Partial Equilibrium R&D Models

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Issues

- We study models where **intentional innovation** drives productivity growth.
- We start with partial equilibrium to see how consumers and firms behave.
- Then we embed this into a GE model.

Background

- Historians often view innovation as the result of research that is not profit driven.
- Economists treat innovation as producing goods that are sold in markets ("blueprints").
- There are historical examples of both types of innovation.
- How important are the 2 cases? – An open question.

How to model innovation

- Current models are somewhat reduced form.
- The issue how existing knowledge feeds into future innovation is treated as a knowledge spillover.
- Knowledge is treated as a scalar - like capital.
- In fact, the only difference between blueprints and machines is **non-rivalry**:
  - blueprints can be used simultaneously in the production of several goods.

How to model innovation

There are N consumption goods (or intermediate inputs).

The goods are imperfect substitutes in preferences (or final output production).

- Therefore downward sloping demand curves

Approach 1: **Quality ladders**

- Each good can be made by many firms.
- Firms can invest to improve quality (equivalently: lower the cost) of 1 good.

Approach 2: **Increasing variety**

- Each firm can invest to create a new variety $(N \to N+1)$
- Then it becomes the monopolist for that variety

The Supply Block: Quality Improvements

Firm

We study the problem of an innovating firm.

The industry faces a **demand curve** $Q = D(p)$.

- Price elasticity: ε_D.

There are infinitely many competitors with marginal cost ψ.

All can make the same good.

No innovation $\implies$ zero profit.

Innovation

Spend μ goods

Reduce marginal cost to ψ/λ.

The innovator becomes a **monopolist**

- subject to a fringe of competitors with MC ψ.

Pricing Decision

- Monopoly price:

$$p^M = \frac{\psi/\lambda}{1 - 1/\varepsilon_D} \tag{1}$$

- Monopoly profit:

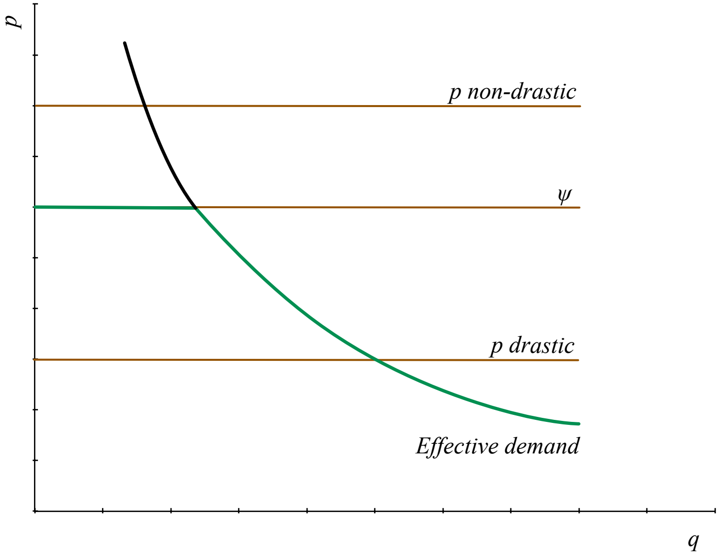
$$\pi_1^I = D\left(p^M\right)\ \left[p^M - \psi/\lambda\right] - \mu \tag{2}$$

- This is also the (private) value of the innovation.

Drastic / non drastic innovation

- A **drastic** innovation gives the innovator a monopoly.
  - The monopoly price is below competitor's marginal cost ψ.
  - It is optimal to set the monopoly price.
- A **non-drastic** innovation leaves the monopoly price above ψ.
  - Charging monopoly prices results in 0 sales.
  - The demand curve facing the firm becomes infinitely elastic at price ψ.
  - It is optimal to set $p = \psi$.

Effective demand curve



Optimality

Is the level of innovation too above or below what maximizes "welfare"?

The answer is usually ambiguous.

1. The innovator does not capture the entire consumer surplus.
2. Business stealing effect: innovation destroys competitor profits.
3. Replacement effect: innovation destroys the innovator's own profit.

The Role of Patents

Innovation only occurs, if the innovator earns monopoly status for some time

- but see Boldrin and Levine (2008)

The trade-off:

- Stronger patents - higher profits - more innovation.
- Stronger patents distort prices.

