

# Example: Overlapping Generations Model

Econ720. Fall 2015. Prof. Lutz Hendricks

---

## 1 Government bonds in an OLG model

Demographics: At each date  $N_t = (1 + n)^t$  households are born.

Preferences are given by

$$(1 - \beta) \ln(c_t^y) + \beta \ln(c_{t+1}^o)$$

Endowments: The initial old are endowed with  $s_0$  units of capital. Each young is endowed with one unit of work time.

Technology:

$$C + K' - (1 - \delta)K = F(K, L) = K^\alpha L^{1-\alpha}$$

Government: The government only rolls over debt from one period to the next:

$$B_{t+1} = R_t B_t$$

Markets: for goods, bonds, labor, capital rental.

**Questions:** (a) Solve the household problem for a saving function.

(b) Derive the FOCs for the firm.

(c) Define a competitive equilibrium. Make sure the number of variables equals the number of independent equations.

(d) Derive the law of motion for the capital stock  $(b_{t+1} + k_{t+1})(1 + n) = \beta(1 - \alpha)k_t^\alpha$ , where  $b = B/L$ .

(e) Derive the steady state capital stock for  $b = 0$ . Why does it not depend on  $\delta$ ?

(f) Derive the steady state capital stock for  $b > 0$ .

(g) Can you show that the capital stock is lower in the steady state with positive debt (crowding out)?

**Answer: Government bonds**

(a) The household solves  $\max(1 - \beta) \ln(w - s) + \beta \ln(R's)$ .

The FOC is  $c'/c = R'\beta/(1 - \beta)$ . Therefore  $s = (w - s)\beta/(1 - \beta)$  and thus  $s = \beta w$ .

(b) Firms: This is standard:

$$\begin{aligned} r &= f'(k) = \alpha k^{\alpha-1} \\ w &= f(k) - f'(k)k = (1 - \alpha)k^\alpha \end{aligned}$$

where  $k = K/L$ .

(c) A CE is a list of sequences  $(c_t^y, c_t^o, s_t, K_t, L_t, b_t, w_t, r_t)$  that satisfy

- the saving function and the 2 household budget constraints
- the 2 firm FOCs
- capital market clearing:  $s_t = (1 + n)(b_{t+1} + k_{t+1})$
- goods market clearing:  $N_t c_t^y + N_{t-1} c_t^o + K_{t+1} = F(K_t, L_t) + (1 - \delta)K_t$ .
- government budget constraint

(d) Law of motion: This follows directly from the capital market clearing condition together with the equilibrium levels of  $w$  and the saving function.

(e) Steady state with  $b = 0$ : From the law of motion:

$$k^{1-\alpha} = \beta(1 - \alpha)/(1 + n)$$

It does not depend on  $\delta$  because of log utility: households save a constant fraction of earnings.

(f) Steady state with  $b > 0$ . Now we need to satisfy the law of motion for  $b$ :  $b'(1 + n) = Rb$ . In steady state:  $R = 1 + n$ . The steady state capital stock therefore satisfies  $\alpha k^{\alpha-1} - \delta = n$  or

$$k^{1-\alpha} = \alpha/(n + \delta)$$

Note that the steady state satisfies the Golden Rule. There is some concern that this steady state may not be stable. Imagine that  $R > 1 + n$ . Then  $b$  rises ( $b' > b$ ). This may reduce the capital stocks and drive up  $R$  even further, etc.

(g) Crowding out: Note that from the law of motion derived in (d):

$$b = \frac{\beta(1 - \alpha)}{1 + n} k^\alpha - k = k \left[ \frac{\beta(1 - \alpha)}{1 + n} k^{\alpha-1} - 1 \right]$$

Therefore  $b > 0$  requires

$$\beta(1 - \alpha)/(1 + n) > \alpha/(n + \delta)$$

which is exactly what was to be shown.