Review Questions: Infinite Horizon Models in Continuous Time

Econ720. Fall 2015. Prof. Lutz Hendricks

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1 Household Behavior

1.1 Wealth tax

[Romer 2.3] Suppose it is known in advance that the government will confiscate some wealth at a future date t_0. Does consumption change disontinuously at t_0? Why or why not? What is the equation governing consumption around t_0?

Consider two cases:

1. The government confiscates half of each household's wealth.
2. The government confiscates half of the average wealth held by all households.

2 Equilibrium

2.1 Exponential utility

[Barro and Sala-i-Martin 2.3] Consider a version of the Ramsey model with exponential utility function

$$u(c) = -(1/\theta)\ e^{-\theta c} \tag{1}$$

where $\theta > 0$.

1. Compute the intertemporal elasticity of substitution. How does it change with c?
2. Find the household's Euler equation.
3. Draw a phase diagram and characterize how the economy converges to a steady state. Assume $g(A) = 0$.
4. Does the economy have a balanced growth path if $g(A) > 0$?

2.2 CES Production function

Consider a Ramsey model with land Λ. Output is produced according to

$$Y = \left[a\ \left(K^{\alpha} L^{1-\alpha}\right)^{\psi} + (1-a)\,\Lambda^{\psi}\right]^{1/\psi} \tag{2}$$

1. Show that this production function has constant returns to scale.
2. Calculate the elasticity of substitution between Λ and $Q = K^{\alpha} L^{1-\alpha}$. Show that it is simply a function of ψ. This is why the production function is called CES or Constant Elasticity of Substitution.
3. Under what conditions on ψ is Y/L constant in steady state? Under what conditions does Y/L decline in the long run? Can Y/L grow in the long run?

2.2.1 Answer: CES production function

1. Easy.

2. The answer is in any micro text book. The elasticity is $1/(1-\psi)$.

3. The easiest method is probably to look at

$$Y/K = \left[a + (1-a)\left(\frac{\Lambda}{K^{\alpha}L^{1-\alpha}}\right)^{\psi}\right]^{1/\psi} \tag{3}$$

If Y/K is constant, then from $g(K) = sY/K - \delta$ we know that K and Y both grow at constant rates.

Case 1: $\psi > 0$. The substitution elasticity is greater than 1. Land is "not essential" in production. As L and K grow,

$$X = \left(\frac{\Lambda}{K^{\alpha}L^{1-\alpha}}\right)^{\psi} \to 0$$

and $Y/K \to a$. There is a steady state (asymptotically) which looks exactly like the one of an economy without land.

Case 2: $\psi < 0$. Land is essential. Over time $X \to \infty$ and $(a + [1-a]X)^{\psi} \to 0$ because $\psi < 0$. $Y/K \to 0$ and $g(K) \to -\delta$. $Y/L \to 0$. [Is there an easier way of doing this?] Persistent growth is not possible.

2.3 Capital adjustment costs

Consider a version of the standard one sector, neoclassical growth model with no technological progress, inelastic labor supply and and zero population growth. We will examine the planner's problem in an economy in which capital is costly to adjust.

The planner maximizes the lifetime utility of the representative household:

$$\int_0^{\infty} u(c_t)e^{-\rho t}dt. \tag{4}$$

where $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$.

There is one final good which is produced using labor and capital using a production technology which can be written in intensive form as:

$$y_t = f(k_t), \tag{5}$$

where $f'(\cdot) > 0$, $f''(\cdot) < 0$, and the usual Inada conditions apply.

Capital is costly to adjust however, so the resource constraint is given by:

$$f(k_t) = c_t + i_t(1 + T(i_t/k_t)), \tag{6}$$

where i_t is investment, and $T(\cdot)$ represents additional costs incurred whenever capital is adjusted. We assume that $T(0) = 0$ and $T'(\cdot) > 0$. Capital depreciates at rate δ, implying that:

$$\dot{k}_t = i_t - \delta k_t. \tag{7}$$

The initial capital stock, k_0, is given.

i) Derive first order conditions for the planner's problem. Interpret your results.

ii) Show that the solution to the planner's problem is consistent with a steady state in which y, c, k and i all grow at rate zero.

iii) Derive an expression characterizing the steady state capital stock. Contrast this to the steady state capital stock in the case where capital can be adjusted costlessly.

