@ the world: *How public debates bias public trust?-*

The Opinion Leaders in a Heterogeneous Society

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Abstract:

1. Introduction:

2. The Key Assumptions

- Communication is the only interaction
- Endogenous reputational concern
- Advisor's preferences over the current decision either coincide with those of the decision maker or are biased in a particular, commonly known direction.
- Heterogeneous social beliefs on the advisor
- The distribution of belief is common knowledge

3. The Two-period Advice Model

In the both periods, a decision maker's optimal decision depends on the stated of the world $\omega t \in \{0,1\}$. Each state occurs with equal probability. The decision maker has no information about the state, but he has access to an opinion leader who is partially informed about the state of the world. She observes a signal $S_t \in \{0,1\}$ and could deduce the true state with probabilityγ. It is assumed that the signal is informative but not perfectly so, that is, $\gamma \in (1/2,1)$. Decision makers are uncertain about the objectives of the advisor but each has a prior belief λ_{1i} about her type. Specifically, with probabilityλ_{1i}, decision maker i believes that the advisor is "neutral", with a utility function identical to the sum of the decision makers'. With probability 1- λ_{1i} , the advisor is "biased", meaning that she always wants him to make the same decision (independent of her information). In our model, we distinguish a heterogeneous society where λ_{1i} is uniformly distributed on [0,1] from a relatively homogeneous one where it is normally distributed on [0,1].(More generally, greater variation, i.e. Var(λ_1), implies social heterogeneity).

The opinion leader has an opportunity to announce her message m1(0 or 1) as a function of the signal she has observed. The decision maker will interpret the message he receives in the light of his belief. Given the opinion leader's message, each decision maker i must choose an action $a_t \in R$ in each period t. After the action is chosen, the state of the world ωt is publicly observed. At the end of first period, the decision maker i rationally updates his belief about the type of the opinion leader, as a function of the initial reputation λ_{1i}, the message sent m1, and the realized state $\omega 1$. Her reputation at the beginning of the second period is written as$\lambda 2i(\lambda 1i, 0,1)$ on decision maker i.

The utility of the decision maker is given by

$$-x_1(a_1-\omega_1)^2-x_2(a_2-\omega_2)^2$$

Where $x_1>0$ *and* $x_2>0$
This implies that the optimal action of a decision maker is to set a_t equal to his expectation of the state of the world (i.e. $E(\omega_t)$) and that he may put different weights on period 1 and period 2 decisions.

The total utility of a neutral and a biased opinion leader is given respectively by
Neutral:

$$\int_0^1 (-x1(\omega 1-a_{1i})^2-x2(\omega 2-a_{2i})^2)f(\lambda_{1i})d\lambda_{1i}$$

Biased:

$$\int_0^1 (y1*a_{1i}+y2*a_{2i})\,f(\lambda_{1i})d\lambda_{1i}$$

Where $y1>0$ *and* $y2>0$, $f(\lambda_{1i})$ is the probability density function of λ_{1i}

The neutral opinion leader is assumed to have preference identical to the sum of that of decision makers', while the biased one always wants a higher action chosen, independent of the state. Both leaders want to maximize their influence on the general public.

This game can be solved by backward induction.

Equilibrium in the Second Period Game

The opinion leader will enter the second period with a reputation λ_{2i} on decision maker i, commonly known to all. Since the second period is the last period, the opinion leader (whether biased or not) will have no reputational concern and will simply seek to achieve her current objective.

Since this game is an example of a *cheap talk* game (see Crawford and Sobel 1982) , there always exists a "babbling equilibrium" where no information is conveyed. But the interesting question, in all cheap talk models, is when there exist equilibria in which costless actions (cheap talk) *do* convey meaning.

