Growth, Death, and Taxes*

by

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Abstract: Growth, Death, and Taxes

Recent theories of endogenous growth suggest that changes in tax rates may permanently affect growth. However, attempts to quantify these growth effects have reached very different conclusions in spite of a common theoretical framework: the neoclassical growth model with human capital accumulation by infinitely lived households. This paper shows that a model which explicitly specifies human capital accumulation over the life-cycle provides sharper answers. In such a model, a plausible range for the growth effects of eliminating taxes in the U.S. is between 0.5 and 1.3 percentage points compared with zero to four percentage points in the infinite horizon model. The much wider range found in the literature is due to two assumptions which are commonly viewed as innocuous simplifications but contrast sharply with traditional human capital theory: that households are infinitely lived and face constant point-in-time returns in human capital accumulation. The widely held view that long, finite horizons are closely approximated by infinite horizons is generally invalid. Abstracting from finite horizons leads to a systematic overstatement of the growth effects of taxes.

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1. Introduction

Recent theories of endogenous growth suggest that changes in tax rates may permanently affect growth. However, attempts to quantify the growth effects of taxes have reached very different conclusions. For example, Lucas (1990) reports that eliminating taxes in the United States would have only a negligible effect on the long-run growth rate, while Jones, Manuelli, and Rossi (1993, henceforth JMR) show that the annual growth rate would rise by 8.3 percentage points. These large differences arise in spite of a shared theoretical framework: the neoclassical growth model with human capital accumulation by infinitely lived households.

Stokey and Rebelo (1995, henceforth SR) show that growth effects in this model are highly sensitive to the parameters governing preferences and human capital production. Their conclusion therefore relies on restricting the values of these parameters: "If human capital's share is large in all sectors, if the sector producing human capital is lightly taxed, and if long-run labor supply is fairly inelastic, then taxing returns in the sectors producing consumption goods and physical capital does not have large growth effects" (p. 520). However, the sensitivity of growth effects in their model together with the considerable uncertainty over the values of these parameters renders this conclusion fragile. Moreover, even if SR's parameter restrictions are maintained, the range of growth effects consistent with the neoclassical model remains very large. Eliminating all taxes may leave the growth rate virtually unchanged or increase growth from its current value of two percent to as much as six percent per year (see section 4 for details).

Although the infinite horizon human capital model does not seem to provide very sharp predictions about the growth effects of taxes, this paper shows that a model which explicitly specifies human capital accumulation over the life-cycle does provide sharp answers. I study the impact of tax changes on long-run growth in a life-cycle model along the lines of Becker (1975) and find that a plausible range for the growth effects of eliminating taxes in the U.S. is between 0.5 and 1.3 percentage points. The much wider range found in the literature is due to two simplifying assumptions: that households are infinitely lived and face constant point-in-time returns in human capital accumulation. These assumptions contrast sharply with traditional human capital theory (e.g., Becker 1975, Mincer 1993) which argues that finite horizons and diminishing returns are essential for capturing the life-cycle patterns of human capital accumulation and earnings found in microeconomic data.[1]

A life-cycle model therefore provides a more natural framework for modeling human capital. Its analysis reveals that the most important determinants of growth effects are precisely the features that have been abstracted from throughout the literature: returns to scale in the production of human capital and the length of the household's planning horizon. In contrast to SR's (p. 538) finding that "introducing diminishing point-in-time returns in the sector producing human capital does not make much difference quantitatively," I show that it is precisely the interaction of long, finite horizons with diminishing returns that makes it hard to generate large growth effects. Once these features are taken into account, the parameters emphasized by SR lose much of their importance, which greatly enhances the robustness of the results.

These findings cast doubt on three widely held views concerning the relationship between infinite horizon and life-cycle models. First, abstracting from life-cycle features is often considered an innocuous simplification. For example, Lucas remarks that infinite horizon and life-cycle models "have very different theoretical structures, yet in practice, for the kind of tax problem under study here, seem to yield quite similar results."[2] However, for the growth effects of taxes, the notion that long, finite horizons are similar to infinite horizons is shown to be generally invalid. The growth effects of tax changes in life-cycle models go to zero as horizons get longer with a striking discontinuity at infinity. Thus, infinite horizons systematically overstate the growth effects of long, finite horizons.

Secondly, life-cycle models are often perceived to behave in very similar ways to infinite horizon models, if generations are altruistically linked. I find that this is not the case for the growth effects of taxes, even if parents can leave financial bequests and influence their children's human capital endowment.

Thirdly, I show that simple parameter adjustments are not sufficient to offset the bias introduced by abstracting from finite lifetimes. In particular, the common practice of increasing depreciation rates in infinite horizon models so as to effectively shorten horizons (e.g., SR, p. 543) actually exacerbates the discrepancy compared with the finite horizon case.

To understand the importance of finite horizons and diminishing returns in human capital accumulation, I develop an overlapping generations model that nests the infinite horizon, constant returns model used previously as a special case. For a version of the model that abstracts from labor-leisure choice, I provide closed form solutions for the growth effects of taxes. For the case of endogenous leisure, I present numerical solutions.[3]

The rest of the paper is organized as follows. Section 2 presents the model framework used throughout this paper. Closed form solutions for the growth effects of taxes are derived in section 3, while section 4 presents numerical simulation results. The final section concludes.

2. Theoretical Framework

The model framework used throughout this paper is a finite horizon version of a neoclassical growth model with human capital along the lines of Lucas (1990) or Stokey and Rebelo (1995). The economy is populated by a large number of competitive firms who produce a single good and by a continuum of households who live for a period of length T. Each household belongs to a dynasty, indexed by the date of birth of a representative member, $i \in [0, t^*)$. The members of dynasty i are born at dates $i + j\, t^*$ (j = 0, 1, 2, …). It is useful to think of household $i + t^*$ as the child of household i and to assume that $t^* = \eta\ T$ with $0 < \eta \leq 1$. The mass of each cohort is normalized to one so that the population size is time-invariant.[4] In order to nest the infinite horizon model as a special case, the assumption that generations are linked by an operative bequest motive is retained throughout. Generations are also linked in their human capital endowments.

2.1 The Household Problem

Each household is endowed with one unit of time that can be used for leisure (l), human capital accumulation (v) or work ($1 - l - v$). Work time earns the after tax wage w, but training time costs $w + q$. To the extent that v represents own time, which is purchased with foregone earnings, its opportunity cost equals the *after-tax* wage rate, so that $q = 0$. If v represents purchased market time, it cost the *pre-tax* wage rate. The wedge between these two wage rates is the tax rate τ_L, which is constant over time: $w = (1 - \tau_L)(w + q)$.

Household i solves the following problem:

$$V(h_0^i, M^i) = \max \int_0^T e^{-\rho t} \frac{[c_t^i\,(l_t^i)^{\omega}]^{1-\sigma}}{1-\sigma}\ dt + e^{-\rho t^*} V(h_0^{i+t^*}, M^{i+t^*}) \qquad \text{(P–1)}$$

subject to $\qquad h_0^i, h_0^{i+t^*}, M^i$ given, $M^{i+t^*} \geq 0$

$$1 - l_t^i - v_t^i \geq 0 \qquad (1)$$

$$\int_0^T e^{-rt} [c_t^i - w(1 - l_t^i - v_t^i)\, h_t^i + q\, v_t^i\, h_t^i + p_x\, x_t^i - \Gamma_t^i]\, dt = e^{-r(T-t^*)} M^i - e^{-rT} M^{i+t^*} \qquad (2)$$

$$\dot{h}_t^i = G(v_t^i, h_t^i, x_t^i, b_t^i) \qquad (3)$$

where subscripts denote ages. This is a conventional setup, studied extensively in the literature, except for denoting the dynastic structure explicitly. The first term in the value function V represents utility derived from own consumption and leisure. In addition, the household values utility of the child as represented by the second term. Market time is restricted to be non-negative by (1). Equation (2) is a present value budget constraint, where M^i denotes the bequest received by generation i and M^{i+t*} is the bequest left to the child which may not be negative. Goods inputs in human capital accumulation (x) cost p_x per unit. Γ denotes lump-sum transfers from the government. Human capital is accumulated according to (3). The functional form assumed for G is standard in this literature,

$$\dot{h}_t^i = B\,(v_t^i\, h_t^i)^{\alpha}\,(x_t^i)^{\varphi}\,(b_t^i)^{1-\alpha-\varphi} - \delta\, h_t^i\,, \tag{4}$$

where $0 < \alpha + \varphi \leq 1$. b_t represents learning ability which the household takes as given. With diminishing returns ($\alpha + \varphi < 1$), time invariant age profiles require that b grows from generation to generation. As is conventional in the literature, I assume that time and human capital enter as "effective time."

