Cash-in-Advance Model

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Cash-in-advance Models

- We study a second model of money.
- Models where money is a bubble (such as the OLG model we studied) have 2 shortcomings:
  1. They fail to explain rate of return dominance.
  2. Money has no transaction value.
- CIA models focus on **transactions demand** for money.

Environment

Demographics:

- a representative household of mass 1
- no firms; households operate the technology

Preferences: $\sum_{t=1}^{\infty} \beta^t u(c_t)$

Endowments at $t = 1$:

- m_{t-1}^d units of money;
- k_1 units of the good

Technologies:

- $f(k_t) + (1-\delta)k_t = c_t + k_{t+1}$

Environment

Transactions technology

- requires that **some goods are purchased with money**.
- $m_t/p_t \geq c_t + k_{t+1} - (1-\delta)k_t$

Government

- costlessly prints τ_t units of money and hands it to households (lump-sum)

Markets:

- goods: price p_t
- money: price 1

Timing within periods

1. Household enters the period with k_t and m^d_{t-1}.
2. He receives money transfer τ_t:

$$m_t = m^d_{t-1} + \tau_t$$

3. He produces and sells his output for money to be received at the "end of the period."
4. He uses m_t to buy goods from other households (c_t and k_{t+1}).
5. He is paid for the goods he sold in step 3, so that his end of period money stock is m^d_t.

Note that money earned in period t cannot be used until $t+1$.

Household problem

We simply add one constraint to the household problem: the CIA constraint.

The household solves

$$\max \sum_{t=1}^{\infty} \beta^t u(c_t)$$

subject to the budget constraint

$$k_{t+1} + c_t + m_t^d / p_t = f(k_t) + (1-\delta)k_t + m_t/p_t$$

and the CIA constraint

$$m_t/p_t \geq c_t + k_{t+1} - (1-\delta)k_t$$

and the law of motion

$$m_{t+1} = m_t^d + \tau_{t+1}$$

Household problem

Remarks

- Exactly what kinds of goods have to be bought with cash is arbitrary.
- The CIA constraint holds with equality if the rate of return on money is less than that on capital (the nominal interest rate is positive).

Houshold: Dynamic Program

Individual state variables: m, k.

Bellman equation:

$$\begin{aligned} V(m,k) &= \max u(c) + \beta V(m', k') \\ &\quad + \lambda(BC) + \gamma(CIA) \end{aligned}$$

We need to impose

$$m_t = m^d_{t-1} + \tau_t$$

Then we can use m_{t+1} as a control (this would not work under uncertainty).

Bellman Equation

$$
\begin{aligned}
V(m,k) &= \max u(c) + \beta V(m',k') \\
&\quad + \lambda[f(k) + (1-\delta)k + m/p - c - k' - (m' - \tau')/p] \\
&\quad + \gamma[m/p - c - k' + (1-\delta)k]
\end{aligned}
$$

$\lambda > 0$: multiplier on budget constraint

γ : multiplier on CIA constraint - could be 0.

First-order conditions

$$
\begin{aligned}
u'(c) &= \lambda + \gamma \\
\beta V_m(\bullet') &= \lambda / p \\
\beta V_k(\bullet') &= \lambda + \gamma
\end{aligned}
$$

Kuhn Tucker:

$$
\begin{aligned}
\gamma[m/p - c - k' + (1-\delta)k] &= 0 \\
\gamma &\geq 0
\end{aligned}
$$

Household Problem

Thus:

$$u'(c) = \beta V_k(\bullet')$$

Envelope conditions:

$$\begin{aligned} V_m &= (\lambda + \gamma)/p \\ V_k &= \lambda[f'(k) + 1 - \delta] + \gamma[1 - \delta] \end{aligned}$$

Eliminate V's

$$
\begin{aligned}
\beta[\lambda' + \gamma']/p' &= \lambda/p \\
(\lambda + \gamma)/\beta &= \lambda' f'(k') + [1-\delta][\lambda' + \gamma'] \\
\beta u'(c') p/p' &= \lambda \\
u'(c) &= \lambda + \gamma
\end{aligned}
$$

Note: there are only 3 independent FOCs
(one for each choice variable)

Household: Solution

A solution to the household problem: $\{c_t, m_{t+1}, k_{t+1}, \lambda_t, \gamma_t\}$ that solve

1. 3 FOCs
2. budget constraint
3. either CIA constraint or $\gamma = 0$
4. transversality conditions

$$\begin{aligned} \lim_{t\to\infty} \beta^t \, u'(c_t) \, k_t &= 0 \\ \lim_{t\to\infty} \beta^t \, u'(c_t) \, m_t/p_t &= 0 \end{aligned}$$

Household: CIA does not bind

With $\gamma = 0$:

$$\begin{aligned} \beta \lambda' / p' &= \lambda / p \\ \lambda / \beta &= \lambda' [f'(k') + 1 - \delta] \\ u'(c) &= \lambda \end{aligned}$$

Standard Euler equation:

$$u'(c) = \beta u'(c') \; [f'(k') + 1 - \delta] \tag{1}$$

"No arbitrage" condition:

$$f'(k') + 1 - \delta = p / p' \tag{2}$$

When does the CIA constraint bind?

