop a small open economy model with nominal wage rigidity and a currency peg which, using an exogenous real interest rate shock instead of a tightening collateral constraint, reaches similar conclusions. The currency peg is the vehicle which generates the deflation needed for nominal wage rigidity to bind. In both studies the resulting involuntary unemployment is persistent.

Finally, Korinek and Simsek (2016) contribute two-fold. First, they show that the deleveraging episode need not be unexpected; agents acting rationally with a foreseeable deleveraging episode may still be incentivized toward excessive leveraging. Secondly - and tangentially - they show an alternative vehicle for transitionary deflation is the introduction of disutility of labor.

The resultant policy implication of Korinek and Simsek (2016) concerns the reduction of available leverage for borrowing households, either ex-post via debt write-downs or ex-ante via stricter constraints or taxes on debt issuance. These interventions offer socially desirable first-order improvements to the unconstrained equilibrium by reducing aggregate demand externalities, particularly in the presence of a liquidity trap.

This result parallels nicely to the leveraged bubble asset literature. In particular, Bengui and Phan (2016) highlight that pledgeability of bubbly assets can cause risk-shifting to credit-constrained borrowers which have higher incentive for equilibrium default. This provides the basis for our paper's deleveraging motivation: the bursting of leveraged bubble assets. In other words, whereas the secular stagnation literature has typically relied on an amorphous deleveraging episode, the leveraged bubble literature provides a more rich explanation for such an episode. This allows for further exploration of potential "macroprudential" policy. (Borio, 2003)

Furthermore, our paper links the recent work of Gali (2014) to the secular stagnation literature. Gali argues against "leaning into the wind" monetary policy, which advocates raising the real interest rate in response to asset bubble growth and fluctuation. Raising the real interest rate can increase the expected size of the bubble as well as its volatility. Our paper adds to this cautionary tale by providing a one-to-one mapping between bubble size and resulting depth of recession.

Finally, our paper follows the framework of Samuelson (1958), Diamond (1965), and Tirole (1985). They jointly represent the foundation of the rational bubble literature, particularly in an overlapping generations model context. Further considerations of this class of models containing financial frictions include Miao and Wang (2011), Farhi and Tirole (2011), and Martin and Ventura (2012).

The remainder of the paper proceeds as follows: Section II develops a baseline model without nominal rigidites. Section III augments this baseline model with nominal rigidities. Section IV discusses global properties of transition following the period in which the bubble bust occurs. Section V highlights the potential for policy intervention to mitigate risk ex ante or to ease ex post, and finally section VI concludes.

II. Environment Without Nominal Rigidities

Consider a closed economy with overlapping generations in discrete, infinite time. There is a single consumption good which is the numeraire. Each generation consists of a unit mass continuum of households, each of which live for three generations. This framework follows from Samuelson (1958) and Eggertsson and Mehrotra (2014). We assume households maximize the following lifetime log utility:

$$log(C_t^y) + \beta log(C_{t+1}^m) + \beta^2 log(C_{t+2}^o)$$

where C_t^y, C_{t+1}^m, and C_{t+2}^o represent consumption when young, middle-aged, and old respectively.

A. Bubbleless Economy

Households face the following budget constraints:

$$C_t^y = Y^y + \frac{D_t^y}{R_t}$$
$$C_{t+1}^m = Y^m - D_t^y + \frac{D_{t+1}^m}{R_t + 1}$$
$$C_{t+2}^o = Y^o - D_{t+1}^m$$

Y^y, Y^m, and Y^o represent general income terms for the time being. They will eventually be replaced with labor earnings and dividends. D_t^y and D_{t+1}^m represent debt accrued (or saved) when young and middle-aged respectively. R_t is the real interest rate in time t.

Further, we assume that households face an exogenous collateral constraint:

$$D_t^y \leq \bar{D}$$

Parameters are assumed such that the collateral constraint will bind $\forall t$.

Definition Assuming $D_0^m = -\bar{D}$, a competitive equilibrium consists of consumption choices $\{C_t^y, C_t^m, C_t^o\}_{t=1}^{\infty}$, investment decisions $\{D_t^y, D_t^m\}_{t=1}^{\infty}$, and interest rates $\{R_t\}_{t=1}^{\infty}$ such that

1) The debt market clears $-D_t^m = D_t^y$
2) Collateral constraint binds $D_t^y = \bar{D}$
3) The interest rates solve the middle-aged to old-aged household Euler equation
4) Given interest rates, binding collateral constraint, and cleared debt market; consumption profiles satisfy the household budget constraints.

A steady state is an equilibrium in which all real variables are time invariant.

