Applying the Solow Model

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Topics

We apply the Solow model to study:

1. Cross-country income differences
2. Cross-country variation in growth rates
3. Growth with non-renewable resources

Why Are Some Countries Rich and Others Poor?

What does the Solow model tells us about why some countries are much richer than others?

How important are:

- physical capital?
- human capital?
- productivity?

To analyze this: assume countries are close to steady state.

Solving for the steady state

- Recall the law of motion

$$\dot{k} = sf(k) - (n+\delta)k \tag{1}$$

  where $f(k) = A^{1-\alpha}k^{\alpha}$.
- For simplicity: we abstract from productivity growth.
- Exercise: repeat the analysis with productivity growth.

Solving for the steady state

- Impose steady state: $\dot{k} = 0$:

$$sy = (n+\delta)k \tag{2}$$

- The capital-output ratio is

$$k/y = \frac{s}{n+\delta} \tag{3}$$

- Apply production function

$$sA^{1-\alpha}k^{\alpha} = (n+\delta)k \tag{4}$$

- Solve for k

$$k = A\left(\frac{s}{n+\delta}\right)^{1/(1-\alpha)} \tag{5}$$

Steady state output

One approach: substitute production function into steady state k.

Easier:

$$y = (y/k) \times k \tag{6}$$

$$= \underbrace{\frac{s}{n+\delta}}_{y/k} \times \underbrace{A\left(\frac{s}{n+\delta}\right)^{1/(1-\alpha)}}_{k}$$

Collect terms:

$$y = A\left[\frac{s}{n+\delta}\right]^{\alpha/(1-\alpha)} \tag{7}$$

Reality check

A key prediction of the model: $k/y = s/(n+\delta)$.

Countries with higher saving rates have higher capital output ratios.

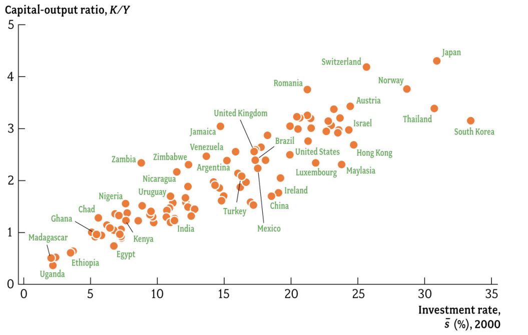


FIGURE 5.3 Explaining Capital in the Solow Model

Macroeconomics, Charles I. Jones

Why does Y/L differ across countries?

- Our static production model answered: K/L and A differ:

$$y = A^{1-\alpha} k^{\alpha} \tag{8}$$

- The Solow model gives a similar answer: s and A differ:

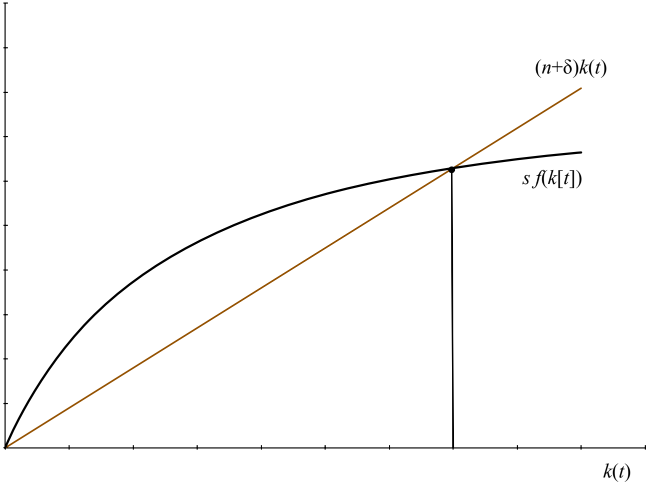
$$y^* = A \left[\frac{s}{n+\delta} \right]^{\alpha/(1-\alpha)} \tag{9}$$

- This is the same answer in disguise: higher s means higher K/L.

Why does Y/L differ across countries?

- But the answer is **quantitatively** different:
  - Production model: Double $A \rightarrow$ raise y by $2^{1-\alpha} = 2^{2/3} = 1.59$.
  - Solow model: Double $A \rightarrow$ double y^*.
- In the Solow model, the contribution of A to output gaps is larger - why?
- Draw a picture...

Saving and Output



Thought experiment: A rises.

How important is K/L for cross-country Y/L gaps?

Our previous answer was: K/L accounts for a factor near 4.

