

# Knowledge Spillovers and Scale Effects

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# Issues

- ▶ What happens when innovation takes labor (a non-reproducible factor)?
- ▶ Then we need a knowledge spillover to sustain growth.
- ▶ It takes some tricks to prevent the model from exhibiting explosive growth.

# Knowledge Spillovers

# Ideas Produced From Labor

The previous model had endogenous growth because ideas were produced with constant return from a **reproducible factor**: ideas (embodied in goods).

If ideas are produced from (non-reproducible) labor: there is no sustained growth.

## Example

Assume  $\dot{N}_t = \eta Z_t^\alpha L_{Rt}^{1-\alpha}$ . Show that the balanced growth rate is 0 unless  $\alpha = 1$ .

# Knowledge Spillovers

We need a mechanism that offsets diminishing returns to ideas in the production of ideas.

Knowledge spillover:  $N$  appears in the innovation production function for  $N$ .

This is an **externality**: firms do not pay for the  $N$  input.

This is possible because  $N$  is non-rival.

The idea: "standing on the shoulders of giants"

Problem: A **knife-edge** parameter assumption is needed for endogenous growth.

- ▶ Some parameters must sum to 1.
- ▶ This is always true because we need **constant returns to reproducible factors**.

## Knowledge spillover model

Keep everything the same, except the production of ideas:

$$\dot{N}_t = \eta N_t L_{Rt} \quad (1)$$

We show later: linearity in  $N$  is required for endogenous growth.

Labor now has 2 uses:

- ▶ produce goods:  $L_E$
- ▶ produce ideas:  $L_R$

Resource constraint:

$$L = L_{Rt} + L_{Et} \quad (2)$$

Note: this does not change the problems of household, final goods firms, or intermediate input firms.

## Balanced growth rate

Euler equation is still:  $g(C) = (r - \rho)/\theta$ .

Interest rate is determined by free entry:  $V = \pi/r$ .

But now the cost of creating a new patent is different:

$$\eta N_t V_t = w_t \quad (3)$$

- hire a unit of labor and produce a flow of  $\eta N_t$  patents per “period”

## Balanced growth rate

Wage rate (unchanged):

$$w_t = \frac{\beta}{1 - \beta} N_t \quad (4)$$

Profits earned by monopolists (unchanged):

$$\pi = \beta L_E \quad (5)$$

Sub wage rate into free entry:

$$\eta N_t \frac{\beta L_E}{r} = w = \frac{\beta}{1 - \beta} N_t \quad (6)$$

$\Rightarrow$

$$r^* = (1 - \beta) \eta L_E^* \quad (7)$$



## Balanced growth rate

Euler equation (unchanged):

$$g^* = g(C) = \frac{(1 - \beta)L_E^* - \rho}{\theta} \quad (8)$$

Almost done - just need to find  $L_E$ .

Balanced growth requires

$$g(C) = g(Y) = g(N) \quad (9)$$

Ideas production function:

$$g(N) = \eta L_R^* = \eta (L - L_E^*) \quad (10)$$

## Balanced growth

Solve for the growth rate.

$$\begin{aligned}g(C) &= \frac{(1-\beta)L_E^* - \rho}{\theta} \\ &= \eta(1-L_E^*)\end{aligned}$$

$\Rightarrow$

$$L_E^* = \frac{\theta\eta L + \rho}{(1-\beta)\eta + \theta\eta} \quad (11)$$

Scale effects: larger economies grow faster.

With population growth, output growth explodes.

# Growth without scale effects

- ▶ The previous models do not have balanced growth paths when there is population growth.
- ▶ The reason is the scale effect:
  - ▶ Larger population  $\rightarrow$  more R&D  $\rightarrow$  faster growth.
- ▶ Diminishing returns to reproducible factors avoid the scale effect, but also kill endogenous growth.

## Growth without scale effects

To avoid scale effects, modify the model as follows.