LEMMA 1: *In steady state the real interest rate in the bubbleless environment R_{nb} solves:*

$$R_{nb} = \frac{Y^o + (1+\beta)\bar{D}}{\beta(Y^m - \bar{D})} \tag{1}$$

PROOF:
See Appendix A

B. Bubble Economy

Households face the following budget constraints:

$$C_t^y = Y^y + \frac{D_t^y}{R_t}$$
$$C_{t+1}^m = Y^m - D_t^y + \frac{D_{t+1}^m}{R_t + 1} + B_{t+1}^N - B_{t+1}s_{t+1}$$
$$C_{t+2}^o = Y^o - D_{t+1}^m + B_{t+2}^o s_{t+1}$$

Following a simplification of Martin and Ventura (2012), the bubble assets satisfy the following equations:

$$B_t^N = nB_t \tag{2}$$
$$B_t = B_t^N + B_t^o \tag{3}$$

where B_t represents the size of the aggregate bubbly assets in real consumption units in time t. B_t^N represents new bubbly assets introduced by the middle-aged at exogenous fraction n of current aggregate bubbly asset size, and B_t^o represents the size – again in real consumption units – of bubbly assets held to old age by the middle-aged in time $t-1$. s_t represents the portfolio size invested in this bubbly asset as a savings vehicle entering old age. A critical piece of intuition is necessary: in this framework bubble size and relative bubble value are synonymous. This means, while interpreted as "size" for the remainder of the paper, B_t and its constituents act as relative prices. They are subject to no-arbitrage conditions and are inextricably linked therein to the real interest rate.

We now assume that the bubble assets are fully pledgeable. This means that the full value of new bubble assets introduced when middle-aged may be used as leverage:

$$D_t^y \leq \bar{D} + B_{t+1}^N$$

which we will again assume to bind $\forall t$. Thus the first-best equilibrium is unattainable in the bubble environment.

LEMMA 2: *In steady state, the real interest rate and aggregate bubble size solve:*

$$R_b = 1 - n \tag{4}$$
$$B = \frac{1}{1-n}\left[\frac{\beta(1-n)Y^m - Y^o}{1+\beta(1-n)} - \left(1 + \frac{\beta}{1+\beta(1-n)}\right)\bar{D}\right] \tag{5}$$

PROOF:

See Appendix A

The critical comparative static to note is that steady state bubble size is decreasing in the exogenous debt limit $\bar{D}$. This result hinges on the savers in the economy, the middle-aged, who are presented with two vehicles for savings: crediting or bubble asset investment. With a low $\bar{D}$, demand for alternative savings is high. This increases the price of the bubble asset in a partial equilibrium framework.

Note that in steady state, the binding collateral constraint operates as if households were able to borrow a fixed fraction of lifetime income. In a stochastic environment this result would carry more weight as the collateral constraint would depend on the expectation of future income. This could easily result in endogenous over-leveraging, such as Bengui and Phan (2016) note increases the risk of equilibrium default.

LEMMA 3: *A necessary and sufficient condition for the existence of a deterministic equilibrium with bubble assets is:*

$$R_{nb} = \frac{Y^o + (1+\beta)\bar{D}}{\beta(Y^m - \bar{D})} < 1 - n$$

PROOF:

See Appendix A

A condition similar to Lemma 3 exists in Samuelson (1958) and Tirole (1985) and implies that the bubbleless equilibrium allocation is Pareto suboptimal. Traditionally in the literature this condition has implied a negative (net) real interest rate. The existence of a bubble requires a more stringent condition, because the (net) real interest rate in the bubble economy must be negative. This ensures that the shrinking of existing bubble asset value is exactly matched by the introduction of new bubble value by the middle-aged, as required by the definition of the steady state. Were the interest rate greater than one, the bubble would grow unbounded.

These conditions on the real interest rate begin to foreshadow the risk of the liquidity trap.

III. Introducing Nominal Rigidities

We now augment the simple endowment economy with a production sector, returns to labor, and firm ownership dividends. We assume that middle-aged households supply a unit of labor inelastically and that ownership of the firms – and thereby dividends due – lies with the old-aged. We abstract from the market for ownership, and instead assume that all old-aged households are endowed with an equal share in the firms. Or, equivalently, each household is endowed with a firm containing identical technology for production.

We assume the firms produce using the following technology:

$$V_t = \max_{L_t^d} P_t \cdot A(L_t^d)^\alpha - W_t L_t^d \tag{6}$$

where V_t represents nominal profits distributed as dividends to the old-aged households, P_t is the price of the consumption good, W_t is the nominal wage paid to middle-aged households, and L_t^d represents labor quantity demanded by the firm given wages. We assume the technology satisfies decreasing returns to scale ($\alpha < 1$) and will normalize total factor productivity A to one for simplicity.

Decreasing returns to scale is an important deviation from traditional firm framework, because of subsequent introduction of downward nominal wage rigidity. Given constant returns to scale, firms would prefer to shut down when faced with a period of artificially inflated nominal wages. Decreasing returns to scale technology ensures that firms may optimally lower labor demand when faced with artificially inflated nominal wages while still operational.

Decreasing returns to scale also implies that V_t will be non-zero even in a perfectly competitive environment.