We can prove that such a non-babbling equilibrium exist in the second period of the game and the opinion leaders' strategy may be summarized by the following table:

	S1=0	S1=1
Neutral	0	1
Biased	1	1

Given the opinion leader's strategy, what inferences will the decision maker draw about the state of the world? If the decision maker i receives message 0, he will be sure that the opinion leader is neutral and is truthfully reporting her signal. Thus he will assign probability 1-γ to state 1 and choose action 1-γ. If he receives message 1, he will be uncertain about the opinion leader's type and will thus assign probability (by Bayes' rule),

$$\frac{\frac{1}{2}[1-\lambda_{2i}+\gamma\lambda_{2i}]}{\frac{1}{2}[1-\lambda_{2i}+\gamma\lambda_{2i}]+\frac{1}{2}[\lambda_{2i}(1-\gamma)]}=\frac{1-\lambda_{2i}+\gamma\lambda_{2i}}{2-\lambda_{2i}}$$

to state 1 and choose action

$$\frac{1-\lambda_{2i}+\gamma\lambda_{2i}}{2-\lambda_{2i}}\ (>1\text{-}\gamma)$$

Thus his action is greater than if he receives m_1=0 and will be increasing in λ_{2i}, the reputation of the opinion leader. The biased opinion leader thus has a strict incentive to announce 1 (independent of the signal she has observed), and the neutral one will have a strict incentive to announce her signal truthfully(since every decision maker will choose a strictly higher action if she announces 1 than if she announces 0). Now the utility function for both types of opinion leaders can be derived:

$$V_G=-x_2\int_0^1\Big[\frac{1}{2}\gamma\Big(\frac{1-\gamma\lambda_{2i}}{2-\lambda_{2i}}\Big)^2+\frac{1}{2}(1-\gamma)\Big(\frac{1-\lambda_{2i}+\gamma\lambda_{2i}}{2-\lambda_{2i}}\Big)^2+\frac{1}{2}(1-\gamma)\gamma^2+\frac{1}{2}\gamma\ (1-\gamma)^2\Big]\,f(\lambda_{2i})d\lambda_{2i}\qquad(1)$$

and

$$V_B=y_2\int_0^1\Big(\frac{1-\lambda_{2i}+\gamma\lambda_{2i}}{2-\lambda_{2i}}\Big)\ f(\lambda_{2i})d\lambda_{2i}\qquad(2)$$

Given social techonology γ, time discount x_2 and y_2, both value functions are continuous and depend ***only*** on the distribution of λ_{2i}(i.e. the type of a society).
Let us take a closer look at two interesting examples of that. Suppose$\gamma=0.75, x2=1, y2=1$, then

if λ_{2i} is uniformly distributed on [0,1], i.e. $f(\lambda_{2i})=1$, that is, in a *heterogeneous* society with *Laplacian* beliefs,

$$V_G = -x_2\left(\frac{\gamma^3}{2} + \left(\frac{5}{2} - 4\ln 2\right)\gamma^2 + 2(2\ln 2 - 1)\gamma + \frac{3}{4} - \ln 2\right) = 0.27$$

$$V_B = y2[(2\ln 2 - 1)\gamma - \ln 2 + 1] = 0.60$$

If λ_{2i} is subject to normal distribution, i.e.

$$f(\lambda_{2i}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\lambda_{2i}-\mu)^2}{2\sigma^2}} \quad ,$$

that is , in a *relatively homogeneous* one, where the expected belief (μ =0.5) implies social shared value, and σ(=1) is the standard deviation of public beliefs,

$$V_G = 0.104$$
$$V_B = 0.228$$

We can see in this special case that both agents would be better off in the second period if their *ex ante* reputation diverges even when its expected value fixed at 50-50. The intuition here is that in a heterogeneous society (1) it's less costly for both opinion leaders to announce 1 because the number of *swing* population (with λ_{2i} = 0.5) falls that needs more reputation investment but rarely supports。 In this sense, if the majority is suspicious about those opinion leaders, they can stay no longer as "the leader" but marginalized in the market of public opinions. (2) Both benefit from a few "big fans" who always do as are told regardless of the realized state and this "*star effect* " may offset the negative effect imposed by rebellious individuals.

In the analysis that follows, it is assumed that the informative equilibrium giving rise to these value functions is played in the second period. If the babbling equilibrium were played in the second period, then there would be no reputational concerns in the first period.

