Global Uncertainty and International Business Cycles

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Abstract

We document that global uncertainty has a significant impact on the business cycle. We define global uncertainty as the component of conditional output volatility that is common across countries and establish a link between global uncertainty and business cycle dynamics. We argue that in spite of the fact that global uncertainty shocks hit all countries simultaneously, their impact is not homogeneous across countries. Our results suggest that macroeconomic quantities and prices significantly respond to volatility shocks. The empirical results rely on an agnostic identification strategy and do not impose any assumption on the order of propagation of shocks to the variables of interest. In addition, we discuss the implications of global uncertainty in an environment populated by agents that fear model misspecification.

1 Introduction

The literature in the effects of macroeconomic uncertainty in the business cycles has grown considerably in recent years. In this paper, we document that global uncertainty has a significant impact on the business cycle.

The uncertainty literature has provided important insights in how aggregate uncertainty can have deleterious effects on economic activity. Bernanke (1983), one of the earliest contributions in the field, argued that higher uncertainty in the presence of capital irreversibilities cause firms to postpone investment in physical capital until uncertainty is resolved. Bloom (2009) showed that economic activity significantly declines in the aftermath of uncertainty-triggering events and proposed a general equilibrium framework in which firms face real rigidities (capital and labor partial irreversibilities, investment adjustmen costs) and freeze investment and hiring decisions. Jones et al. (2005) found that uncertainty might actually lead to higher average growth rates. Justiniano and Primiceri (2008) highlight that time-varying volatility considerably improves the adequacy of DSGE models to the data.

More recently, the role of macroeconomic uncertainty in open-economy settings has also been the subject of important contributions to the literature. Novy and Taylor (2014) document that international trade significantly falls when uncertainty is high due to a disproportionate decline in the demand for foreign capital goods. Colacito et al. (2015), on the other hand, focus on the transmission of volatility shocks across countries and its implications to quantity and prices.

We also focus on the open-economy effects of uncertainty shocks but, unlike Novy and Taylor (2014), we abstract from investment and, naturally, the demand for capital goods. We document that even in the absence of the most sensitive component of output to business conditions, uncertainty has significant effects on the business cycle.

We define global uncertainty as the component of conditional output volatility that is common across countries and empirically establish a link between global uncertainty and business cycle dynamics. We argue that in spite of the fact that global uncertainty shocks hit all countries simultaneously, their impact is not homogeneous across countries. The heterogeneity in the cross-country exposure to global uncertainty enables countries to share global risk and attain a smoother consumption profile.

Our results suggest that macroeconomic quantities and prices significantly respond to volatility shocks. When global uncertainty rises, trade balance in countries highly exposed to that source of risk immediately deteriorates and interest rates fall within a few quarters. Countries that are less exposed to global uncertainty have a trade balance improvement and their interest rate does not fall significantly. Also, the data suggest that currencies in high-exposure countries appreciate relative to those of low-exposure ones. The empirical results rely on an agnostic identification strategy and do not impose any assumption on the order of propagation of shocks to the variables of interest.

Our contribution to the literature is twofold. From an empirical perpective, we estimate a process for the global component of volatility and find evidence that the exposure level to this global source of risk is significantly heterogeneous across countries. We also document that the business cycles are significantly affected by global uncertainty. From a theoretical perspective, we extend the work Colacito and Croce (2012) as to incorporate a component of output volatility that is common across agents.

This manuscript is organized as follows: Section 2 decribes the model, Section 3 discusses the estimation of conditional volatility processes for country-level output growth, the global component of volatility and empirical plausibility of the theoretical model.

2 The model

The world is populated by two agents, home and foreign, who fear model misspecification (Hansen and Sargent, 2008). In every period, the home country receives an endowment of good X whereas the foreign country receives an endowment of good Y, but both agents derive utility from consumption of goods X and Y. A consumption aggregator tilts utility towards the domestic good so the consumption profiles show a realistic degree of home bias. We denote agent i's time-t demand for good X by x_{it} and agent j's demand for good Y by y_{jt}. Then, at time t, the home and foreign agents derive utility according to

$$\begin{aligned} C_{ht} &= x_{ht}^{\alpha}\, y_{ht}^{1-\alpha} \\ C_{ft} &= x_{ft}^{1-\alpha}\, y_{ft}^{\alpha} \\ U_{it} &= (1-\delta)\, log\, C_{it} + \delta\, \theta\, log\left[E_t\left(exp\left\{\frac{U_{i,t+1}}{\theta}\right\}\right)\right] \end{aligned}$$

where δ is the subjective discount factor and parameter α denotes the magnitude of home bias. Parameter θ determines agents' sensitivity to model misspecification (see Appendix A for details on model misspecification and robust optimal choice) and is typically negative. Higher values of θ indicate the agent is more sensitive to model misspecification.

We denote the state of nature at time t by s_t and the history of realized states up to period t by s^t.

Agents save through contingent claims to future consumption. Let $q_t(s_{t+1}|s^t)$ be the time-t price of an asset that entitles its holder to one unit of good X in period $t+1$ and $a_i(s_{t+1}|s^t)$ agent i's holdings of such asset. If $p_t(s^t)$ is the relative price of good Y in terms of good X, then the budget constraints are

$$\begin{aligned} x_{ht} + p_t(s^t) y_{ht}(s^t) + \sum_{s_{t+1}} q_t(s_{t+1}|s^t) a_h(s_{t+1}|s^t) &\leq X_t(s^t) + a_h(s_t) \\ x_{ft} + p_t(s^t) y_{ft}(s^t) + \sum_{s_{t+1}} q_t(s_{t+1}|s^t) a_f(s_{t+1}|s^t) &\leq p_t(s^t) Y_t(s^t) + a_f(s_t) \end{aligned} \tag{1}$$

and the net supply of contingent claims is zero in every state: $a_h(s_{t+1}|s^t) + a_f(s_{t+1}|s^t) = 0$.

Endowment streams are subject to time-varying uncertainty, and we define uncertainty as the conditional volatility of the endowment processes. Innovations to the conditional volatility have from two components: global and domestic. The domestic component is country-specific and resembles a white noise process. The global component of volatility, on the other hand, hits all countries simultaneously and is highly persistent over time. The data suggest that countries are heterogeneously exposed to global uncertainty, so we define β_i to be agent i's exposure to global uncertainty and $\beta_i \neq \beta_j,\ i \neq j$.

To describe the endowment dynamics more concisely, let X_t and Y_t denote the endowments received by the home and foreign countries, respectively. Also, assume that endowment uncertainty follows a stochastic volatility process with mean μ_σ, persistence ϕ and variance σ_σ^2. Global uncertainty η_t is assumed to follow an $AR(1)$ process with mean zero, persistence ρ_η and variance σ_η^2. We can summarize endowment dynamics as follows:

$$\begin{aligned} log\ (X_{t+1}) &= log\ (X_t) + \mu_x + e^{\frac{\sigma_{xt}}{2}}\, \varepsilon_{x,t+1} \\ log\ (Y_{t+1}) &= log\ (Y_t) + \mu_y + e^{\frac{\sigma_{yt}}{2}}\, \varepsilon_{y,t+1} \\ \sigma_{xt} &= \mu_\sigma + \phi\ (\sigma_{x,t-1} - \mu_\sigma) + \beta_h\, \eta_t + \sigma_\sigma\, \varepsilon_{x,t}^\sigma \\ \sigma_{yt} &= \mu_\sigma + \phi\ (\sigma_{y,t-1} - \mu_\sigma) + \beta_f\, \eta_t + \sigma_\sigma\, \varepsilon_{y,t}^\sigma \\ \eta_t &= \rho_\eta\, \eta_{t-1} + \sigma_\eta\, \varepsilon_{\eta,t} \end{aligned} \tag{2}$$

Decisions are made in a frictionless environment and markets are complete, so the optimal allocations are the solution to the planner's problem:

$$\begin{aligned}
&\underset{\{x_{ht},x_{ft},y_{ht},y_{ft}\}_{t=0}^{\infty}}{\text{Max}} && \mu\, U_{h0} \;+\; (1-\mu)\, U_{f0} \\
&\text{subject to} && x_{ht} \,+\, x_{ft} \;=\; X_t \\
& && y_{ht} \,+\, y_{ft} \;=\; Y_t, \;\; \forall t.
\end{aligned}$$

where μ and $1-\mu$ are the respective Pareto weights. As pointed out by Anderson (2005) and Colacito and Croce (2012), the allocations can be characterized in terms of "pseudo-Pareto" weights that summarize the history of realized states s^t. Let $\mu_t^h(s^t)$ and $\mu_t^f(s^t)$ be the path-dependent pseudo-Pareto weights for the home and foreign countries respectively and $\varphi(s^t) = \dfrac{\mu_t^h(s^t)}{\mu_t^f(s^t)}$. Then the allocation of consumption goods is

$$\begin{aligned}
x_{ht}(s^t) &= \frac{\alpha\, \varphi_t(s^t)}{1-\alpha+\alpha\, \varphi_t(s^t)} X_t(s^t) \quad , \quad x_{ft}(s^t) = \frac{1-\alpha}{1-\alpha+\alpha\, \varphi_t(s^t)} X_t(s^t) \\
y_{ht}(s^t) &= \frac{(1-\alpha)\, \varphi_t(s^t)}{\alpha+(1-\alpha)\varphi_t(s^t)} Y_t(s^t) \quad , \quad y_{ft}(s^t) = \frac{\alpha}{\alpha+(1-\alpha)\varphi_t(s^t)} Y_t(s^t)
\end{aligned}$$

and the law of motion for $\varphi(s^t)$ is given by

$$\varphi_t\left(s_t|s^{t-1}\right) = \varphi_{t-1}(s^{t-1})\, \frac{exp\left(\dfrac{U_{h,t}(s_t)}{\theta}\right)}{E_{t-1}\left[exp\left(\dfrac{U_{h,t}}{\theta}\right)\right]}\, \frac{E_{t-1}\left[exp\left(\dfrac{U_{f,t}}{\theta}\right)\right]}{exp\left(\dfrac{U_{f,t}(s_t)}{\theta}\right)}.$$

The detailed solution to the planner's problem is available in Appendix B.

Note that an increase in $\varphi_t(\cdot)$ implies that a larger share of world consumption goods is diverted to the home country. The mechanism works as follows: given history of realized states up to $t-1$, the ratio of $exp\left\{\dfrac{U_{h,t}(s_t)}{\theta}\right\}$ and $exp\left\{\dfrac{U_{f,t}(s_t)}{\theta}\right\}$ (scaled by previous expectations about current utility) indicates which of the agents has a better draw in time t, tilting $\varphi_t(s_t)$ towards the agent with the relatively worse draw. Also note that $\varphi_{t-1}(s^{t-1})$ and $E_{t-1}\left[exp\left(\dfrac{U_{i,t}(s_t)}{\theta}\right)\right]$ are known quantities as of the beginning of period t so all new information about $\varphi_t(\cdot)$ must come from $U_{h,t}(s_t)$ and $U_{f,t}(s_t)$.

To illustrate how the mechanism works, suppose the home country gets a bad draw, then the ratio of pseudo-Pareto weight rises and temporarily shifts world consumption towards the home country, partially offsetting the welfare loss from the bad draw and allowing the home country to have a less volatile consumption profile. The mechanism becomes quite clear when we consider a Taylor expansion of the allocations around $\varphi = 1$:

$$\begin{aligned} \Delta\, log\, C_{ht} &= 2\alpha(1-\alpha)(\varphi_t - \varphi_{t-1}) + \alpha\, \Delta\, log\, X_t + (1-\alpha)\, \Delta\, log\, Y_t \\ \Delta\, log\, C_{ft} &= -2\alpha(1-\alpha)(\varphi_t - \varphi_{t-1}) + (1-\alpha)\, \Delta\, log\, X_t + \alpha\, \Delta\, log\, Y_t. \end{aligned} \tag{3}$$

The perhaps most important feature of the model is that pseudo-Pareto weights respond to second-moment shocks, in addition to fluctuations in the level of endowments. Because agents are utility-risk averse, a rise in endowment volatility decreases continuation utility. To counteract the welfare loss caused by the volatility shock, in the current period, the ratio of pseudo-Pareto weights slants world consumption towards the agent in the relatively worse state. From a robust control perspective, a rise in volatility makes agents assign higher probability to bad states which, in turn, lowers expected utility. As discussed in Appendix A, agents that fear model misspecification distort the benchmark state density towards bad states and the magnitude of the distortion depends on how bad the drawn state is. Specifically, the Radon-Nikodym (R-N) derivative that distorts the benchmark state density is identical to marginal utility, which is high in bad states (see Appendix B).

