Overlapping Generations Model
Bequests and Altruism

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Econ720

August 31, 2015

Topics

We introduce intergenerational links into the OLG model:

- parents leave bequests to their children

The main **goal** is to learn the model setup.

A key result:

- when parents leave bequests, they behave as if they lived forever
- some view this as micro-foundation for models where households live forever

We study whether bequests solve the **dynamic inefficiency** problem

- The answer is no
- Bequests can only increase the capital stock

Bequest Motives

- Why do parents leave bequests to their children?
- Empirically, we don't know (a possible research question).
- Theoretically, there are various ways of modeling bequests:
  1. **Altruism**: parents value their children's utility.
  2. **Warm glow**: parents value the bequest itself (a reduced form).
  3. **Strategic**: parents promise bequests so kids behave well.

OLG Model With Altruism

Model Elements

- We study the standard endowment economy, just with different preferences.
- Demographics: Each household has $(1+n)$ children when old.
- Endowments: e_1 when young, e_2 when old.
- Technology: nada.
- Markets: goods, bonds

Preferences

- The household values own consumption according to

$$u(c_t^y, c_{t+1}^o)$$

- The household also values the utility of the child.
- Preferences are defined recursively:

$$V(t) = u(c_t^y, c_{t+1}^o) + \beta V(t+1)$$

Household

Expanding this we find that the parent values utility of all future generations:

$$\begin{aligned} V(t) &= u(c_t^y, c_{t+1}^o) + \beta[u(c_{t+1}^y, c_{t+2}^o) + \beta V(t+2)] \\ &= u(c_t^y, c_{t+1}^o) + \beta u(c_{t+1}^y, c_{t+2}^o) \\ &\quad + \beta^2[u(c_{t+2}^y, c_{t+3}^o) + \beta V(t+3)] \end{aligned}$$

and therefore

$$V(t) = \sum_{j=0}^{\infty} \beta^j u(c_{t+j}^y, c_{t+j+1}^o) \tag{1}$$

Household

This looks like

- the **planner**'s welfare function,
- the utility function of a household who **lives forever**.

Household problem

- Period budget constraints are

$$\begin{aligned} c_t^y + s_t &= e_1 + b_t \quad (2) \\ c_{t+1}^o + (1+n)\, b_{t+1} &= e_2 + R_{t+1} s_t \quad (3) \end{aligned}$$

- b_{t+1} is the bequest left to each child by cohort t.
- Present value budget constraint (set $n = 0$ for simplicity):

$$\begin{aligned} b_t &= \underbrace{c_t^y - e_1 + (c_{t+1}^o - e_2)/R_{t+1}}_{z_t} + b_{t+1}/R_{t+1} \\ &= z_t + b_{t+1}/R_{t+1} \end{aligned}$$

Budget constraint

Successively replace the b_{t+j} with $z_{t+j} + b_{t+j+1}/R_{t+j+1}$ to obtain

$$b_t = \sum_{j=0}^{J} \frac{z_{t+j}}{D_{t,j}} + \frac{b_{t+J+1}}{D_{t,t+J+1}}$$

where

$$D_{t,j} = \prod_{i=1}^{j} R_{t+i}$$

is a discount factor.

Budget constraint

Take $J \to \infty$ and assume that

$$\lim_{J\to\infty} \frac{b_{t+J}}{D_{t,t+J}} = 0$$

We discuss (much) later why we might want to assume this.

Then the present value budget constraint becomes

$$\begin{aligned} & \textstyle\sum_{j=0}^{\infty} \frac{c^y_{t+j} + c^o_{t+j+1}/R_{t+j+1}}{D_{t,j}} \\ = \quad & b_t + \textstyle\sum_{j=0}^{\infty} \frac{e_1 + e_2/R_{t+j+1}}{D_{t,j}} \end{aligned}$$

Budget constraint

This is a common result:

Present value of spending = [Present value of income] + [Initial assets]

This looks like the budget constraint of an infinitely lived household.

Infinitely lived dynasty

The parent therefore behaves exactly like an infinitely lived individual

- maximizing a single utility function over an infinite horizon
- subject to a single present value budget constraint.

This only works if

- households can borrow and lend at the same interest rate;
- bequests can be negative or are always intended to be positive
- parents are altruistic (not warm glow etc)

Exercise

Show that the equilibrium allocation is the same as the planner's allocation.

Implications

Why is this important?

- If we think bequests are positive, we can ignore finite lifetimes and write down models with a single, infinitely lived household.

