Aggregate Uncertainty: Krusell and Smith

Econ720

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A Bewley Model of the Wealth Distribution

- We study Krusell and Smith (1998 JPE).
- The problem: In models with aggregate uncertainty, the entire (wealth) distribution is a state variable.
- KS propose an important method for solving models with aggregate uncertainty and heterogeneity.
- It contains an important finding: **approximate aggregation**
  - First moments are often enough to approximate the entire distribution of the state vector.
  - The aggregate law of motion for K looks a lot like an individual's decision rule.

The Question

- Economists commonly use models with representative households.
- Are these **good approximations** for models with heterogeneous agents?

Contributions

1. In the standard Real Business Cycle (RBC) framework, the representative agent is a good approximation.
2. A method for computing models with heterogeneity and aggregate uncertainty.

The approach

- Compute a standard RBC model (representative agent)
- Add uninsured employment risk.
- Compare: how good is the representative agent approximation?

Result: Approximate Aggregation

- Aggregate consumption and saving resemble those of a representative agent.
- Therefore: it is enough to keep track of mean wealth, instead of keeping track of the wealth distribution, in order to forecast future prices.

Intuition

- The distribution of wealth is unimportant, if most agents have the same marginal propensity to consume out of aggregate shocks.
- This is true in the model because agents achieve good self-insurance (consumption policy functions are roughly linear).
- Only for the very poor does self-insurance fail. But the very poor account for only a small fraction of aggregate consumption.
- A point made in passing: **preference heterogeneity** permits the model to match the U.S. wealth distribution.

The Model

Demographics:

- a unit mass of infinitely lived households
- households are ex ante identical (Bewley model)

Preferences:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \; U(c_t)$$

Technologies

$$\bar{k}' = (1-\delta)\bar{k} + \bar{y} - \bar{c}$$

$$\bar{y} = z\ \bar{k}^{\alpha}\ \bar{l}^{1-\alpha}$$

- $\bar{k}, \bar{l}$: aggregate capital and labor inputs.
- z is a two-state Markov process.
- takes on values z_g, z_b
- $\Pr(z' = z) = P_z$

Endowments

- $\bar{k}_0$ units of capital at $t = 0$
- $\varepsilon_{i,t}$ units of labor time
- $\Pr(\varepsilon_{i,t} = 0 | z = z_g) = u_g$ and $\Pr(\varepsilon_{i,t} = 0 | z = z_b) = u_b$

Markets

- Goods (numeraire)
- Capital rental: r
- Labor rental: w
- Households hold capital, but cannot borrow: $k_{i,t} \geq 0$.

Household Problem

- Individual state: k, ε.
- Aggregate state: z, Γ.
- Γ is the distribution of households over (k, ε).
- Bellman equation:

$$v(k, \varepsilon, z, \Gamma) = \max \; u(c) + \beta \; E \; v\left(k', \varepsilon', z', \Gamma'\right)$$

subject to

$$\begin{aligned} k' &= r\left(\bar{k}, \bar{l}, z\right) k + w\left(\bar{k}, \bar{l}, z\right) \varepsilon + (1-\delta) k - c \\ \Gamma' &= H(\Gamma, z, z') \end{aligned}$$

- H is the law of motion for the distribution, given z (basically due to household saving decisions).

Recursive Competitive Equilibrium

Objects:

- Household value function v and decision rule $k' = f(k, \varepsilon, z, \Gamma)$.
- Price functions $r(.)$ and $w(.)$.
- Law of motion for the distribution of (k, ε): H.

These satisfy:

- v,f solve the household problem.
- r, w are consistent with firm profit maximization.
- H is "consistent with" household decision rules f.
  - see Bewley slides

Computation

- Problem: The distribution Γ cannot be described with a finite number of parameters.
- KS's idea: Only keep track of a small number of moments of the distribution: ℓ.
  - e.g.: mean, variance, percentile values, ...
- Guess a law of motion for ℓ: $\ell' = h(z, \ell)$.
- Solve the household problem, given h rather than H.
- As long as ℓ contains $\bar{k}$, the household can compute prices.

Approximate Household Problem

- Bellman equation:

$$v(k,\varepsilon,z,\ell) = \max u(c) + \beta E v\left(k',\varepsilon',z',\ell'\right)$$

subject to

$$\begin{aligned} k' &= r(z,\ell)k + w(z,\ell)\varepsilon + (1-\delta)k - c \\ \ell' &= h(z,\ell) \end{aligned}$$

Algorithm

- Start from an arbitrary guess for h, such as $\ell' = \ell$.
- Solve the household problem, given h.
- Simulate many household histories.
- Update the guess for h from the household solution.
- Iterate until the guesses for h converge.

Computation

The key problem:

- How to represent the distribution using a small vector ℓ?
- How to find the law of motion h from simulated household histories?

Krusell and Smith approximate Γ using the first J **moments**: mean, standard deviation, etc.