Figure 1 shows the movement in R-N derivatives (marginal utility) for the home and foreign agent when a volatility shock of size 1 s.d. hits the home country. Values above 1 indicate marginal utility above expectations, i.e., drawn state is worse than agents expected.

Note that a volatility shock to the home country is bad news for both countries, since both agents like the home good. However, because the home agent's preferences are strongly

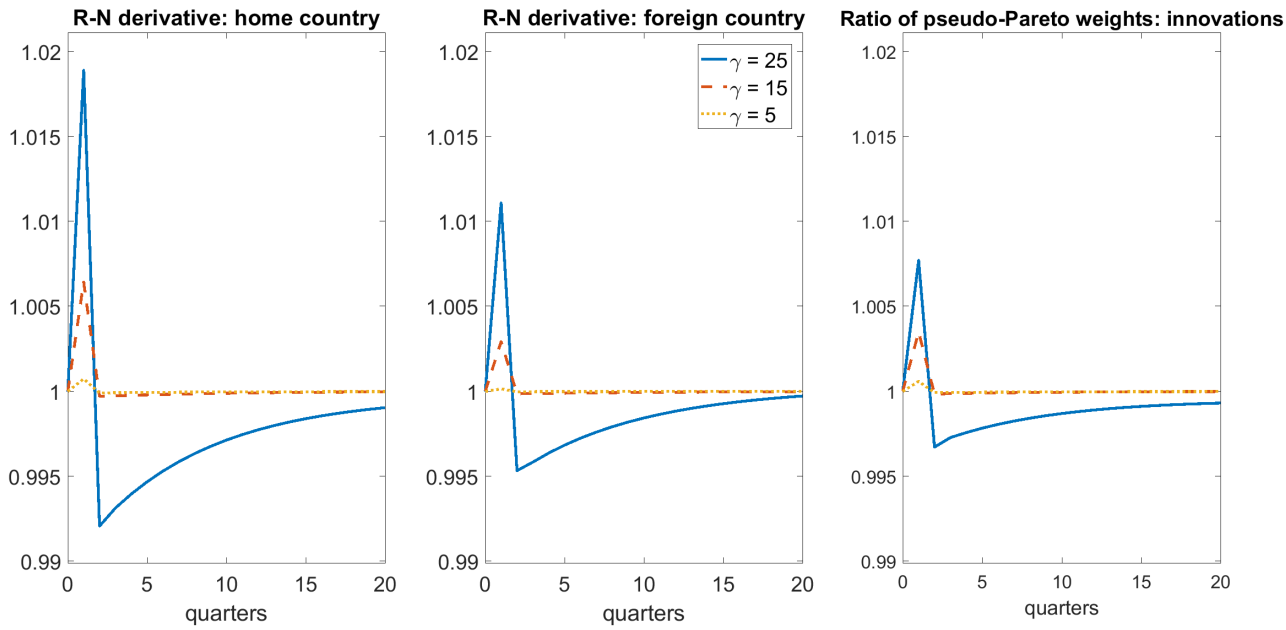

Figure 1: Left and center panels show marginal utility $exp\left\{\frac{U_t(s_t|s^{t-1})}{\theta}\right\}\Big/E_{t-1}\left[exp\left\{\frac{U_t}{\theta}\right\}\right]$ for the home and foreign countries upon the realization of a 1 s.d. volatility shock to the home country. Right panel shows the ratio of marginal utilities and confirms that at the moment of the shock, home country is in a relatively worse state. The risk-sharing mechanism slants world consumption towards the home country when times are bad (ratio of marginal utilities sharply above 1 for one period) and the home country slowly pays the foreign country back in the following periods (ratio of marginal utilities slightly below 1 for many periods). Higher values of parameter γ indicate higher sensitivity to model misspecification.

tilted towards the domestic good ($\alpha = 0.95$), she is in a relatively worse state compared to the foreign agent, as the left and center panels show.

The right panel shows that upon the realization of a relatively bad state for the home agent, a larger share of world consumption is sent to the home country (value sharply above 1). In the following periods, world consumption is slightly tilted towards the foreign country as the home country slowly repays the foreign country for the consumption goods sent in bad times (value slightly below 1). From Eq. 3 and Figure 1, the risk-sharing mechanism entails that an increase in volatility in the home country leads to positive (negative) consumption growth in the home (foreign) country, even if the level of endowments is unchanged.

The risk-sharing mechanism presented here also mitigates the welfare loss caused by shocks to the level of endowments. When a negative output shock hits the home country, its marginal utility is high and a larger share of world consumption is "sent" to the home agent. In the following quarters, the home country slowly repays the foreign agent. We will focus on the effects of volatility shocks as Colacito and Croce (2012) provides a comprehensive treatment of the effects of endowment shocks on macro quantities and prices.

2.1 Asset prices

The previous section described how the risk-sharing mechanism relies on marginal utilities to allocate consumption across agents. Not surprisingly, agents that fear model misspecification will price volatility risk since their marginal utility is high in high volatility states.

First we must define the stochastic discount factors in terms of the home and foreign good. Let $M^h_{x,t+1}$ and $M^f_{y,t+1}$ denote the stochastic discount factors for the home and foreign agents in terms of goods X and Y, respectively. For a general state s_{t+1} given history s^t,

$$M^h_{x,t+1} = \frac{\frac{\partial U_{ht}}{\partial x_{h,t+1}}}{\frac{\partial U_{ht}}{\partial x_{ht}}} \quad \text{and} \quad M^f_{y,t+1} = \frac{\frac{\partial U_{ft}}{\partial y_{f,t+1}}}{\frac{\partial U_{ft}}{\partial y_{ft}}}.$$

Recall from the budget constraints that we assume good X is the numeraire. Hence, if we write the pricing kernels in terms of goods rather than consumption itself then pricing contigent claims to future output is straightforward. From Eq. 1, the contingent claim (Arrow security) entitles its holder to one unit of good X if state s_{t+1} realizes and commands price $q_t(s_{t+1}|s^t)$. From the fundamental asset pricing equation,

$$q_t\left(s_{t+1}|s^t\right) = E_t\left[M^h_{x,t+1}\,\mathcal{I}(s_{t+1}|s^t)\right]$$

where $\mathcal{I}(s_{t+1}|s^t)$ is the indicating function that assigns 1 if state s_{t+1} occurs and zero otherwise.

If, given history s^t, state s_{t+1} occurs with probability $\pi(s_{t+1}|s^t)$, then expression above simplifies to

$$q_t\left(s_{t+1}|s^t\right) = \delta\left(\frac{x_{h,t+1}}{x_{h,t}}\right)^{-1} \frac{exp\left\{\dfrac{U_{t+1}(s_{t+1}|s^t)}{\theta}\right\}}{E_t\left[exp\left\{\dfrac{U_{t+1}(\cdot|s^t)}{\theta}\right\}\right]}\pi\left(s_{t+1}|s^t\right).$$

The term $\delta\left(\frac{x_{h,t+1}}{x_{h,t}}\right)^{-1}$ is identical to the stochastic discount factor under time-separable preferences: assets that pay off in states in which consumption is low command higher prices. Under non-time-separable preferences as considered in this paper, agents who fear misspecification have an additional precautionary savings motive and the increased demand for claims that pay off in bad states is reflected in their higher prices.

The risk-free asset is a portfolio of contingent claims for all possible states. Thus, the price $Q_t(s^t)$ of the risk-free asset is given by

$$Q_t^x(s^t) = \sum_{s_{t+1}} q_t\left(s_{t+1}|s^t\right) = E_t\left(M_{x,t+1}^h\, 1\right)$$

since the risk-free asset pays one unit of X regardless of the state in $t+1$. Likewise, an asset that pays one unit of good Y with certainty in the next period costs $Q_t^y = E_t\left(M_{y,t+1}^f\, 1\right)$, in units of good Y. Returns from the risk-free assets are $\frac{1}{Q_t^x}$ and $\frac{1}{Q_t^y}$, in units of good X and Y respectively.

Figure 2 shows the response of risk-free returns in the home and foreign countries to 1 s.d. volatility shocks originated the home (left) and foreign (center) countries and a 1 s.d. shock to the global component (right). The left and center panel show that the demand for safe assets increases in the country that receives a volatility shock and the lower risk-free returns reflect the increase in precautionary savings.

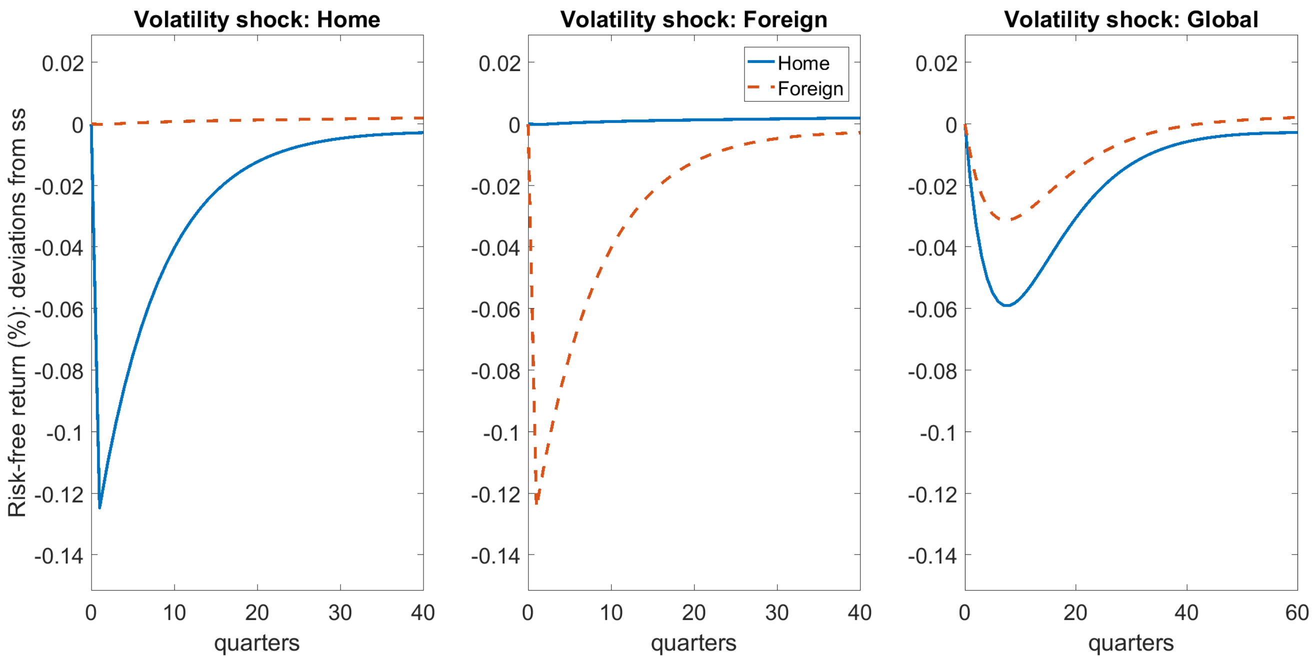

Figure 2: Risk-free returns: deviations from steady state upon arrival of 1 s.d. shock to the home (left), foreign (center) and global (right) volatility processes. Home country volatility is assumed to be more exposed to the global component ($\beta_h = 1.3$ and $\beta_f = 0.7$). Common component in volatility induces co-movement in interest rates across countries.