Equilibrium in the First period (with reputational concern)

The first –period game is the same as the second-period game except that now the opinion leader has reputational concerns arising from the second stage of the game. In particular, the neutral opinion leader optimizes when she receives signal s1:

$$U_G = \int_0^1 -x1(\omega 1 - a_{1i})^2 f(\lambda_{1i})d\lambda_{1i} + Ev_G\big(\lambda_{2i}(\lambda_{1i}, m1, \omega 1)\big)_{|S1}$$

While the biased one optimizes

$$U_B = \int_0^1 (y1 * a_{1i})\, f(\lambda_{1i})d\lambda_{1i} + Ev_B\big(\lambda_{2i}(\lambda_{1i}, m1, \omega 1)\big)_{|S1}$$

Where $\lambda_{2i}(\lambda_{1i}, m1, \omega 1)$ is the equilibrium posterior probability assigned to the opinion leader being neutral. Once again, there will be a babbling equilibrium of the first period game: If the opinion leader randomized between messages

independently of the signals, the decision maker would learn about *neither* the state of the world *nor* the type of the opinion leader, and again she would have no incentive to send informative messages. The purpose of the following analysis is to characterize informative equilibria and to identify when they exist.[1]
The argument is structured as follows. It is first assumed that there exists an equilibrium in which the neutral opinion leader always tells the truth. Then it is possible to characterize how the biased one must behave in such an equilibrium. This in turn implies certain reputational incentives for the neutral opinion leader. Now it is possible to check for which parameters truth-telling first proposed for the neutral leader is indeed optimal

Suppose that the neutral advisor always told the truth. Would it be a best response for the bad advisor also to always tell the truth? No. To see this, assume that such an equilibrium existed, then no reputation is updated at the beginning of second period, leaving the distribution of public belief unaffected, i.e. $\lambda_{2i} = \lambda_{1i}$, the bad agent would then have a strict incentive to announce m1=1 just like she would do in the second period, contradicting our earlier assumption that she tells the truth. Thus the biased advisor cannot always tell the truth. By a similar logic, it is clear that the biased opinion leader must announce 1 strictly more (on average) often than the neutral one. If not, announcing 1 would (in equilibrium) reduce (or at least not increase) the likelihood the leader was neutral. But since announcing 1 maximizes the action of the decision maker, it would therefore be strictly optimal for the biased leader to announce 1(contradicting our premise that the biased opinion leader announced 1 no more than the neutral one).   This strategy can be summarized in the following table:

	S1=0	S1=1
Good	0	1
Bad	0 with probability 1-v	1
	1 with probability v	

Now by Bayes' rule, the decision maker i's posterior belief about the type of the advisor will be

$$\lambda_{2i}(\lambda_{1i},1,1)=\frac{\lambda_{1i}\gamma}{\lambda_{1i}\gamma+(1-\lambda_{1i})[\gamma+(1-\gamma)v]}$$

$$\lambda_{2i}(\lambda_{1i},1,0)=\frac{\lambda_{1i}(1-\gamma)}{\lambda_{1i}(1-\gamma)+(1-\lambda_{1i})[1-\gamma+\gamma v]}$$

$$\lambda_{2i}(\lambda_{1i},0,1)=\frac{\lambda_{1i}}{\lambda_{1i}+(1-\lambda_{1i})[1-v]}$$

$$\lambda_{2i}(\lambda_{1i},0,0)=\frac{\lambda_{1i}\gamma}{1+(1-\lambda_{1i})(1-v)}$$

[1] Stephen Morris (2001) *Political Correctness*

Observe that certain statements (e.g. 0 in this case) will lower the reputation of the speaker independent of whether they turn out to be true.

