Problem Set 7: Asset Pricing

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1 Lucas Fruit Trees With Crashes

Demographics: There is a single, representative household who lives forever.

Preferences: $U = \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $u(c) = c^{1-\sigma} / (1-\sigma)$.

Endowments: The agent is endowed at $t = 0$ with 1 tree. In each period, the tree yields stochastic consumption d_t, which cannot be stored. d_t evolves as follows:

- If $d_t = d_{t-1}$, then $d_{t+1} = d_t$ forever after.
- If $d_t \neq d_{t-1}$, then $d_{t+1} = \gamma d_t$ with probability π and $d_{t+1} = d_t$ with probability $1 - \pi$. $\gamma > 1$.

In words: d grows at rate $\gamma - 1$ until some random event occurs (with probability $1 - \pi$), at which point growth stops forever.

Markets: There are competitive markets for consumption (numeraire) and trees (price p_t). Assume that p_t is *cum dividend*, meaning that d_t accrues to the household who buys the tree in t and holds it into $t + 1$.

Questions:

1. State the household's dynamic program.
2. Derive the Euler equation.
3. Define a recursive competitive equilibrium. Key: what is the state vector?
4. Characterize the stochastic process of p_t. Is p_t a Markov process? Hint: there are 2 phases: before and after dividends have stopped growing. Assume that p/d is constant during the phase with growth.
5. What happens to the stock market when the economy stops growing? Does it crash? Under what condition?