One potential problem:

- We set up the parent's problem as if he could choose the child's actions.
- Why can we do that?

When Are Bequests Positive?

And do they help with dynamic inefficiency?

When are bequests positive? I

Bequests are positive, if a small bequests raises parental utility.

Consider the following perturbation of the optimal plan with $b = 0$:

1. Reduce old age consumption by ε. The utility loss is $-u_2(t)\varepsilon$.
2. Give $\varepsilon/(1+n)$ to each child as a bequest.
3. Assume the child eats the bequest when young [what if not?] and gains
$$\beta u_1(t+1) \cdot \varepsilon/(1+n) \tag{4}$$
4. The household wants to leave a bequest if
$$\beta u_1(t+1) \cdot \varepsilon / (1+n) > u_2(t) \cdot \varepsilon \tag{5}$$

When are bequests positive? II

5. Apply the parent's FOC to express both gain and loss in terms of u_1. The FOC is

$$u_1(t) = (1 + r_{t+1}) u_2(t)$$

Thus the parent increases his bequest if

$$\beta u_1(t+1) \cdot \varepsilon / (1+n) > u_1(t)/(1 + r_{t+1}) \cdot \varepsilon$$

6. In steady state this reduces to $\beta / (1+n) > 1/(1 + r_{t+1})$ or $(1+r) > (1+n)/\beta$.

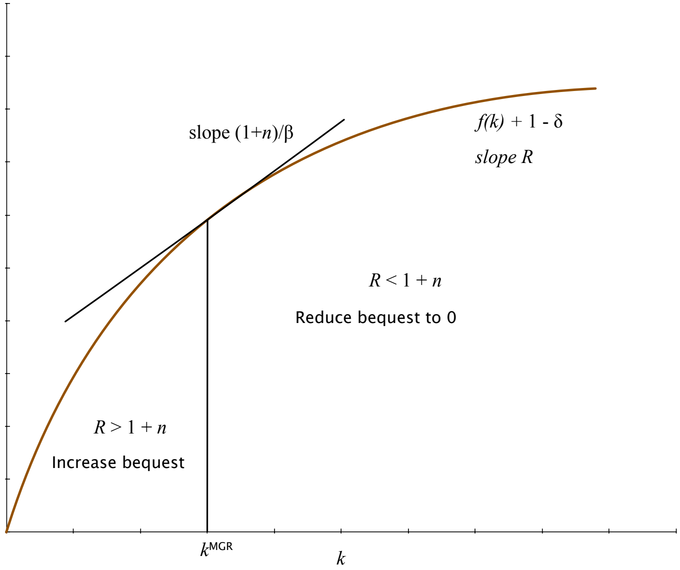
$1 + r = (1+n)/\beta$ is the **modified golden rule** (the planner's FOC).

Dynamic inefficiency

This means:

- A situation where $R > (1+n)/\beta$ can never be an equilibrium.
  - Every parent would want to increase his bequest until the MGR holds with equality
  - Then the economy is dynamically *efficient*.
- If without bequests $R < (1+n)/\beta$, households don't want to leave bequests and the bequest motive is irrelevant.
  - Dynamic inefficiency remains.

Dynamic inefficiency



Summary

If the bequest motive is operative $(b > 0)$, then:

- The economy attains the modified golden rule.
- Therefore it is dynamically efficient.
- The market equilibrium coincides with the planner's solution (show this!).
- Ricardian equivalence holds even across generations. (We haven't shown that, but it follows directly from the fact that there is a present value budget constraint that holds across generations.)

If the bequest motive is not operative, it does not matter.

Applications of OLG Models

Two main reasons for using OLG models:

1. Demographic structure matters:
   1.1 Social security and tax analysis (e.g., many papers by Auerbach and Kotlikoff (1987))
   1.2 Human capital: schooling followed by on-the-job learning (e.g., many papers by Heckman and his students)
   1.3 Income or wealth inequality (e.g., Huggett (1996); Huggett, Ventura, and Yaron (2011)

   These are usually computational many-period models.
2. Analytical tractability:
   With log utility consumption becomes independent of r_{t+1}.
   Easy dynamics because agents behave as if not foward looking.
   E.g., Aghion, Howitt, and Violante (2002), Krueger and Ludwig (2007)

Reading

- Acemoglu (2009), ch. 5.3, 9.

References I

Acemoglu, D. (2009): *Introduction to modern economic growth*. MIT Press.

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