2.3.1 Answer: capital adjustment costs

i) Set up the present value Hamiltonian:

$$H = u(c_t)e^{-\rho t} + \lambda_t[f(k_t) - c_t - i_t(1 + T(i_t/k_t))] + \mu_t(i_t - \delta k_t) \tag{8}$$

The first order conditions: $H_c = 0$, $H_i = 0$, $H_k = -\dot{\mu}$, $H_\mu = \dot{k}$, $H_\lambda = 0$ and the transversality condition ($\lim_{t\to\infty} k_t\mu_t = 0$).

$$H_c = 0 : \qquad u'(c_t)e^{-\rho t} = \lambda_t \tag{9}$$

$$H_i = 0 : \qquad \lambda_t[1 + T(i_t/k_t) + (i_t/k_t)T'(i_t/k_t)] = \mu_t \tag{10}$$

$$H_k = -\dot{\mu}_t : \qquad \lambda_t[f'(k_t) + T'(i_t/k_t) \cdot (i_t/k_t)^2] - \mu_t\delta = \dot{\mu}_t \tag{11}$$

Solution: $c\,(t)\,, i\,(t)\,, k\,(t)\,, \mu\,(t)\,, \lambda\,(t)$ that satisfy:

- 3 first-order conditions and TVC, k_0 given.
- Resource constraint.
- Law of motion for k.

ii) Manipulating 9 in the usual fashion yields:

$$\theta\dot{c}_t/c_t = \dot{\lambda}_t/\lambda_t + \rho \tag{12}$$

Manipulating 10 and 11 gives:

$$\dot{\mu}_t/\mu_t = \hat{r}(k_t, i_t/k_t) - \delta \tag{13}$$

and

$$\dot{\lambda}_t/\lambda_t = \dot{\mu}_t/\mu_t - A(i_t/k_t)[\ (\dot{i}_t)/i_t - \dot{k}_t/k_t] \tag{14}$$

where

$$\hat{r}(i_t/k_t) = \frac{f'(k_t) + T'(i_t/k_t) \cdot (i_t/k_t)^2}{1 + T(i_t/k_t) + (i_t/k_t) \cdot T'(i_t/k_t)}$$

$$A(i_t/k_t) = \frac{2T'(i_t/k_t) + (i_t/k_t) \cdot T''(i_t/k_t)}{1 + T(i_t/k_t) + (i_t/k_t)T'(i_t/k_t)}$$

We can then combine these equations to eliminate the multipliers λ and μ and write our system in terms of k, c, i and their growth rates:

$$\theta\dot{c}_t/c_t = \hat{r}(k_t, i_t/k_t) - \delta - A(i_t/k_t)[(\dot{i}_t)/i_t - \dot{k}_t/k_t] \tag{15}$$

$$\dot{k}_t/k_t = i_t/k_t - \delta \tag{16}$$

$$f'(k_t)\dot{k}_t/k_t = (c_t/k_t)\dot{c}_t/c_t - [(i_t/k_t)(1 + T(i_t/k_t) + (i_t/k_t)T'(i_t/k_t)](\dot{i}_t)/i_t \tag{17}$$

Are these equations consistent with a zero growth steady state (where $\gamma_c = \gamma_i = \gamma_k = 0$)? Yes: From 16, $i_t/k_t = \delta$. Plugging this into 15 gives:

$$\theta\dot{c}_t/c_t = \hat{r}(k, \delta) - \delta = 0, \tag{18}$$

which implies a steady state value of k. Finally, 17 is also satisfied. The resource constraint gives the steady state value for c, once steady state values for k and i are calculated.

iii) $\hat{r}(k,\delta)-\delta=0$ implies a steady state value for k. Substituting back in for $\hat{r}(\cdot)$ and manipulating gives:

$$f'(k^*)=\delta[1+T(\delta)+\delta(1-\delta)T'(\delta)] \tag{19}$$

Note that k^* in this case is smaller than the steady state capital stock in the case where there is no adjustment cost for capital. This can be seen from the facts that the restrictions on $T(\cdot)$ imply that $T(\delta)+\delta(1-\delta)T'(\delta)$ is positive, $f''<0$, and steady state capital is given by: $f'(k^*)=\delta$.

