McCall Model

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Econ720

November 22, 2016

Motivation

- We would like to study basic labor market data:
  - unemployment and its duration
  - wage heterogeneity among seemingly identical workers
  - job to job transitions
  - how do policies affect those variables?
- Frictionless models of the labor market cannot talk about these issues.
- We need models in which workers must search for jobs.

Search Models

- Unemployment is a productive activity: search for a new job.
- Types of models:
  1. Decision theoretic (McCall model).
  2. Matching: A matching function creates new jobs.
  3. Search: Random encounters and bargaining.

McCall Model

- A partial equilibrium model of a worker searching for a job.
- The worker lives forever, in discrete time.
- Preferences:

$$\sum_{t=0}^{\infty} \beta^t y_t$$

- y_t is income.
- When employed: $y = w$. When unemployed: $y = c$.

Timing

- Enter the period as unemployed worker.
- Draw a wage offer w from the distribution $F(W) = \Pr(w \leq W)$.
- Support: $[0, B]$.
- Choose whether to accept or reject.
- If accept: work forever at wage w with lifetime income $\frac{w}{1-\beta}$.
- If reject: start over next period.

Bellman equation

Before knowing today's wage offer: value is a constant Q

After learning the wage offer w, value is $v(w)$

Therefore:

$$
\begin{aligned}
Q &= c + \beta \mathbb{E} v\left(w'\right) \\
&= c + \beta \int_0^B v\left(w'\right) dF\left(w'\right)
\end{aligned}
$$

Value **after** learning wage offer:

$$
v(w) = \max\left\{\frac{w}{1-\beta}, Q\right\}
$$

Reservation wage property

Accept all offers with

$$\frac{w}{1-\beta} \geq Q \tag{1}$$

The reservation wage makes the worker indifferent between accepting and rejecting:

$$v(\bar{w}) = \max\left\{\frac{\bar{w}}{1-\beta}, Q\right\} = \frac{\bar{w}}{1-\beta} = Q \tag{2}$$

Note: For $w < \bar{w}$ the worker still gets $v(\bar{w}) = Q$.

Reservation wage

Write the reservation wage as (proof below):

$$\begin{aligned}\bar{w} - c &= \beta \int_{\bar{w}}^{B} \frac{w' - \bar{w}}{1-\beta} dF\left(w'\right) \\ &= \beta E\left\{\frac{w' - \bar{w}}{1-\beta} | w' \geq \bar{w}\right\} \Pr\left(w' \geq \bar{w}\right)\end{aligned}$$

In words:

- the surplus from working now $(\bar{w} - c)$ equals
- the surplus from searching: the expected lifetime wage gain from perhaps finding a better job

Proof

Write the indifference condition as

$$\frac{\bar{w}}{1-\beta} - c = Q - c = \beta \int_0^B v\left(w'\right) dF\left(w'\right)$$

$$= \underbrace{\beta \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} dF\left(w'\right)}_{\text{reject}} + \underbrace{\beta \int_{\bar{w}}^B \frac{w'}{1-\beta} dF\left(w'\right)}_{\text{accept}}$$

Simplify:

$$\begin{aligned}
\frac{\bar{w}}{1-\beta} - c &= \beta \int_0^B \frac{\bar{w}}{1-\beta} dF\left(w'\right) + \beta \int_{\bar{w}}^B \frac{w' - \bar{w}}{1-\beta} dF\left(w'\right) \\
&= \beta \frac{\bar{w}}{1-\beta} + \beta \int_{\bar{w}}^B \frac{w' - \bar{w}}{1-\beta} dF\left(w'\right)
\end{aligned}$$

Implications: Unemployment Benefits

What is the effect of more generous unemployment benefits (higher c)?

Optimality: $c = \bar{w} -$ expected surplus or

$$c = \bar{w} - \beta \mathbb{E} \left\{ \frac{w' - \bar{w}}{1 - \beta} | w' \geq \bar{w} \right\} \Pr \left(w' \geq \bar{w} \right) \tag{3}$$

Expected surplus shrinks when $\bar{w}$ rises.

RHS increases in $\bar{w}$.

Higher $c \rightarrow$ higher reservation wage $\rightarrow$ longer unemployment.

More dispersed wage offers

- Result: A mean preserving spread in the wage offer distribution raises the reservation wage and ex ante utility.
- Intuition:
  - Making bad wage offers worse is costless - they are rejected anyway.
  - Making good wage offers better is valuable.
- Proof: Ljunqvist & Sargent.

Extension: Job separations

Each period the worker is fired with probability α.

A fired worker must wait 1 period before drawing a new wage.

Now we have 3 states the worker can be in:

1. unemployed, waiting for a wage offer: v_U
2. unemployed with a wage offer: $v(w)$
3. employed: $v_E(w)$

Value functions

Value when unemployed without offer:

$$v_U = c + \beta \int v(w') dF(w')$$

- unemployed today; eat c
- get an unknown wage offer tomorrow

Value when unemployed with an offer:

$$v(w) = \max \{v_E(w), v_U\}$$

- all of this is the same as in basic McCall model

Value when employed at wage w:

$$v_E(w) = w + \beta(1-\alpha) v_E(w) + \beta \alpha v_U$$

Firing: Reservation wage

Reservation wage makes the worker indifferent between accepting an rejecting an offer:

$$v(\bar{w}) = v_E(\bar{w}) = v_U$$
$$\bar{w} + \beta(1-\alpha)v_E(\bar{w}) + \beta\alpha v_U = v_U$$

With $v_E(\bar{w}) = v_U$:

$$\frac{\bar{w}}{1-\beta} = v_U = c + \beta \int v(w')dF(w')$$

Firing: Implications

- How does the firing probability affect unmployment?
- The reservation wage equations are the "same" with and without firing:

$$\frac{\bar{w}}{1-\beta} = c + \beta \int v(w')dF(w')$$

- The value function is lower with firing
  - because quitting is never optimal
- Therefore $\bar{w}$ is lower with firing.
- If jobs do not last as long, there is no point holding out for the perfect offer.