In the Solow model:

$$\frac{y^*_{US}}{y^*_{poor}} = \left(\frac{\bar{A}_{US}}{\bar{A}_{poor}}\right)\left(\frac{s_{US}}{s_{poor}}\right)^{1/2} \tag{10}$$

$$32 = 16 \times 2 \tag{11}$$

Why factor 2 for saving rates?

- Because s (or K/Y) differs by a factor near 4.
- The ratio of saving rates is taken to the power $\alpha/(1-\alpha)$

How important is K/L for cross-country Y/L gaps?

This is a central and robust result:

> ***Capital accumulation accounts for only a small fraction of cross-country income gaps.***

Exercise

How would this result change for higher values of α?

Consider a 4-fold increase in s.

Calculate the effect on y and graph it.

Long-run Growth

Long-run Growth

What does the Solow model imply for long-run growth?

Main result

The principle of transition dynamics

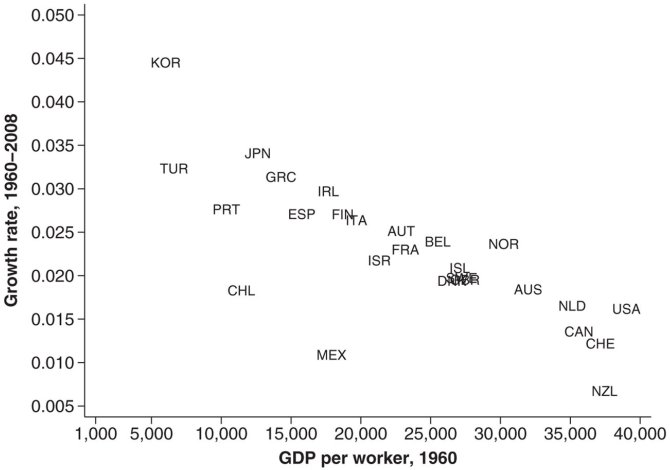
Countries grow faster when they are far below their steady state.

What is the evidence for this?

- One exercise: if countries have similar steady states, their income levels should converge over time
- initially poor countries should grow faster

Convergence: Evidence

FIGURE 3.5 CONVERGENCE IN THE OECD, 1960–2008



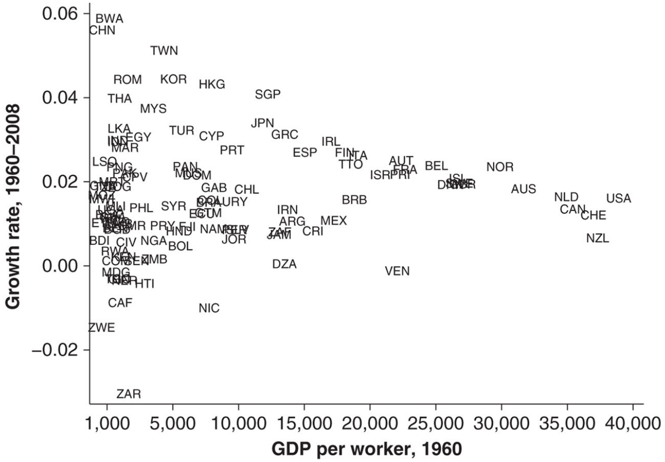
Among OECD countries: those that were initially poor grew faster.

Empirical Evidence

- Should we conclude that transitional growth explains cross-country differences in output growth?
- No!
- Figure 5.8 only shows OECD countries - mostly rich Western European countries + North America.

Empirical Evidence

FIGURE 3.6 **THE LACK OF CONVERGENCE FOR THE WORLD, 1960–2008**



No luck for a broad set of countries.

Empirical Evidence

- But figure 5.9 is the wrong experiment!
- The Solow model does not say: "poor countries grow faster"
- It says: "countries that are poor **relative to their steady states** grow faster."
- That is true in the data.

Empirical Evidence

Exercise

For a set of countries gather data on s, n.

Compute steady state output: y^*

Compute output in 1960 relative to steady state: y/y^*

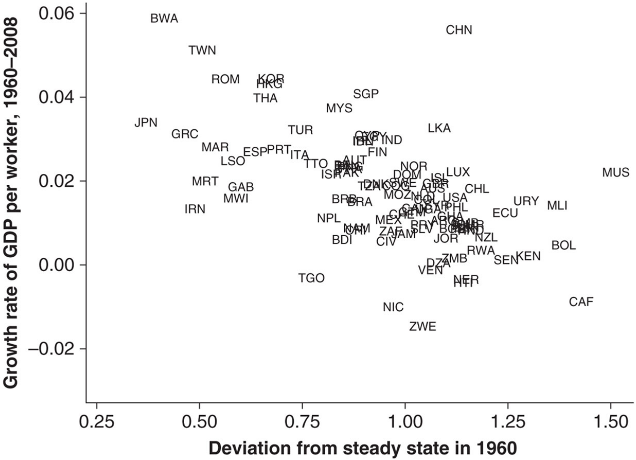
Compute average growth 1960-2000

Plot average growth against y/y^*

What do you expect to find?