Essentially, the adjustment costs make it more costly to purchase new capital, implying that less will be accumulated. This is true even in the steady state, where the capital stock is held constant, because new capital is constantly being accumulated to replace previously depreciated capital.

3 Government Capital and Congestion

A representative household has preferences $\int_0^\infty e^{-\rho\, t}u(c_t)\, dt$. The budget constraint is

$$\dot{k}_t=(1-\tau_t)\, f(k_t, G_t/K_t)-c_t$$

where k is the household's own capital stock, K is the aggregate capital stock, G is government spending, and τ is an income tax rate. Government spending increases productivity: $f_2>0$. The government adjusts the tax rate to finance an exogenous G_t/K_t stream.

(a) Define a solution to the household problem.

(b) Define a competitive equilibrium.

(c) Assume that the utility function is $u(c)=c^{1-\sigma}/(1-\sigma)$ and that the production function is of the form $f(k,G/K)=k\, h(G/K)$ with $h'>0$ and $h''<0$. Derive a system of 3 equations that solves for the balanced growth values of c/k, τ, and the balanced growth rate g.

(d) Now consider the planner's problem, who optimally chooses government spending (and of course capital and consumption). Derive the planner's balanced growth rate and compare it with the competitive one. Is there a tax rate that implements the planner's allocation? What is the intuition for the finding?

3.1 Answer: Government Capital and Congestion

(a) The household problem is entirely standard. It is solved by and Euler equation of the form

$$g(u'(c))=\rho-(1-\tau)\, f_1(k,G/K)$$

and the budget constraint. In addition there are two boundary conditions: k_0 given and $\lim e^{-\rho t}\, u'(c_t)\, k_t=0$.

(b) A competitive equilibrium is a set of functions (c_t,k_t,K_t,τ_t,G_t) which solve the 2 household equations, $K_t=k_t$, the government budget constraint $G_t=\tau_t\, f(K_t,G_t/K_t)$, and goods market clearing: $f(K_t,G_t/K_t)=c_t+\dot{K}_t+G_t$.

(c) The balanced growth path $(c/K,g,\tau)$ is characterized by

$$\begin{aligned} h(G/K) &= c/K+g+G/K \\ G/K &= \tau\, h(G/K) \\ g &= \frac{(1-\tau)\, h(G/K)-\rho}{\sigma} \end{aligned}$$

The closed form solution for the growth rate is

$$g^{CE}=\frac{h(G/K)-G/K-\rho}{\sigma}$$

(d) The planner's Hamiltonian is

$$H = u(c) + \lambda\,[K\,h(G/K) - c - G]$$

First-order conditions are

$$\begin{aligned} u'(c) &= \lambda \\ h'(G/K) &= 1 \\ \dot{\lambda} &= \rho\,\lambda - \lambda\,[h(G/K) - h'(G/K)\,G/K] \end{aligned} \tag{20}$$

The planner choses the growth rate

$$g^{PL} = \frac{h(G/K) - G/K - \rho}{\sigma}$$

where G/K is set according to (20). Note that the competitive allocation is identical to the one chosen by the planner, if the tax rate is set to match the planner's G/K. To see this, note that the planner's allocation is characterized by the growth equation and the feasibility constraint, both of which are also part of the equilibrium definition. The intuition is that the income tax internalizes the congestion externality arising from investment.

4 Comparative Dynamics

4.1 Productivity slowdown

[Romer 2.6] Consider a Ramsey model on its balanced growth path. Suppose there is a permanent fall in $g\,(A)$.