The right panel shows that interest rates in both countries fall when a volatility shock hits the global economy. The magnitude of the drop, however, is not the same across countries. Motivated by the data, we assume that one of the countries is more exposed to global uncertainty and calibrate $\beta_h = 1.3$ and $\beta_f = 0.7$ in Eq. 2. The stronger precautionary savings motive in the more exposed country causes a more pronounced increase in the price of the risk-free asset and consequent decline in interest rate.

Because output volatility in the more exposed country rises more when global volatility rises, the risk-sharing mechanism described previously will slant world consumption towards the high-exposure country (home country in Figure 2). Assuming the output level remains unchanged, the increased consumption will initially lead to a deterioration of trade balance (increase in imports/ decrease in exports) and subsequent reversion in the direction of the trade flows as the high-exposure country must repay the low-exposure agent for the goods sent in bad times.

So far we have discussed the model-implied responses of macro quantities (consumption and trade balance dynamics) and asset prices (co-movements in interest rates and increased demand for safe assets) to fluctuations in global uncertainty. Another noteworthy feature of the global component of uncertainty is the induced cross-country correlation of output volatility without imposing any covariance structure on the idiosyncratic country-specific shocks. Despite our assumption that country-specific innovations to the volatility process are uncorrelated, volatility itself is correlated across countries. The magnitude of these correlations depend on the exposure to the global factor. The dynamics proposed in Eq. 2 imply that the correlation between output log-volatilities of countries i and j is given by

$$Corr\left(\sigma_{it}, \sigma_{jt}\right) = \frac{\beta_i \beta_j \sigma_\eta^2}{\left(\beta_i^2 \sigma_\eta^2 + \sigma_\sigma^2\right)^{\frac{1}{2}} \left(\beta_j^2 \sigma_\eta^2 + \sigma_\sigma^2\right)^{\frac{1}{2}}}. \tag{4}$$

We will revisit Eq. 4 later with estimated values for the exposure parameters and discuss the model-implied correlations of output volatility across countries.

Next section discusses the estimation of the stochastic processes driving output volatility and the empirical plausibility of the theoretical model proposed in this section.

3 Parameter estimation and empirical plausibility of the model

This section discusses the estimation of the stochastic processes presented in the previous section and whether the model implications are supported by the data. We first estimate the stochastic volatility process for the each country's output growth. Then, we extract the common component of country-level filtered volatilities via a Kalman filter. Later, we check if macroeconomic data corroborate the model-implied responses of consumption, foreign trade and interest rates.

The sample consists of quarterly data for Australia, Canada, Denmark, France, Germany, Greece, Italy, Japan, Mexico, the Netherlands, Norway, New Zealand, South Africa, Spain, Sweden, Switzerland, Turkey, UK and the U.S. from 1960-2016. Data for Argentina, Brazil, Chile, Colombia, India, Korea and Poland are available since 1995. All macroeconomic variables are measured in real terms and extracted from OECD website.

The estimation of stochastic volatility follows Kim et al. (1998). Specifically, let y_{it} be country i's real output per capita at time t. Then we can denote the stochastic volatility specification by

$$\begin{aligned} \Delta\,\tilde{y}_{i,t+1} &= exp\left(\frac{h_{it}}{2}\right)\varepsilon^{y}_{i,t+1} \\ h_{it} &= \mu_i + \phi_i\,(h_{i,t-1}-\mu_i) + \sigma_{\sigma,i}\,\varepsilon^{\sigma}_{it} \end{aligned} \tag{5}$$

where $\varepsilon^{\sigma}_{it} \sim N(0,1)$ and $\varepsilon^{\sigma}_{it} \sim N(0,1)$ are endowment and volatility shocks, respectively. Also, we assume $E\,(\varepsilon^{y}_{it}\,\varepsilon^{\sigma}_{it}) = 0$. Parameters μ_i, ϕ_i and $\sigma_{\sigma,i}$ are the mean, persistence and volatility of log-volatility of output growth.

Eq. 5 is estimated via Markov Chain Monte Carlo (MCMC). To be consistent across countries, all priors are vague. The prior distribution for the persistence ϕ, for example, is uniform on $[-1,1]$. Prioris for the conditional mean and volatility are $N(-10,3)$ and

$Gamma(0.5, 5)$. To give the reader a sense of magnitude, the priori for the conditional mean implies an annualized output volatility of $200\, exp\left\{\frac{-10}{2}\right\} = 1.35\%$. Figure C.5 displays the empirical priori and posteriori densities for U.S. data. The dashed lines indicate that the priori distributions do not play a major role in the posterioris, as the the posteriori distributions look considerably "tight." This precision is evident in the credible bounds for the estimated parameters in Tables 1, 2 and C.6.

In general, output volatility in developed countries is lower in the 1985-2016 period (see Figures C.7, C.8 and C.9 in Appendix C), with scandinavian countries being notably exceptions. Output volatility in Denmark and Sweden has fluctuated in the same range throughout the sample while Norway's output becomes, on average, more volatile in the post-1980s period. Some other noteworthy events are evident in Figures C.7, C.8 and C.9: the social unrest of the "May 1968" strikes and riots in France (annualized output volatility reached astounding 10%), the "tequila" crisis in Mexico in 1994, the severe economic crisis in Korea in 1998, the collapse of the Turkish lira in 1994 and the turbulent times after the end of military government in Greece (output volatility consistently above 5%). Conditional volatility of output growth in the U.S. is displayed by itself in Figure C.6. Visual inspection suggest that there are two regimes in place, a high-volatility one from 1960 to mid 1980s and a low-volatility regime from the mid-1980s to 2016, as documented by Cogley and Sargent (2005) and Justiniano and Primiceri (2008), among others.

The months following the 2008 financial crisis are, perhaps, the most noticeable event as output volatility spiked for almost all countries. Interestingly, the volatility levels reached in 2009 are hardly unprecedented when when we consider each country individually. From a global perspective, however, it is the only volatility spike in which virtually all countries are affected. That kind of synchronicity is unprecedented. Of our sample, Australia and New

$200\, e^{\frac{\mu}{2}}$					
Country	Australia	Canada	Denmark	France	Germany
Estimate	1.65	1.5	1.79	1.1	1.73
Credible interval	(0.61,3.66)	(1.19,1.95)	(1.39,2.24)	(0.88,1.38)	(1.46,2.05)
Greece	Italy	Japan	Mexico	Netherlands	Norway
3.36	1.66	2.15	1.47	1.82	1.85
(1.34,6.28)	(1.33,2.07)	(1.81,2.58)	(0.99,2.1)	(1.09,2.96)	(1.07,2.96)
New Zealand	South Africa	Spain	Sweden	Switzerland	Turkey
1.84	1.69	1.41	2.04	1.52	1.97
(1.19,2.61)	(1.05,2.57)	(0.81,2.35)	(1.45,2.74)	(1.12,2.09)	(1.14,3.26)
UK	U.S.	Argentina	Brazil	Chile	Colombia
1.36	1.35	3.7	2.35	2.13	1.64
(1,1.86)	(1.06,1.75)	(3,4.49)	(1.9,2.86)	(1.72,2.59)	(1.29,2.02)
India	Korea	Poland			
1.39	2.62	1.56			
(0.66,2.63)	(2.09,3.29)	(1.2,2.04)			

Table 1: Conditional mean of the log-volatility processes. Values are annualized in % terms. Credible intervals are the 2.5- and 97.5-percentiles from the posteriori distributions.

Zealand are the only countries in which output volatility did not rise in 2009.

Output volatility is highly persistent in developed countries, with Germany having the lowest estimated autocorrelation $\hat{\phi} = 0.72$. Also, estimated autocorrelations above 0.9 are not uncommon, with Australia having the most persistent output volatility at 0.992. Under such conditions, the Bayesian approach is particularly convenient because it naturally avoids the nonstationarity issues caused by unit roots. In the context of this paper, we sample from

ϕ					
Country	Australia	Canada	Denmark	France	Germany
Estimate	0.992	0.879	0.849	0.764	0.719
Credible interval	(0.967,0.999)	(0.743,0.962)	(0.676,0.949)	(0.599,0.889)	(0.425,0.897)
Greece	Italy	Japan	Mexico	Netherlands	Norway
0.973	0.844	0.785	0.892	0.925	0.961
(0.935,0.995)	(0.66,0.961)	(0.502,0.933)	(0.805,0.96)	(0.779,0.989)	(0.884,0.994)
New Zealand	South Africa	Spain	Sweden	Switzerland	Turkey
0.899	0.954	0.933	0.912	0.881	0.907
(0.782,0.97)	(0.871,0.992)	(0.865,0.982)	(0.755,0.979)	(0.75,0.962)	(0.835,0.964)
UK	U.S.	Argentina	Brazil	Chile	Colombia
0.87	0.864	0.457	0.467	0.437	0.229
(0.713,0.959)	(0.652,0.963)	(-0.086,0.823)	(-0.409,0.891)	(-0.253,0.865)	(-0.594,0.813)
India	Korea	Poland			
0.899	0.769	0.512			
(0.682,0.983)	(0.51,0.909)	(-0.087,0.917)			

Table 2: Persistence of the log-volatility processes. Credible intervals are the 2.5 and 97.5-percentiles from the posteriori distributions.

the posteriori distribution that draws $\phi = 1$ with probability zero. The credible bounds in Tables 1, 2 and C.6 are the 2.5- and 97.5-percentiles from the posteriori distributions.

Emerging countries are a rather heterogeneous group. Output growth in Latin America is, on average, more volatile than in developed countries, whereas India, Colombia and Poland are on par with Norway and Sweden, for instance. Output volatility in Latin America is, generally speaking, less persistent than Greece, Korea and India, for example. While the short time series for emerging countries play a role in the wide credible intervals, India has one of the shortest time series in our sample of countries but its persistent parameter is estimated with precision comparable to developed countries with greater data availability. Hence, these results suggest that there is considerably more noise in the data for Latin America countries.

Section 3.1 discusses the extraction of the common component of the filtered volatilities obtained from the procedure discussed in this section.

3.1 The global component of output volatility

From the stochastic volatility specification, conditional volatilities are gaussian processes. In this sense, the common component across countries - the global component of volatility - can be extracted via a Kalman filter. One can think of the global component as a latent factor and, as such, agents may be more or less exposed to it.

We use the filtered volatilities from the previous section in the observation equation and assume the global component evolves according to an $AR(1)$ process. Note that the conditional volatility itself is a latent process, hence the necessity of estimating the country-level volatilities before the extraction of the common factor.

Parameters of the Kalman filter are estimated via maximum likelihood but we restrict the sample to the period 1996-2016 so all countries except Colombia have available data.

Regarding the observation equation, some features are worth mention. First, Table 1 suggests that conditional volatility means and autocorrelations vary considerably across countries. With a sample size of 24 countries in the cross-section, the estimation of country-specific parameters for the mean, persistence and volatility of conditional volatility is a challenging task. Plus, we have consistent estimators for the mean and persistence from the previous step. Hence, we subtract the mean from the filtered volatilities h_{it}'s and fix the autocorrelations at the estimated values displayed in Table 2. The volatility of idiosyncratic shocks estimated in the previous step (Table C.6), on the other hand, is likely biased upwards since the specification Eq. 5 ignores the global source of volatility shocks. We use the estimated $\sigma_{\sigma,i}$'s as starting values in the likelihood maximization procedure.

In the theoretical model discussed previously, we conjectured that the home and foreign endowment streams were heterogeneously exposed to global volatility. This heterogeneity was crucial in enabling the volatility-risk-sharing mechanism, as consumption goods flow from the less exposed to the more exposed agent when global volatility rises. Were both agents equally exposed to global uncertainty, there would be no room for risk-sharing as both countries would be equally affected. In this context, some of the model implications discussed before - trade flows towards high-exposure countries and interest rates comovements[1] - are a consequence of the heterogeneity in the exposure to global component.

