## Search Models of Money

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#### Search Models

- Matching models have one drawback:
  - the matching function is a black box
- Search models model explicitly how matches come about.
- ▶ Their drawback: hard to take to the data.
- Applications:
  - Money the main / original motivation for search
  - Housing
  - ▶ Labor market

## Search Models of Money

- Most models of money either have no notion of liquidity (OLG model) or they assume liquidity (CIA).
- Search models offer a micro-foundation for liquidity.
- Caveats:
  - ▶ The institutional arrangement is medieval.
  - Money is literally currency, not credit.
  - Some recent work tries to relax this.

#### The Idea

- People produce and consume many goods.
- ▶ To consume what they want, people need to search for sellers.
- ▶ Without money: trade is hard.
  - ▶ the double coincidence of wants problem.
- ▶ One solution: designate one good to be "money."
- Money means: the good is accepted by all traders.
  - a social convention.

## Kiyotaki-Wright Model

- ▶ There is a unit mass of ex ante identical, infinitely lived agents.
- ▶ There is a unit mass of goods types.
- Goods are indivisible.
  - ▶ The model gets harder when goods are divisible.

#### Preferences

- ▶ Each agent can consume fraction  $x \in (0,1)$  of the goods.
- $\triangleright$  Each good can be consumed by fraction x of households.
- Consumption yields utility *U*.
- ▶ Not consuming yields 0.
- Agents cannot consume goods they produce themselves.

#### Technology

- ► An agent can produce a good at utility cost 0.
- ▶ The type of the good is random.
- ▶ An agent can only hold one unit of the good at a time.
- ▶ This is a shortcut for a model with
  - costly production
  - costly storage of inventory

# Non-monetary Equilibrium Timing

- Agents start holding one good each.
- **Each** agent meets another agent with probability  $\theta$ .
- ▶ If two agents meet, they can exchange goods, if both agree.
- ► Trade costs **ε**.
  - ► Ensures that trade only occurs if both parties can consume.

#### Symmetric Equilibrium

- ▶ A strategy is a decision whether or not to trade in a meeting.
- Obviously, agents trade when the opponent holds the good they can consume.
- Agents do not trade otherwise.
- ► There is no gain from changing the type of good held.
- ▶ There is no way to compensate the party who cannot consume.
  - ► This changes when goods are divisible.

#### Symmetric Equilibrium

Value before meeting an agent

$$V_c^n = \theta x^2 (U - \varepsilon) + \beta V_c^n$$
 (1)

$$= \frac{\theta x^2 (U - \varepsilon)}{1 - \beta} \tag{2}$$

- ▶ This is a measure of equilibrium welfare.
- ▶ With probability  $\theta$ : meet an agent.
- ▶ With probability  $x^2$ : both hold goods that the other can eat.
- Otherwise: get 0 this period.

## Monetary Equilibria

- At the beginning of time,  $\overline{M}$  agents are offered a unit of "money."
- ▶ If they accept, they have to discard the good they carry.
- ▶ Agents can only carry 1 unit of a good or 1 unit of money.

## Decision to hold money

- ▶ Money is a bubble: its value derives from the expectation that money will be valued tomorrow.
- There is always a non-monetary equilibrium.
- $ightharpoonup \Pi$  is the probability that agents will accept money tomorrow.
- ▶ The state of the economy is  $(\Pi, \overline{M})$ .

#### Decision to hold money

Value function for an agent holding a good:

$$V_{c} = ([1 - \theta] + \theta M [1 - x] + \theta [1 - M] [1 - x^{2}]) (0 + \beta V_{c}) + \theta (1 - M) x^{2} (U - \varepsilon + \beta V_{c}) + \theta Mx \max_{\pi} [\pi \beta V_{m} + (1 - \pi) \beta V_{c}]$$

 $\pi$ : the agent can play mixed strategies

## Agent holding a good

With prob  $(1 - \theta)$ : meet nobody and get  $\beta V_c$ . With prob.  $\theta$  meet somebody.

- With prob.  $\theta M$  he holds money
  - With prob  $\theta Mx$  he likes my good.
    - ▶ Then trade with prob.  $\pi$  and get  $\beta V_m$ .
  - If he does not like my good:  $\beta V_c$ .
- ▶ With prob.  $\theta(1-M)$  he holds a good.
  - With prob  $\theta(1-M)x^2$ : trade and get  $U-\varepsilon+\beta V_c$ .
  - Otherwise, no trade and get  $\beta V_c$ .