The traditional human capital literature typically ignores altruistic linkages and assumes diminishing returns in G (e.g., Heckman 1976, Haley 1976). In contrast, the endogenous growth literature typically assumes infinite horizons and constant returns in G, such that growth is sustainable (e.g., Lucas 1990, Stokey and Rebelo 1995).

2.2 Firms

Firms rent capital (K) and labor (L) from households to produce a single good according to a constant returns to scale technology $Y = F(K, L)$. Labor input is measured in efficiency units. Profit maximization requires that pre-tax factor prices equal marginal products:

$$r^* = F_K \tag{5}$$

$$w^* = F_L, \tag{6}$$

where the subscripts denote partial derivatives.

2.3 Government

The government collects revenues from labor income $(\tau_L\, F_L\, L)$ and capital income $(\tau_K\, F_K\, K)$.[5] These are used to finance government consumption $(\overline{C})$ and lump-sum transfers to households $(\overline{\Gamma})$. The budget constraint is therefore

$$\overline{C} + \overline{\Gamma} = \tau_L\, F_L\, L + \tau_K\, F_K\, K\,. \tag{7}$$

It is assumed that $\overline{C} = \bar{c}\,Y$ is a constant fraction of output, while $\overline{\Gamma}$ is adjusted so as to satisfy (7). Households receive per capita transfers of $\Gamma = \overline{\Gamma} / N$, where N is the constant population size, and face after-tax factor prices of $p_x = 1$ and

$$r = (1 - \tau_K)\, r^* - \delta_K \tag{8}$$

$$w = (1 - \tau_L)\, w^*. \tag{9}$$

2.4 The Human Capital Endowment

To close the model, it is necessary to make a number of choices that are implicit in infinite horizon models: (*i*) How is the human capital endowment of new generations (h_0) determined? (*ii*) How is learning productivity (b_0) determined and (*iii*) at what rate (γ) does it grow over the life-cycle?

Turning to the first question, for growth to be sustainable, the human capital endowment of new generations must grow over time. Four modeling approaches have been used in the literature. (*i*) Human capital may be *heritable* just like physical capital ($h_0^i = h_T^{i-T}$). That is, parents transfer their human capital to their children when they die. While perhaps difficult to justify empirically, this assumption is necessary to derive a reduced form infinite horizon representation along the lines of Barro (1974).[6] (*ii*) Parents may invest in the human capital endowment of their children as in Ehrlich and Lui (1990). (*iii*) New generations may start out with a fraction of average human capital in the population at the time of birth ($h_0^i = \varepsilon\, \bar{h}_i$) as in Azariadis and Drazen (1990) or (*iv*) with a fraction of the parent's human capital stock ($h_0^{i+t^*} = \varepsilon\, h_{t^*}^i$) as in Becker and Tomes (1986) or Bils and Klenow (1997). This does not amount to assuming that human capital is heritable. In particular, parents do not believe that they can affect the endowment of their offspring by accumulating more human capital themselves. The latter two specifications are motivated by evidence for intergenerational links in human capital (e.g., Hanushek 1986).

The second major choice is how learning productivity (b_0) is determined. Two specifications have been used in the literature. There is evidence in favor of the assumption that a higher initial stock of human capital improves the ability to learn later in life (Welch 1970). This would suggest setting $b_0^i = h_0^i$. Alternatively, some authors assume that learning is facilitated by the current stock of knowledge held by other agents in the economy, perhaps because of learning-by-watching as in Jovanovic and Nyarko (1995). This would suggest setting $b_0^i = \bar{h}_i$ as in Azariadis and Drazen (1990). Both specifications will be explored.

Finally, the rate at which learning productivity grows over the life-cycle (γ) must be determined (recall that $b_t^i = b_0^i e^{\gamma t}$). It is desirable to nest two important special cases: first, a fixed, generation specific learning productivity (b_t^i is independent of age; $\gamma = 0$) as suggested by Welch (1970), and secondly, an average human capital spillover $(b_t^i = \overline{h}_{i+t}; \gamma = g)$. I therefore specify γ in a flexible way: $\gamma = \zeta g$ with $0 \le \zeta \le 1$.

Note that the intergenerational transmission of human capital must be *linear*, if growth is to be sustained. In addition, learning productivity must depend linearly on human capital to overcome the effects of diminishing returns. Without this linearity, it would be necessary to introduce technical change in the production of human capital as in Laitner (1993).

In what follows, it is assumed that the model is closed by assuming that children start out with a fraction of parental human capital ($h_0^{i+t^*} = \varepsilon h_{t^*}^i$) and that learning productivity is given by $b_0^i = h_0^i$. This specification nests the infinite horizon assumption that human capital is inherited as a special case. Alternative ways of closing the model lead to similar findings.[7] Moreover, allowing parents to invest in their children's human capital endowment does not alter the results, if this investment is not sensitive to taxes for an *arbitrarily short* period at the beginning of the child's life.[8] I conclude that the findings presented below are not sensitive to how the intergenerational transmission of human capital is modeled.

2.5 Definition of Equilibrium

On a balanced growth path, the life-cycle profiles of all generations can be derived from those of the cohort born at date 0, which are denoted without a superscript (i.e., $x_s = x_s^0$). A balanced growth equilibrium consists of life-cycle profile for (c, l, v, h, x) and a bequest M for the representative household born at date 0, a set of factor prices (r, w, r^*, w^*), a level of transfers $\overline{\Gamma}_0$, aggregates (K_0, L_0, Y_0), and a growth rate (g) such that:

1. The representative household solves (P−1).
2. Factor prices are consistent with profit maximization by firms [(5) and (6)] and tax rules [(8) and (9)].
3. The government budget constraint (7) is satisfied.
4. The markets for labor, capital and goods clear.
5. The growth rate is consistent with the condition for intergenerational transmission of human capital: $e^{gt^*} = \varepsilon h_{t^*} / h_0$.

2.6 The Infinite Horizon Case

By modeling agents as infinitely lived, a number of implicit choices are made in closing the model. First, human capital must be heritable: $h_0^{i+t^*} = h_{t^*}^i$. Otherwise, the incentive for human capital investment declines with age as the time period over which new human capital generates payoffs gets shorter with age. Secondly, children must be born when the parent dies: $t^* = T$. Otherwise, investments before age t^* generate an additional return because they also enhance the child's endowment. Thirdly, learning productivity must grow over the life-cycle at the growth rate of per capita output ($\gamma = g$). If $\gamma < g$, the age-profile of human capital follows a hump-shape (as in the life-cycle model). For infinite horizons to be a useful abstraction, none of these choices must be important for the growth effects of taxation.

3. Analytical Results

This section compares the growth effects of taxes in infinite horizon and overlapping generations versions of the model. In order to be able to derive closed form solutions, two simplifications are made. First, goods inputs are dropped from human capital accumulation ($x = 0$; $\varphi = 0$). For the qualitative properties of the model, this is unimportant. Secondly, leisure is dropped from the utility function ($l = 0$, $\omega = 0$). This greatly simplifies the analysis because the household problem can then be solved in two steps: First, choose a time path v_t for each generation so as to maximize the present value of earnings of this generation. Then choose consumption paths for all generations belonging to the dynasty consistent with this present value.

For taxes to have any effect on human capital accumulation, it is essential to allow for some training inputs that are not tax-deductible. In actual tax systems, these inputs are time and goods purchased in the market. Since goods inputs are abstracted from in this section, the distortionary effects of taxes are captured by assuming that time inputs are not tax-deductible.

It is a common feature of infinite horizon models that the household sector alone determines the balanced growth rate. Since aggregate consumption growth obeys the Euler equation $g = (r - \rho)/\sigma$, the production side of the economy becomes irrelevant. In particular, the technology for producing physical capital and consumption goods or how those sectors are taxed do not affect long-run growth.[9] Since generations are altruistically linked, this remains true in the present model despite the fact that individuals are finitely lived. Thus, capital taxation can be ignored without loss of generality, and it is convenient to denote the wage tax rate simply by τ.

3.1 Optimal Human Capital Accumulation

It is useful to first characterize the household problem with finite horizons ($T < \infty$) and diminishing returns in training ($\alpha < 1$) and then to compare the result with the other possibilities. Because leisure is exogenous, the household chooses an earnings profile that maximizes the present value of earnings net of training costs over the life-cycle. Thus, the household solves

$$\max \int_0^T e^{-rt} \left[w_t h_t - (w_t + q_t) v_t h_t \right] dt \tag{P–2}$$

subject to

$$\begin{gathered} h_0 \text{ given} \\ 0 \le v_t \le 1 \\ \dot{h}_t = G(h_t, v_t, b_t) \end{gathered}$$

This problem statement remains valid even if horizons are infinite or returns to scale are constant, but not for the case studied in the literature (constant returns *and* infinite horizons). It is useful to restate the problem in terms of the normalized variable $z \equiv h / b$. Maximizing the Hamiltonian then yields the following necessary conditions:

$$\begin{gathered} z_0 \text{ given; } \lambda_T = 0 \\ \dot{z} = B(v\,z)^{\alpha} - (\delta + \gamma)\,z \\ \dot{\lambda} = (r + \delta - \alpha\,B v^{\alpha}\, z^{\alpha-1})\lambda - w + (w+q)\,v \\ \lambda - \frac{w+q}{\alpha\,B}(v\,z)^{1-\alpha} \ge 0 \end{gathered} \tag{10}$$

where the last expression holds with equality if $v < 1$. Training time will always be positive because of diminishing returns to v.