Conditional Convergence

FIGURE 3.8 "CONDITIONAL" CONVERGENCE FOR THE WORLD, 1960–2008



Source: Jones (2013)

Convergence Clubs?

- One hypothesis one often reads:
  - Countries that share similar "fundamentals" (e.g. OECD countries) converge to similar steady states.
  - For them the principle of transition dynamics explains post-war growth.
  - Other countries have different fundamentals, so the poorer ones need not grow faster.
- This is the **wrong explanation** for post-war growth.

Convergence Clubs?

- The Solow model makes a quantitative prediction about growth rates.
- If you work out the numbers, countries should converge fairly quickly to their steady states (perhaps within 20 years).
- Then they all should grow at almost the same rates.

Fact

The Solow model cannot explain why countries grow at different rates for long periods of time.

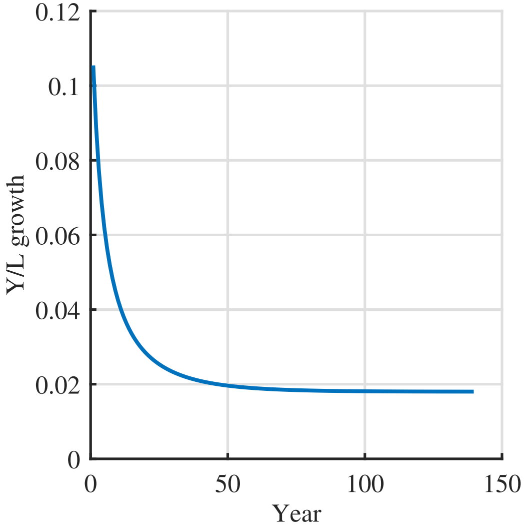
- Post-war growth may look like Solow convergence to a common steady state, but it is not.
- Post-war growth is driven by growth in TFP, not by growth in K/L.

Exercise

- Take a spreadsheet.
- Fix parameters at plausible empirical values: $\alpha = 1/3$, $d = 0.08$, $s = 0.2$.
- Compute the steady state.
- Fix K_0 at some multiple of K^*.
- Compute the transition path for K_t by iterating over $K_{t+1} = sY_t + (1-d)K_t$.
- Plot the growth rate of Y_t against time.
  - You should see that growth is very high initially, if K_0 is small. But growth slows dramatically very quickly.
- Now plot the growth rate of Y_t against over 40 years against initial Y_t - this is the model analogue of figure 5.8.
  - You should see that the model relationship is much flatter than the data relationship.
  - The model fails to explain large variation in 40 year average growth rates.

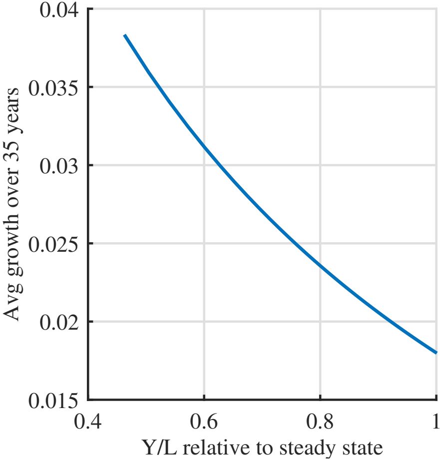
Simulating the Solow Model

Example: Start an economy from 1/10th of steady state k^*.



Simulating the Solow Model

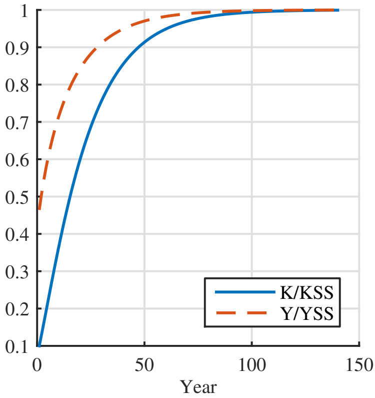
Growth varies less with y/y^* than in the data.



This suggests that part of growth is not transitional in the data.

Simulating the Solow Model

Speed of convergence



Convergence is too fast.