- The CIA constraint binds unless the return on money equals that on capital
  - i.e. the nominal interest rate is zero.
- No arbitrage:

$$1+i=(1+r)(1+\pi)=[f'(k)+1-\delta]\; p'/p=1$$

- Holding money has no opportunity cost.
- The presence of money does not distort the intertemporal allocation.
- We have the standard Euler equation.

Binding CIA constraint

Euler equation:

$$u'(c) = \beta^2 u'(c'')(p'/p'')f'(k') + (1-\delta)\beta u'(c') \tag{3}$$

Today:

- Give up $dc = -\varepsilon$.

Tomorrow:

- $dk' = \varepsilon$.
- Eat the undepreciated capital: $dc' = (1-\delta)\varepsilon$.
- Produce additional output $f'(k')\varepsilon$.
- Save it as money: $dm'' = f'(k')\varepsilon\, p'$.

The day after:

- Eat an additional dm''/p''.

Household Problem

- Why isn't there a simple Euler equation for the perturbation:
  1. $dc = -\varepsilon$. $dm' = p\varepsilon$.
  2. $dc' = \varepsilon \; p/p'$.
- Answer:
- Therefore, the Euler equation for this perturbation is:

$$\begin{aligned} u'(c) &= \lambda + \gamma \\ &= \beta u'(c') \; p/p' + \gamma \end{aligned}$$

Equilibrium

Government

- The government's only role is to hand out lump-sum transfers of money.
- The money growth rule is

$$\tau_t = g m_{t-1}^d$$

- Money holdings in period t are

$$\begin{aligned} m_t &= m_{t-1}^d + \tau_t \\ &= (1+g) m_{t-1}^d \end{aligned}$$

Market clearing

- Goods: $c + k' = f(k) + (1 - \delta)k$.
- Money market:

Equilibrium

An **equilibrium** is a sequence
that satisfies

Steady State

Binding CIA constraint

- In steady state all real, per capita variables are constant $(c, k, m/p)$.
- This requires $\pi = g$ to hold real money balances constant.
- The Euler equation implies

$$1 = \beta^2 (1+\pi)^{-1} f'(k') + (1-\delta)\beta$$

- Using $1+\pi = 1+g$ this can be solved for the capital stock:

$$f'(k_{ss}) = (1+g)[1-\beta(1-\delta)]/\beta^2 \tag{4}$$

- Higher inflation reduces k_{ss}.

Steady State

Assuming that the CIA constraint binds:

$$f(k) = m/p \tag{5}$$

Goods market clearing with constant k implies

$$c = f(k) - \delta k \tag{6}$$

A steady state is a vector $(c, k, m/p)$ that satisfies (4) through (6).

Properties of the Steady State

CIA binding

Definition

Money is called **neutral** if changing the level of M does not affect the real allocation.

It is called **super neutral** if changing the growth rate of M does not affect the real allocation.

Money is not super neutral

- Higher inflation (g) implies a lower k.
- Inflation increases the cost of holding money, which is required for investment (inflation tax).

Properties of the Steady State

CIA binding

Exercise:

- Show that super-neutrality would be restored, if the CIA constraint applied only to consumption $(m/p \geq c)$.
- What is the intuition for this finding?

CIA binding

The velocity of money is one

- Higher inflation reduces money demand only be reducing output.
- This is a direct consequence of the rigid CIA constraint and probably an undesirable result.
- Obviously, this would not be a good model of hyperinflation.
- This limitation can be avoided by changing the transactions technology (see RQ).

Steady State

CIA constraint does not bind

$$
\begin{aligned}
f'(k)+1-\delta &= (1+g)^{-1} && (7)\\
&= 1/\beta && (8)\\
f(k)-\delta k &= c && (9)
\end{aligned}
$$

Result: A steady state only exists if $\beta = 1+g$.

Then: The steady state coincides with the (Pareto optimal) non-monetary economy.

Steady State

CIA constraint does not bind

- Why is there no steady state with $1+g<\beta$?
- $\beta R = \beta/(1+g) > 1$.
- The household would choose unbounded consumption. Cf.

$$u'(c) = \beta R \, u'(c') \tag{10}$$

Optimal Monetary Policy

- **The Friedman rule maximizes steady state welfare.**
- Friedman Rule: Set nominal interest rate to 0.
- Proof: Under the Friedman rule, the steady state conditions of the CE coincides with the non-monetary economy's.
- Intuition:
  - It is optimal to make holding money costless b/c money can be costlessly produced.
  - This requires that the rate of return on money $\frac{1}{1+\pi}$ equal that on capital.

Is this a good theory of money?

Recall the central questions of monetary theory:

1. Why do people hold money, an asset that does not pay interest (rate of return dominance)?
2. Why is money valued in equilibrium?
3. What are the effects of monetary policy: one time increases in the money supply or changes in the money growth rate?

Is this a good theory of money?

Positive features:

1. Rate of return dominance.
2. Money plays a liquidity role.

Drawbacks:

1. The reason why money is needed for transactions is not modeled.
2. The form of the CIA constraint is arbitrary (and important for the results).
3. The velocity of money is fixed.

Reading

- Blanchard & Fischer (1989), 4.2.