Figure 1 shows a phase diagram (see Appendix 7.1 for details). The shaded region can never be entered without violating the transversality condition. For low enough z_0 there may be a corner ($v = 1$), which can only occur at the beginning of life. At some point the household switches to part-time training. Eventually, training time becomes sufficiently small that z falls over time. The optimal path has a "turnpike" property: a steady state level z_{SS} exists that is never reached. But for long horizons, the household spends almost all of its time close to the steady state.[10]

[INSERT FIGURE 1 HERE]

3.2 Characterization of Growth Effects

Combining the household first-order conditions with the balanced growth conditions of section 2.5 allows to solve for the growth effect of a tax change, which I define as $-dg/d\tau$. To make models comparable, B is chosen in all cases to generate an initial balanced growth rate of g^*. In what follows I focus on the generic case where ζ and α are strictly below one. The special cases $\zeta = 1$ and $\alpha = 1$ are considered below. The relevant equilibrium conditions are $z_0 = 1$ and

$$e^{(g-\gamma)t^*} = \varepsilon\, z_{t^*} / z_0 . \tag{11}$$

For long horizons, the household spends an arbitrarily long time arbitrarily close to the turnpike z_{SS}. In particular, if children are born before the parent dies ($\eta < 1$), the parent will be close to the turnpike at age t^*. This allows to set $z_{t^*} = z_{SS}$ in (11) which yields an implicit equation for the growth rate.[11]

$$\varepsilon \frac{B}{\gamma+\delta}\left[\frac{(1-\tau)\alpha B}{r+\delta}\right]^{\frac{\alpha}{1-\alpha}} = z_0\, e^{(g-\gamma)t^*} . \tag{12}$$

Taking logs and differentiating provides a closed form solution for the growth effect

$$-\frac{d\,g}{d\,\tau} = \frac{1/(1-\tau)}{\dfrac{\sigma}{r^*+\delta} + \dfrac{1-\alpha}{\alpha}\left(t^*(1-\zeta) + \dfrac{1}{g^*+\delta/\zeta}\right)}, \tag{13}$$

where $r^* = \rho + \sigma\, g^*$. Note that z_0 may be treated as a constant in (13) because learning ability is determined by $b_0 = h_0$. The main results of this paper follow directly from this equation.

To understand (13) it is useful to compare it with the well-known expression from the infinite horizon, constant returns model used throughout the literature (e.g., Stokey and Rebelo 1995, p. 524)

$$-\frac{d\,g}{d\,\tau} = \frac{1/(1-\tau)}{\sigma/(r^*+\delta)} . \tag{14}$$

There are two additional terms in the denominator of (13), reflecting the impact of diminishing returns and finite horizons (via t^*), both of which reduce the growth effect. The growth effect declines with longer horizons and eventually falls to *zero* as $t^* \to \infty$.

The first major result is therefore that focusing on the infinite horizon, constant returns case, as the literature has done throughout, necessarily overstates the growth effects of taxes.

Moreover, due to the interaction term $(1-\alpha)/\alpha \cdot t^*$ in (13), the bias is largest in the empirically most relevant case of strongly diminishing returns and long, but finite horizons.[12]

The intuition why more diminishing returns lead to smaller growth effects is straightforward and also applies to the case of infinite horizons. An increase in the tax rate induces the household to accumulate less human capital. But with diminishing returns, reducing the amount of time spent in training raises its marginal product. Thus, a given adjustment causes a larger increase in the rate of return to human capital investments. The same argument underlies the role of ζ in (13). If learning productivity grows more rapidly with age (high ζ), the effects of diminishing returns are reduced.

The second main finding is that incorporating diminishing returns into the standard infinite horizon model does not resolve the problem of overstated growth effects. It is commonly thought that abstracting from finite horizons is an innocuous simplification with little consequence for the quantitative properties of the model.[13] The underlying presumption is that, as horizons get longer, life-cycle models become more similar to infinite horizon models. Given that actual households live for many decades, it would seem safe to model their behavior as if they lived forever. This intuition is very misleading for the growth effects of taxes.

To show this, Appendix 7.5 proves that the growth effects with infinite horizons and diminishing returns are given by[14]

$$-\frac{dg}{d\tau} = \frac{1/(1-\tau)}{\frac{\sigma}{r+\delta} + \frac{1-\alpha}{\alpha}\frac{1}{g+\delta}} \tag{15}$$

so that infinite horizons overstate the growth effects of taxes, if and only if

$$\frac{1-\alpha}{\alpha}\frac{1}{g+\delta} < \frac{1-\alpha}{\alpha}\left((1-\zeta)t^* + \frac{1}{g+\delta/\zeta}\right).$$

In contrast to the common presumption, the bias does not vanish as $t^* \to \infty$. Quite to the contrary: the extent to which infinite horizon models overstate the growth effects of taxes *increases* for longer horizons. For short t^*, the infinite horizon model *under*states growth effects. However, growth effects in the life-cycle model are declining in t^* and fall below the infinite horizon level for long t^*, approaching zero in the limit. Thus, longer horizons imply smaller growth effects, but with a discontinuity at infinity. Again, due to the interaction term $(1-\alpha)/\alpha \cdot t^*$, infinite horizons overstate the growth effects of taxes especially for the empirically relevant case of diminishing returns and long, finite horizons.

Note that what matters for the magnitude of growth effects is the length of a generation, t^*, not the household's planning horizon, T. Once the household lives long enough to come close to the turnpike human capital level, further increases in T will alter the growth effect only little, if t^* is held fixed.

Importantly, these results do not depend on the assumption that households are close to the turnpike most of the time. For the special case $t^* = T$ and $\alpha = 0.5$ it is possible to derive analytical solutions for the growth effect without this assumption, which have the same properties as (13) (see Appendix 7.2). Results also do not depend on the assumption that parents fail to internalize the intergenerational link in human capital (Appendix 7.3).

In sum, abstracting from finite horizons and diminishing returns to scale in the production of human capital leads to a systematic overstatement of the growth effects of taxes. Moreover, the bias is largest in the empirically relevant case which combines strongly diminishing returns with long, but finite lifetimes. To illustrate the magnitudes involved, Figure 2 shows how the growth effect of a small tax change varies with α and T.[15] Lifetimes have a strong impact on growth effects with a striking discontinuity at infinity, suggesting that the assumptions of constant returns and infinite horizon lead to a substantial bias in the predicted growth effects.

[INSERT FIGURE 2 HERE]

3.3 Discussion

The main finding of the previous analysis is that the interaction of long, finite horizons with diminishing returns in human capital accumulation leads to small growth effects of taxes. In a special case, the intuition for this finding can be illustrated in a simple graph. In a life-cycle model, the balanced growth rate is determined by the growth of the human capital endowment from generation to generation. Assume, as in infinite horizon models, that children are born when parents die and inherit all of their parents' human capital. The balanced growth rate of the economy then equals the average growth rate of human capital over a household's life-cycle: $e^{gT} = h_T / h_0$. This is illustrated in Figure 3 which shows age profiles of $\ln(h)$ for parent and child. The growth rate is the slope of the line connecting the endowments of successive generations.

[INSERT FIGURE 3 HERE]

Eliminating taxes induces the parent to accumulate more human capital, so that the age profile of human capital becomes steeper. Since the change in the growth rate is determined by the increase in the average growth rate of human capital over the parent's life-cycle, longer horizons imply that the same increase in the growth rate requires a larger level change in the parent's human capital: $dg = d\ln(h_T / h_0)/T$. With realistic lifetimes, even moderate changes in long-run growth require large changes in life-cycle human capital accumulation. For example, if the parent lives for T = 60 years, increasing the growth rate by 1 percent requires that h_T / h_0 almost *doubles* ($e^{0.6} = 1.82$). But such large level changes are difficult to induce because the rate of return to human capital accumulation is depressed by diminishing returns to scale in the production of human capital.

The intuition is more general than this example. A tax change alters the turnpike level of z by an amount that is independent of the household's horizon. But (13) translates this into a change in the growth rate that depends inversely on the horizon. Any deviation from constant returns to private inputs therefore qualitatively changes the outcome: infinite horizons are now a poor approximation for long, finite horizons. Note that this reasoning carries over to any specification in which the household problem exhibits a turnpike property with human capital growing over the life-cycle at a rate below g. In particular, Hendricks (1999*c*) shows that these findings are robust to the details how human capital is transmitted across generations.