In the data, the "**half-life**" is about 30 years – 10 years in the model.

Convergence is even faster when the saving rate is endogenous.

Did we just invalidate the Principle of Transition Dynamics?

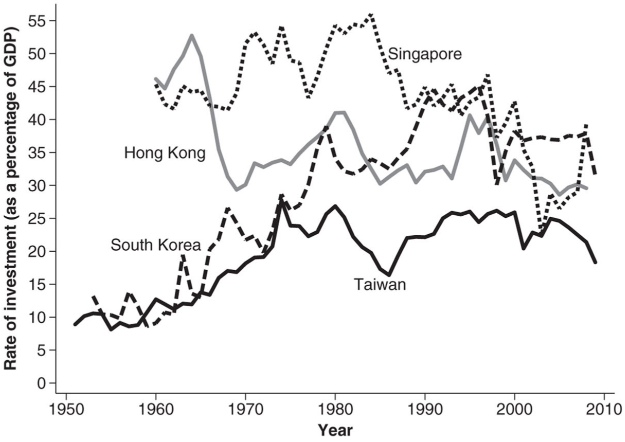
- No, we did not.
- Countries grows faster when their capital stocks are low.
- But this does not account for the observed differences in long-run (40 year) growth rates across countries.
- It does account for growth rates at business cycle frequencies.

The Tigers

- There are a few countries that sustained growth by capital accumulation for a long period of time.
- How?
- It cannot work with a constant saving rate s - the Solow model shows this.
- Such countries must have saving rates that **rise over time**.
- Examples are: South Korea, Singapore, Hong-Kong.

The Tigers

FIGURE 2.14 **INVESTMENT RATES IN SOME NEWLY INDUSTRIALIZING ECONOMIES**



Source: Jones (2013)

Exercise: Rising saving rate

- Simulate the Solow model with a saving rate that rises from 10% to 40% (Singapore).
- Start the model in steady state: $K_0 = K^*$.
- Show that the growth rate of y stays positive for a long time.
- You could now compare that growth path with data for Singapore and convince yourself that a large share of Singapore's spectacular growth since 1960 is indeed due to capital accumulation (as shown by Alwyn Young).

Non-renewable Resources

Non-renewable Resources

What happens when production uses essential resources that are in fixed supply?

Modify the Solow model as follows:

1. The economy is endowed with a resource stock R_0.
2. It digs up R at a rate of E:

$$\dot{R} = -E \tag{12}$$

3. The rate of extraction is constant:

$$E = s_E R \tag{13}$$

4. E is used in production:

$$Y = BK^{\alpha}E^{\gamma}L^{1-\alpha-\gamma} \tag{14}$$

Everything else is unchanged

The Solow Law of Motion

$\dot{R} = -E = -s_E R$ implies

$$R(t) = R_0 e^{-s_E t} \tag{15}$$

The stock is depleted at a constant exponential rate.

Therefore, resource input is declining exponentially:

$$E(t) = s_E R(t) = s_E R_0 e^{-s_E t} \tag{16}$$

In the limit, $E(t) \to 0$, which does not look promising

Balanced Growth Path

From $\dot{K} = sY - \delta K$, it follows that K/Y converges to a constant. Output is given by

$$Y^{1-\alpha} = B\left(K/Y\right)^{\alpha} \underbrace{\left(s_E R_0 e^{-s_t t}\right)^{\gamma}}_{E} L^{1-\alpha-\gamma} \tag{17}$$

Take growth rates:

$$(1-\alpha)\,g(Y) = g(B) - \gamma s_E + (1-\alpha-\gamma)\,n \tag{18}$$

Or in per capita terms:

$$g(y) = \frac{g(B)}{1-\alpha} - \frac{\gamma}{1-\alpha}\left(s_E + n\right) \tag{19}$$

Interpretation: faster resource extraction permantly slows down growth.

Intuition

Output per worker is

$$y = Bk^{\alpha}\left(E/L\right)^{\gamma} \tag{20}$$

Population growth has the same effect as in the Solow model: capital dilution.

E shows up as negative productivity growth

$$y = \left(B\left(E/L\right)^{\gamma}\right)k^{\alpha} \tag{21}$$

with growth rate of the productivity term given by

$$g\left(B\left(E/L\right)^{\gamma}\right) = g\left(B\right) - \gamma\left(s_E + n\right) \tag{22}$$

Therefore: non-renewable resources have the same effect as slower productivity growth.

How Big Is the Drag on Growth?

We need parameter values for α, γ, s_E.