We now follow Eggertsson and Mehrotra (2014) and augment the simple endowment economy by introducing downward nominal wage rigidity. The rigidity takes the following form:

$$W_t = \max\{\gamma W_{t-1} + (1-\gamma) P_t w_t^f, P_t w_t^f\} \tag{7}$$

where γ represents the degree of rigidity. If $\gamma = 1$, nominal wages are fully rigid and equal to the previous period's wage. If $\gamma = 0$ then wages are fully flexible in every period. $w_t^f = \alpha$ represents the real wage which would satisfy full employment demand by the firm in time period t. It is derived from the firm's first-order condition in real terms:

$$w_t = \alpha (L_t^d)^{\alpha - 1} \tag{8}$$

and setting $L_t^d = 1$.

Note that this implies, in real terms, that wages are sticky iff $w_{t-1} \geq \Pi_t w_t^f$. $\Pi_t = \frac{P_t}{P_{t-1}}$ is inflation in the price of the consumption good and w_{t-1} represents real wages paid in the previous period. Finally, combining this condition with the firm's first-order condition leads to a compact representation of labor demand:

$$L_t^d = \begin{cases} 1 & \text{if} \quad W_t = P_t w_t^f \\ \left(\frac{P_t w_t^f}{W_t}\right)^{\frac{1}{1-\alpha}} & \text{if} \quad W_t > P_t w_t^f \end{cases}$$

A. Monetary Policy

Again following Eggertsson and Mehrotra (2014), we assume there exists a central bank which employs the following monetary policy rule:

$$1 + i_t = \max \left\{ 1, (1 + i_t^*) \left(\frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \right\} \tag{9}$$

subject to an exogenous, time-invariant inflation target Π^*. i_t is the (net) nominal interest rate set by the central bank and is subject to the zero lower bound. i_t^* is a time-varying target nominal interest rate set according to $(1+i_t^*) = R_t^f \Pi^*$, where R_t^f is the interest rate which corresponds to the full employment equilibrium in period t. The latter is called the "natural rate of interest" and will be expanded upon in a later section.

This monetary policy satisfies a Taylor Rule which targets the natural rate of interest. Other Taylor Rules exist, such as ones which also target output gap or that employed in Gali (2014) which seeks to minimize bubble size deviation. However, our result only requires this basic targeting scheme. The central bank seeks to use the nominal interest rate to balance fluctuations in interest rate from its exogenously set target rate, and does so in a greater than one-to-one ratio ($\phi_\pi > 1$).

B. Bubbleless Economy

As before, households maximize the following lifetime log utility:

$$log(C_t^y) + \beta log(C_{t+1}^m) + \beta^2 log(C_{t+2}^o)$$

However, the budget constraints have been augmented:

$$\begin{aligned}
C_t^y &= \frac{D_t^y}{R_t} \\
C_{t+1}^m &= \frac{W_{t+1}}{P_{t+1}} L_{t+1}^d - D_t^y - \frac{x_{t+1}}{P_{t+1}} + \frac{D_{t+1}^m}{R_{t+1}} \\
C_{t+2}^o &= \frac{V_{t+2}}{P_{t+2}} + \frac{(1 + i_{t+1}) x_{t+1}}{P_{t+2}} - D_{t+1}^m \\
D_t^y &\leq \bar{D}
\end{aligned}$$

We assume for simplicity that $Y^y = 0$ so that the only consumption when young is debt-driven. x_t represents nominal bonds in zero supply for which the (net) nominal return is i_t as determined by the monetary policy rule.

Definition A competitive equilibrium is defined as a set of consumption bundles $\{C_t^y, C_t^m, C_t^o\}_{t=1}^{\infty}$, investment decisions $\{D_t^y, D_t^m, x_t\}_{t=1}^{\infty}$, labor demand $\{L_t^d\}_{t=1}^{\infty}$, prices $\{P_t, \Pi_t, W_t, w_t^f\}_{t=1}^{\infty}$, dividends $\{V_t\}_{t=1}^{\infty}$, and interest rates $\{i_t, i_t^*, R_t, R_t^f\}_{t=1}^{\infty}$ which satisfy:

1) Debt market clears $D_t^y = -D_t^m$
2) Collateral constraint binds $D_t^y = \bar{D}$
3) Nominal rigidity (7)
4) Firm profit maximization (6) & (8)
5) Monetary policy rule (9)
6) Household maximization problem
7) Household budget constraints
8) No arbitrage condition $\Pi_{t+1}R_t = 1 + i_t$
9) $w_t^f = \alpha \ \forall \ t$
10) $R_t^f = \frac{v_{t+1}+(1+\beta)\bar{D}}{\beta(w_t^f-\bar{D})}$ [2]

Definition A steady state is a competitive equilibrium in which all real variables, interest rates, and inflation are time-invariant. A full-employment steady state is a steady state which corresponds to a full-employment equilibrium.

A certain departure from traditional framework will be key moving forward. Typically, steady state inflation is defined to be unity, because prices are the object being held time-invariant. Our definition of steady state merely defines inflation to be time-invariant in steady state, not necessarily equal to unity. Thus we allow prices for the consumption good to fluctuate in steady state.