Twist the twisted

Definition1: We say that the random variable x(stochastically) dominates the independent random variable y on the interval [a,b） if , whenever P(x∈[a,b))>0 and P(y∈[a,b)), we have

$$P(x \in [\theta,b)|\ x \in [a,b))-P(y \in [\theta,\ b)|\ y \in [a,b)) \geq\ 0$$

In this case we write $x \geq\ y$ on [a,b)

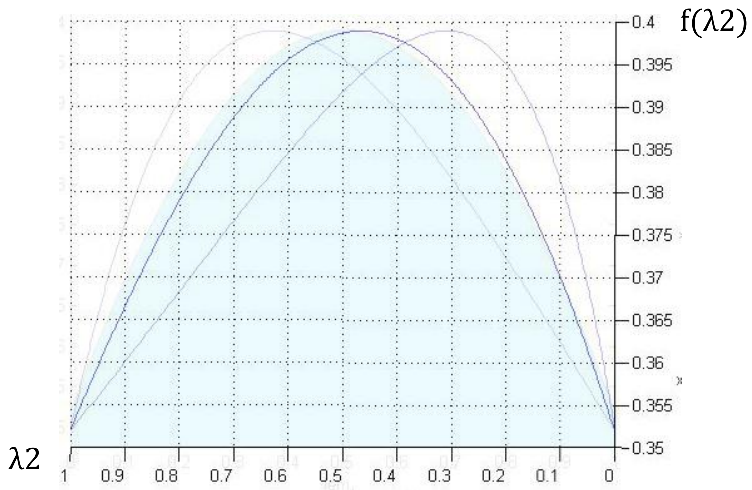
Definition2: We say that the random variable x uniformly dominates y on the interval [A,B) if x dominates y on every subinterval [a,b）∈[A,B). In this case, we write $x \geq_U y$ On [A,B)[2]

Proposition1: The posterior distribution of public beliefs satisfies the following properties in any informative equilibrium:

(1) In a relatively *homogeneous* society(i.e. normally distributed on[0,1]), the distribution of ex ante beliefs strictly dominates (more highly concentrated reputation) that of ex post beliefs on [0,1) when the opinion leader announces 0 independent of whether they turn out to be true; i.e.

$$\lambda 2(\lambda 1,0,1) > \lambda 2(\lambda 1,0,0) \geq \lambda 1 \geq \lambda 2(\lambda 1,1,1) > \lambda 2(\lambda 1,1,0)$$

It is illustrated in figure 1&2, where the shaded area captures the prior distribution.



(Fig.1)

Thus, in a homogeneous society, as analyzed in Morris model, both types of

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[2] Pradeep Dubey and John Geanakoplos (2004) *Grading Exams:100,99,...,1 Or A,B,C*

opinion leaders have a strict reputational incentives to announce 0, whatever signal they observe, in order to look like a neutral advisor. In equilibrium, such reputational incentives may lead to the loss of information.[3]

(2) When social beliefs fully diverge (i.e. uniformly distributed on [0,1]), the distribution of social beliefs remain the same ex post as ex ante. Opinion leaders' statements can no longer bias the public belief. They may still be better off to announce 0 in terms of higher ex post reputation on individuals with certain prior belief λ_{1i}, but not the society as a whole. Thus, they have more (aggregated) incentives to make an informative signal compared to the case in (1).

Political radicalism

Proposition 2: In any informative equilibrium**,** given reputational concern (y1 fixed) the biased opinion leader announces 1 more often in a heterogeneous society than in a homogeneous one.

Let f(v(y_1)) = $V_u - V_N$, where V_u and V_N denote a biased agent's optimal probability of lying in a heterogeneous and homogeneous society, respectively. if v'(y1) reaches to its maximum value, that is, when the equilibrium strategy v responds sensitively to the marginal change in time discount y1, f'(v(y_1))=0 will hold, which implies a most distinct gap of signaling of the biased opinion.