Key assumption: factors (including E) are paid their marginal products.

Then: α is the capital share (as before); γ is the share of renewables.

Empirical estimates (Nordhaus et al., 1992):

- $\alpha = 0.2$
- $\gamma = 0.1$
- there is also a fixed factor (land) with a share of 0.1.
- $s_E = 1/200$
- $n = 0.01$

The growth drag is then

$$\frac{0.1n + (\gamma + n)\, s_E}{1 - \alpha} = 0.3\% \tag{23}$$

Resource Prices

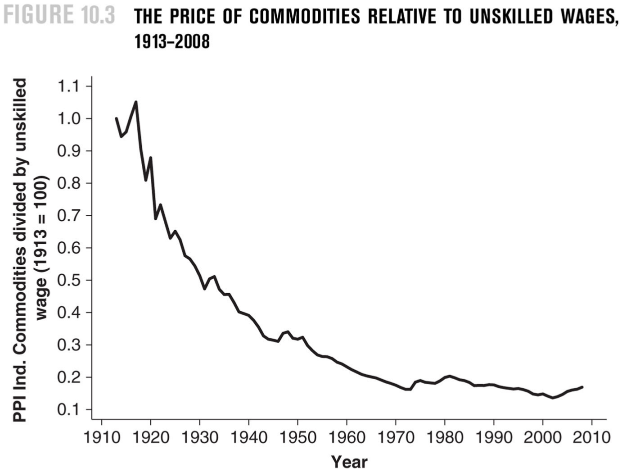
If this model is correct, the relative price of resources should rise over time.

Intuition:

- the income share of resources is constant (γ)
- $\gamma/(1-\alpha) = (P_E E)/(wL)$ should be constant
- E/L is falling, so P_E/w should be rising

Evidence: resource prices are **falling** instead.

Resource Prices



Source: Jones (2013)

Implication: the share of renewables γ must be falling over time.

Why Is the Renewables Share Declining?

One possibility: renewables and other inputs are **highly substitutable**.

- using less E then reduces its income share (its price does not rise much)
- buth then its price has been falling, not rising

Resource conserving technical change

- even though E declines over time, its efficiency rises
- directed technical change

Conclusion:

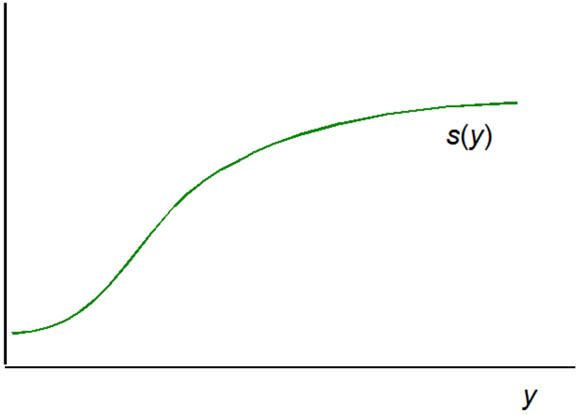
- the direct growth drag from non-renewables is not likely large

Discussion

What is missing in this discussion?

Exercise: The Saving Rate Depends on Income

- Consider an alternative version of the Solow model.
- The saving rate depends on income.
- What happens?



Conclusion: Is the Solow Model Useful?

- As a model of growth or large cross-country income differences, the model is a failure.
- But its failure contains important insights:
  1. Capital does not drive growth.
  2. Capital does not drive large fractions of cross-country income gaps.
- Both findings are surprising - and often not understood in the policy debate.

Conclusion: Is the Solow Model Useful?

- But the main significance of the Solow model itself is as a **building block** for macro models.
- We always have to keep track of how capital is accumulated.
- A Solow block is therefore part of virtually every model.
- The same logic extends to other accumulated factors: human capital, knowledge capital, organization capital.
- The Solow transition dynamics is an important piece for understanding business cycle dynamics.

Reading

- Jones (2013), ch. 2, 3
- Non-renewable resources: Jones (2013), ch. 10.

Advanced Reading:

- Hall and Jones (1999)

References I

Hall, R. E. and C. I. Jones (1999): "Why do some countries produce so much more output per worker than others?" *Quarterly Journal of Economics*, 114, 83–116.

Jones, Charles; Vollrath, D. (2013): *Introduction To Economic Growth*, W W Norton, 3rd ed.

Nordhaus, W. D., R. N. Stavins, and M. L. Weitzman (1992): "Lethal model 2: the limits to growth revisited," *Brookings Papers on Economic Activity*, 1–59.