A Simple Two Period OLG Model

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Contents

Introduction

We now compute the 2 period OLG model in general equilibrium.

We pretend that we want to use the model to study fiscal policy.

Environment

Demographics: There are $N_t = (1+n)^t$ young born at t.

Technology: $F(K, L) = AK^{\alpha}L^{1-\alpha} = K' - (1-\delta)K + C$

Markets:

- Labor: w^G (before tax)
- Capital rental: r^G (before tax)
- Goods: numeraire

Firms

First-order conditions (standard):

- $w_t^G = (1-\alpha)\, A\, (K/L)^{\alpha}$
- $r_t^G = \alpha\, A\, (K/L)^{\alpha-1}$.

Government

Taxes income when young at rate τ_w and capital income at rate τ_r.

Depreciation is not tax deductible.

After tax prices are

- $r = (1 - \tau_r)\ r^G - \delta$.
- $w^y = (1 - \tau_w)\ w^G$.

Pays transfers at fixed level, w^o.

Expenditures balance the budget.

$G_t + N_{t-1} w_t^o = \tau_w N_t w_t^G + \tau_r r^G\ N_{t-1} s_{t-1}$.

Equilibrium definition

Sequences $\left\{G_t, K_t, L_t, s_t, c_t^y, c_t^o, r_t, w_t^y, r_t^G, w_t^G\right\}$ that satisfy

Household: Euler equation and 2 budget constraints.

Firms: First-order conditions.

Government: Budget constraint.

Market clearing:

- Goods $F(K_t, L_t) + (1-\delta) K_t = G_t + N_{t-1} c_t^o + N_t c_t^y + K_{t+1}$
- Capital $K_t = N_{t-1} \, s_{t-1}$,
- Labor $L_t = N_t$.

Identities: Price relationships.

Steady State Conditions

Constants $\{g, k, c^y, c^o, r, w^y\}$ that solve

$$r = (1 - \tau_r)\ f'(k) - \delta \tag{1}$$

$$w^y = (1 - \tau_w)\ [f(k) - f'\,(k)\, k] \tag{2}$$

$$u'(c^y) = \beta\,(1 + r)\, u'(c^o) \tag{3}$$

$$c^o = w^o + (1 + r)\ (w^y - c^y) \tag{4}$$

$$k\,(1 + n) = w^y - c^y \tag{5}$$

$$g + w^o/(1 + n) = \tau_w\ [f(k) - f'\,(k)\, k] + \tau_r\, f'\,(k)\ k \tag{6}$$

Computing the Steady State:

Guess k.

Compute r, w^y, c^y, c^o from (1) through (4).

Iterate until deviation from (5) is close to zero.

Program: `bg_comp_olg2d.m`.

Calibrating the Model

Calibration means:

- Choose n model parameters to exactly match n observations.

There are more general notions of calibration.

Fixed Parameters

- $n = 0.01$ per year.
- $\alpha = 0.36$ (Cooley and Prescott 1995).

Calibrated parameters

β matches $K/Y = 2.9$ per year.

Set A to match $w^y = 1$: $A = \left(\frac{w^y}{[1-\alpha]\,[1-\tau_w]}\right)^{1-\alpha}\,\left(\frac{Y}{K}\right)^{\alpha}$

δ matches $r = 0.05$ per year.

Calibration Programs

A simple approach:

- Guess parameter values.
- For each: solve the steady state conditions.
- Iterate until deviations from calibration targets are small.

This is inefficient because the values of certain endogenous quantities are known during the calibration computations.
In this model: K/Y implies k.

A better approach: Write separate programs for calibration and for computation of equilibrium.

Calibration Algorithm

Fix k at the value implied by the target level for K/Y: $k = (AK/Y)^{1/(1-\alpha)}$

Compute r, w^y from marginal products; (1) and (2).

Guess β.

Solve household problem for c^y.

Iterate until deviation from capital market clearing (5) is small.

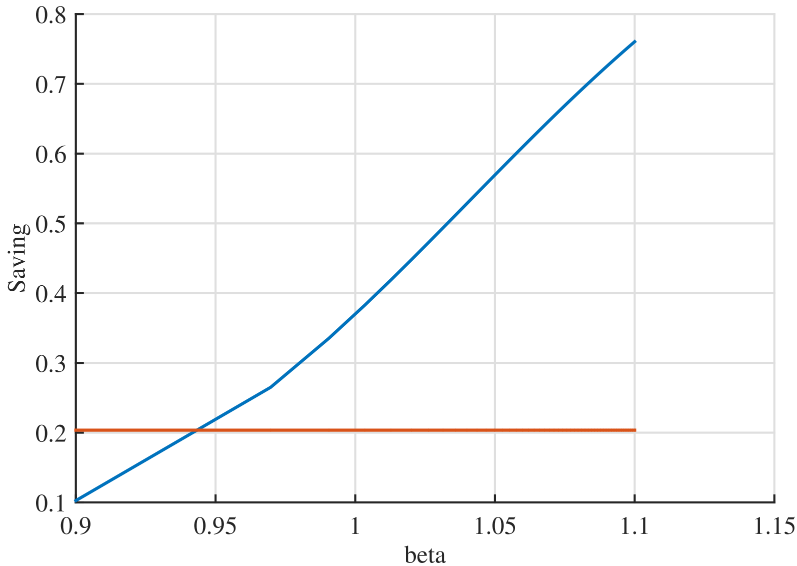
Program: `cal_comp_olg2d.m`.