LEMMA 4: *In equilibrium, the real interest rate solves:*

$$R_t = \frac{v_{t+1} + (1+\beta)\bar{D}}{\beta(w_t - \bar{D})} \tag{10}$$

In a full employment steady state, the real interest rate solves:

$$R_{nb} = \frac{v^f + (1+\beta)\bar{D}}{\beta(w^f - \bar{D})} = \frac{(1-\alpha) + (1+\beta)\bar{D}}{\beta(\alpha - \bar{D})} \tag{11}$$

[2] Imposes full employment in period t (not necessarily in period $t+1$) on the household inter-temporal Euler. Note that $v_{t+1} \equiv \frac{V_{t+1}}{P_{t+1}}$

PROOF:

See Appendix B.

For the transition dynamics observed in later sections, this is the state in which we assume the economy begins prior to bubble creation.

C. Bubble Economy

The households face the same lifetime log utility maximization problem, but are now faced with further augmented budget constraints:

$$
\begin{aligned}
C_t^y &= \frac{D_t^y}{R_t} \\
C_{t+1}^m &= \frac{W_{t+1}}{P_{t+1}} L_{t+1}^d - D_t^y + B_{t+1}^N - \frac{x_{t+1}}{P_{t+1}} + \frac{D_{t+1}^m}{R_{t+1}} - B_{t+1} s_{t+1} \\
C_{t+2}^o &= \frac{V_{t+2}}{P_{t+2}} + \frac{(1+i_{t+1}) x_{t+1}}{P_{t+2}} + B_{t+2}^o s_{t+1} - D_{t+1}^m \\
D_t^y &\leq \bar{D} + B_{t+1}^N
\end{aligned}
$$

Once again, the market for bubble assets meets the following specifications:

$$
\begin{aligned}
B_t^N &= n B_t \\
B_t &= B_t^N + B_t^o
\end{aligned}
$$

Given the no arbitrage conditions gleaned from household first order conditions, we reach the following mapping from monetary policy to bubble growth via the real interest rate:

$$
\frac{1+i_t}{\Pi_{t+1}} = R_t = \frac{B_{t+1}^o}{B_t}
$$

As in Gali (2014), the central bank's monetary policy has the ability to affect bubble growth. In Gali's case the control manifested in manipulating the second-order properties of bubble asset innovation. Our monetary policy modeling instead exerts control over the loss (gain) in existing bubble value carried to the next period. Since bubble asset innovation, $B_{t+1}^N = \frac{n}{1-n} B_{t+1}^o$, depends on a fixed fraction of this existing bubble asset value, monetary policy also exerts control over first-order properties of innovation.

Gali's cautionary tale about "leaning into the wind" policy also applies here. An artificial increase in the real interest rate will accelerate bubble growth, both inter-temporally and in subsequent bubble asset innovation B_{t+1}^N. As will be shown, an increase in the bubble size leads to a potentially deeper, persistent recession following a burst event. Thus monetary policy which seeks to encourage aggregate savings and credit availability may further expose the economy in the long run.

We define a competitive equilibrium in the same manner as before, adding only that the bubble asset market must clear according to (2) and (3). Further we augment the definition of the steady state such that the aggregate size of the bubble B_t is time-invariant.

LEMMA 5: *In the full employment "bubbly" steady state, real interest rate and aggregate bubble size solve:*

$$R_b = 1 - n \tag{12}$$

$$B = \frac{\alpha\beta(1-n) - (1-\alpha)}{1+\beta} - \left[1 + \frac{\beta(1-n)}{1+\beta}\right]\bar{D} \tag{13}$$

Recall that $\bar{D}$ represents the degree to which households are allowed to leverage non-bubble future income to finance debt-driven consumption when young. Suppose $\bar{D}$ is set according to some government policy, and is thereby ex-ante flexible. A relaxation in the constraint of these assets for use as collateral will decrease the steady state size of the bubble. As will be shown, this policy vehicle could have positive implications for the economy in insuring against bubble-burst events.

LEMMA 6: *A necessary and sufficient condition for the equilibrium existence of a bubble asset is*

$$R_{nb} = \frac{(1-\alpha) + (1+\beta)\bar{D}}{\beta(\alpha - \bar{D})} < 1 - n$$

D. Aggregate Demand and Aggregate Supply

To show the existence of the secular stagnation equilibrium, we visualize aggregate demand curves and aggregate supply curves in the inflation-output space. These will be piece-wise functions representing periods of sticky wages and flexible wages for aggregate supply and the traditional backward-bending aggregate demand curve in the presence of the zero lower bound.

