Overlapping Generations Model

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Introduction

Two approaches for modeling the household sector

1. households live forever (**infinite horizon**)
   tractable
2. households live for finite number of periods (**overlapping generations**)
   can talk about questions where demographics matter

OLG Applications

Fiscal policy analysis

- often models where households live many periods

Economic growth

Business cycles (when stochastic)

Many others...

What we do in this section

- How to set up and solve an OLG model
- Show that the world is **not efficient**: households may save too much.
- "Social security" can prevent overaccumulation
- We can make households "infinitely lived" by adding altruistic **bequests**.

What we don't do in this section

- We sidestep some technical issues:
  - why is there a representative household?
  - why is there a representative firm?
- We come back to those later.

An OLG Model Without Firms

Steps

We go through the standard steps:

1. Describe the economy: demographics, endowments, preferences, technologies, markets
2. Solve each agent's problem
3. Market clearing
4. Competitive equilibrium

Demographics

- At each date a cohort of size

$$N_t = N_0(1+n)^t$$

  is born.
- Each person lives for two periods.
- Therefore, at each date there are N_t young and N_{t-1} old households.

Endowments, Preferences

- Endowments
  - Young households receive endowments w_t.
- Preferences: $u(c_t^y) + \beta u(c_{t+1}^o)$.

Technology

- Endowments can be stored.
- Storing s_t today yields $f(s_t)$ tomorrow.
- Resource constraint:

$$N_t c_t^y + N_{t-1} c_t^o + N_t s_t = N_t w_t + N_{t-1} f(s_{t-1}) \tag{1}$$

Resource constraints

Technological constraints that describe the set of feasible choices.

Often identical to market clearing conditions.

Markets

Goods are traded in spot markets.

Households can issue one period bonds with interest rate r_{t+1}.

We are done with the description of the environment.

Next step: solve the household problem.

A Missing Market

Even though there is a bond market, **intergenerational** borrowing and lending is not possible.

The reason: the young at t cannot borrow from the old because the old won't be around at $t+1$ to have their loans repaid.

- If households live for more periods, the problem becomes weaker, but does not go away.

An asset that stays around forever solves this problem

- e.g., money, land, shares

Household Problem

- The budget constraints are

$$\begin{aligned} w_t &= c_t^y + s_{t+1} + b_{t+1} \\ c_{t+1}^o &= f(s_{t+1}) + b_{t+1}(1 + r_{t+1}) \end{aligned}$$

- Lifetime budget constraint:

$$w_t - c_t^y - s_{t+1} = [c_{t+1}^o - f(s_{t+1})]/[1 + r_{t+1}]$$

Lagrangian

$$
\begin{aligned}
\Gamma &= u(c_t^y) + \beta u(c_{t+1}^o) \\
&\quad + \lambda_t \{[w_t - c_t^y - s_{t+1}] - [c_{t+1}^o - f(s_{t+1})]/[1 + r_{t+1}]\}
\end{aligned}
$$

FOCs:

$$
\begin{aligned}
u'(c_t^y) &= \lambda_t \\
\beta u'(c_{t+1}^o) &= \lambda_t/(1 + r_{t+1}) \\
f'(s_{t+1}) &= 1 + r_{t+1}
\end{aligned}
$$

Euler equation

$$u'(c_t^y) = \beta\,(1 + r_{t+1})\,u'(c_{t+1}^o)$$

Interpretation:

Give up 1 unit of consumption when young and buy a bond.

Marginal cost: $u'\left(c_t^y\right)$

Marginal benefit:

$(1 + r_{t+1})$ units of consumption when old

valued at $\beta u'\left(c_{t+1}^o\right)$

Household Solution

A vector $(c_t^y, c_{t+1}^o, s_{t+1}, b_{t+1})$ which satisfies

- 2 FOCs (an EE and the foc for *s*)
- 2 budget constraints.

This is (unsurprisingly) the same as in the two-period model.

Equilibrium

A CE is an allocation $\{c_t^y, c_t^o, s_t, b_t\}$ and a price system $\{r_t\}$ that satisfy:

- 4 household conditions
- bond market clearing: $b_t = 0$;
- goods market clearing (same as resource constraint):

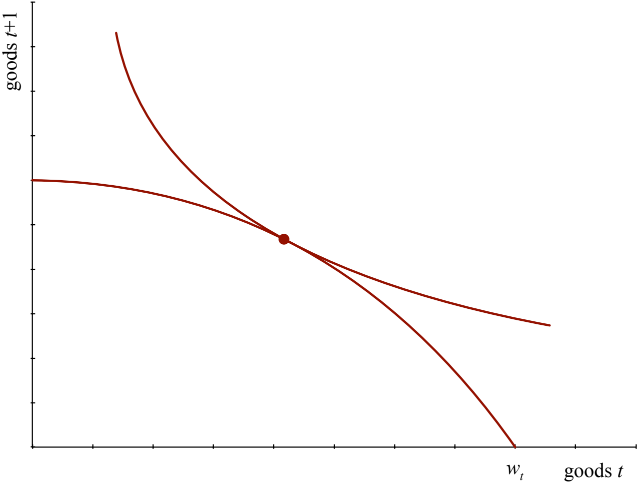
$$N_t c_t^y + N_{t-1} c_t^o + N_t s_t = N_t w_t + N_{t-1} f(s_{t-1}) \qquad (2)$$

We are done with the definition of equilibrium.

Next step: characterize equilibrium.

Characterization

There is no trade in equilibrium $(b_t = 0)$



A Production Economy

A Production Economy

- The model is modified by adding firms who rent capital and labor from households.
- The endowment w is now interpreted as labor earnings.
- Households supply one unit of labor inelastically to firms when young.
- Capital depreciates at rate δ.