To check the accuracy of the approximation:

- Verify that the forecast errors are "small."
- Verify that increasing J has little effect on the equilibrium properties.

This is a form of **bounded rationality**: Households only use the first J moments and forecast them using only today's moments.

Details

- Assume that ℓ'_j is a linear function of ℓ, conditional on z.
- Simulate a large number of households from their decision rules.
- Compute a history ℓ_t.
- Estimate the coefficients by running a regression of ℓ' on ℓ.
- Iterate until regression coefficients converge.
- To check the accuracy of the approximation: Try alternative functional forms for h.

Details

- Start with $J = 1$ moments: $\ell = \bar{k}$.
  - If $z = z_g$: $\ln\left(\bar{k}'\right) = a_0 + a_1 \ln\left(\bar{k}\right)$.
  - If $z = z_b$: $\ln\left(\bar{k}'\right) = b_0 + b_1 \ln\left(\bar{k}\right)$.

Parameters

- Choose standard RBC parameters:
- Preferences: $\beta = 0.99, \ \sigma = 1$.
- Technology: $\alpha = 0.36$. $z_g = 1.01$ and $z_b = 0.99$ based on size of aggregate output fluctuations.
- Unemployment rates: $u_g = 0.04$ and $u_b = 0.1$.
- P_z: match length of business cycles.
- P_ε: Labor endowments match length of unemployment spells.

Results

- Solve for $J = 1$ and $\ell = \bar{k}$.
- Forecasting equations are of the form: $\ln\left(\bar{k}'\right) = a_0 + a_1 \ln\left(\bar{k}\right)$.
- Goodness of fit:
  - $R^2 = 0.999998$.
  - Variance of error term: $\sigma^2 = 0.00003$.
- The log-linear forecasting equation is nearly perfect.
- The welfare gains from better forecasts are negligible.

Approximate aggregation

- Individual decision rules are nearly linear.
- All agents have nearly identical marginal propensities to consume.
- Redistributing wealth has essentially no effect on aggregate consumption.

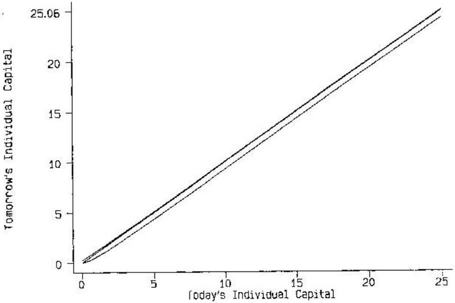

FIG. 2.—An individual agent's decision rules (benchmark model, aggregate capital = 11.7, good aggregate state).

Why approximate aggregation?

- Why are decision rules nearly linear?
- Most agents are rich enough to almost completely smooth shocks.
- One reason: aggregate capital is (by construction) 3 times larger than output.
- Another reason: agents live forever.
- Only a small number of poor agents cannot self-insure. But they account for a tiny fraction of aggregate wealth.

How important is heterogeneity for business cycles?

- The experiment: Compare two identical economies, except that one has complete markets (therefore no heterogeneity).
- Finding: heterogeneity has little effect on the model's business cycle properties.

How important is heterogeneity for business cycles?

Aggregate Time Series

Model	Mean(k_t)	Corr(c_t, y_t)	Standard Deviation (i_t)	Corr(y_t, y_{t-4})
Benchmark:				
Complete markets	11.54	.691	.031	.486
Incomplete markets	11.61	.701	.030	.481
$\sigma = 5$:				
Complete markets	11.55	.725	.034	.551
Incomplete markets	12.32	.741	.033	.524
Real business cycle:				
Complete markets	11.56	.639	.027	.342
Incomplete markets	11.58	.669	.027	.339
Stochastic-β:				
Incomplete markets	11.78	.825	.027	.459

Preference heterogeneity

- The question:
  - The baseline model has far too little wealth heterogeneity.
  - Does approximate aggregation still hold when there is a realistic amount of wealth heterogeneity?
- The approach:
  - Add enough preference heterogeneity to the model to roughly replicate the observed distribution of wealth.
  - Check that the mean is enough to forecast prices very accurately.
- Model
  - Allow for 3 arbitrary values of β: 0.986, 0.989, 0.993.
  - Agents switch β values stochastically, on average every 50 years (once per generation).

Results

Approximate aggregation is still very good:

- $R^2 = 0.99999$
- $\sigma^2 = 0.00006$.

The model matches wealth distribution statistics.

- But: this does not show that preference heterogeneity is important in the data.
- An open question!

Summary

- The main contribution of Krusell and Smith is the **method** for computing economies with heterogeneity and aggregate uncertainty.
- The finding that **approximate aggregation** holds seems robust for frictionless business cycle models (the RBC type), but we don't know whether it holds more generally.

Reading

Krusell, Per; Anthony A. Smith (1998). "Income and wealth heterogeneity in the macroeconomy." *Journal of Political Economy* 5: 867-96.