1. How is the $\dot{k} = 0$ locus affected?
2. How is the $\dot{c} = 0$ locus affected?
3. What happens to the balanced growth values of c and k?
4. What happens to c at the time of the change? Provide intuition.
5. Derive an expression for the change in the saving rate ($[y - c]/y$) on the balanced growth path as a function of the change in $g\,(A)$. Can you tell whether it is positive or negative? Provide intuition.

4.1.1 Answer: Productivity slowdown

1. From

$$c = f\,(k) - (n + \delta + g)\,k$$

we see that the $\dot{k} = 0$ locus shifts up.

2. From

$$f'\,(k) = \rho + \delta + \theta g \tag{21}$$

we see that $\dot{c} = 0$ shifts right.

3. Balanced growth: Draw the phase diagram to see that k and c both rise. The effect is actually (almost) the same as that of a drop in δ or n.

4. From the law of motion

$$g\,(c) = \frac{f'\,(k) - \delta - \rho}{\theta} - g$$

we find that $g\,(c)$ rises by as much as the drop in g. The level change is hard to find from the equations. My intuition is that c initially falls. This is in part an income effect (lower future output) and in part a substitution effect (the household postpones consumption more).

5. The steady state saving rate is

$$s = \frac{y - c}{y} = (n + \delta + g)\,\frac{k}{y} \tag{22}$$

From (21) we see that

$$g \downarrow \Rightarrow k \uparrow \Rightarrow k/y \uparrow \Rightarrow s \uparrow$$

At the same time the direct effect of lower g on s in (22) is negative. The result is ambiguous.

Intuition: There is an income effect that lowers consumption. But there is also a substitution or rate of return effect: future investment is less productive, so households have a smaller incentive to save.

4.2 Changes in balanced growth

Show how the following shocks affect the balanced growth path:

1. A rise in θ.
2. A fall in A.
3. An increase in the depreciation rate δ.

4.3 Useful government purchases

Assume that households obtain utility from government purchases $G(t)$, so that the utility function is given by

$$U = \int_0^\infty e^{-\rho t}\ \left[c(t) + G(t)\right]^{1-\theta}\ dt\ /\ (1-\theta) \tag{23}$$

If the economy is initially on the balanced growth path, what are the effects of a temporary increase in $G(t)$? Hint: first think about how the household would respond if the interest rate did not change. Then ask how the path of $k(t)$ is affected.

4.4 Taxes and Dynamics in a Cass-Koopmans Model

Consider a standard Cass-Koopmans model with a lump-sum tax. The households solves

$$\max \int_0^\infty e^{-\rho\, t}\, u(c_t)\, dt$$

subject to

$$\dot{k}_t = r_t\, k_t + w_t - c_t - \tau_t$$

and k_0 given. Firms produce output using technology $F(K, L)$. Capital depreciates at rate δ. The government uses the tax revenue to finance goverment spending: $G_t = \tau_t$.

(a) Define a solution to the household problem.

(b) Define a competitive equilibrium.

(c) Draw a phase diagram and show that the economy is saddle-path stable. Assume that government spending is constant and that the utility function is given by $u(c) = c^{1-\sigma}/(1-\sigma)$.

(d) Consider a permanent, unannounced increase in G. Show its effects in the phase diagram for $k_0 = k_{ss}$ and for $k_0 < k_{ss}$, where k_{ss} is the pre-change steady state capital stock. Plot the time path of consumption. Assume that the change in G is not too large; otherwise you get into trouble with the nonnegativity constraint on consumption.

(e) Now consider a *temporary*, unannounced increase in G. That is, $G_t = G^* + \Delta G$ for $0 \le t \le T$, but $G_t = G^*$ for $t > T$. Show its effects in the phase diagram for $k_0 = k_{ss}$ and for $k_0 < k_{ss}$. Plot the time path of consumption.