To make the estimation process more clear, let the global component be denoted by η_t and β_i be country i's exposure to η_t. Also, let v_t denote the innovations to the global component and w_{it} be the idiosyncratic innovations to country i's volatility. The Kalman filter is built for gaussian processes, so $v_t \sim N(0,1)$ and $w_{it} \sim N(0,1)$ are white noise innovations. If the persistence parameters driving the state and observation processes are ρ and ϕ_i for $i = 1, ..., n$, then we can express the system in its state-space representation

[1] If the exposure to the global component is identical across countries then interest rates should move in lockstep upon the realization of a global volatility shock. The heterogeneity in the exposure allows for different magnitudes in the fluctuations of interest rates.

$$
\begin{aligned}
\boldsymbol{\eta}_t &= \mathbf{F}\,\boldsymbol{\eta}_{t-1} + \mathbf{R}\,\mathbf{v_t} \\
\mathbf{h_t} &= \Phi\,\mathbf{h}_{t-1} + \mathbf{H}'\boldsymbol{\eta}_t + \boldsymbol{\Gamma}\,\mathbf{w_t}
\end{aligned}
$$

where

$$
\boldsymbol{\eta}_t = \begin{bmatrix} \eta_t \\ \eta_{t-1} \end{bmatrix}, \mathbf{F} = \begin{bmatrix} \rho & 0 \\ 1 & 0 \end{bmatrix}, \mathbf{v_t} = \begin{bmatrix} v_t \\ 0 \end{bmatrix}, \mathbf{h_t} = \begin{bmatrix} h_{1t} \\ h_{2t} \\ \vdots \\ h_{nt} \end{bmatrix}, \boldsymbol{\Phi} = \begin{bmatrix} \phi_1 & 0 & \dots & 0 \\ 0 & \phi_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \phi_n \end{bmatrix}, \mathbf{H}' = \begin{bmatrix} \beta_1 & 0 \\ \beta_2 & 0 \\ \vdots & \vdots \\ \beta_n & 0 \end{bmatrix}
$$

and

$$
\mathbf{w_t} = \begin{bmatrix} w_{1t} \\ w_{2t} \\ \vdots \\ w_{1t} \end{bmatrix}, \mathbf{R} = \begin{bmatrix} \sigma_\eta & 0 \\ 0 & 0 \end{bmatrix}, \Gamma = \begin{bmatrix} \sigma_{w,1} & 0 & \dots & 0 \\ 0 & \sigma_{w,2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \sigma_{w,n} \end{bmatrix}.
$$

Parameter estimates are displayed in Table 3. Australia, Argentina, Brazil, Chile, Greece, Norway, New Zealand, Poland, and South Africa have unmistakably low exposure to global uncertainty. The low exposure for Australia and New Zealand are not particularly surprising since the two countries have relatively sheltered economies from global economic turmoil. This fact is in line with the literature in currency risk premia (see Lustig et al. (2011) and Lustig and Verdelhan (2007), for example). As mentioned before, output volatility in Australia and New Zealand did not rise even in the aftermath of the 2008 global financial crisis (see Appendix C).

On the higher end of the exposure spectrum are France, Germany, Spain, Switzerland, and the UK. Surprisingly, the largest economies in the sample, Japan and the U.S., are only moderately exposed to global uncertainty.

Global component of volatility	
Persistence	Volatility
0.853	0.061
(0.066)	(0.002)

Exposure to global volatility shocks (β_i)					
Argentina	Australia	Brazil	Canada	Chile	Denmark
0.211	0.186	0.439	1.053	0.345	0.885
(0.0178)	(0.009)	(0.018)	(0.0251)	(0.018)	(0.0229)
France	Germany	Greece	India	Italy	Japan
1.976	1.625	0.341	0.875	1.144	0.934
(0.0357)	(0.0325)	(0.0133)	(0.0215)	(0.0258)	(0.0254)
Korea	Mexico	Netherlands	Norway	New Zealand	Poland
1.214	1.424	1.209	0.091	0.469	0.594
(0.027)	(0.03)	(0.0276)	(0.0016)	(0.0138)	(0.0191)
S. Africa	Spain	Sweden	Switzerland	UK	U.S.
0.256	1.533	1.042	1.293	1.842	1.165
(0.0132)	(0.025)	(0.0235)	(0.029)	(0.0301)	(0.0261)

Volatility of idiosyncratic shocks (σ_i^w)					
Argentina	Australia	Brazil	Canada	Chile	Denmark
0.243	0.026	0.135	0.082	0.207	0.124
France	Germany	Greece	India	Italy	Japan
0.24	0.199	0.079	0.221	0.122	0.158
Korea	Mexico	Netherlands	Norway	New Zealand	Poland
0.266	0.193	0.106	0.07	0.18	0.361
S. Africa	Spain	Sweden	Switzerland	UK	U.S.
0.079	0.169	0.084	0.129	0.161	0.119

Table 3: Maximum likelihood estimates and standard errors of the global volatility process and country-specific exposures.

The values in Table 3 suggest that global uncertainty is highly persistent - estimated autocorrelation $\hat{\rho} = .853$ implies that a volatility shock lingers for almost two years - but relatively small in magnitude - estimated short-run volatility of $\sigma_v = 0.061$. Note, however, that the country-specific exposure amplifies, or dampens, if $\beta < 1$, the magnitude of global volatility shocks. On impact, a global volatility shock will have its magnitude almost doubled in France, for example, but will hit Australia with less than a fifth of its original magnitude. Additionally, the impact in subsequent periods will also depend on the country-specific persistence of conditional volatility. To illustrate the point, on impact, the magnitude of a 1 s.d. global uncertainty shock in France is roughly ten times as large as the impact of the same shock in Australia ($\beta_{FRA}/\beta_{AUS} = 10.62$) but only 6.17 times as large after four periods and 3.45 times as large after eight periods. Recall that v_t denotes the innovations to the global volatility process and h_{it} denotes country i's log-volatility at time t. Thus $\dfrac{\partial\, h_{i,t+j}}{\partial\, v_t}$ is the response of country i's conditional volatility j periods ahead to global volatility shock at time t. On impact, country i's impulse-response is $\dfrac{\partial\, h_{it}}{\partial\, v_t} = \beta_i\, \sigma_\eta$. However, j periods ahead, we have that $\dfrac{\partial\, h_{i,t+j}}{\partial\, v_t} = \beta_i\, \sigma_\eta\, \left(\phi_i^j + \rho_i^j\right)$.

Note that under the high estimated persistence levels, the magnitude of the response in the periods following a global volatility shock will be larger than the response on impact. This will be the case for as long as $\phi_i^j + \rho_i^j > 1$. In addition, the magnitude of the response to global shocks will decay slower than that of idiosyncratic shocks, i.e., conditional volatility will take longer to return to steady state after a global shock. Figure 2 shows how interest rates take considerably longer to return to steady state after a global uncertainty shock compared to after a country-specific shock, even if the magnitude of the deviation is smaller in the former.

At this point, with estimates for the exposure levels and volatility of idiosyncratic shocks

	Germany	Japan	UK	U.S.
Australia	0.18	0.14	0.23	0.21
Canada	0.27	0.21	0.35	0.32
France	0.20	0.15	0.26	0.23
Germany		0.15	0.26	0.23
Japan	0.15		0.19	0.17
Mexico	0.18	0.14	0.24	0.21
Norway	0.04	0.03	0.05	0.04
New Zealand	0.07	0.05	0.09	0.08
Switzerland	0.23	0.18	0.30	0.27
UK	0.26	0.19		0.29
Brazil	0.09	0.07	0.11	0.10
India	0.10	0.08	0.13	0.12
Korea	0.12	0.09	0.15	0.14

Table 4: Model-implied correlations of conditional volatility.

for all countries at hand, we can compute the model-implied correlations of output volatility with parameters estimated from the data. Note that these correlations *assume* that idiosyncratic shocks are uncorrelated across countries. In this sense, discrepancies between empirical correlations computed from conditional volatility series and implied by the model are due to non-orthogonality in cross-country idiosyncratic volatility shocks. Accounting for correlation of the idiosyncratic shocks would greatly increase the number of parameters since each pair of countries would have a specific correlation. For the sample of countries with data since 1960 alone there would be $\binom{24}{2} = 276$ additional parameters, which is clearly unfeasible with roughly fifty years of quarterly data. Hence, the implied correlations displayed in Table 4 should be viewed as the correlations of conditional volatility that can be accounted for by means of loadings on a common source of risk.

Naturally, cross-country interdependence that emerges on a local or regional scale will not be captured by the global uncertainty channel as it ignores bilateral trade agreements, geographical proximity and cultural similarities. Nevertheless, the implied correlation between U.S. and Mexico and U.S. and Canada output volatilities is sizable at 0.21 and 0.32, respectively.

As expected, highly exposed countries tend to be more correlated. Note, however, that the size of the correlation will also depend on the magnitude of the volatility of global shocks relative to that of idiosyncratic shocks, i.e., the noise-to-signal ratio. The high correlation between Australia's output volatility and the other large economies stems from the remarkably small dispersion of its idiosyncratic fluctuations ($\sigma^w_{AUS} = 0.026$). Conversely, France's exceptionally high exposure is not reflected in abnormally high correlations as the volatility of its idiosyncratic shocks is roughly four times as large as that of the global uncertainty series.

Colacito et al. (2016) find that average correlation of output volatility among G-7 countries is 0.32 (0.30 among G-17 countries). According to Table 4, the global component implies correlations that range from 0.15 (Japan and Germany) to 0.35 (Canada and UK). Implied correlations for all G7 countries are higher than 0.15, suggesting that global uncertainty can account for a significant portion of cross-country correlation of output volatility.

Next section discusses the response of macroeconomic variables to global uncertainty shocks. To avoid the idiosyncratic noise from country-specific data, we assign countries to groups according to their level of exposure to the global component (β's from Table 3). Three groups are considered: High, Medium and Low. The cutoffs are the 30th and 70th percentile of the βs point estimates. To remain in the low or high exposure groups, however, country i's exposure must be significantly lower (higher) than the lower (upper) cuttoffs at

Group allocation					
High		Medium		Low	
France	Spain	Canada	Japan	Argentina	New Zealand
Germany	Switzerland	Denmark	Netherlands	Australia	Norway
Mexico	UK	India	Sweden	Brazil	Poland
		Italy	USA	Chile	S. Africa
		Korea		Greece	

Table 5: Countries assigned to groups according to their exposure to the global volatility component.

the 90% confidence level. Otherwise, countries are assigned to the medium exposure group. Table 5 displays the resulting group allocation.

We then compare if the responses observed in the data are compatible with those predicted by the theoretical model.

3.2 VAR evidence: identification via sign restrictions

To asses the impact of global volatility shocks on business cycles, we run a vector autoregression (VAR) for each group of countries with the variables $\left[\hat{\eta}_{t|t},\, h_{it},\, \Delta y_{it},\, \Delta c_{it},\, \Delta \frac{X_{it}-Z_{it}}{Y_{it}},\, r_{it},\, \Delta\, e_{it}\right]$, where $\hat{\eta}_{t|t}$ is the filtered global volatility component, h_{it} is country i's log-volatility, Δy_{it} is the growth rate of real output per capita, Δc_{it} is the growth rate of consumption per capita, $\Delta \frac{X_{it}-Z_{it}}{Y_{it}}$ is the growth rate of trade balance, r_{it} is the real interest rate in annual terms (%) and $\Delta\, e_{it}$ is the growth of real exchange rate measured in units of local currency per U.S. dollar[2]. Countries in each group are pooled together and the country-level variables are centered.

The reduced-form VAR, however, does not yield unique impulse-responses (IRFs) so additional assumptions regarding the autoregressive structure are required in order to identify the response of business cycle variables. Unfortunately, identifying assumptions are not innocuous from an economic perspective and may impose certain dynamics to the data that will lead to misleading conclusions. For instance, it is unclear that the ordering in the propagation of shocks imposed by a Cholesky identification is correct and we cannot formally test if the identification strategy is appropriate. To make matters worse, tests on the shape and significance of impulse-responses will take the identification structure as given so conclusions drawn from the impulse-responses may be tainted by identification assumptions that were unreasonable to begin with.