3.4 Two Special Cases

The previous analysis has shown that long, finite horizons are generally not well approximated by infinite horizons. This section considers two special cases where this finding is overturned: constant returns in the production of human capital (α = 1) and the case where learning productivity grows with age at rate g (ζ = 1).

3.4.1 Constant Returns to Scale in the Production of Human Capital

Constant returns in the production of human capital fundamentally change the nature of the household problem. With finite lifetimes it can be shown that the household follows a "bang-bang" solution. At the beginning of life there may be a phase of full-time training (v_t = 1). At some time $s \geq 0$ the household switches to full-time work (v_t = 0) without an intermediate phase of part-time work and training. It is shown in Appendix 7.4 that the growth rate is implicitly determined by

$$1 - \frac{r+\delta}{(1-\tau)B} = e^{-(1-[g+\delta]/B)(r+\delta)T}. \tag{16}$$

Implicitly differentiating and noting that, as $T \to \infty$, the right-hand-side of (16) approaches zero, implies that the growth effect approaches

$$\lim_{T\to\infty} -\frac{d\,g}{d\,\tau} = \frac{r+\delta}{(1-\tau)\,\sigma},$$

which coincides with the expression for the infinite horizon case (14).

This finding is surprising, given that optimal human capital accumulation differs fundamentally in the two cases. An infinitely lived household accumulates human capital at a time-invariant rate (on a balanced growth path). Human capital then enters the model perfectly symmetrically with physical capital.[16] This is immediate, if it is assumed that human capital is produced by firms and sold to the household. The model then becomes indistinguishable from a model with two capital goods (see Hendricks 1999*b* for an example). In contrast, with finite horizons human capital is chosen to maximize the present value of earnings, holdings of physical capital are chosen residually to finance the optimal consumption stream, and training follows a bang-bang solution. Still, the growth effects coincide for long horizons. However, as pointed out above, the hypothesis of constant returns to scale is empirically strongly rejected.

3.4.2 Growing Learning Productivity

The second special case in which infinite horizons closely approximate long, finite horizons has learning productivity growing with age at *exactly* rate g ($\zeta = 1$ or $\gamma = g$). Growth in learning productivity then exactly offsets the effect of diminishing returns on the household's incentives for human capital accumulation. With $\zeta = 1$, (13) becomes independent of t^* and reduces to (15). That is, regardless of the length of horizon, if households are close to the steady state at age t^* and $\zeta = 1$, growth effects are exactly the same as in the infinite horizon case.

It is interesting to see why this result holds. Appendix 7.5 shows that the growth rate with infinite horizons and diminishing returns is implicitly determined by

$$\left(\frac{B}{g+\delta}\right)^{1-\alpha}\left(\frac{(1-\tau)\alpha B}{r+\delta}\right)^{\alpha} = 1. \qquad (17)$$

Differentiation of (17) yields (15). Note that the left-hand-side of (17) equals $z_{SS}^{1-\alpha}$ (see equation (19) in the Appendix) so that growth effects in the infinite horizon model are determined by the *turnpike* level of h of the finite horizon model. This result appears surprising at first as there is of course no turnpike in the infinite horizon model. The key intuition stems from the fact that the turnpike level of h is *growing* at rate g, if $\gamma = g$. The infinitely lived household is essentially on the turnpike all the time. However, this is strictly a knife-edge result, which is destroyed if ζ deviates from one by even an arbitrarily small amount.

4. Simulation Results

For the special case discussed in the previous section it is possible to derive closed form solution for the growth effects of taxes. This is no longer the case in a more general setting that allows for endogenous labor-leisure choice and goods inputs in the production of human capital. This section therefore presents numerical simulations based on a realistically calibrated version of the model described in section 2.[17]

4.1 Parameters

The parameter choices are largely based on Jones, Manuelli, and Rossi (1993), although some changes are made to address Stokey and Rebelo's (1995) criticisms. Parameters that are common to the infinite horizon and life-cycle versions of the model are summarized in Table 1. Preference parameters are conventional. The discount factor (ρ) is chosen so that the observed capital-output ratio (K / [$Y - X$]) equals 2.5 and σ is set to 1.1 (JMR). The leisure coefficient (η) is chosen to generate a sensible labor supply response to tax changes. Since there is little evidence on the response of aggregate labor supply to large tax changes, two values are considered. In baseline I (ω = 0.44), eliminating all taxes increases aggregate hours in the IH model by 10 percent, while the increase is 15 percent in baseline II (ω = 0.695).

[INSERT TABLE 1 HERE]

The production function for goods is of the form $Y = AK^{\theta} L^{1-\theta}$ with θ = 0.36 (JMR) and δ_K = 0.06 (SR, p. 543). A is normalized to one. Policy variables follow JMR (τ_L = 0.31, τ_K = 0.21, $\overline{C}/Y = 0.2$). Time inputs in training are tax-deductible. Transfers balance the government budget.

For the production function for human capital, there are two specifications. Based on estimates of Haley (1976) and Heckman (1976), the first specification has diminishing returns to scale for private inputs of 0.7.[18] Mincer (1993) estimates that the cost shares of goods and time are approximately equal, which motivates $\alpha = \varphi$ = 0.35. Following JMR, the second specification has constant returns with φ = 0.479 and $\alpha = 1-\varphi$. The life-cycle depreciation of human capital is set to 0.02 with a correction of 0.04 in the IH case (SR, p. 543).

Several parameters are specific to each model. In the IH case, children inherit all of their parents' human capital (ε = 1) and B is chosen to match g^*. In the life-cycle model, ε is chosen to match the growth rate of g^* = 0.02, while B replicates observed wage growth between the ages of 20 and 45. Households live for T = 55 years and have children at age t^* = 28. Parents are not altruistic towards their children.[19] I examine the cases γ = 0 and the knife-edge case γ = g.

Two alternative sets of parameters are explored that are designed to generate reasonable upper and lower bounds of growth effects. The *conservative* specification has $\sigma = 2$, $\delta_h = 0.01$ (0.04 in the IH case), and $\rho = 0.5$. The *aggressive* specification has $\delta_h = 0.04$ (0.08 in the IH case),[20] and ρ replicates the observed leisure share. In the IH model, the target leisure share is $l = 0.71$, while in the life-cycle case the average fraction of time spent on leisure between the ages of 25 and 60 is 0.56. The latter figure is lower because it excludes the very old and young.

4.2 Results

The top panel of table 2 shows the growth effects resulting from eliminating all taxes for the four sets of parameters (columns) and the four models (rows). Four main conclusions emerge. First, growth effects in the IH model are larger than in the life-cycle model by a factor between two and six. In the life-cycle model, growth effects never exceed 1.3 percent (1.7 percent in the knife-edge case $\gamma = g$), compared with 8.33 percent in the IH case. It is often believed that adjusting the depreciation rate of human capital is sufficient to offset the effects of longer horizons in the IH model. However, since higher depreciation rates imply larger growth effects, such an adjustment exacerbates the discrepancy by increasing a growth effect which is already overstated in the IH case.

Second, large growth effects are possible in the IH model with sensible changes in labor supply. SR's conclusion that large growth effects are implausible in IH models is largely based on the notion that the implied labor supply responses are unreasonable. The literature has typically chosen ρ to replicate the empirical fraction of time spent on leisure. The labor supply changes shown in the bottom panel of table 2 confirm that the changes in labor supply, defined as total hours worked ($1-l$), in that case are indeed very large (see column 4; the increase of 45.3 percent is close to JMR's finding as is the growth effect of 8.33 percent). However, even if parameters are chosen to generate more moderate labor supply responses of 10 percent (baseline I) or 15 percent (baseline II), the growth rate changes substantially (3.6 and 4.15 percent, respectively).[21] This means that, instead of doubling every 35 years, per capita income doubles every 12 years. Note that labor supply responses are generally smaller in the OLG model, even if ω is chosen to replicate the observed leisure share.

[INSERT Table 2 HERE]

Third, growth effects in the life-cycle model are much less sensitive to changes in parameters. Moving from the conservative to the aggressive specification increases growth effects by a factor of two (in the knife-edge case, 2.5) in the life-cycle model, but by a factor of four in the IH model with diminishing returns and seven in the case of constant returns. The responses of labor supply are also less sensitive. Even if ρ is chosen to replicate the observed leisure share, labor supply in the OLG model increases by only 7.6 percent in the OLG model compared with 45.3 percent in the IH model with constant returns.

Fourth, in contrast to SR's (p. 538) finding, returns to scale in human capital accumulation are quantitatively important in the IH model. Abstracting from diminishing returns overstates growth effects by almost two in the baseline specification. In the life-cycle model, constant returns are obviously not an option because of the implied counterfactual age profiles.