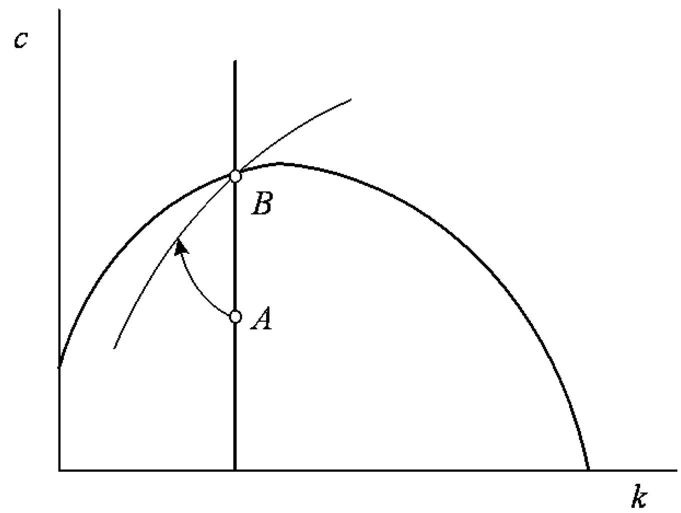

Figure 1: Temporary tax change

4.4.1 Answer: Taxes and Dynamics in a Cass-Koopmans Model

(a) This is standard: The functions $c(t)$ and $k(t)$ solve the Euler equation $g(u') = \rho - r$ and the budget constraint. The TVC is $\lim e^{-\rho t} u'(c_t)\, k_t = 0$.

(b) The firm's FOCs are $r = f'(k) - \delta$ and $w = f(k) - f'(k)\, k$. A CE consists of functions $\{c(t), k(t), r(t), w(t), \tau(t)\}$ that solve 2 household equations, 2 firm FOCs, goods market clearing ($\dot{k} = f(k) - \delta\, k - c - G$), and the government budget constraint.

(c) Nothing new here. Note that the Euler equation is $g(c) = (r - \rho)/\sigma$.

(d) The effect of higher G on the phase diagram is to shift the $\dot{k} = 0$ locus down by ΔG. k_{ss} remains unchanged because the $\dot{c} = 0$ locus does not shift. c_{ss} drops to the new saddle path. See Figure 1.

If $k_0 = k_{ss}$, then consumption jumps to the new steady state level. The economy is immediately in the new steady state.

If $k_0 < k_{ss}$, then consumption jumps to the new saddle-path and then grows along it.

(e) The first important point to note is that from $t = T$ onwards the economy looks exactly as if no change had ever occurred. That is, c and k must be on the saddle path through B in the figure below. Of course, we don't know at which level of k. The issue is therefore what happens before date T. The second important point is that c cannot jump, except at $t = 0$.

What happens at date 0? The $\dot{k} = 0$ locus shifts down. Consumption must drop; otherwise the TVC would be violated. To see this, draw the new phase diagram. If c did not drop initially, then k would start falling, causing $g(c)$ to rise. The economy would move north-west in the phase diagram. It could never reach the (unchanged) saddle-path at date T.

First, consider the case $k_0 = k_{ss}$. One candidate path that doesn't violate any optimality conditions has c_0 dropping by less than ΔG. Then k initially falls. The higher $f'(k)$ makes c grow. At date T, c will hit the saddle path. This is the only path that does not violate boundary conditions. If c were to drop by ΔG, then k would remain constant until T. But then c would have to jump at T to get to the saddle path. If c were to drop by more than ΔG, then the economy would drift into the lower right quadrant.

Next, consider the case where $k_0 < k_{ss}$. As in the case where the economy starts in the steady state, c_0 drops. It cannot drop below the new saddle-path because then it would not reach the old saddle-path at T. Being above the new saddle-path, the economy must move north-west and reach the old saddle-path at T.