The Demand Block

Modeling the Demand Side

- The trick in all R&D models:
  a demand side that generates a **constant price elasticity**
- This makes the monopoly price essentially exogenous
  $p_M = MC/(1-1/\varepsilon_D)$

Dixit Stiglitz Model

- The world is static.
- There are N consumption goods c_i with prices p_i.
- There is one "other" consumption good y with price 1.
  - It purpose is to absorb income effects.
- Household income is m.

Preferences

- Preferences: $u(C, y)$
- C is a CES composite consumption good:

$$C = \left(\sum_{i=1}^{N} c_i^{\theta} \right)^{1/\theta} \tag{3}$$

- $\theta = (\varepsilon - 1)/\varepsilon > 0$.
- Elasticity of substitution $\varepsilon > 1$.
- The trick: constant substitution elasticity implies constant price elasticity.

Love for variety

A key implication: simply having more varieties increases welfare.

Assume you have $\bar{C}$ units of "stuff" that can be made (1-for-1) into any variety:

$\sum_{i=1}^{N} c_i = \bar{C}$.

Consider the symmetric case: $c_i = \bar{C}/N$.

Then

$$
\begin{aligned}
C &= \left(\sum_{i=1}^{N} [\bar{C}/N]^{\theta} \right)^{1/\theta} \\
&= \left(N \, \left[\bar{C}/N\right]^{\theta} \right)^{1/\theta} && (4) \\
&= N^{(1-\theta)/\theta} \, \bar{C} && (5)
\end{aligned}
$$

Spreading $\bar{C}$ over more varieties (N) increases utility.

Demand functions

The household's demand functions are iso-elastic.

The household solves:

$$\max u(C, y)$$

subject to

$$\sum_{i=1}^{N} p_i c_i + y = m \qquad (6)$$

Given m, this is just a CES cost minimization problem.

Demand functions

$$\max u\left(\left[\sum_{i=1}^{N} c_i^{\theta}\right]^{1/\theta}, m-\sum p_i c_i\right)$$

FOC

$$\begin{aligned}
\frac{\partial u}{\partial y} &= \frac{\partial u}{\partial c_i}\frac{1}{p_i} \\
&= \frac{\partial u}{\partial C}\frac{1}{\theta}\left[\sum_{i=1}^{N} c_i^{\theta}\right]^{1/\theta-1} \theta \ \frac{c_i^{\theta-1}}{p_i}
\end{aligned}$$

Demand functions

A useful feature:

$$[c_i/c_j]^{-1/\varepsilon} = p_i/p_j \tag{7}$$

Equal for all goods:

$$c_i^{-1/\varepsilon}/p_i \tag{8}$$

Demand function:

$$c_i = Xp_i^{-\varepsilon} \tag{9}$$

for some endogenous constant X (which we need to find).

Price elasticity is constant at ε.

Demand functions

Claim:

The demand functions take the form

$$c_i/C = (p_i/P)^{-\varepsilon} \tag{10}$$

where C is the composite consumption good

$$C = \left[\sum_{i=1}^{N} c_i^{\theta}\right]^{1/\theta} \tag{11}$$

and P is the "ideal price index" for the household (the cost minimizing cost of C:

$$P = \left(\sum p_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \tag{12}$$

Note: This is just a CES cost function.

Finding X

Now we have a simple two good problem:

$$\max u(C,y) \tag{13}$$

subject to

$$PC + y = m \tag{14}$$

FOC:

$$u_y/u_C = 1/P \tag{15}$$

Example: $u(C,y) = \alpha \ln(C) + (1-\alpha)\ln(y)$.

- $1/P = \frac{1-\alpha}{\alpha}\frac{C}{y}$
- with budget constraint: $y = (1-\alpha)m$ and $PC = \alpha m$.

Ideal price index

- Another way of thinking about the household problem:
  1. For given C, find the cost minimizing c_i. Define the price index as
  $$PC = \sum p_i c_i \tag{16}$$
  2. max $u(C, y)$ subject to $PC + y = m$.
- The cost minimizing price index is
$$P = \left(\sum p_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \tag{17}$$

Ideal price index I

Proof:

$$\min \sum_i p_i c_i + \lambda \left[\left(\sum_j c_j^\theta \right)^{1/\theta} - C \right] \tag{18}$$

FOC:

$$p_i = \lambda \left(\sum_j c_j^\theta \right)^{(1/\theta)-1} c_i^{\theta-1} \tag{19}$$

$$= \lambda C^{1-\theta} c_i^{\theta-1} \tag{20}$$

Solve for λ:

$$c_i = (\lambda / p_i)^{1/(1-\theta)} C \tag{21}$$

Ideal price index II

$$\left(\sum c_i^{\theta}\right)^{1/\theta} = C\lambda^{1/(1-\theta)}\left(\sum p_i^{\theta/(1-\theta)}\right)^{1/\theta} \tag{22}$$

$$\lambda = \left(\sum p_i^{\theta/(1-\theta)}\right)^{(1-\theta)/\theta} \tag{23}$$

Substitute and simplify.