Output volatility is unquestionably counter-cyclical, but whether surges in volatility cause or are caused by recessions is still an ongoing debate in the literature. Ludvigson et al. (2015), for example, discusses the difficulties in determining the direction of causality

[2] $\Delta\, e_i > 0$ indicates a depreciation of country i's currency against the dollar.

and follows a more skeptical approach to identifying uncertainty shocks. We, too, follow a skeptical approach to identifying the volatility shocks as the literature on the direction of causality between volatility and real activity has not reached a consensus yet.

We follow the identification strategy proposed by Uhlig (2005) and impose restrictions on the sign of the responses to global volatility shocks.

Another reason to pool countries with similar exposure level together is that the shape of the model-implied responses for one country will depend on its exposure level *relative* to the other. For instance, upon the realization of a global volatility shock, the risk-sharing mechanism implies that the trade balance deteriorates in the more exposed and improves in the less exposed country. In the theoretical two-agent economy, the role of each country in the risk-sharing scheme is well defined by the exposure level. In practice, the 24 countries in our sample do not comprise the entire world economy and pairwise comparisons are a bit less straightforward. France and Germany, for example, are both highly exposed to global volatility shocks and are also important trade partners to each other. On the other side of the exposure spectrum, Australia and New Zealand are also important trade partners with New Zealand being more exposed to global uncertainty than Australia. In the event of a global uncertainty shock, the model suggests that both France and Germany see an inflow of consumption goods from abroad (trade balance deterioration) whereas low-exposure Australia and New Zealand both experience an outflow of consumption goods (trade balance improvement), even if in a two-country world with, say, France and Germany only, goods would flow from Germany to France.

The same can be said about consumption. The shape of the response of consumption to a global volatility shock depends on the exposure level *relative* to the other countries. Hence, it would not be reasonable to force the response of consumption and foreign trade before hand. In addition, imposing the sign of the responses *ex-ante* defeats the purpose of checking the plausibility of the model , which is to check if the patterns predicted by the model emerge naturally in the data. Also, because the model is silent with respect to the response of output, no restrictions were made on the shape of output's response.

In the previous section, we found that the estimated exposure to global uncertainty is positive for all countries. From the stochastic volatility specification, if $\hat{\beta}_i$ is greater than zero, then a positive global volatility shock must raise conditional volatility of output in country i. Therefore, the only restriction made on the sign of the responses is that an increase in global volatility leads to an increase in local volatility.

The idea of sign responses is to characterize the set of possible rotation matrices that, when paired with the reduced-form VAR estimates, deliver impulse-responses that comply with the sign restrictions (Uhlig, 2005). Unlike the fully identified impulse-responses obtained with a Cholesky rotation matrix, in which one can draw confidence bounds around unique impulse-responses, the identification via sign restriction considers all possible impulse-responses from the reduced-form VAR that are admissible under the sign restrictions. The penalty function approach penalizes responses that do not conform to the sign restrictions whereas the pure-sign-restriction approach discards responses that violate the restrictions altogether. Following Uhlig (2005), we use the penalty function approach with the same choice of penalty parameter (100) and keep all 1000 random draws. We plot the sets that contain 90% of admissible responses of output, consumption, foreign trade, interest and exchange rates in 3 and 4.

The model assumes volatility and output are uncorrelated, but the data suggests otherwise. Global uncertainty has a strong and significant effect in depressing output in the high and medium exposure groups (see Figure 3). Interestingly, output per capita in the

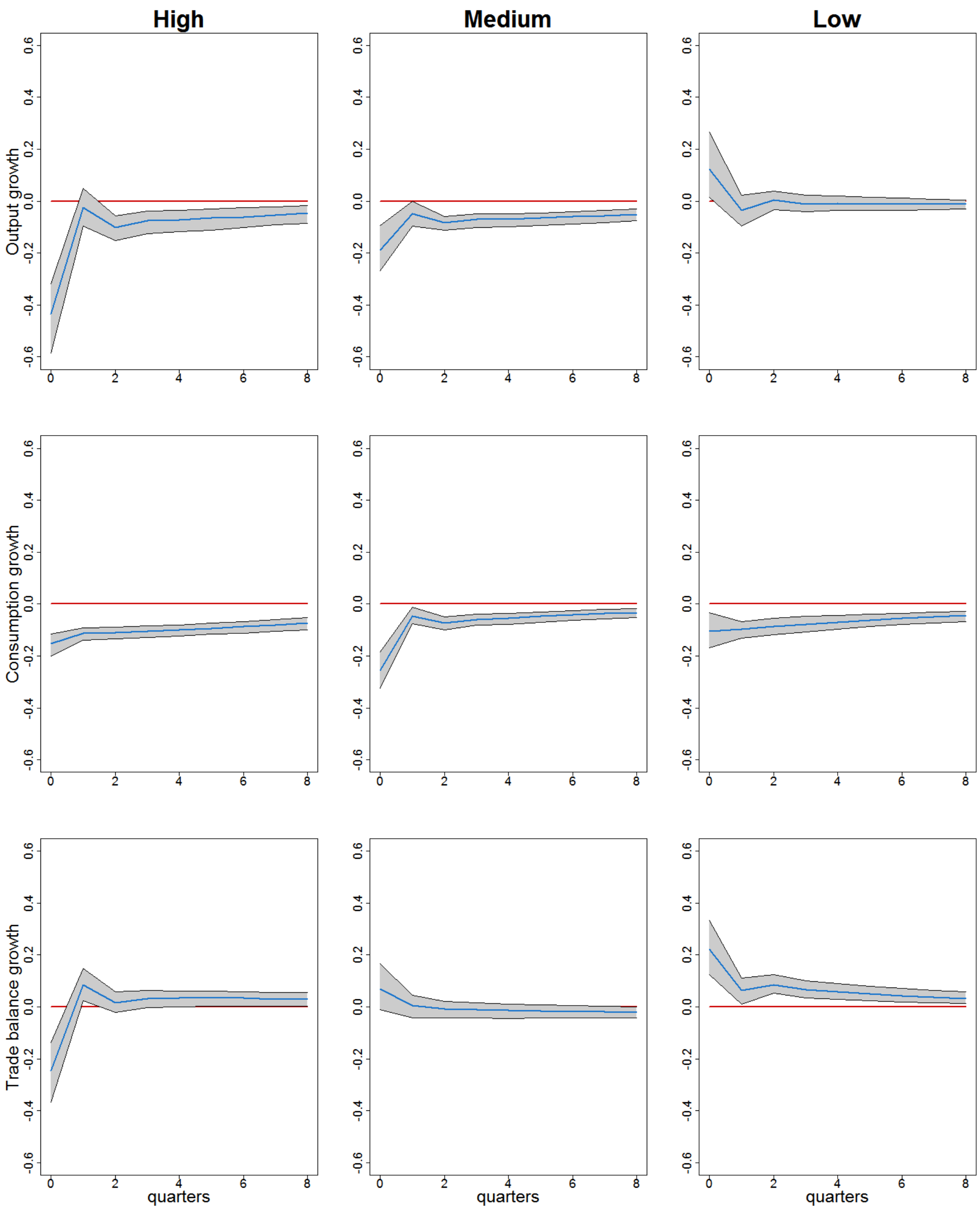

Figure 3: Impulse-responses of output, consumption and trade balance growth to a global volatility shock. Trade balance is measured as a share of GDP. Shaded area indicates 90% of admissible responses.

low exposure group significantly increases when a global volatility shock arrives.

The risk-sharing mechanism does predict movement in consumption when global uncertainty rises: if output remains unchanged and consumption goods flow from the low-exposure towards the high-exposure agent, then consumption in the former must fall and rise in the latter. The data indicate that output does not remain unchanged when global volatility rises, so if the risk-sharing mechanism is in effect, then the magnitude of the decline in consumption in the high exposure country must be smaller than that of output.

Consider the responses of output and consumption in Germany and Switzerland (Figure 3). Upon the realization of a volatility shock, output declines in the high-exposure group, about 0.4%, but the inflow of consumption goods from the rest of the world allows consumption to fall by only 0.2%. In the next period, the direction of the net flows of consumption goods is reverted and the high-exposure countries repay the rest of the world.

The opposite is true for the low exposure countries. The risk-sharing mechanism predicts that low-exposure countries send consumption goods towards their high-exposure counterparts and that this movement in trade flows is reflected in trade balance improvements. That is precisely what the data show (see, again, Figure 3). Consumption falls while output remains constant or even increases leading to a positive and significant trade balance improvement.

The model predicts that the demand for risk-free assets in highly exposed countries increases upon the arrival of a global volatility shock and gradually[3] drives interest rates down. As the exposure level decreases, the magnitude of the increase in marginal utility caused by a rise in global volatility gets smaller, which, in turn, diminishes the size of the decline in interest rate. Figure 4 indicates that the data support the model predictions, that is, interest rates significantly fall in high-exposure countries and the magnitude of the decline in less exposed countries is not significantly different from zero.

[3]Recall that the impulse-response of local volatility to a global volatility shock, j periods ahead is $\dfrac{\partial\, h_{i,t+j}}{\partial\, v_t} = \beta_i\, \sigma_\eta\, \left(\phi_i^j + \rho_i^j\right)$.

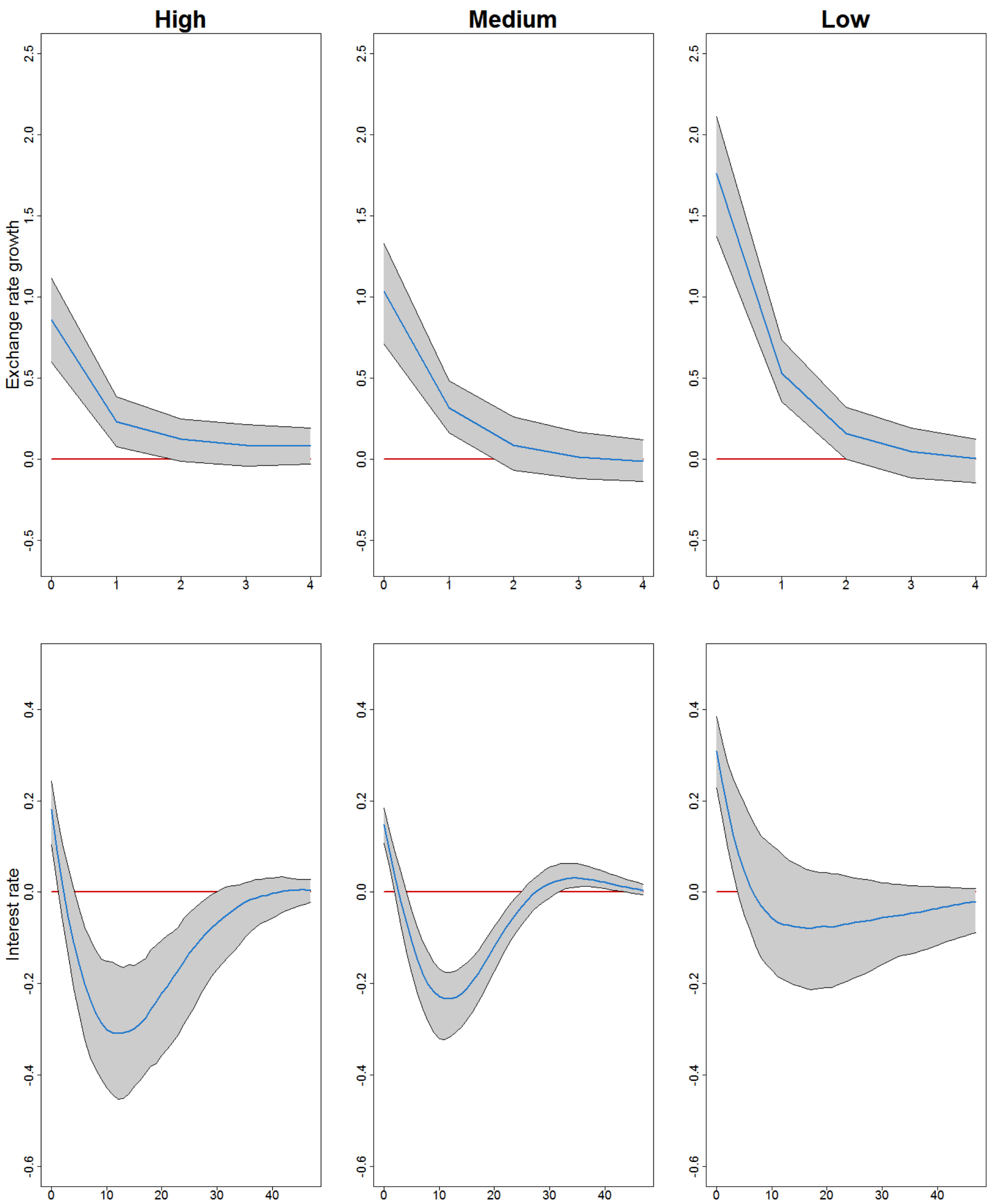

Figure 4: Impulse-responses of interest rate and exchange rate growth to a global volatility shock. Shaded area indicates 90% of admissible responses.