Assume that $w_{t-1} = w^f = \alpha$ so that the economy was in a full employment equilibrium in the prior period. Aggregate supply is derived from the condition for nominal wage rigidity and firm equations:

$$Y_t = \begin{cases} 1 & \text{if} \quad \Pi_t \geq 1 \\ \left[\frac{\gamma}{\Pi_t} + (1-\gamma)\right]^{\frac{\alpha}{\alpha-1}} & \text{if} \quad \Pi_t < 1 \end{cases}$$

Aggregate demand is derived from the household inter-temporal Euler equation. It is upward sloping when the zero lower bound is in effect and downward sloping otherwise. As in Eggertsson and Mehrotra (2014), the kink point at which this change occurs is critical. [3]

$$\Pi_t^{kink} = (R_t^f)^{\frac{-1}{\phi_\pi}}(\Pi^*)^{\frac{\phi_\pi - 1}{\phi_\pi}} \tag{14}$$

$$R_t^f = \begin{cases} \dfrac{v_{t+1} + (1+\beta)\bar{D}}{\beta(\alpha - \bar{D})} & \text{if} \quad B_t = 0 \\ \dfrac{v_{t+1} + (1+\beta)\bar{D} + B_{t+1}^o}{\beta(\alpha - \bar{D} + B_t^N - B_t)} & \text{if} \quad B_t > 0 \end{cases} \tag{15}$$

The kink point is decreasing in the natural rate of interest. This gives the first indication of a self-fulfilling prophecy explanation: the natural rate of interest is increasing in firm dividends the following period. These dividends are increasing in the scope of production. Therefore expectation of a strong economy in the subsequent period relaxes the risk of liquidity trap in the current period in the sense that the economy can withstand larger inflation deviations below target. Intuitively, this means that demand for savings in the current period decreases when expecting large non-savings income the subsequent period. In order to incentivize lending to the young therefore, returns to savings must increase.

The natural rate of interest is also increasing in bubble size, which lends credence to assertions in Summers (2013) and Krugman (2013) that successive bubbles have helped to prop the economy up and away from a liquidity trap. High speculative returns to bubble assets increases the incentive to invest savings and therefore artificially inflates the natural rate of interest. However, this short term benefit belies potential long run costs.

Aggregate demand satisfies the following piece-wise function (derived in Appendix C):

Bubbleless Economy

$$Y_t = \begin{cases} \dfrac{v_{t+1} + (1+\beta)\bar{D}}{\alpha\beta}\Pi_{t+1} + \frac{1}{\alpha}\bar{D} & \text{if} \quad \Pi_t < \Pi_t^{kink} \\ \frac{1}{\alpha}\left[\Pi_{t+1}(\Pi^*)^{\phi_\pi - 1}(\alpha - \bar{D})\Pi_t^{-\phi_\pi} + \bar{D}\right] & \text{if} \quad \Pi_t \geq \Pi_t^{kink} \end{cases} \tag{16}$$

Bubble Economy

(17)

$$Y_t = \begin{cases} \dfrac{v_{t+1} + (1+\beta)\bar{D} + B_{t+1}^o}{\alpha\beta}\Pi_{t+1} + \frac{1}{\alpha}\left(\bar{D} - B_t^N + B_t\right) & \text{if} \quad \Pi_t < \Pi_t^{kink} \\ \frac{1}{\alpha}\left[\Pi_{t+1}(\Pi^*)^{\phi_\pi - 1}(\alpha - \bar{D} + B_t^N - B_t)\Pi_t^{-\phi_\pi} + \bar{D} - B_t^N + B_t\right] & \text{if} \quad \Pi_t \geq \Pi_t^{kink} \end{cases}$$

[3] This is the point at which $1 = (1 + i_t^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}$

Notice that the aggregate demand function subject to the zero lower bound is not an explicitly mapping from current inflation to current output. If the previous period $t-1$ was also subject to the zero lower bound, then we need only backtrack the equation above by one period for the explicit mapping to emerge. [4] This indeterminacy of inflation will be solved if there exists a zero lower bound equilibrium in period t; the combination of aggregate supply and aggregate demand will pin down inflation.

For now, we follow a simplification used by Eggertsson and Mehrotra (2014) which abstracts from transition periods and focuses solely on resulting steady states. In other words, we compare aggregate demand and aggregate supply for the full employment bubbleless steady state and the full employment bubble steady state.

Aggregate demand then simplifies to:

Bubbleless Economy

$$Y = \begin{cases} \dfrac{(1+\beta)\bar{D}\Pi + \beta\bar{D}}{\alpha\beta - (1-\alpha)\Pi} & \text{if} \quad \Pi < \Pi^{kink} \\ \frac{1}{\alpha}\left[(\Pi^*)^{\phi_\pi - 1}(\alpha - \bar{D})\Pi^{1-\phi_\pi} + \bar{D}\right] & \text{if} \quad \Pi \geq \Pi^{kink} \end{cases} \tag{18}$$

Bubble Economy

$$Y = \begin{cases} \dfrac{\left[(1+\beta)\bar{D} + (1-n)B\right]\Pi + \beta\left[\bar{D} + (1-n)B\right]}{\alpha\beta - (1-\alpha)\Pi} & \text{if} \quad \Pi < \Pi^{kink} \\ \frac{1}{\alpha}\left[(\Pi^*)^{\phi_\pi - 1}(\alpha - \bar{D} - (1-n)B)\Pi^{1-\phi_\pi} + \bar{D} + (1-n)B\right] & \text{if} \quad \Pi \geq \Pi^{kink} \end{cases} \tag{19}$$