The demand functions $c_i/C = (p_i/P)^{-\varepsilon}$ emerge.

QED

Digression: An Alternative Derivation

By definition:

$$PC = \sum p_i c_i \tag{24}$$

We need to express C and $\sum p_i c_i$ as functions of prices to solve for P.

First-order conditions determine relative demands:

$$c_i / c_1 = p_i^{-\varepsilon} / p_1^{-\varepsilon} \tag{25}$$

Sub into expression for

$$\begin{aligned} \sum p_i c_i &= c_1 \sum p_i (c_i / c_1) \\ &= c_1 p_1^{\varepsilon} \sum p_i^{1-\varepsilon} \end{aligned}$$

Alternative Derivation

Sub the same into expression for

$$\begin{aligned} C &= c_1 \left(\sum (c_i/c_1)^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \\ &= c_1 \left(\sum (p_i/p_1)^{1-\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \\ &= c_1 p_1^{\varepsilon} \left(\sum p_i^{1-\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \end{aligned}$$

Take the ratio:

$$P = \frac{PC}{C} = \frac{c_1 p_1^{\varepsilon}}{c_1 p_1^{\varepsilon}} \frac{\sum p_i^{1-\varepsilon}}{\left(\sum p_i^{1-\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}}$$

Simplify to get the solution for P.

Alternative Derivation

The demand functions take the form

$$c_i/C = (p_i/P)^{-\varepsilon} \tag{26}$$

Proof:

$$p_i c_i = p_i c_1 \ (p_i/p_1)^{-\varepsilon}$$

$$\begin{aligned} \sum p_i c_i &= PC = c_1 p_1^{\varepsilon} \ \sum p_i^{1-\varepsilon} \\ &= c_1 p_1^{\varepsilon} \ P^{1-\varepsilon} \end{aligned}$$

$$PC \ P^{\varepsilon-1} = c_1 p_1^{\varepsilon}$$

Rearrange. QED.

Household summary

- Assume a Dixit-Stiglitz composite consumption good in preferences.
- Then demand is isoelastic.
  - the elasticity is determined by the elasticity of substitution across varieties in C.
- The cost of the optimal bundle C is given by P.
- The household reduces to a 2 good problem with standard solution.

Firms

- Each firm has a monopoly over a variety i.
- The demand elasticity is ε.
- Optimal monopoly pricing implies a constant markup over marginal cost:

$$p_i = \frac{\psi}{1 - 1/\varepsilon} \tag{27}$$

- Assumption: The firm is small enough to neglect its effect on C and P.

Equilibrium

- Assume symmetry.
- Price index:

$$\begin{aligned} P &= \left(\sum p_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \\ &= N^{\frac{1}{1-\varepsilon}} \frac{\psi}{1-1/\varepsilon} \end{aligned}$$

- More goods of the same price $\rightarrow$ it costs less to achieve the same utility.

Equilibrium: Profits

$$
\begin{aligned}
\pi_i &= c_i \, (p_i - \psi) \\
&= C \; P^{\varepsilon} \; p_i^{-\varepsilon} \; (p_i - \psi) \\
&= C \; N^{\varepsilon/(1-\varepsilon)} \; \frac{\varepsilon}{\varepsilon - 1} \psi
\end{aligned} \tag{28}
$$

More varieties can increase profits:

- Direct effect: P falls - more competitors erode profits.
- "Aggregate demand externality": C may rise (depends on preferences)
  - Higher N raises marginal utility for a given variety.
  - Innovators impose pecuniary externality on competitors.

Continuum of varieties

- Nothing changes when i is continuous.
- Replace all $\sum$ with $\int$.

Reading

- Acemoglu (2009), ch. 12.
- Romer (2011), ch. 3.1-3.4.
- Jones (2005)

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