Finally, the growth effects predicted by the life-cycle model are consistent with the empirical evidence presented in Engen and Skinner (1996). Using a variety of methods and data sets, Engen and Skinner estimate the change in the growth rate resulting from a reduction in all marginal tax rates by 0.05. The estimates lie consistently in the range between 0.2 and 0.3 percent. The growth effects predicted by the models for the same experiment are shown in table 3, where lump-sum transfers are adjusted to balance the government budget. For the IH model, the range of predicted growth effects (0.27 to 2.41 percent) is so wide as to be virtually uninformative. For the OLG model, in contrast, *all* values are close to Engen and Skinner's range (0.1 to 0.41 percent) despite the extreme range of parameters covered by the experiments.[22]

[INSERT TABLE 3 HERE]

In sum, abstracting from finite horizons and diminishing returns in human capital accumulation leads to a substantial overstatement of the growth effects of taxes and of their sensitivity to changes in parameter values. The growth effects predicted by life-cycle models, in contrast, are consistently close to the values suggested by empirical evidence. The extensive sensitivity analysis conducted in Hendricks (1999*a*) shows that these findings are robust under a wide range of parameters and assumptions.

5. Conclusion

Recent theories of endogenous growth suggest that changes in tax rates may permanently affect growth. However, attempts to quantify such growth effects in neoclassical growth models with human capital have reached very different conclusions. Growth effects were found to be highly sensitive to parameters governing preferences and human capital production, and the range of growth effects consistent with empirically plausible parameter values is very wide.

In deriving these results, the literature has relied throughout on models with infinitely lived agents. A more natural alternative, adopted by virtually the entire human capital literature, is a life-cycle model along the lines of Becker (1975) which allows to capture the characteristic life-cycle phases of human capital accumulation found in the data. The main reason why this alternative has not been pursued is a widely held belief that, in Lucas's words, the two frameworks "have very different theoretical structures, yet in practice, for the kind of tax problem under study here, seem to yield quite similar results" (Lucas 1990, pp. 295-6).

In contrast, this paper shows that the growth effects of taxes depend critically on two features that have been abstracted from in spite of being central to traditional human capital theory: finite horizons and diminishing returns in the production of human capital. Incorporating these features into a standard growth model leads to much sharper predictions about the growth effects of taxes. Eliminating all taxes in the U.S. increases the growth rate by between 0.5 and 1.3 percentage points, compared with a range of zero to four percent in the infinite horizon model used in the literature. The growth effects of smaller tax changes are robustly within the range suggested by the evidence of Engen and Skinner (1996).

The very different structure of the life-cycle model is reflected in very different determinants of growth effects. The main features responsible for smaller and more robust growth effects in the life-cycle model are precisely the ones that have been neglected in the literature: long, but finite horizons and diminishing returns to scale in the production of human capital. Once these are taken into account, the parameter choices found essential by SR are only of secondary importance, which greatly enhances the robustness of the result.

These results suggest that the relationship between life-cycle and infinite horizon models needs to be reconsidered. Abstracting from finite horizons is a common device for enhancing the tractability of models employed throughout macroeconomics. Examples include studies of the effects of taxation on aggregate human capital levels (Trostel 1993), or of the convergence properties of the neoclassical growth model (Mankiw 1995) and of endogenous growth models (Mulligan and Sala-i-Martin 1993; Ortigueira and Santos 1997). In light of the results presented here, it appears important to investigate the robustness of the findings in these areas when finite lifetimes are modeled explicitly.

6. References

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7. Appendix

7.1 The Household Problem With Finite Horizons and Diminishing Returns

It is useful to rewrite problem (P–2) in terms of stationary variables. Let $z = h/b$. The household then solves

$$\max \int_0^T e^{-rt} b_t [w z_t - (w+q) v_t z_t] dt \qquad \text{(P–3)}$$

subject to z_0 given

$$\dot{z}_t = B v_t^{\alpha} z_t^{\alpha} - (\gamma + \delta) z_t . \qquad (18)$$

Recall that γ is the constant growth rate of b_t. The law of motion (18) then follows from dividing (4) by b_t together with $\dot{z} = \dot{h}/b - \gamma z = (\dot{h}/h - \gamma) z$. The Hamiltonian is

$$\mathrm{H} = [w - (w+q) v_t] z_t + \lambda_t [B v_t^{\alpha} z_t^{\alpha} - (\gamma + \delta) z_t]$$

with an effective discount factor of $(r - \gamma)$. The necessary first order conditions are

$$-(w+q) z_t + \lambda_t G_v(t) \geq 0 \qquad (= 0, \text{if } v_t < 1)$$

$$\dot{\lambda}_t = (r - \gamma) \lambda_t - w + (w+q) v_t - \lambda_t [G_z(t) - \gamma]$$

with the boundary condition $\lambda_T = 0$. Applying functional form, this implies (10) in the text.

The first order conditions are sufficient, if the maximized Hamiltonian is concave in the state variable (Leonard and van Long 1992, theorem 4.6.4). Here, the value of v which maximizes the Hamiltonian is given by $(v_t z_t)^{1-\alpha} = \alpha B \lambda_t /(w+q)$, assuming that $\alpha < 1$. The maximized Hamiltonian is therefore

$$H_{\max} = w z_t - (\alpha B \lambda_t /[w+q])^{1/(1-\alpha)} + \lambda_t [B (\alpha B \lambda_t /[w+q])^{\alpha/(1-\alpha)} - (\gamma + \delta) z_t],$$

which is linear in h_t and therefore concave.

7.1.1 Phase Diagram

It is useful to draw the phase diagram in the $(z^{1-\alpha}, \lambda/[w+q])$-plane (see Figure 1). Consider first region II where $v < 1$. This requires $\lambda/(w+q) \leq z^{1-\alpha}/(\alpha B)$. The first-order condition for

v holds with equality: $v = [\lambda\alpha B/(w+q)]^{1/(1-\alpha)} z^{-1}$. Substituting this into the law of motion for λ and simplifying yields

$$\dot{\lambda} = (r+\delta)\lambda - w.$$

The stationary locus for λ is horizontal at $\lambda/(w+q) = (1-\tau)/(r+\delta)$. Substituting the optimal v into the law of motion for z yields

$$\dot{z} = B[\alpha B\lambda/(w+q)]^{\alpha/(1-\alpha)} - (\gamma+\delta)z$$

which implies a stationary level of z given by

$$z^{1-\alpha} = \left[\frac{B}{\gamma+\delta}\right]^{1-\alpha}\left[\alpha B\frac{\lambda}{w+q}\right]^{\alpha}.$$

This is shown as the convex $\dot{z}=0$ locus in the phase diagram.

Next consider region I. With v = 1 the laws of motion simplify to $\dot{\lambda} = (r+\delta-\alpha B z^{\alpha-1})\lambda + q$ and $\dot{z} = Bz^{\alpha} - (\gamma+\delta)z$. Therefore, z is increasing (decreasing) whenever $z^{1-\alpha}$ is less (greater) than B / ($\gamma + \delta$). This is illustrated by the $\dot{z}=0$ locus. It follows that the shaded region cannot be entered without violating the transversality condition. It is therefore sufficient to study the evolution of λ for $z^{1-\alpha} \le (1-\tau)\alpha B/(r+\delta)$. In this region $r+\delta-\alpha B z^{\alpha-1} < 0$ so that the expression for $\dot{\lambda}=0$ has a positive solution

$$\frac{\lambda}{w+q} = \frac{\tau}{\alpha B z^{\alpha-1} - r - \delta}.$$

This schedule intersects the region boundary where the $(\dot{\lambda}=0)$ schedule for phase II does. To the left, it lies below the region boundary so that λ is always falling. A number of implications are immediate. Phase I only occurs if the initial endowment is below $z_0^{1-\alpha} < (1-\tau)\alpha B/(r+\delta)$ and the horizon is sufficiently long. If phase I occurs, it does so at the beginning of life.

The steady state level of z is given by

$$z_{SS}^{1-\alpha} = \left[\frac{B}{\gamma+\delta}\right]^{1-\alpha}\left[\frac{\alpha B}{r+\delta}(1-\tau)\right]^{\alpha}. \tag{19}$$

It can never be reached, but for long horizons the household spends most of his time arbitrarily close to the steady state. This might be dubbed an *asymptotic turnpike*, in contrast to the model of Sheshinski (1968) in which the household actually reaches and stays on the turnpike for some time. This turnpike property turns out to be crucial for the growth effects of taxation.

The empirically interesting case has endowments below the steady state level, so that age-earnings profiles slope upward at the beginning of life. In this case two paths are possible: (1) For short horizons or large endowments, the household accumulates human capital as a part-time activity for his entire life. (2) If the horizon is longer or the endowment smaller, there will be an additional phase of full-time training at the beginning of life.