Innovation:

$$\dot{N}_t = \eta N_t^\phi L_{Rt} \quad (12)$$

$$0 < \phi \leq 1 \quad (13)$$

Demographics:

$$L_t = e^{nt} \quad (14)$$

$$= L_{Rt} + L_{Et} \quad (15)$$

## Balanced growth

From the innovation technology:

$$g(N) = \eta N_t^{\phi-1} L_{Rt} \quad (16)$$

Constant growth requires constant  $N^{\phi-1} L_R$  and

$$g(N) = \frac{n}{1-\phi} \quad (17)$$

The growth rate is "**semi-endogenous**:" endogenous, but not responding to changes in agents' choice variables.

There are still scale effects:

- Larger economies tend towards higher levels of output per person.

## Avoiding scale effects

It is possible to write down models that have endogenous growth, but no scale effects (growth does not increase with  $L$ ).

The idea: Prevent innovator profits from increasing with  $L$ .

One approach: the number of products increases with  $L$  exactly so that the market size for each variety remains the same (Young, 1998).

Avoiding scale effects requires knife-edge assumptions like this.

Final Example:  
Durable Intermediate Inputs

# Environment

We study a final example where intermediates are durable (the model has capital).

Demographics: There is a representative household who lives forever.

Preferences:

$$\int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1-\sigma} dt \quad (18)$$

Endowments: The household works one unit of time at each instant.



## Technologies: Final goods

$$Y_t = AL_t^\alpha \int_0^{N_t} x_{j,t}^{1-\alpha} dj = C_t + \int_0^{N_t} I_{j,t} dj + Z_t$$

where

- ▶  $I_{j,t}$  is investment in intermediates
- ▶  $Z_t$  is investment in R&D

## Technologies: Intermediates

- ▶ Upon invention, the inventor is endowed with  $x_0$  units of  $x_j$ .
- ▶ Additional units are accumulated according to

$$\dot{x}_{j,t} = \eta I_{j,t}^\varphi - \delta x_{j,t} \quad (19)$$

- ▶  $0 < \varphi < 1$
- ▶ Diminishing returns imply smooth adjustment of  $x$  over time.
- ▶ Intermediates are *rented* to final goods firms at price  $R_{j,t}$ .

## Technologies: R&D

New varieties are invented according to:

$$\dot{N}_t = Z_t / \beta \quad (20)$$

where  $Z$  denotes goods devoted to R&D.

# Market arrangements

- ▶ Final goods: price 1
- ▶ Labor:  $w_t$
- ▶ Each intermediate input producer has a permanent monopoly for his variety.
- ▶ Free entry into the market for innovation

# Household

- ▶ Standard, with complicated budget constraint.
- ▶ Euler:

$$\dot{c}_t/c_t = \frac{r_t - \rho}{\sigma} \quad (21)$$

- ▶ TVC  $\lim_{t \rightarrow \infty} e^{-\rho t} u'(c_t) a_t = 0$ .

## Final goods firm

$$\max AL_t^\alpha \int_0^{N_t} x_{j,t}^{1-\alpha} dj - w_t L_t - \int_0^{N_t} R_{j,t} x_{j,t} dj \quad (22)$$

$\{y_t, L_t, x_{j,t}\}$  solve the production function and the FOCs

$$w_t = \alpha y_t / L_t \quad (23)$$

$$R_{j,t} = (1 - \alpha) AL_t^\alpha x_j^{-\alpha} \quad (24)$$

## Intermediate input producer

Now a truly dynamic problem ( $j$  index suppressed)

$$V = \max \int e^{-rt} [R(x_t)x_t - I_t] dt$$

subject to

$$\dot{x} = \eta I_t^\varphi - \delta x \quad (25)$$

Hamiltonian:

$$H = R(x)x - I + \mu [\eta I^\varphi - \delta x] \quad (26)$$

## Intermediate input producer

FOCs:

$$\begin{aligned}\partial H / \partial I &= -1 + \mu \eta \phi I^{\phi-1} = 0 \\ \dot{\mu} &= (r + \delta) \mu - R'(x)x - R(x)\end{aligned}$$

Solution:  $\{I_t, x_t, \mu_t\}$  that solve 2 FOCs and law of motion for  $x$ .