Stochastic Wages

Model With Stochastic Wages

Based on Rogerson et al. (2005).

Timing:

- Enter the period either as
  - unemployed: value V_U or as
  - employed: value $V(w)$.
- If **unemployed**:
  - earn c today
  - draw a wage offer w' for next period with probability α
  - if accept: get $V(w)$ tomorrow
  - if reject: get V_U tomorrow

Timing

- If **employed**:
  - earn w today and eat it
  - draw a new wage w' for tomorrow with probability λ.
  - if accept: $V(w')$
  - if reject (or no offer): unemployed tomorrow

All wage offers are drawn from the same distribution:
$F(W) = \Pr(w' \leq W)$ with support $[0, B]$.

Value of a wage offer

Consider an unemployed (or employed) worker who is about to receive a wage offer.

His value is

$$\hat{Q} = \int \max\left\{V(w'), V_U\right\} dF(w') \tag{4}$$

Independent of current w (in case of employed)

- because that offer is lost

Call the reservation wage $\bar{w}$.

- it is the same for employed or unemployed

Value of a wage offer

$$
\begin{aligned}
\hat{Q} &= \int \max\{V(w'), V_U\}\, dF(w') &(5)\\
&= \int \max\{V(w') - V_U, 0\}\, dF(w') + V_U &(6)\\
&= \underbrace{\int_{\bar{w}}^{B} \{V(w') - V_U\}\, dF(w')}_{Q} + V_U &(7)
\end{aligned}
$$

In words:

- you always get at least V_U (because you can always take that option)
- if $w' > \bar{w}$, you also get a surplus Q

Unemployed Worker

Before receiving offer

$$
\begin{aligned}
V_U &= c + \beta\left[\alpha\hat{Q} + (1-\alpha)V_U\right] && (8)\\
&= c + \beta\left[\alpha\left(Q + V_U\right) + (1-\alpha)V_U\right] && (9)\\
&= c + \beta\alpha Q + \beta V_U && (10)
\end{aligned}
$$

Get c today.

With probability α get to choose between work and unemployment tomorrow.

Therefore

$$(1-\beta)V_U = c + \beta\alpha Q \qquad (11)$$

Employed Worker

Bellman equation for a worker with wage w:

$$V(w) = w + \beta \left[\lambda \hat{Q} + (1-\lambda) V(w)\right] \tag{12}$$

Get w today.

With probability λ, face the same choice as an unemployed worker with offer w'.

Simplify:

$$V(w) = w + \beta\lambda (Q + V_U) + \beta (1-\lambda) V(w) \tag{13}$$

Reservation Wage

Evaluate $V(w)$ at $w = \bar{w}$ and use $V(\bar{w}) = V_U$:

$$V(\bar{w}) = \bar{w} + \beta\lambda(Q + V_U) + \beta(1-\lambda)V_U \tag{14}$$

Therefore

$$(1-\beta)V_U = \bar{w} + \beta\lambda Q \tag{15}$$

Reservation Wage

We now have

$$(1-\beta)V_U = \bar{w} + \beta\lambda Q \tag{16}$$
$$= c + \beta\alpha Q \tag{17}$$

Perhaps easier:

$$\bar{w} - c = \beta(\alpha - \lambda)Q \tag{18}$$

Reservation Wage

If $\alpha = \lambda$: $\bar{w} = c$.

- Accept any job that pays more than unemployment benefits
- The reason is that the continuation value does not depend on employment status.

If $\alpha > \lambda$: $\bar{w} > c$.

- Being unemployed has a search value. So the agent holds out for better wage offers.

Reservation Wage

Add and subtract $V_U - V(w)$ in equation for $V(w)$:

$$(1-\beta)V(w) = w + \beta\lambda Q + \beta\lambda[V_U - V(w)] \tag{19}$$

Substitute out Q from equation for reservation wage

$$(1-\beta)\,V_U = \bar{w} + \beta\lambda Q \tag{20}$$

to obtain

$$(1-\beta)[V(w) - V_U] = w - \bar{w} + \beta\lambda[V_U - V(w)] \tag{21}$$

Solve for

$$V(w) - V_U = \frac{w - \bar{w}}{1 - \beta + \beta\lambda} \tag{22}$$

If we specified the distribution F, we could use this to evaluate Q and solve for everything else.

Applications

Life-cycle earnings profiles and occupational mobility:

- Kambourov and Manovskii (2009, 2008)

Business cycle models that match labor market facts:

- Jovanovic (1987)

What is missing?

- Not satisfactory: The job finding rate / wage offer distribution should be endogenous.
  - Think about analyzing policies...
- Matching and search models address this.
  - by introducing endogenous supply of jobs
  - and wage bargaining.

Reading

- Ljungqvist and Sargent (2004), ch. 6.3
- Krusell (2014), ch. 11
- Williamson (2006), "Notes on macroeconomic theory," ch. 7, works out a similar model with exogenous job separations.

References I

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