Deviations from Calibration Targets

`cal_comp_olg2d` searches for β that makes the two lines cross.

The horizontal line is $k(1+n)$.

The upward sloping line is $(w^y - c^y)$.



Calibration Results

```
Deviation from calibration targets: -0.000000
beta   = 0.799.  Annual beta = 0.993
cY/wY = 0.796.  cY/W = 0.699
k      = 0.151
.
```

To verify the code, compute the steady state and check that k has the calibrated value.

Fiscal Policy

How large are the effects of taxes on output, capital, etc.?

A large, quantitative literature investigates questions of this type.

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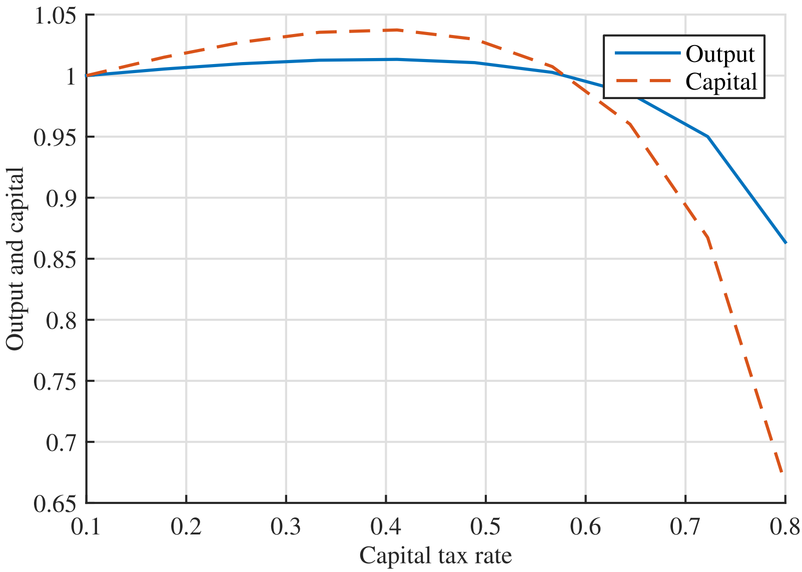
An experiment for purposes of illustration:

- Vary capital incomes tax rate between 10% and 90%.
- Government spending balances the budget.
- `tax_exper_olg2d.m`

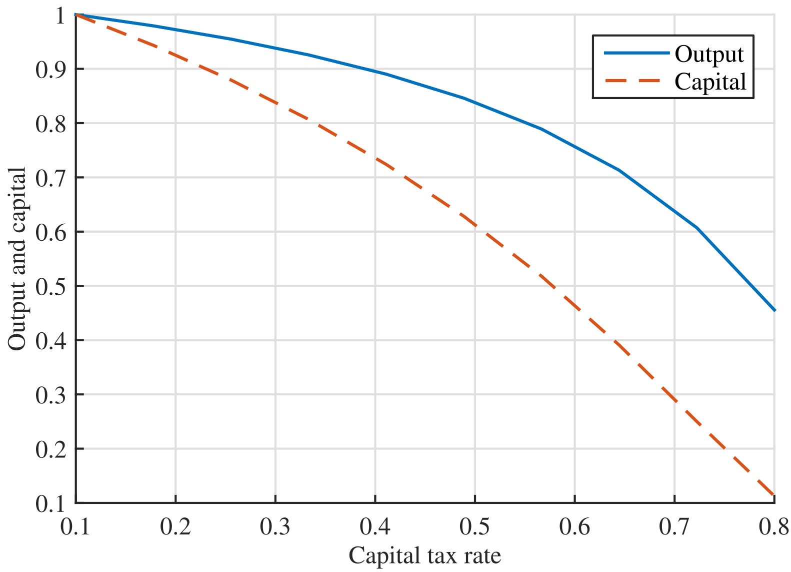
Capital Tax Experiment



Surprise: Capital taxes increase savings and output (up to a point).

Why?

The Same Experiment With Log Utility



Take-away Points

1. Even this little model requires quite a bit of code
   Structure and organization matter
2. Don't hard code functional forms / budget constraints
   For speed: have 2 versions
3. Plan ahead for model versions (calNo) and counterfactual experiments (expNo)

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