Boundary conditions:

- ▶  $x(0) = 0$  given,
- ▶  $\lim_{t \rightarrow \infty} e^{-rt} \mu_t x_t = 0$ .



# Free entry of innovators

Technology:

$$\dot{N} = \beta^{-1} Z \quad (27)$$

Free entry:

- ▶ Spend  $\beta \, dt$  to obtain  $dN = \beta / \beta \, dt$  new patents worth  $V \, dt$ .
- ▶ Equate cost and profits:  $\beta = V$

# Equilibrium

Objects:  $\{R_{j,t}, x_{j,t}, N_t, I_{j,t}, \mu_{j,t}, y_t, L_t, r_t, c_t, w_t\}$

Equilibrium conditions:

- ▶ Household: 2
- ▶ Final goods: 3
- ▶ Intermediate goods: 3
- ▶ Free entry:  $\beta = V = \int e^{-rt} [R(x_t)x_t - I_t] dt$
- ▶ Goods market clearing:  $y = c + NI + \dot{N}\beta$ .
- ▶ Labor market clearing:  $L = 1$ .
- ▶ Asset market clearing: This depends, as usual, on the structure of asset markets.
  - ▶ If households own the firms  $a_t = \int_0^{N_t} V_{j,t}$ .
  - ▶ Alternatively, one could assume that innovators issue bonds.

## Case $\varphi = 1$

Assume that the same equilibrium conditions hold for  $\varphi = 1$  (not obvious).

Then:  $x$  and  $\mu$  must be constant over time.

Intermediate input firms:

- ▶  $\partial H / \partial I = 0 \implies \mu = (\eta \varphi)^{-1}$
- ▶  $\dot{\mu} = (r + \delta) \mu - R'(x)x - R(x)$
- ▶ Demand function:  $R'(x) = \alpha R(x) / x$
- ▶ Therefore:  $R'(x)x - R(x) = -(1 - \alpha) R(x) = (r + \delta) \mu$

With a linear technology, the best approach is to build all  $x$  in one shot, then keep  $x$  constant.

## Symmetric equilibrium I

With  $\varphi = 1$  there is a symmetric equilibrium because it does not take time to build up the stock of  $x_j$ .

Start from the Euler equation:  $g(c) = (r - \rho) / \sigma$ .

Free entry pins down  $r$ :

$$\beta = V = \int_0^\infty e^{-rt} [R(x_t)x_t - I_t] dt - \underbrace{\frac{x - x_0}{\eta}}_{I_0} \quad (28)$$

Assume  $x_0 = 0$ .

Stationary  $x$ :  $I_t = x\delta/\eta$ .

FOC of intermediate firm and demand for  $x$ :  $R = \frac{r+\delta}{\eta(1-\alpha)}$

## Symmetric equilibrium II

Therefore:

$$\beta = \frac{1}{r}x \left[ \frac{r + \delta}{\eta(1 - \alpha)} - \frac{\delta}{\eta} \right] - \frac{x - x_0}{\eta} \quad (29)$$

$$= x \frac{r + \delta}{r\eta} \left[ \frac{1}{1 - \alpha} - 1 \right] \quad (30)$$

Demand for intermediates:

$$R = A(1 - \alpha)L^\alpha x^{-\alpha}$$

Now we have 3 equation in  $(R, r, x)$  that could, in principle, be solved for the equilibrium values.

# Reading

- ▶ Acemoglu (2009), ch. 13.
- ▶ Romer (2011), ch. 3.1-3.4.
- ▶ Jones (2005)

# References I

- Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.
- Jones, C. I. (2005): "Growth and ideas," *Handbook of economic growth*, 1, 1063–1111.
- Romer, D. (2011): *Advanced macroeconomics*, McGraw-Hill/Irwin.
- Young, A. (1998): "Growth without scale effects," *The Journal of Political Economy*, 106, 41.