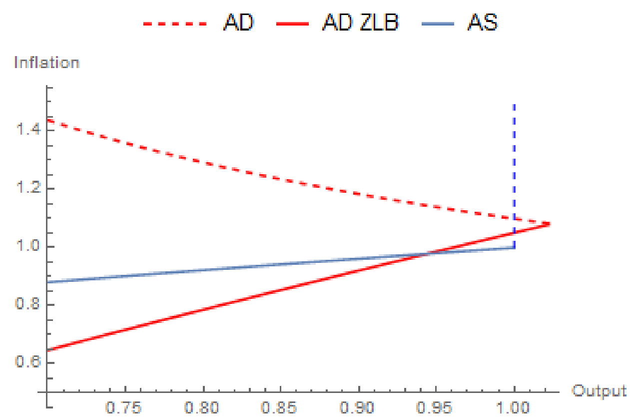
Figure 3 provides a visual representation of aggregate demand and supply curves faced in a bubbleless steady state. Intersections of the aggregate demand and aggregate supply curves denote feasible steady state pairs of $\{\Pi, Y\}$. Unlike in Eggertsson and Mehrotra (2014)[5], we have calibrated the model such that a full employment steady state exists in the post-deleveraging-event case.[6] In fact, there are three such possible steady state pairs. The highest intersection represents the full employment steady state not subject to a liquidity trap. Inflation will be equal to the target inflation rate, and monetary policy is able to correct for any variation. The intersection of the vertical AS piece and the upward sloping aggregate demand curve represents an indeterminate full employment steady state

[4] $Y_t = \frac{1}{1-\alpha}\left[\dfrac{\beta}{\Pi_t}(\alpha Y_{t-1} - \bar{D}) - (1+\beta)\bar{D}\right]$

[5] Eggertsson and Mehrotra (2014) abstract from the ex-ante nature of the deleveraged steady state. They speak simply of a deleveraging event large enough to derive a unique persistent unemployment steady state. In the case of a bubble boom-burst, there must exist full employment steady states and the persistent unemployment steady state simultaneously.

[6] See Table 1 for a list of parameter values used in the subsequent figures.

Figure 3. : Aggregate Demand and Supply Curves in a Bubbleless Steady State



which *is* subject to a liquidity trap.[7] Finally, the intersection between the upward sloping aggregate demand and supply curves represents the potential secular stagnation steady state. This allows us to perform the following thought experiment: the economy begins in a bubbleless full employment steady state following which the economy experiences a bubble boom-burst even. However, rather than transitioning back to the original full employment steady state, the nature of the bubble boom-burst drives the economy to the secular stagnation steady state.

The presence of a bubbly asset in full employment steady state changes the slope of both pieces of the aggregate demand function. However, more importantly, it acts as a curve shift, generating greater aggregate demand for every value of inflation. In other words, the curve shifts to the right as shown in Figure 4.

When the bubble bursts unexpectedly bursts, the economy will transition back to the curves in Figure 3 and will assume one of the three steady state intersections. Which intersection is assumed depends on the transition path. In particular, for the economy to deviate from the stable full employment steady state, the shock to aggregate demand must be deep enough to force the economy into a liquidity trap in transition.

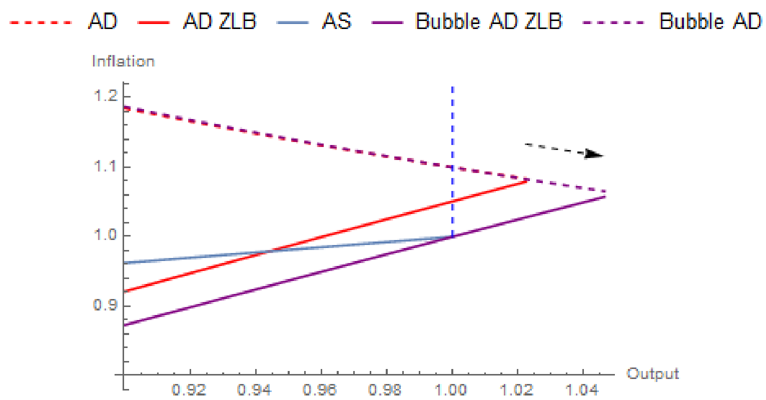
E. Characterizing the Secular Stagnation Steady State

Before turning to transition dynamics, we first characterize the properties of the secular stagnation steady state. This steady state corresponds to the intersection of the upward sloping aggregate demand and supply curves in Figure 3. The economy reaches this steady state under two important conditions.

The first condition is that the economy experiences a liquidity trap. This occurs

[7]For further discussion of this particular steady state and its indeterminacy, see Eggertsson and Mehrotra (2014) and Benhabib, Schmitt-Grohé and Uribe (2001).