Figure 4 also suggests that currencies highly exposed to global volatility depreciate less than those of low exposure countries. If the complete markets assumption holds then the agent in relatively worse conditions (higher marginal utility) sees a currency appreciation.

All currencies are quoted in U.S. dollars and it is a widely documented fact that the demand for U.S. dollars increases disproportionately during times of economic turmoil. Therefore, it is not surprising that all three groups experience an exchange rate depreciation against the dollar.

The magnitude of the depreciation in the low exposure group, however, is significantly larger than in the high exposure group. Put differently, highly-exposed currencies relatively appreciate against their low-exposure counterparts upon the realization of a global volatility shock, which is in line with the model predictions. Also, these results corroborate the findings of the currency premium literature (see Lustig et al. (2011) and Lustig and Verdelhan (2007), among others).

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Appendix A Preference for robustness

Hansen and Sargent (2008) put forth the idea of an agent that has a benchmark model of how the economy works but is concerned that her model may be misspecified. In this sense, her decision must be optimal under the benchmark model but also under the set of alternative "reasonable" models.

We denote the agent's benchmark model by the probability measure $p(s_{t+1}|s^t)$ that governs state transitions, conditional on history s^t. One can also think of the benchmark model as the probability measure under which the agent is utility-risk neutral. Choosing the optimal consumption path under fear of model misspecification boils down to maximizing lifetime utility under the worst-case scenario:

$$\begin{aligned} \max \min_{m(s_{t+1})} \quad & U_t = (1-\delta)u(C_t) + \delta \int \left[U_{t+1}\left(s_{t+1}|s^t\right) - \theta\, log\, \left(m(s_{t+1}|s^t)\right)\right] m\left(s_{t+1}|s^t\right) p\left(s_{t+1}|s^t\right) ds_{t+1} \\ \text{s.t.} \quad & \int m\left(s_{t+1}|s^t\right) p\left(s_{t+1}|s^t\right) ds_{t+1} = 1 \end{aligned}$$

The worst-case scenario is defined endogenously by the Radon-Nikodym derivative $m(s_{t+1}|s^t)$ that minimizes expected future utility. The intuition is that $m(\cdot)$ distorts the benchmark measure to assign higher probability to bad states[4]. Also, parameter θ determines the size of the set of models considered by the agent and, in practice, measures the sensitivity to model misspecification since an agent that is less confident about the benchmark model will consider a wider set of alternatives.

The R-N derivative that implies the worst-case scenario minimizes continuation utility

[4]High marginal utility states - see Appendix B

(Bidder and Smith, 2012):

$$\begin{aligned}
&\underset{m(s_{t+1}|s^t)}{\text{Min}} && \int \left[U_{t+1}\left(s_{t+1}|s^t\right) - \theta\, log\, \left(m(s_{t+1}|s^t)\right)\right] m\left(s_{t+1}|s^t\right) p\left(s_{t+1}|s^t\right) ds_{t+1} \\
&\text{s.t.} && \int m\left(s_{t+1}|s^t\right) p\left(s_{t+1}|s^t\right) ds_{t+1} = 1
\end{aligned} \tag{A.1}$$

First order necessary conditions for states $\bar{s}_{t+1}$ and s_{t+1} imply that

$$\begin{aligned}
U_{t+1}(\bar{s}_{t+1}) - \theta\, log\,(m(\bar{s}_{t+1})) &= U_{t+1}(s_{t+1}) - \theta\, log\,(m(s_{t+1}))\,, \quad \forall\, s_{t+1} \\
\Rightarrow \theta\,[log\,(m(s_{t+1})) - log\,(m(\bar{s}_{t+1}))] &= U_{t+1}(s_{t+1}) - U_{t+1}(\bar{s}_{t+1}) \\
log\,\left(\frac{m(s_{t+1})}{m(\bar{s}_{t+1})}\right) &= \frac{U_{t+1}(s_{t+1}) - U_{t+1}(\bar{s}_{t+1})}{\theta} \\
\frac{m(s_{t+1})}{m(\bar{s}_{t+1})} &= \frac{exp\left\{\dfrac{U_{t+1}(s_{t+1})}{\theta}\right\}}{exp\left\{\dfrac{U_{t+1}(\bar{s}_{t+1})}{\theta}\right\}}
\end{aligned}$$

where the history s^t is not explicitly displayed to ease notation.

Impose constraint $\int m\left(s_{t+1}|s^t\right) p\left(s_{t+1}|s^t\right) ds_{t+1} = 1$:

$$\frac{1}{m(\bar{s}_{t+1})} = \frac{E_t\left[exp\left\{\dfrac{U_{t+1}(s_{t+1})}{\theta}\right\}\right]}{exp\left\{\dfrac{U_{t+1}(\bar{s}_{t+1})}{\theta}\right\}} \Rightarrow m(\bar{s}_{t+1}) = \frac{exp\left\{\dfrac{U_{t+1}(\bar{s}_{t+1})}{\theta}\right\}}{E_t\left[exp\left\{\dfrac{U_{t+1}(s_{t+1})}{\theta}\right\}\right]}.$$

Plugging $m(\cdot)$ back into the objective function in Eq. A.1 yields:

$$\begin{aligned}
& \int \left[U_{t+1}(s_{t+1}|s^t) - \theta\, log\left(\frac{exp\left\{\dfrac{U_{t+1}(s_{t+1})}{\theta}\right\}}{E_t\left[exp\left\{\dfrac{U_{t+1}(s_{t+1})}{\theta}\right\}\right]}\right)\right] \frac{exp\left\{\dfrac{U_{t+1}(s_{t+1})}{\theta}\right\}}{E_t\left[exp\left\{\dfrac{U_{t+1}(s_{t+1})}{\theta}\right\}\right]} p(s_{t+1}|s^t) ds_{t+1} \\
\Rightarrow\ & \int \left[U_{t+1}(\cdot) - U_{t+1}(\cdot) + \theta\, log\left(E_t\left[exp\left\{\frac{U_{t+1}(s_{t+1})}{\theta}\right\}\right]\right)\right] \frac{exp\left\{\dfrac{U_{t+1}(s_{t+1})}{\theta}\right\}}{E_t\left[exp\left\{\dfrac{U_{t+1}(s_{t+1})}{\theta}\right\}\right]} p(s_{t+1}|s^t) ds_{t+1} \\
\Rightarrow\ & \theta\, log\left(E_t\left[exp\left\{\frac{U_{t+1}(s_{t+1})}{\theta}\right\}\right]\right) \frac{1}{E_t\left[exp\left\{\cdot\right\}\right]} \int exp\left\{\frac{U_{t+1}(s_{t+1})}{\theta}\right\} p(s_{t+1}|s^t) ds_{t+1} \\
\Rightarrow\ & \theta\, log\left(E_t\left[exp\left\{\frac{U_{t+1}(s_{t+1})}{\theta}\right\}\right]\right) \frac{1}{E_t\left[exp\left\{\cdot\right\}\right]} E_t\left[exp\left\{\cdot\right\}\right] \\
=\ & \theta\, log\left(E_t\left[exp\left\{\frac{U_{t+1}(s_{t+1})}{\theta}\right\}\right]\right).
\end{aligned}$$

Therefore, maximizing lifetime utility in Eq. A.1 is equivalent to maximizing

$$\text{Max} \quad U_t = (1-\delta)\, u(C_t) + \delta\, \theta\, log\left(E_t\left[exp\left\{\frac{U_{t+1}}{\theta}\right\}\right]\right).$$

We use $u(C_t) = log(C_t)$ throughout this paper for simplicity. Hence, utility at time t is given by

$$U_t = (1-\delta)\; log(C_t) + \delta\, \theta\, log\left(E_t\left[exp\left\{\frac{U_{t+1}}{\theta}\right\}\right]\right).$$

Appendix B Solution to the planner's problem

The planner's problem is

$$\begin{aligned} \underset{\{x_{ht}, x_{ft}, y_{ht}, y_{ft}\}_{t=0}^{\infty}}{\text{Max}} \quad & \mu\, U_{h0} \;+\; (1-\mu)\, U_{f0} \\ \text{subject to} \quad & x_{ht} \;+\; x_{ft} \;=\; X_t \\ & y_{ht} \;+\; y_{ft} \;=\; Y_t, \;\; \forall t. \end{aligned}$$

where

$$\begin{aligned} C_{ht} &= x_{ht}^{\alpha}\, y_{ht}^{1-\alpha} \\ C_{ft} &= x_{ft}^{1-\alpha}\, y_{ft}^{\alpha} \\ U_{it} &= (1-\delta)\, log\, C_{it} + \delta\, \theta\, log\left[E_t\left(exp\left\{\frac{U_{i,t+1}}{\theta}\right\}\right)\right]. \end{aligned}$$

Let $\mu_t^h \;=\; \mu\, \dfrac{\partial\, U_{h0}}{\partial\, U_{h1}} \dfrac{\partial\, U_{h1}}{\partial\, U_{h2}} \cdots \dfrac{\partial\, U_{h,t-1}}{\partial\, U_{ht}}$ and $\mu_t^f \;=\; (1-\mu)\, \dfrac{\partial\, U_{f0}}{\partial\, U_{f1}} \dfrac{\partial\, U_{f1}}{\partial\, U_{f2}} \cdots \dfrac{\partial\, U_{f,t-1}}{\partial\, U_{ft}}$ be the pseudo-Pareto weights for the home and foreign agents respectively. Equilibrium allocations $x_{ht}, x_{ft}, y_{ht}, y_{ft}$ solve the (intratemporal) optimality conditions:

$$\begin{aligned} \mu_t^h\, \frac{\alpha}{x_{ht}} &= \mu_t^f\, \frac{1-\alpha}{x_{ft}} \\ \mu_t^h\, \frac{1-\alpha}{y_{ht}} &= \mu_t^f\, \frac{\alpha}{y_{ft}}. \end{aligned}$$

and resource constraints

$$\begin{aligned} x_{ht} \;+\; x_{ft} &= X_t \\ y_{ht} \;+\; y_{ft} &= Y_t. \end{aligned}$$

Denote the ratio of pseudo-Pareto weights by $\varphi_t(s^t) \;=\; \dfrac{\mu^h(s^t)}{\mu^f(s^t)}$. Then the solution to the system of equations above is given by

$$
\begin{aligned}
x_{ht}(s^t) &= \frac{\alpha\,\varphi_t(s^t)}{1-\alpha+\alpha\,\varphi_t(s^t)} X_t(s^t) \quad , \quad x_{ft}(s^t) = \frac{1-\alpha}{1-\alpha+\alpha\,\varphi_t(s^t)} X_t(s^t) \\
y_{ht}(s^t) &= \frac{(1-\alpha)\,\varphi_t(s^t)}{\alpha+(1-\alpha)\varphi_t(s^t)} Y_t(s^t) \quad , \quad y_{ft}(s^t) = \frac{\alpha}{\alpha+(1-\alpha)\varphi_t(s^t)} Y_t(s^t).
\end{aligned}
$$