Model Elements

- Unchanged: demographics, preferences
- Endowments:
  - at $t = 0$ each old household owns k_0 units of capital
  - each young has 1 unit of work time
- Technology

$$F(K_t, L_t) + (1 - \delta) K_t = C_t + K_{t+1} \tag{3}$$

  - constant returns to scale
  - Inada conditions
- Markets:
  - goods (numeraire), capital rental (q), labor rental (w)

Notes

Representative household

- All households are the same.
- So we talk as if there were only 1 household, who behaves competitively.

The household owns everything

- The firm rents capital from the household in each period
- That makes the firms' problem static (easy)
- It is usually convenient to pack all dynamic decisions into 1 agent
- In this model, who owns the capital makes no difference - why not?

Households

- Budget constraints:

$$\begin{aligned} w_t &= c_t^y + s_{t+1} + b_{t+1} \\ c_{t+1}^o &= e^o + (s_{t+1} + b_{t+1})(1 + r_{t+1}) \end{aligned}$$

- e^o: any other income received when old (currently 0)
- There are no profits b/c the technology has constant returns to scale.

Lifetime budget constraint

- Combine the 2 budget constraints:

$$w_t - c_t^y = \left(c_{t+1}^o - e^o\right)/[1 + r_{t+1}]$$

or

$$W_t = w_t + \frac{e^o}{1 + r_{t+1}} = c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} \tag{4}$$

- W_t: present value of lifetime earnings

Permanent Income Hypothesis

The lifetime budget constraint only depends on W_t, not on timing of income over life.

Therefore, the optimal consumption path only depends on W_t.

This is a somewhat general implication that has been tested many times.

A recent example:

- Hsieh, C. T. (2003). Do consumers react to anticipated income changes? Evidence from the Alaska permanent fund. American Economic Review, 397-405. [Nice example of using a natural experiment to test a theory.]

Overall, the evidence seems favorable.

Lagrangian

$$\begin{aligned}\Gamma &= u(c_t^y) + \beta u(c_{t+1}^o) \\ &\quad + \lambda\{W_t - c_t^y - c_{t+1}^o/[1 + r_{t+1}]\}\end{aligned}$$

FOCs:

$$\begin{aligned}u'(c_t^y) &= \lambda \\ \beta u'(c_{t+1}^o) &= \lambda/(1 + r_{t+1})\end{aligned}$$

Households

Euler:

$$u'(c_t^y) = \beta(1 + r_{t+1})u'(c_{t+1}^o)$$

Solution: A vector $(c_t^y, c_{t+1}^o, s_{t+1}, b_{t+1})$ that satisfies 2 budget constraints and 1 EE.

We lack one equation! Why?

Graph: Household Solution

Assuming:

- log utility: $c^o_{t+1}/c^y_t = \beta(1+r_{t+1})$
- many periods

Firms

Firms maximize current period profits taking factor prices (q, w) as given.

$$\max F(K, L) - wL - qK$$

FOCs:

$$\begin{aligned} q &= F_K(K, L) \\ w &= F_L(K, L) \end{aligned}$$

Firms: Intensive form

It is almost always convenient to write the production function in **intensive form**:

$$\begin{aligned} F(K,L) &= LF(K/L,1) \\ &= Lf(k^F) \end{aligned}$$

where $k^F = K/L$ and

$$f(k^F) = F(k^F,1)$$

This, of course, requires constant returns to scale.

Firms: Intensive form

Now the factor prices are

$$F_K = Lf'(k^F)(1/L)$$

and

$$\begin{aligned} F_L &= f(k^F) + Lf'(k^F)(-K/L^2) \\ &= f(k^F) - f'(k^F)k^F \end{aligned}$$

Therefore:

$$\begin{aligned} q &= f'(k^F) \\ w &= f(k^F) - k^F f'(k^F) \end{aligned}$$

Important: q is the rental price of capital, which differs from the interest rate r.

The solution to the firm's problem is a pair (K, L) so that the 2 FOCs hold.

Market clearing

Capital rental: $K_{t+1} = N_t s_{t+1}$

Labor rental: $L_t = N_t$

Bonds: $b_t = 0$

Goods: resource constraint

Competitive Equilibrium

An allocation: $(c_t^y, c_t^o, s_t, b_t, K_t, L_t)$

Prices: (q_t, r_t, w_t)

That satisfy:

- the household EE and budget constraints (3 equations)
- the firm's FOCs (2 equations)
- the market clearing conditions (4 equations)

We have 9 objects and 9 equations – one is missing.

We need an accounting identity linking r and q:

- The household receives $1 + r_{t+1} = q_{t+1} + 1 - \delta$ per unit of capital.
- Therefore, $r = q - \delta$.

Reading

- Acemoglu (2009), ch. 9.
- Krueger, "Macroeconomic Theory," ch. 8
- Ljungqvist and Sargent (2004), ch. 9 (without the monetary parts).
- McCandless and Wallace (1991)and De La Croix and Michel (2002) are book-length treatments of overlapping generations models.

References I

Acemoglu, D. (2009): *Introduction to modern economic growth*. MIT Press.

De La Croix, D., and P. Michel (2002): *A theory of economic growth: dynamics and policy in overlapping generations*. Cambridge University Press.

Ljungqvist, L., and T. J. Sargent (2004): *Recursive macroeconomic theory*.

McCandless, G. T., and N. Wallace (1991): *Introduction to dynamic macroeconomic theory: an overlapping generations approach*. Harvard University Press.