Figure 4. : Transitioning to Bubble Steady State



because the bursting of the bubble causes a rapid reduction in the natural rate of interest such that the central bank cannot adjust the nominal interest rate low enough to achieve the natural rate of interest given inflation. As will be shown, larger bubbles cause further reduction in the the natural rate of interest making the liquidity trap more likely.[8]

The second condition is that the economy simultaneously face a deflationary episode. For Schmitt-Grohé and Uribe (2015), this deflationary episode was a result of a currency peg, for instance. This deflationary episode not only aids in causing the liquidity trap as mentioned above, but also triggers nominal wage rigidity. With artificially high nominal wages, firms optimally choose to hire less labor, thus realizing loss in output. The crux of this second condition is that, barring high inflation in subsequent periods, the effect is persistent. In fact, prolonged periods of deflation will further exacerbate the artificial inflation of wages.

Steady state inflation and output are determinate in this case:

LEMMA 7: *Secular stagnation steady state output Y_{stag} and inflation Π_{stag} solve:*

$$Y_{stag} = \frac{(1+\beta)\Pi_{stag} + \beta}{\alpha\beta - (1-\alpha)\Pi_{stag}}\bar{D} \tag{20}$$

$$Y_{stag} = \left[\frac{\Pi_{stag} - \gamma}{(1-\gamma)\Pi_{stag}}\right]^{\frac{\alpha}{1-\alpha}} \tag{21}$$

The key result from this solution is the one-to-one mapping between the steady state size of the bubble and the secular stagnation loss in output. From (13) we

[8]Here "more likely" refers to a larger range of realized inflation for which monetary policy is ineffective in achieving the natural rate of interest.

have that the steady state bubble size is decreasing in the collateral constraint. From the characterization above, we have that secular stagnation output is increasing in the collateral constraint. In other words, large steady state bubbles are linked to deeper recessions in a secular stagnation steady state.

PROPOSITION 8: $\frac{\partial Y_{stag}}{\partial B} < 0$ *which implies that large bubble asset value prior to a burst event is related to deeper loss in output (and employment) following its burst.*

This result may also be considered from the perspective of the bubbleless real interest rate. Low bubbleless real interest rates belie large bubbles and thereby increased risk in the event of a burst event. In fact, below the $1-n$ threshold, low bubbleless real interest rates *guarantee* the emergence of a bubble as a Pareto-improving market innovation and therefore expose the economy to risky leveraging decisions.

IV. Global Transition Dynamics

The key to understanding how and when the secular stagnation equilibrium might arise lies in the transition path. The economy would have to experience secondary effects from the bubble bursting in addition to the rapid reduction in the natural rate of interest. These secondary effects would likely stem from deflationary periods. The collapse in aggregate demand is not sufficient in the case that the bubbleless economy has, ex-ante, a stable full employment steady state. We highlight a few vehicles by which these secondary effects may occur in transition.

Assume that the bubble asset bursts in time period T and that bursting is defined as $B_{T+k} = 0 \; \forall \; k \in [0, \infty)$. Also, assume that the economy was in a full employment bubble steady state in period $T-1$.

In the period the bubble bursts, the old-aged lose a fraction of their savings income: that saved by purchasing bubble assets in period $T-1$. This is the first depression of aggregate demand. The middle-aged also lose two savings vehicles: raw bubble asset purchasing and an artificial reduction in credit demand due to the lowered collateral constraint. The middle-aged are also over-leveraged, in that they borrowed when young against the expectation of new bubble asset endowment which fails to manifest. Finally, the young are unable to borrow as much as previous cohorts, reducing aggregate demand further.

The tightening of the collateral constraint is what begins to drive the real interest rate downward. Excess demand for savings via the debt channel, because it is now the sole channel, depresses the return to savings. Middle-aged households re-derive their inter-temporal Euler equation, for which the natural rate satisfies the following inequality:

$$R_T^f = \frac{(1-\alpha)Y_{T+1} + (1+\beta)\bar{D}}{\beta(\alpha - \bar{D} - nB)} < R_b \tag{22}$$

Discussing a transition path with any degree of implied ex-ante causality is moot in this model. The particular weakness of this model formulation is that the inflation in the period the bubble bursts cannot be determined. As mentioned before, Schmitt-Grohé and Uribe (2015) addressed this issue in a small open economy issue by utilizing a currency peg. The appendix of Korinek and Simsek (2016) seems to suggest incorporating elastic labor supply may help to pin down inflation in the transition period.[9] The downside to this approach is that unemployment can no longer be considered involuntary from the perspective of the households. Markets will correct so that any equilibrium unemployment is voluntary.

What follows are a series of routes with the potential to derive, ex-post at least, the transitional inflation and thereby close the analysis of transitional dynamics.

A. *Self-Fulfilling Prophecy*

The first potential explanation for a transition to secular stagnation concerns the natural rate of interest. In general, lower natural rates of interest lead to shifts in the aggregate demand curve to the left. Thus, if $R_T^f > R_{nb}$, aggregate demand will shift left to a midpoint between those seen in Figure 4 and the economy will remain at the full employment equilibrium, barring explicit deflation.