Careful examination of the pseudo-Pareto weights reveals that $\mu^h_{t+1} = \mu^h_t \dfrac{\partial U_{h,t}}{\partial U_{h,t+1}}$ and $\mu^f_{t+1} = \mu^f_t \dfrac{\partial U_{f,t}}{\partial U_{f,t+1}}$. Note that the innovations to the pseudo-Pareto weights are identical to the Radon-Nikodym derivative from the previous section (up to a constant δ):

$$
\frac{\partial U_{it}\left(s^t\right)}{\partial U_{i,t+1}\left(s_{t+1}|s^t\right)} = \delta \, \frac{1}{E_t\left[exp\left\{\dfrac{U_{i,t+1}}{\theta}\right\}\right]} \; exp\left\{\frac{U_{i,t+1}\left(s_{t+1}|s^t\right)}{\theta}\right\}.
$$

Therefore, the law of motion for the ratio of pseudo-Pareto weights is given by

$$
\varphi_{t+1}\left(s_{t+1}|s^t\right) = \varphi_t\left(s^t\right) \frac{exp\left\{\dfrac{U_{h,t+1}\left(s_{t+1}|s^t\right)}{\theta}\right\}}{E_t\left[exp\left\{\dfrac{U_{h,t+1}}{\theta}\right\}\right]} \; \frac{E_t\left[exp\left\{\dfrac{U_{f,t+1}}{\theta}\right\}\right]}{exp\left\{\dfrac{U_{f,t+1}\left(s_{t+1}|s^t\right)}{\theta}\right\}}.
$$

Appendix C   Stochastic volatility results

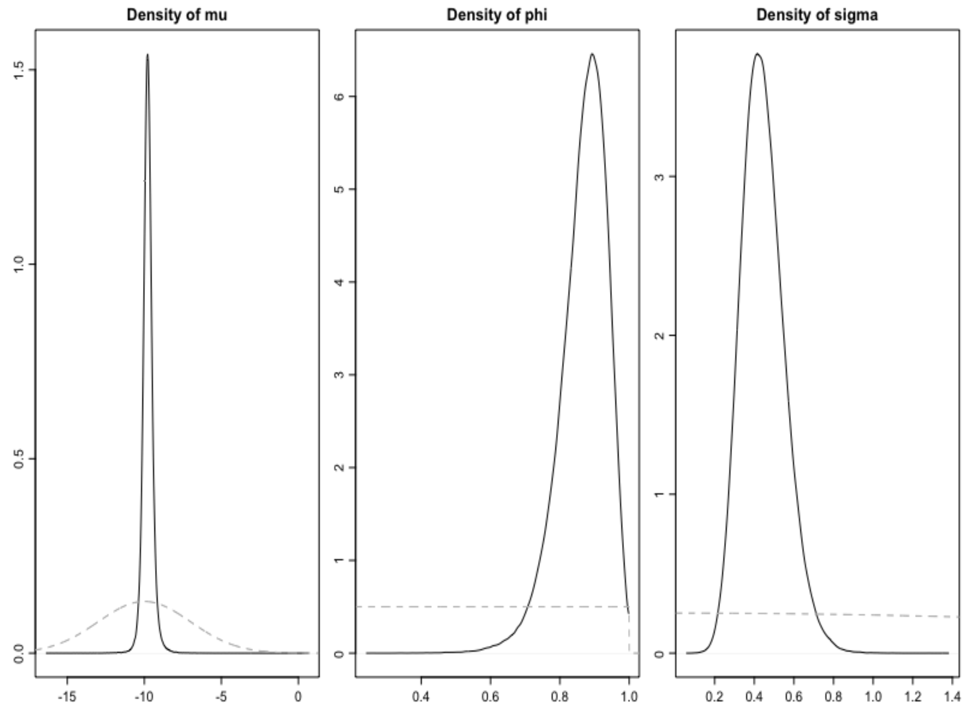

Figure C.5: Priori and posteriori densities for mean, persistence and volatility of log-volatility of U.S. output growth. Dashed (solid) lines show the priori (posteriori) distributions.

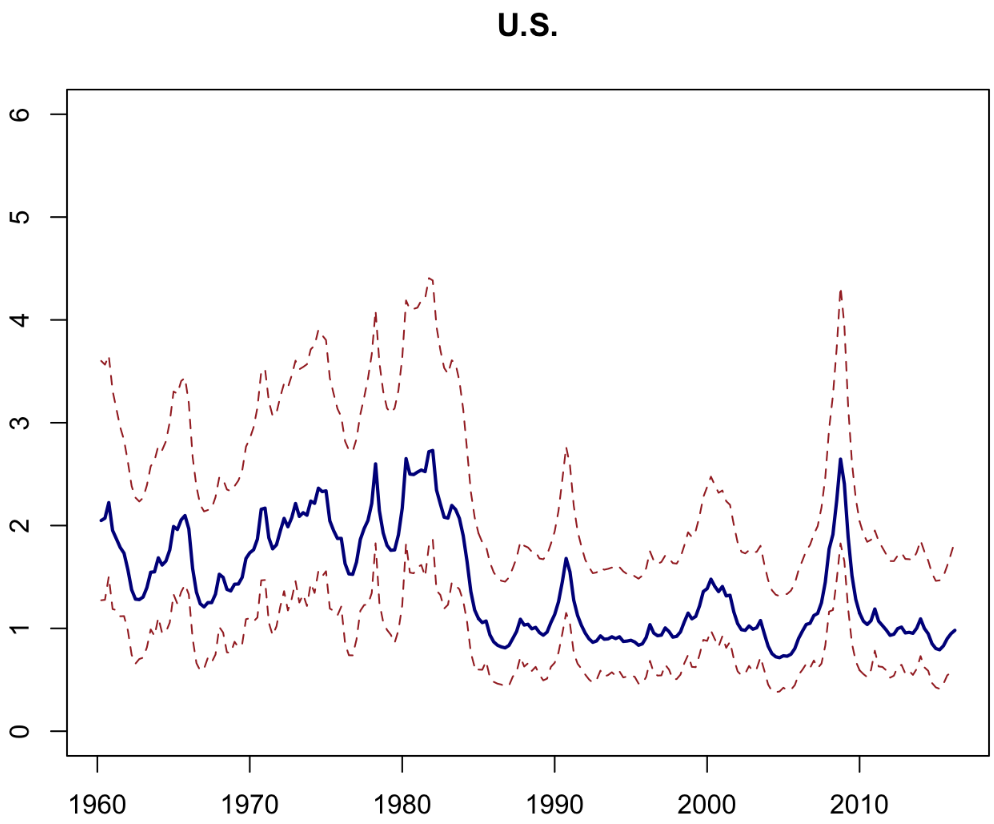

Figure C.6: Annualized volatility of real output growth (%, per capita).

σ_σ					
Country	Australia	Canada	Denmark	France	Germany
Estimate	0.143	0.434	0.517	0.801	0.638
Credible interval	(0.081,0.258)	(0.279,0.63)	(0.288,0.795)	(0.575,1.056)	(0.408,0.909)
Greece	Italy	Japan	Mexico	Netherlands	Norway
0.327	0.486	0.516	0.63	0.524	0.29
(0.241,0.448)	(0.263,0.734)	(0.298,0.812)	(0.472,0.819)	(0.28,0.852)	(0.163,0.49)
New Zealand	South Africa	Spain	Sweden	Switzerland	Turkey
0.58	0.296	0.553	0.399	0.556	0.805
(0.403,0.822)	(0.173,0.476)	(0.406,0.733)	(0.246,0.667)	(0.366,0.798)	(0.629,1.016)
UK	U.S.	Argentina	Brazil	Chile	Colombia
0.605	0.496	0.653	0.485	0.59	0.508
(0.386,0.906)	(0.285,0.818)	(0.263,1.084)	(0.119,0.886)	(0.194,1.055)	(0.052,1.212)
India	Korea	Poland			
0.619	0.694	0.823			
(0.363,1.026)	(0.447,1.01)	(0.378,1.287)			

Table C.6: Estimates for the volatility of the volatility processes.

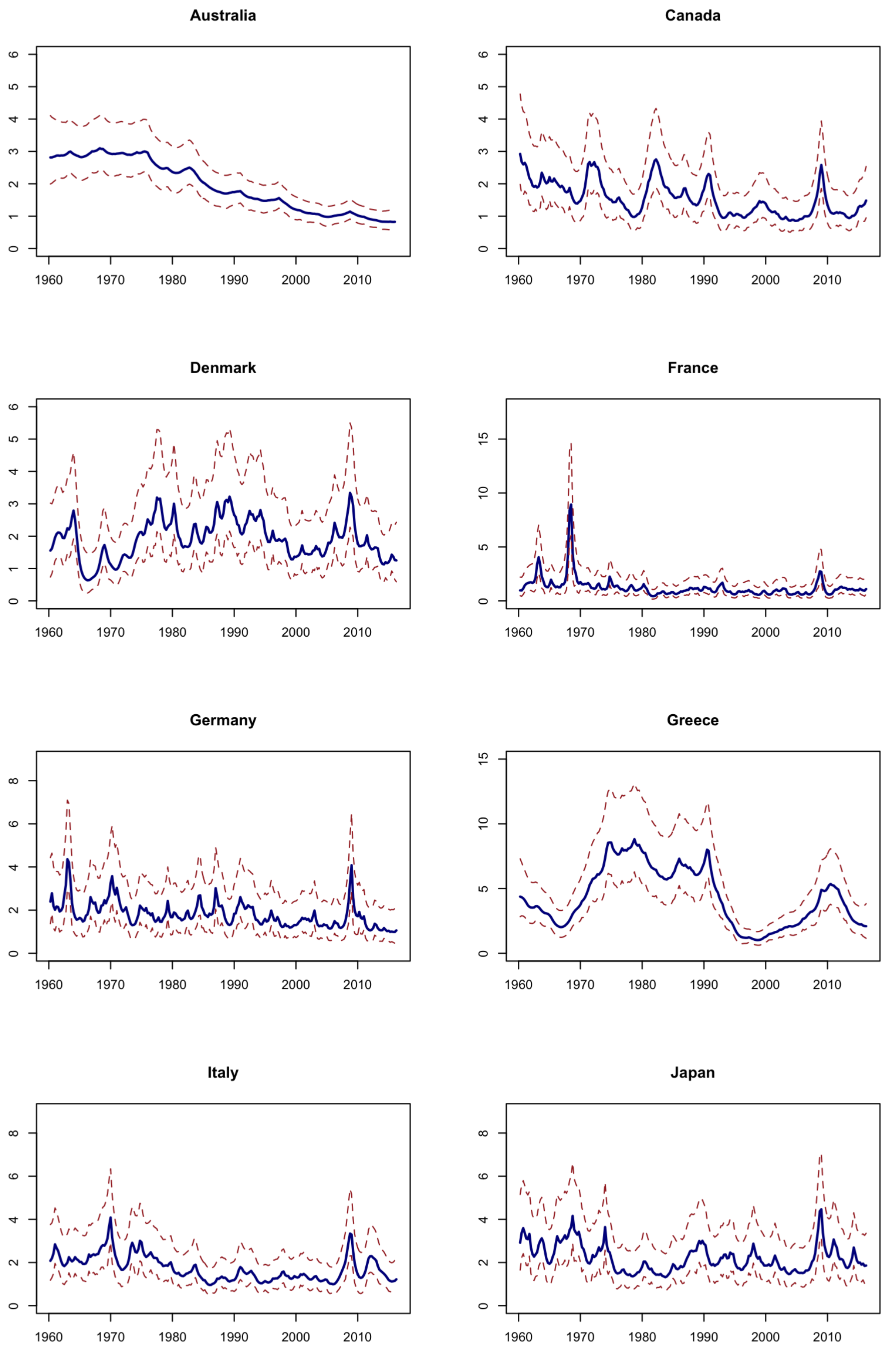

Figure C.7: Annualized volatility of real GDP growth (%).