7.1.2 Solving the Household Problem

A solution to the household problem involves time paths for z_t, v_t and λ_t that satisfy the boundary conditions z_0 given and $\lambda_T = 0$. By a change of variable it is possible to explicitly integrate the law of motion for z in phase I:[23]

$$z_t^{1-\alpha} = \frac{B}{\gamma+\delta} + e^{-(1-\alpha)(\gamma+\delta)t}\left[z_0^{1-\alpha} - \frac{B}{\gamma+\delta}\right].$$

Integrating during phase II implies

$$\lambda_t = \frac{w}{r+\delta}\left[1 - e^{-(r+\delta)(T-t)}\right].$$

Substituting this into the law of motion for z and integrating yields

$$z_t = e^{-(\gamma+\delta)(t-s)}\left[z_s + C_1 \int_s^t e^{(\gamma+\delta)(\nu-s)}\,[1-e^{-(r+\delta)(T-\nu)}]^{\alpha/(1-\alpha)}\,d\nu\right], \tag{20}$$

where $C_1 \equiv B\left[(1-\tau)\,\alpha\, B/(r+\delta)\right]^{\alpha/(1-\alpha)}$ and s denotes the switching time which is determined by equating λ_s for both phases. For phase I,

$$\frac{\lambda_s}{w+q} = \frac{z_s^{1-\alpha}}{\alpha B} = \frac{e^{-(1-\alpha)(\gamma+\delta)s}}{\alpha B}\left[z_0^{1-\alpha} - \frac{B}{\gamma+\delta}\right] + \frac{1}{\alpha(\gamma+\delta)}. \tag{21}$$

For phase II,

$$\frac{\lambda_s}{w+q} = \frac{1-\tau}{r+\delta}\left[1 - e^{-(r+\delta)(T-s)}\right]. \tag{22}$$

A closed form solution for s does not exist.

7.2 Growth Effects With α = 0.5

A special case allows to explicitly solve for the growth effects of taxation without the assumption that households spend most of their time close to the turnpike. Assume that $\alpha = ½$, $t^* = T$, and that v is interior at all times. Integrating the law of motion for z yields

$$z_T = z_0 \, e^{-(\gamma+\delta)T} + C_1 \, Q_T \, ,$$

where
$$Q_T = \frac{1-e^{-(\gamma+\delta)T}}{\gamma+\delta} - \frac{1-e^{-(r+\gamma+2\delta)T}}{r+\gamma+2\delta} > 0 \, .$$

The equilibrium condition with $t^* = T$ is then

$$e^{(g-\gamma)T} \, z_0 = e^{-(\gamma+\delta)T} \, z_0 + C_1 \, Q_T \, . \tag{23}$$

Assume that γ is a general function of g that satisfies $0 \le \gamma < g$, $\zeta = d\gamma / d g$ and $0 \le \zeta \le 1$. Implicit differentiation then implies

$$-\frac{d g}{d \tau} = \frac{1/(1-\tau)}{\dfrac{\sigma}{r+\delta} - \dfrac{\partial Q}{\partial g}\dfrac{1}{Q} + \dfrac{z_0 \, T}{C_1 \, Q}\left[(1-\zeta) \, e^{(g-\gamma)T} + \zeta \, e^{-(\gamma+\delta)T}\right]}$$

Using (23) to substitute out $C_1 \, Q$ and simplifying yields

$$-\frac{d g}{d \tau} = \frac{1/(1-\tau)}{\dfrac{\sigma}{r+\delta} - \dfrac{\partial Q}{\partial g}\dfrac{1}{Q} + \dfrac{(1-\zeta) \, e^{(g-\gamma)T} - \zeta \, e^{-(\gamma+\delta)T}}{e^{(g-\gamma)T} - e^{-(\gamma+\delta)T}} T} \, .$$

As $T \to \infty$, $(\partial Q / \partial g) / Q$ remains bounded. Therefore, growth effects approach zero except in the knife-edge cases $\zeta = 1$ or $\gamma = g$. The assumption that human capital is transferred when parents are close to the turnpike is therefore not essential.

The intuition rests on the fact that the deviation of z_T from z_{SS} is bounded. Note that $C_1 = (\gamma+\delta) \, z_{SS}$ and thus, for long T, $z_T \approx z_{SS} \, (\gamma+\delta) \, Q_T$. Since the change in Q_T is bounded, so is the change of z_T / z_{SS}. Since the relative change of z_{SS} is independent of horizon (even adjusting B to maintain the same g^* for the initial tax rate), it follows that the change in z_T is bounded as well. But for long horizons, a bounded change in z translates into small growth effects.

In the special case when γ is exogenous ($\zeta = 0$),

$$\frac{\partial Q}{\partial g}\frac{1}{Q}=\frac{\sigma}{(r+\gamma+2\delta)^2}\frac{1-[1+(r+\gamma+2\delta)T]\,e^{-(r+\gamma+2\delta)T}}{\dfrac{1-e^{-(\gamma+\delta)T}}{\gamma+\delta}-\dfrac{1-e^{-(r+\gamma+2\delta)T}}{r+\gamma+2\delta}}>0$$

and it is again the case that as $T \to 0$ growth effects approach the infinite horizon, constant returns level . Growth effects are smaller than in the infinite horizon case iff

$$\frac{(g+\delta)T}{1-e^{-(g+\delta)T}}-(g+\delta)\frac{\partial Q}{\partial g}\frac{1}{Q}>1\,.$$

Since $\partial Q/\partial g \cdot 1/Q$ is bounded above, it is apparent that growth effects fall below the infinite horizon level for some finite T and continue to decline beyond it. The intuition is rather similar to the case analyzed in the main text. Because of the turnpike property, z_T is bounded. A longer horizon implies a smaller average growth rate of z over the parent's lifetime and therefore a smaller per period growth rate of the endowment across generations.

7.3 Internalized Intergenerational Spillovers

The results derived here do not depend on the assumption that parents fail to internalize the intergenerational spillover in human capital. To see this, consider a version of the model that is as close as possible to the infinite horizon setup. Children are born at $t^* = T$ and inherit a fraction ε of their parents' human capital. Also assume that $\gamma=\zeta\, g$ with $\zeta < 1$ ($\zeta = 1$ is the infinite horizon case). The household now solves (P–3) with a modified objective function:

$$V(z_0,b_0)=\max\int_0^T e^{-rt}\,b_0\,e^{\gamma t}\,[w-(w+q)\,v_t\,]\,z_t\,dt+e^{-rT}\,V(z_0',b_0')\,.$$

The household takes the learning productivity of the child as given, $b_0'=b_0\,e^{gT}$, but knows that it can influence the child's human capital endowment according to

$$z_0'=\frac{h_0'}{b_0'}=\frac{\varepsilon\, h_T}{b_0\,e^{gT}}=\varepsilon\, z_T\, e^{(\gamma-g)T}\,.$$

Thus,

$$V(z_0,b_0)/b_0=\max\int_0^T e^{-(r-\gamma)t}\,[w-(w+q)\,v_t\,]\,z_t\,dt+e^{(g-r)T}\,V(z_0',b_0')/b_0'\,,$$

where primes indicate variables pertaining to the child. Expanding the right hand side shows that $V(z_0,b_0)/b_0$ is independent of b_0. That is, maximized earnings are homogeneous of degree one in h_0 and b_0.

The necessary conditions for this problem are the same as for (P–3), except for the transversality condition that determines the terminal value of λ, which becomes

$$e^{(\gamma-r)T}\,\lambda_T = e^{(g-r)T}\,\frac{\partial V(\varepsilon\, z_T\, e^{(\gamma-g)T}, b_0')/b_0'}{\partial\, z_T}.$$

Since the derivative of the value function is λ_0, $e^{(\gamma-r)T}\,\lambda_T = e^{(g-r)T}\,\lambda_0'\,\varepsilon\, e^{(\gamma-g)T}$ and therefore $\lambda_T = \varepsilon\,\lambda_0'$. In this stationary problem the shadow prices do not vary by generation, so that the transversality condition becomes $\lambda_T = \varepsilon\,\lambda_0$.

The phase diagram in Figure 1 is unchanged. However, any optimal path now has the property that $\lambda_T = \varepsilon\,\lambda_0$. Assume, as in the infinite horizon case, $\varepsilon = 1$. For long enough horizons, any optimal path must therefore cross the region boundary so as to enter the shaded area. The household then stays close to the turnpike most of the time. Near the end of its life, λ increases and the household accumulates additional human capital (z rises). The intuition is that the child will be more productive in accumulating human capital because $b_0' > b_T$, which makes it more valuable to acquire human capital shortly before death than in the middle of life.

To establish that growth effects fall to zero as $T \to \infty$, it is sufficient to show: (*i*) The relative change of the steady state level z_{SS} is uniformly bounded for all T; (*ii*) for long horizons, z_T / z_{SS} is uniformly bounded. The balanced growth condition $e^{(g-\gamma)t^*} = \varepsilon\, z_{t^*}/z_0$ together with $t^* = T$ then implies $dg\,(1-\zeta) = d\ln(z_T)/T$, which falls to zero as $T \to \infty$.