However, note that if Y_{T+1} is expected or known to be low enough, the economy could face a situation in which $R_T^f < R_{nb}$ enough such that the recession equilibrium is the unique equilibrium in the transition period. This reversed inequality will be the case if the expected deviation from full employment output is greater than a small fraction of steady state bubble size:

$$1 - Y_{T+1} > \left[\frac{n\beta}{1-\alpha} R_{nb}\right] B \tag{23}$$

Under the parameterization used for the aggregate demand and supply figures, this means the expected output loss could be as small as 0.01% of full employment output.

B. *Backward Induction*

In this section we will condition on the result and then proceed by backward induction to solve for all of the transitional variables.

Suppose the economy takes k periods after the bubble bursts to transition to the secular stagnation equilibrium. Explicitly, $Y_{T+k} = Y_{stag}$ and $\Pi_{T+k} = \Pi_{stag}$ as solved in Lemma 7, and all other allocations and prices follow. As long as nominal rigidity is binding, we have the following relationship $\forall\ j \in [1, k]$:

[9] As of the turning in of this draft, this extension was not pursued thoroughly enough to include. It is on the agenda.

$$\Pi_{T+k-j} = \frac{\gamma w_{T+k-j-1}}{w_{T+k-j} - (1-\gamma)w^f}$$

Using the household inter-temporal Euler and the firm equations, we can derive a first-order difference equation in real wages $\forall\ j \in [2, k-1]$:

$$\frac{(1-\alpha)\left(\frac{w_{T+k-j+1}}{\alpha A}\right)^{\frac{\alpha}{\alpha-1}} + (1+\beta)\bar{D}}{\beta\left[\alpha\left(\frac{w_{T+k-j}}{\alpha A}\right)^{\frac{\alpha}{\alpha-1}} - \bar{D}\right]} = \frac{1}{\Pi_{T+k-j+1}} = \frac{w_{T+k-j+1} - (1-\gamma)w^f}{\gamma w_{T+k-j}}$$

with the terminal condition

$$w_{T+k-1} = (\alpha A)^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)Y_{stag} + (1+\beta)\bar{D}}{\beta}\Pi_{stag} + \bar{D}\right]^{\frac{\alpha-1}{\alpha}}$$

Finally, to close out the backward induction, the bubble bursting period differs slightly:

$$\frac{(1-\alpha)\left(\frac{w_{T+1}}{\alpha A}\right)^{\frac{\alpha}{\alpha-1}} + (1+\beta)\bar{D}}{\beta\left[\alpha\left(\frac{w_T}{\alpha A}\right)^{\frac{\alpha}{\alpha-1}} - \bar{D} - nB\right]} = \frac{1}{\Pi_{T+1}} = \frac{w_{T+1} - (1-\gamma)w^f}{\gamma w_T}$$

$$\Pi_T = \frac{\gamma}{\frac{w_T}{w^f} - (1-\gamma)} < 1$$

C. *Disutility of Labor*

Work in progress.

V. Policy Implications

This section needs to be fleshed out. Potential avenues:

- Revisit "leaning into the wind" policy à la Gali (2014)
- Austerity/Fiscal Policy
- Credit availability
- Credit channel implications like Kiyotaki-Moore model
- Inflation targeting

VI. Conclusion

Our paper formalizes the use of a bubble asset and its boom-bust cycle to simulate a debt-deleveraging episode using an overlapping generations model and nominal wage rigidities. We also provide some tools for global transitional analysis following the bust. We show that low natural rates of interest in the bubbleless economy incentivize the creation of bubble assets, which if heavily leveraged expose the economy to the risk of secular stagnation. In this secular stagnation equilibrium, the zero lower bound will bind, nominal wages will remain artificially inflated above optimal levels, and output and employment will remain below full employment level.

We also show that the larger the bubble prior to a bust episode, the deeper the resulting recession will be. Thus any policy which inadvertantly or otherwise causes accelerated growth of a bubble asset may increase aggregate demand in the short run but accentuates potentially severe long run risk. These policies include the recently popular "leaning into the wind" approach of raising interest rates in an attempt to slow bubble growth and tightening of collateral constraints in a policy sense or structural sense. Instead, the government may alleviate the secular stagnation equilibrium by setting a higher inflation target or through fiscal spending to bolster aggregate demand.

In short, bubble boom-bust cycles in an environment with downwardly sticky nominal wages have the potential to generate serious, persistent recessions. This warrants some form of policy intervention to mitigate the risk.

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Table 1—: Parameterization for AD and AS Figures

Description	Parameter	Value
Labor share	α	0.9
Discount rate	β	0.95
Nominal rigidity	γ	0.3
Taylor coefficient	ϕ_π	3
Gross inflation target	Π^*	1.1
Bubble innovation rate	n	0.0001
Collateral constraint	$\bar{D}$	0.25 [10]

Without Nominal Rigidities

Proofs go here.

Nominal Rigidities

Proofs go here.

Aggregate Demand & Aggregate Supply

Proofs go here.

[10] $\bar{D}$ calibrated to satisfy a number of inequalities which guarantee equilibrium bubble existence and plurality of bubbleless steady states, given the other parameters.