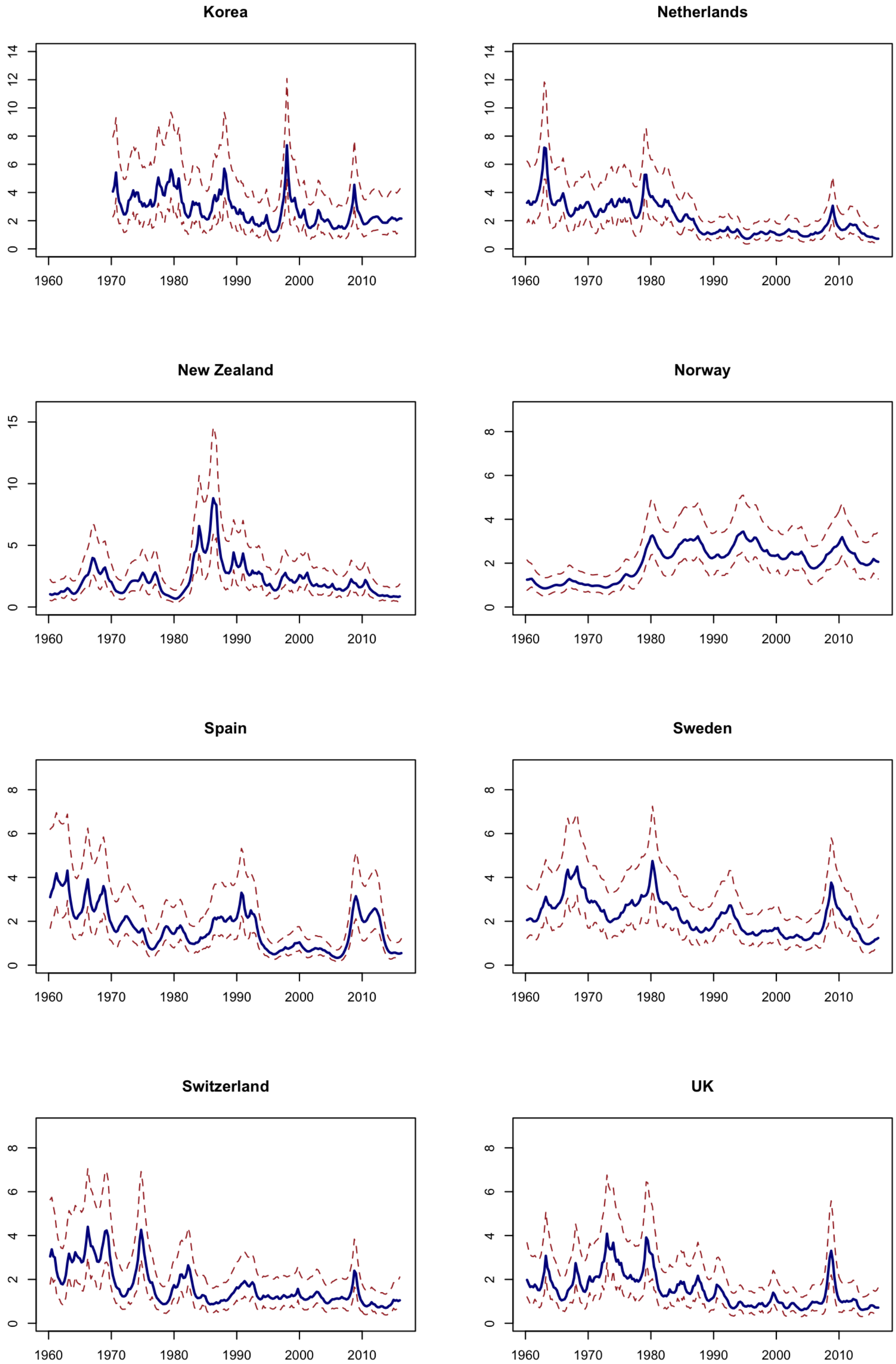

Figure C.8: Annualized volatility of real GDP growth (%).

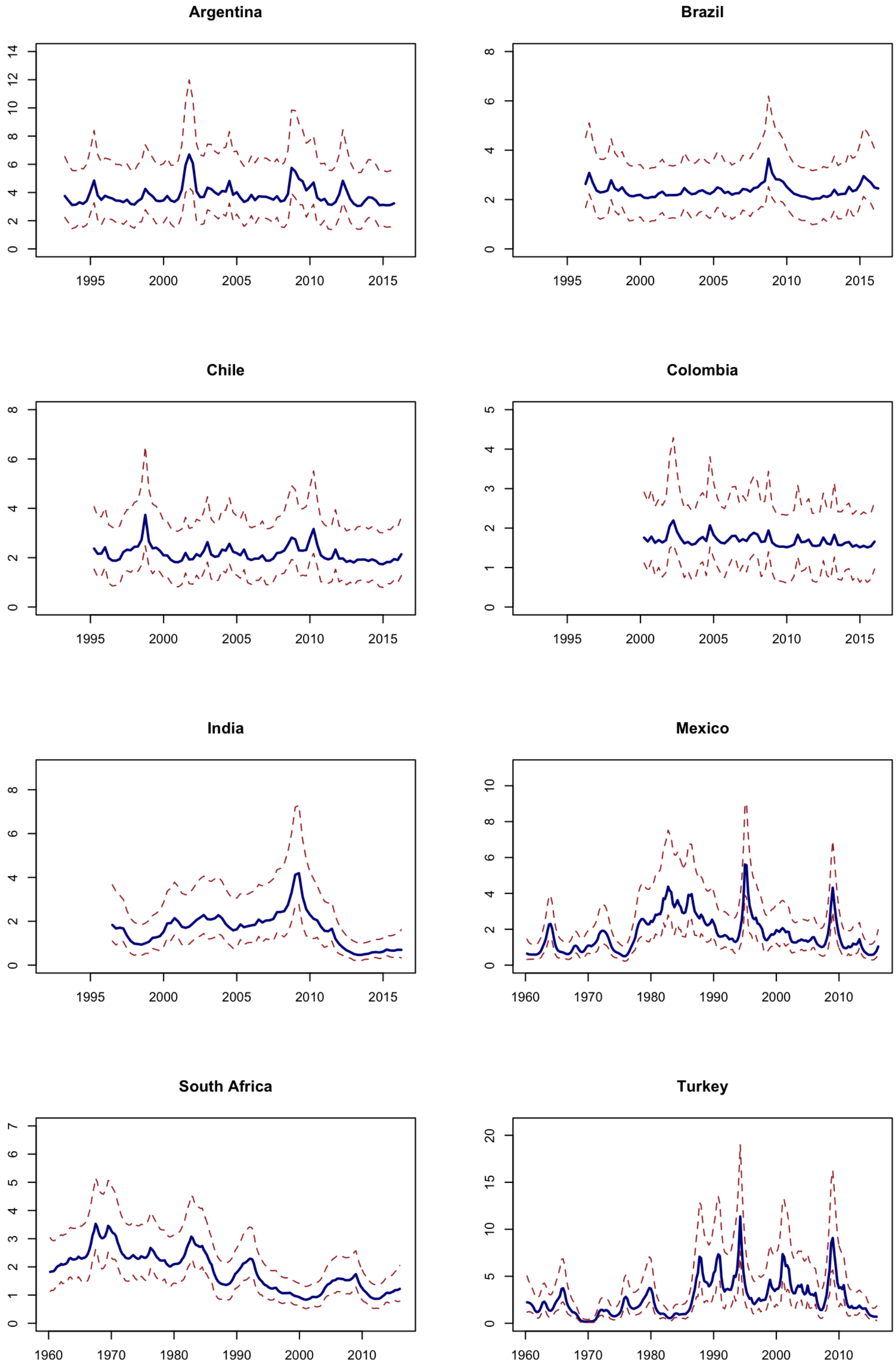

Figure C.9: Annualized volatility of real GDP growth (%).

Appendix D Composition of trade flows

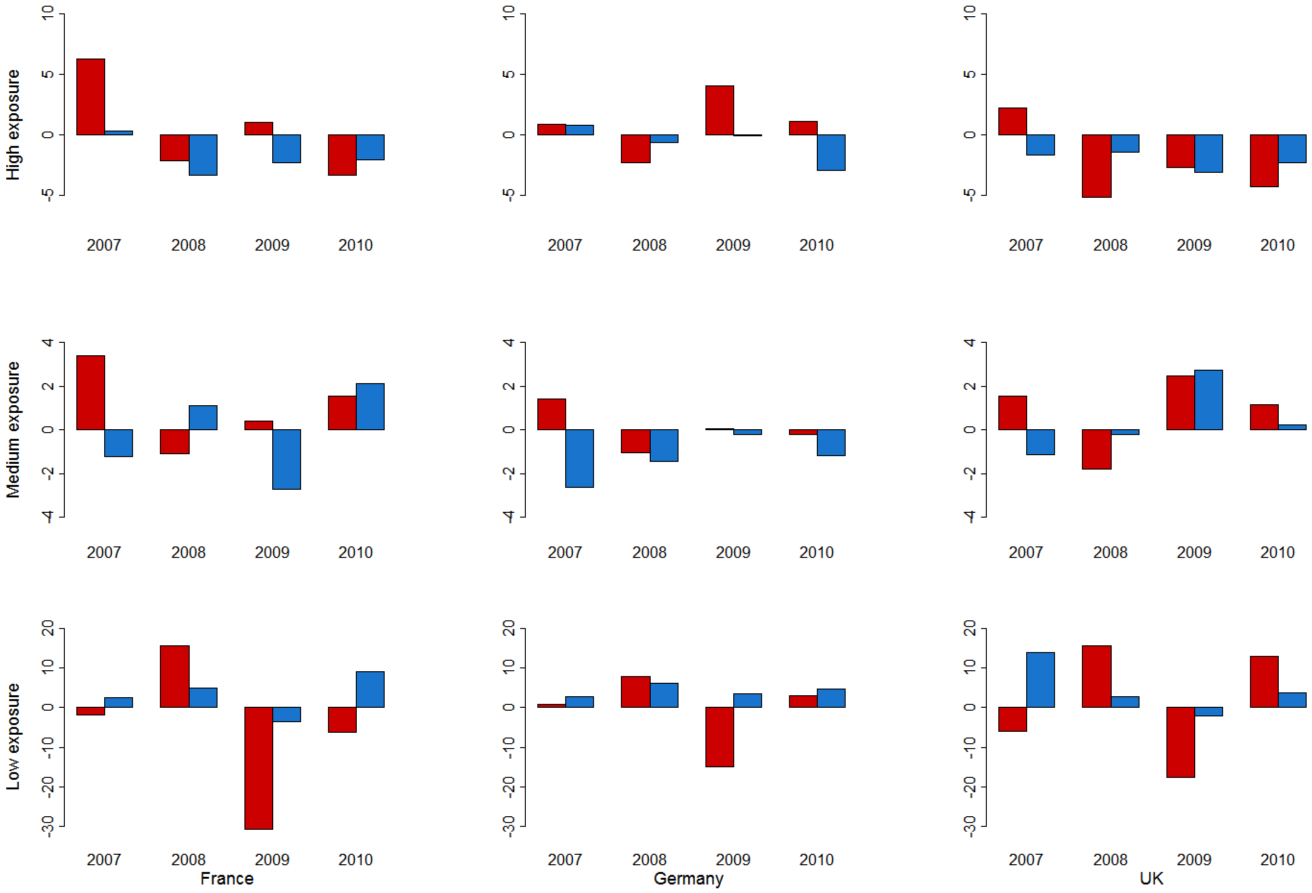

Figure D.10: Growth of imports (red) and imports (blue) by origin/destination in excess of total imports/exports.