4.5 Dynamics of the Baby Boom

Consider a standard Cass-Koopmans model with population growth. Household preferences are

$$\max \int_0^\infty e^{-(\rho-n)\,t}\, u(c_t)\, dt$$

where n is the population growth rate, c is per capita consumption, and $\rho \in (n,1)$ is a discount factor. Assume that the utility function is given by $u(c) = c^{1-\sigma}/(1-\sigma)$. The resource constraints are given by

$$\begin{aligned} \dot{k} &= x - (n+\delta)\,k \\ f(k) &= c + x \end{aligned}$$

(a) Define a solution to the planner's problem.

(b) Derive two equations characterizing the steady state levels of k and c.

(c) Draw a phase diagram and show that the economy is saddle-path stable.

(d) Consider a permanent, unannounced increase in population growth n. Show its effects in the phase diagram for $k_0 = k_{ss}$ and for $k_0 < k_{ss}$, where k_{ss} is the pre-change steady state capital stock. Plot the time path of consumption. Assume that the change in n is not too large; otherwise you get into trouble with the nonnegativity constraint on consumption.

(e) Now consider a *temporary*, unannounced increase in n. That is, $n_t = n + \Delta n$ for $0 \le t \le T$, but $n_t = n$ for $t > T$. Show its effects in the phase diagram for $k_0 = k_{ss}$ and for $k_0 < k_{ss}$. Plot the time path of consumption. Here are a couple of hints: (1) Where will the equilibrium point be a $t = T$? (2) At what dates is it possible for consumption to jump? (3) How then does the economy get to the place where it should be at $t = T$?

(f) Consider an anticipated, permanent increase in n. That is, agents learn at date 0 that n will increase at date T. Show its effect in the phase diagram for $k_0 = k_{ss}$. Very briefly explain the intuition why the paths of c and k are optimal.

4.5.1 Answer: Dynamics of the Baby Boom

(a) This is standard: The functions $c(t)$ and $k(t)$ solve the Euler equation

$$\begin{aligned} g(c) &= \frac{f'(k) - \rho - \delta}{\sigma(c)} \\ \dot{k} &= f(k) - (n+\delta)\,k - c \end{aligned}$$

The TVC is $\lim e^{-(\rho-n)\,t}\, u'(c_t)\, k_t = 0$.

(b) Steady state:

$$\begin{aligned} f'(k_{ss}) &= \rho + \delta \\ c_{ss} &= f(k_{ss}) - (n+\delta)\,k_{ss} \end{aligned}$$

(c) The phase diagram is perfectly standard. We did this in class.

(d) Permanent baby boom: A higher n shifts the $\dot{k} = 0$ locus down, but leaves the $\dot{c} = 0$ locus unchanged. If $k_0 = k_{ss}$, the economy jumps to the new steady state. If $k_0 < k_{ss}$, consumption drops to the new saddle path and then moves along it.

(e) The first important point to note is that from $t = T$ onwards the economy looks exactly as if no change had ever occurred. That is, c and k must be on the saddle path through B in the figure below. Of course, we don't know at which level of k. The issue is therefore what happens before date T.

The second important point is that c cannot jump, except at $t = 0$.

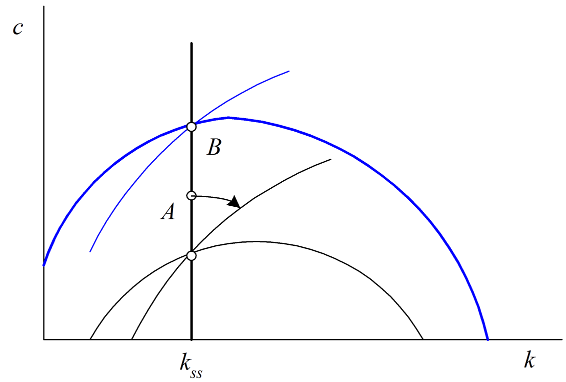

Figure 2: Announced baby boom

The third important point is that, prior to T, the economy is governed by the "arrows" that belong to the phase diagram with the lower steady state (not B), but the economy does not follow the saddle path because the boundary conditions do not require this.