The first part of the argument follows immediately from (19). The relative change of z_{SS} is

$$d\ln(z_{SS}) = -d\ln(\gamma+\delta) + \frac{\alpha}{1-\alpha}\, d\ln([1-\tau]/[r+\delta]),$$

which is bounded because the change in the growth rate that determines γ and r is bounded above by the maximum feasible growth rate. For long horizons, a uniform lower bound for z_T / z_{SS} is 1 from the phase diagram. An upper bound can be derived from the maximum attainable level of z that is achieved by setting $v = 1$ forever: $z^{1-\alpha} \le z_{\max}^{1-\alpha} = B/(\gamma+\delta)$. Thus,

$$(z_T / z_{SS})^{1-\alpha} \le \left[\frac{r+\delta}{(1-\tau)\,\alpha\,(\gamma+\delta)}\right]^{\alpha}.$$

Since r is uniformly bounded via the maximum sustainable growth rate, so is z_T / z_{SS}. Therefore, growth effects fall to zero for long horizons.

7.4 Growth Effects With Finite Horizons and Constant Returns

The household solves problem (P–2), which is stated in its generic form because the first-order conditions will be valid for the case of diminishing returns as well. Write the Hamiltonian as

$$\mathrm{H}_t = w_t\, h_t - (w_t + q_t)\, h_t + \lambda_t G(h_t, v_t, b_t)\,. \tag{24}$$

The necessary conditions for an optimal path are

$$\lambda_T = 0 \tag{25}$$

$$\dot{\lambda}_t = (r_t - G_h(t))\,\lambda_t - w_t + (w_t + q_t)\, v_t \tag{26}$$

and v_t maximizes the Hamiltonian:

$$\lambda_t\, G_v(t) - (w_t + q_t)\, h_t \begin{cases} > 0 & \Rightarrow \quad v_t = 1 \\ = 0 & if \quad 0 < v_t < 1 \\ < 0 & \Rightarrow \quad v_t = 0 \end{cases} \tag{27}$$

To ease notation, functions are written with time as their argument. Let s denote the age at which the household switches from schooling to work. The case $s = 0$ is, of course, uninteresting and will be ignored in what follows. With linear G, (26) and (27) simplify to

$$\dot{\lambda}_t = (r + \delta - B\, v_t)\,\lambda_t - w_t + (w_t + q_t)\, v_t$$

$$\lambda_t\, B - (w + q) \begin{cases} > 0 \Rightarrow v_t = 1 \\ < 0 \Rightarrow v_t = 0 \end{cases}$$

Switching occurs when $\lambda_s = (w + q)/B$. During Phase I,

$$0 \le t \le s;\ v_t = 1;\ h_t = h_0 e^{(B-\delta)t}$$

$$\dot{\lambda}_t = (r + \delta - B)\,\lambda_t + q\,.$$

During Phase II: $$s < t \le T; v_t = 0; h_t = h_s e^{-\delta(t-s)}$$

$$\dot{\lambda}_t = (r + \delta)\,\lambda_t - w\,, \tag{28}$$

which integrates to $\lambda_t = e^{(r+\delta)(t-s)}[\lambda_s - \frac{w}{r+\delta}(1 - e^{-(r+\delta)(t-s)})]$. Imposing $\lambda_T = 0$ and substituting out λ_s implies

$$0 = e^{(r+\delta)(T-s)}\left[\frac{w+q}{B} - \frac{w}{r+\delta}(1-e^{-(r+\delta)(T-s)})\right].$$

Since $(1-\tau) = w/(w+q)$, the switching time obeys

$$s = T + \frac{\ln\{1-(r+\delta)/[(1-\tau)B]\}}{r+\delta} \tag{29}$$

To close the model, assume that each generation inherits the human capital stock of the previous one at death (exactly analogous to the infinite horizon case). Then $h_T / h_0 = e^{gT}$. Since $h_T = h_0 e^{Bs-\delta T}$, we have $(g+\delta)T = Bs$ or

$$s/T = (g+\delta)/B. \tag{30}$$

Equations (29) and (30) together with $r = \rho + \sigma g$ imply expression (16) in the text. To derive the growth effect, note that the exponential term (and its derivative) become negligible in the limit, so that only the term $(r+\delta)/[(1-\tau)B]$ needs to be implicitly differentiated taking into account that $r = \rho + \sigma g^*$.

The growth effect then coincides with the infinite horizon case. In this well-known case it is easy to show that the balanced growth rate is determined by

$$g = \frac{(1-\tau)B - \delta - \rho}{\sigma}.$$

If the growth effect of taxation is measured by $-\, d\,g\,/\,d\,\tau$, this obviously equals $B\,/\,\sigma$. For comparisons across models, it is necessary to "calibrate" B to match a target growth rate g^*:

$$B = \frac{\sigma g^* + \delta + \rho}{1-\tau}$$

The growth effect then becomes

$$-\frac{dg}{d\tau} = \frac{\sigma g^* + \delta + \rho}{(1-\tau)\sigma} = \frac{r+\delta}{(1-\tau)\sigma} \tag{31}$$

which exhibits the well-known sensitivity to the choice of σ.

7.5 Growth Effects With Infinite Horizons and Diminishing Returns

The infinite horizon formulation gives little choice about how to specify b: it must grow at the same rate as h in order to facilitate balanced growth. Assume that b equals average human capital in the economy, $\bar{h}$, which in equilibrium must equal h, but the household takes it as given. The asymmetry between human and physical capital occurs again, this time because of diminishing returns in G: the household chooses a path of human capital so as to maximize the present value of net earnings. Thus he solves (P–2) with $T = \infty$ and $b = \bar{h}$.

The Hamiltonian is given by $\mathrm{H} = wh - (w+q)vh + \lambda[B(vh)^{\alpha}\,\bar{h}^{1-\alpha} - \delta h]$. Maximization implies the first order conditions

$$\lambda = v^{1-\alpha}(w+q)/(\alpha B) \tag{32}$$

$$\dot{\lambda} = [r + \delta - \alpha B v^{\alpha} z^{\alpha-1}]\lambda - w + (w+q)v \tag{33}$$

$$\dot{z} = B(vz)^{\alpha} - (g+\delta)z\,, \tag{34}$$

where $z \equiv h/\bar{h}$ equals one in equilibrium. On a balanced growth path, it follows that λ is constant as well. Solving (34) for v yields $v^{\alpha} = (g+\delta)/B$. Imposing $\dot{\lambda} = 0$ in (33) implies

$$\lambda = \frac{w - (w+q)v}{r + \delta - \alpha B v^{\alpha}}\,. \tag{35}$$

Equating (32) and (35) and substituting in the solution for v yields (15) in the text.

[1] The micro econometric evidence presented in Haley (1976) and Heckman (1976) also clearly rejects the hypothesis of constant returns to scale in the production of human capital.

[2] Lucas (1990, pp. 295-6). See also Trostel (1993, p. 331) and Mankiw (1995, p. 279).

[3] While a sizeable literature studies the growth effects of taxes in infinite horizon models (SR provide a survey), little work has been done in the context of overlapping generations models. Rangazas (1996) investigates the growth effects of redistributional taxes in an economy where credit constraints prevent efficient amounts of schooling investment. Most other finite horizon work addresses level effects, as opposed to growth effects (e.g., Davies and Whalley 1991; Perroni 1995).

[4] Allowing for population growth would not affect any of the qualitative results.

[5] If time inputs in training are not tax-deductible, the government collects in addition $\tau_L F_L \int v h$.

[6] See section 2.6 for details.

[7] Hendricks (1999*c*) studies the cases $h_0^{i+t*} = \varepsilon h_{t*}^i$ with $b_0^i = \bar{h}_i$ and $h_0^i = \varepsilon \bar{h}_i$ with $b_0^i = h_0^i$. The fourth combination of possibilities ($h_0^i = \varepsilon \bar{h}_i$ with $b_0^i = \bar{h}_i$) coincides with the third.

[8] Capital markets are perfect in this paper. Once the endowment is in place, both parents and children would choose the level of human capital investment that maximizes lifetime earnings of the child net of education costs (see Becker 1975).

[9] See Stokey and Rebelo (1995) for more detail and for an analysis of models where this is not the case. Of course, the production technology for capital goods must satisfy conditions that facilitate sustained growth.

[10] It is interesting to note that the optimal path changes again qualitatively when the technology is of the form $G = Bvh^{\alpha}b^{1-\alpha} - \delta h$. In that case, the turnpike is actually reached in finite time, and the household remains there until a fixed interval before death at which point it switches to a corner solution with $v = 0$. Sheshinski (1968) shows a similar result in a model where human capital is not self-productive.