# Simplify

$$V_c = \theta (1-M)x^2 (U-\varepsilon+\beta V_c)$$
  
+\theta M x \text{ max} \left[\pi\beta V\_m + (1-\pi)\beta V\_c\right]   
+\left[1-\theta \left[1-M]x^2 - \theta Mx\right]\beta V\_c

## Agent holding money

$$V_m = [(1-\theta) + \theta M + \theta (1-M)(1-x)](0 + \beta V_m) + \theta (1-M)x\Pi (U-\varepsilon + \beta V_c)$$

With prob  $(1 - \theta)$ : meet nobody and get  $\beta V_m$ . With prob  $\theta$  meet someone.

- ▶ With prob  $\theta M$  he holds money. No trade. Get  $\beta V_m$ .
- ▶ With prob  $\theta(1-M)$  he holds a good.
  - ▶ With prob  $\theta(1-M)x$  I like the good. Trade with prob.  $\Pi$ .
  - ▶ With prob  $\theta(1-M)(1-x)$  I don't like the good. No trade and get  $\beta V_m$ .

## Decision to hold money

- ▶ Benefits of money:
  - ► Trade with person holding a good I like who does not like my good.
- Drawback of money:
  - ► Trade with person who likes my good only with prob. П.
- ▶ No change when:
  - meet somebody who holds money or a good I don't like.

#### Decision to hold money

- ▶ If  $\Pi < x$ :
  - Money is less liquid than goods.
  - $ightharpoonup V_m < V_c$ .
  - Set  $\pi = 0$  and never accept money.
- ▶ If  $\Pi > x$ :
  - Money is liquid.
  - $ightharpoonup V_m > V_c$ .
  - Set  $\pi = 1$  and always accept money.
- $\blacktriangleright$  If  $\Pi = x$ :
  - ▶ Indifferent between money and goods.
  - Any  $\pi$  is optimal.

#### Equilibrium

- ▶ Equilibrium requires  $\pi = \Pi$ .
- ► Three equilibria:
  - 1.  $\Pi = 0$ . Back to the non-monetary equilibrium.
  - 2.  $\Pi = 1$  and  $M = \overline{M}$ .
  - 3. Mixed strategy equilibrium:  $\Pi = x$  and any  $M \in [0, \overline{M}]$ .
    - Now M is indeterminate because agents are indifferent between money and goods.

#### Remarks

- ► Money is purely a way of **breaking a symmetric equilibrium**.
- Any good could take the role of money.
- ► The question is: how to coordinate agents' expectations about the future acceptability of "money"?

#### Welfare

- ▶ To find welfare, impose  $\pi = \Pi$  on the value functions and solve.
- ▶ Result: The mixed monetary equilibrium  $(\Pi = x)$  has lower welfare than the non-monetary equilibrium.
- ▶ Intuition: Money does not facilitate transactions.
- ► The welfare cost stems from the fact that some goods were lost in order to hold money.
- Equivalent to an economy in which some goods can no longer be consumed.

## Monetary equilibrium

- ▶ Result: The monetary equilibrium raises welfare iff x < 0.5.
- Intuition: Trading is always easier.
- But some "goods" cannot be consumed (those held by monetary traders).
- ► For the liquidity benefit to win, barter trade must be sufficiently hard.

#### Extensions

- Goods are divisible:
  - Alberto Trejos and Randall Wright. "Search, Bargaining, Money, and Prices." The Journal of Political Economy Vol. 103, No. 1 (Feb., 1995), pp. 118-141.
- Money and goods are divisible / policy analysis:
  - Ricardo Lagos and Randall Wright. "A Unified Framework for Monetary Theory and Policy Analysis." The Journal of Political Economy Vol. 113, No. 3 (Jun., 2005), pp. 463-484.

#### How Useful Is This?

- ▶ People disagree...
- ▶ The big question:
  - ► How does it fit a modern economy where most money is credit?

## Reading

▶ Ljunqvist & Sargent, "Recursive Methods," ch. 26.8.