What happens at date 0? First, consider the case $k_0 = k_{ss}$. The $\dot{k} = 0$ locus shifts down. We need to find a consumption point c_0 such that the economy hits the saddle path for $n_t = n$ exactly at date T. The only point with this property is a point like A. If c_0 were above B, then the economy would move away from the saddle path to the north-west. If c_0 were below the steady state for the dynamical system that governs behavior until date T, then the economy would move away from the saddle path towards the south-east. In either case, the economy could not connect with the saddle path at T.

Intuition: c drops because more k needs to be provided for the new agents. k falls because it is not worthwhile to build a lot of k given that the higher n is temporary. c rises because the interest rate is high.

Next consider the case $k_0 < k_{ss}$. Now a similar argument establishes that the economy must jump down to a point north of the temporary saddle path. It then moves north-west to connect with the final saddle path at T.

(f) The phase diagram is the same as before. Until T the economy is governed by the old phase diagram. At T it must connect with the new saddle. Therefore, c drops at date 0, then fall over time. k rises above the steady state. At T, k and c begin to decline along the new saddle. The intuition: agents build up capital, anticipating that more people will be around in the future.

5 Heterogeneity

5.1 Cass-Koopmans Model with Two Types of Households[1]

Consider a Cass-Koopmans model with two types of households, $i = 1, 2$. The population of each type is constant (N^i). Wages differ across types (w^i). Preferences are $\int_0^\infty e^{-\rho\, t} u(c_t^i)\, dt$, where c^i denotes consumption per capita of type i and $0 < \rho < 1$ is a discount factor. Households hold capital as their only asset.

A single representative firm rents capital and labor from households. The technology is

$$Y_t = F(K_t, L_t) = K_t^\theta\, [(L_t^1)^{1-\theta} + (L_t^2)^{1-\theta}] = \dot{K}_t + \delta\, K_t + C_t.$$

Here L^i denotes labor input of household type i, K denotes aggregate capital input, and C is aggregate consumption.

(a) State the household problem and solve for the FOCs. Define a solution to the household problem for each type.

(b) Solve the firm's problem.

(c) Define a competitive equilibrium.

(d) Show that the economy has a continuum of steady states. Provide a set of conditions that characterize the steady state values of c^i and k^i.

[1] Based on a problem due to Paul Zak.

(e) What is the intuition for the multiplicity of steady states. Is there multiplicity of equilibrium paths for a given set of initial conditions as well?

5.1.1 Answer: Cass-Koopmans Model with Two Types of Households

(a) Each household solves the same problem as in a Crusoe economy. The solution is a pair of functions c_t^i and k_t^i that satisfy the Euler equation $g(u_c^i) = \rho - r$ and the budget constraint $\dot{k}_t = r_t\, k_t^i + w_t^i - c_t^i$ with initial condition k_0^i given and TVC.

(b) The firm's FOC are standard: $q = F_k$ and $w^i = F_{L^i}$.

(c) A CE consists of functions $c^i, k^i, w^i, r, q, K, L^i$ that satisfy

- 4 household conditions
- 3 firm conditions
- $K = \sum k^i\, L^i$ and $L^i = N^i$
- $r = q - \delta$
- $F(K, L^1, L^2) - \delta\, K = \dot{K} + \sum L^i\, c^i$

(d) The steady state is characterized by $F_K = \rho + \delta$, which determines K. The wage rates can then be determined from $w^i = (1-\theta)\,(K/L^i)^\theta$. We then have an additional 3 equation that determine 4 variables: $c^i = (\rho+\delta)\, k^i + w^i$ and $K = \sum k^i\, L^i$. The latter condition gives the set of feasible k^i. For any k^i pair we pick, the budget constraints tell us the corresponding steady state consumption levels.

(e) Intuition: The equilibrium is unique. But suppose we start in steady state and transfer a unit of capital from agent 1 to agent 2. If both agents adjust their constant consumption levels according to their budget constraints, then prices will be unchanged forever. But then no agent has an incentive to deviate from the constant consumption level and we have another steady state. The point is that we can redistribute capital without changing any steady state variable, except for the distribution of consumption across agents.