[11] Since the difference $z_{t^*} - z_{SS}$ is uniformly bounded in t^*, the assumption that z_{t^*} is close to the turnpike is not essential, but it greatly simplifies the algebra. Appendix 7.2 relaxes this assumption.

[12] The two classical studies by Haley (1976) and Heckman (1976) both find returns to scale of around 0.55.

[13] See, for example, Lucas (1990, pp. 295-6), Trostel (1993, p. 331), or Mankiw (1995, p. 279).

[14] As always, B is “calibrated” so as to maintain the growth rate g^* in the initial steady state.

[15] The parameters underlying this example are $\rho = 0.01$, $\sigma = 2$, $\delta = 0.03$, $\tau = 0.4$, $g^* = 0.02$, $t^* = T$.

[16] Endogenous leisure breaks the symmetry, but tends to be quantitatively unimportant.

[17] For computational reasons, the model is solved in discrete time. A complete description of parameter choices, data sources, and the algorithm is provided in Hendricks (1999*b*).

[18] This is above the point estimates reported in Haley (1976) and Heckman (1976) and therefore likely to understate the importance of diminishing returns.

[19] It is shown in Hendricks (1999*a*) that altruism makes little difference, even if parents internalize the intergenerational link in human capital.

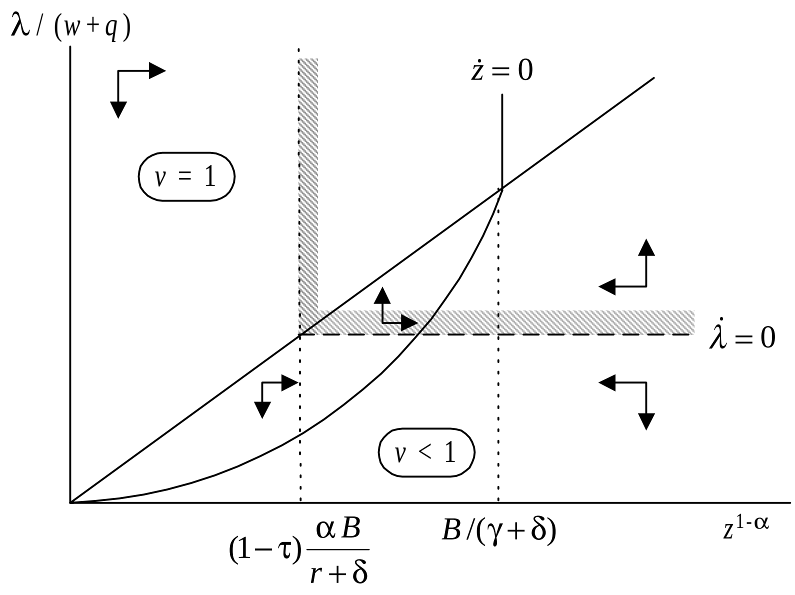
[20] These are the upper bounds of the ranges given in SR, p. 543.

[21] SR (p. 543) show that growth effects of 2.5 percent are plausible, even if labor supply is assumed to be fixed.

[22] In the assessment of this finding it should be kept in mind, however, that the findings in other empirical studies are often mixed. For example, in Easterly and Rebelo (1993) the association between tax rates and growth is almost never statistically significant. In the structural model of Perotti (1996) the association is even positive (Benabou 1996, table 2 summarizes additional evidence).

[23] I am grateful to Andras Löffler for pointing out this solution method.

Figures



Review of Economic Dynamics

"Growth, Death, and Taxes" by Lutz Hendricks

Figure 1

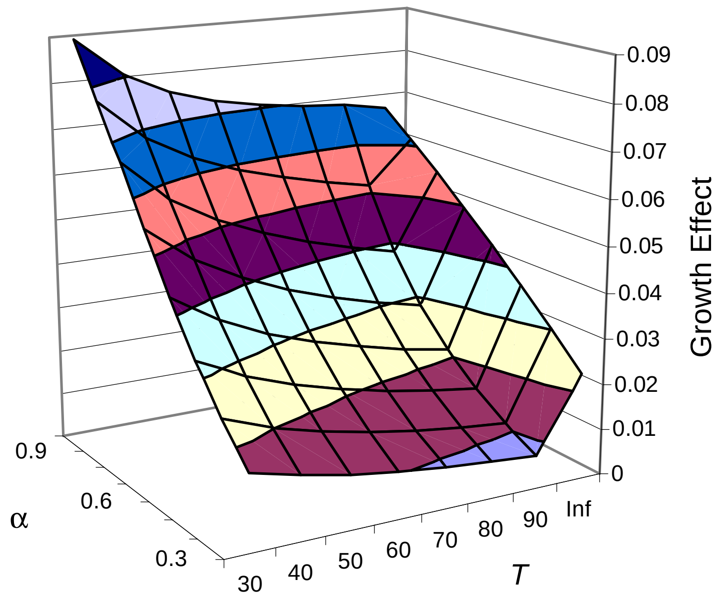


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Figure 2

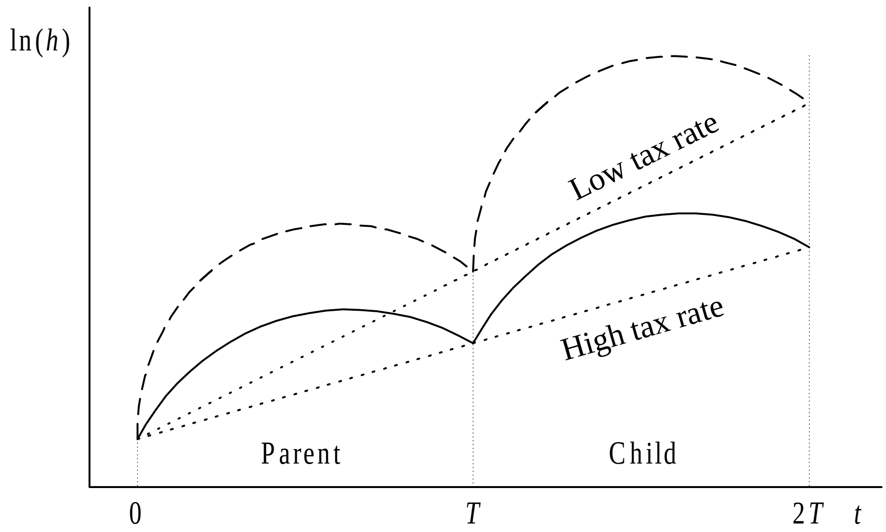

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Figure 3

Figure Legends

Figure 1. Phase diagram

Figure 2. Growth effects with $\gamma = 0$

Figure 3. Growth effects with finite horizons

Tables

Table 1. Baseline parameters

Preferences		Technology	
$\sigma = 1.1$	JMR	$A = 1$	Normalization
ω	Matches leisure share or labor supply response	$\theta = 0.36$	JMR
ρ	Matches capital-output ratio of 2.5	$\delta_K = 0.06$	SR, p. 543
Human Capital		**Goverment**	
$\alpha = \varphi = 0.35$	Haley (1976), Heckman (1976), Mincer (1993)	$\tau_L = 0.31$ $\tau_K = 0.21$	JMR
$\delta_h = 0.01$	SR, p. 543 (0.04 in IH model)	$\overline{C} / Y = 0.2$	JMR

Table 2. Growth effects of eliminating all taxes

Growth Effects	Conservative	Baseline I	Baseline II	Aggressive
OLG, $\gamma = 0$	0.64	0.97	1.08	1.29
OLG, $\gamma = g$	0.68	1.14	1.29	1.70
IH, $\alpha = \varphi = 0.35$	0.90	2.20	2.44	3.87
IH, $\alpha = \varphi = 0.5$	1.14	3.60	4.15	8.33
Changes in labor supply				
OLG, $\gamma = 0$	6.0	5.5	7.3	7.6
OLG, $\gamma = g$	6.7	7.3	9.5	11.5
IH, $\alpha = \varphi = 0.35$	5.0	8.5	12.1	25.9
IH, $\alpha = \varphi = 0.5$	5.9	10.0	15.0	45.3

Note: The table shows percentage changes of the steady state growth rates and levels of hours worked after eliminating all taxes for infinite horizon (IH) and overlapping generations (OLG) models. The initial tax rates are shown in table 1.

Table 3. Growth effects of 5 percent tax cut

Model	Conservative	Baseline I	Baseline II	Aggressive
OLG, $\gamma = 0$	0.10	0.20	0.25	0.31
OLG, $\gamma = g$	0.10	0.22	0.29	0.41
IH, $\alpha = \varphi = 0.35$	0.27	0.53	0.62	1.10
IH, $\alpha = \varphi = 0.5$	0.36	0.87	1.05	2.41

Note: The table shows percentage changes of the steady state growth rate after reducing all taxes by 0.05. The initial tax rates are shown in table 1. Empirical estimates by Engen and Skinner (1996) for this experiment are changes in the growth rate between 0.2 and 0.3 percentage points.