More specifically, if γ =3/4 and $y_2 = x_2 = 1$, the largest "gap" between V_u and V_N emerges when y_1=1/10, as is depicted in Fig.2 and Fig.3:

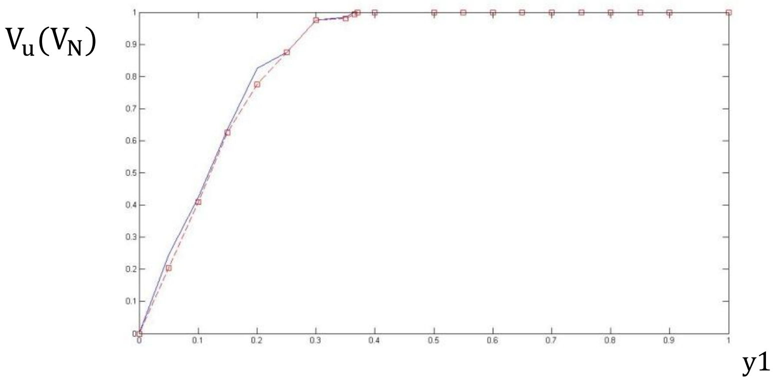

Fig.2

[3] Stephen Morris (2001) *Political Correctness*

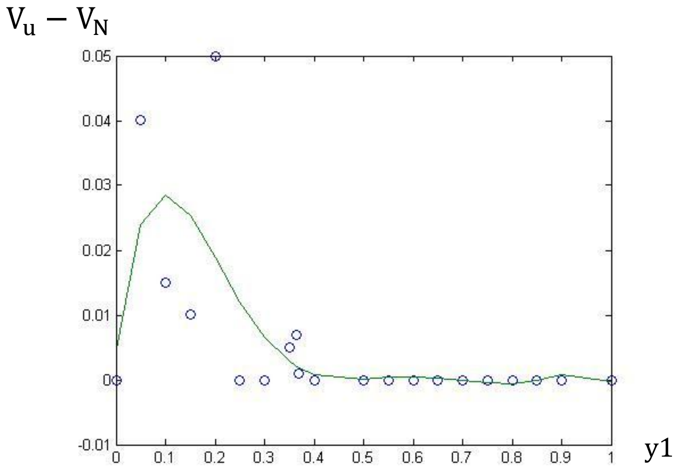

Fig.3

In other words, even with reputational updating, biased advisors would convey radical opinions to the public when beliefs diverge because they care less about their reputation since it would after all not change drastically. This is bad for any individual decision maker, but not quite for the society because an opinion leader's true type is revealed in the long run.

So far, it was assumed that the neutral opinion leader told the truth. If she observes signal 0, however, she has an unambiguous incentive to tell the truth, since this will lead the decision maker to choose a low action and it will enhance her reputation. But if she observes signal 1, she has a current incentive to tell the truth(announces 1) and future reputational concern to lie (announces 0).

Neutralize the neutral

Proposition3: If the second period is sufficiently important (i.e. x2 is large relative to λ2), then no information is conveyed in the first period. Given x1 fixed, however, a neutral opinion leader is more likely to tell the truth to a pooled population (where social belief uniformly distributed).

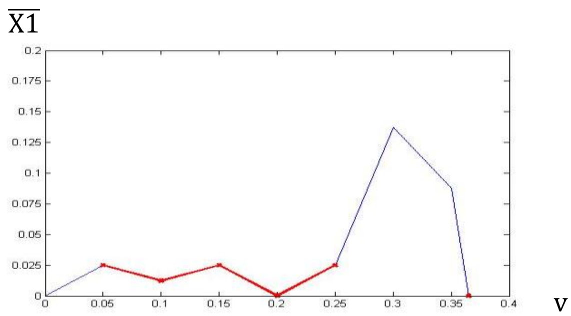

Fig.4

Thus reputational concerns have a smaller impact on opinion leader's actions when public belief is more divergent. On the one hand, it is good for the public because individuals may distinguish the biased advisor and make more informative decisions. On the other hand, this raises the question of whether free access for both principles and agents to the distribution of public belief (via, for example, Twitter and Facebook) will make it more or less likely that informative equilibria exist.

4. Welfare Analysis

The power of "heterogeneity"

Public heterogeneity leads to more radical opinion leaders and are often labeled as a source of conflict. Does that mean that it would be socially desirable to prevent public debates and promote homogeneity to achieve higher public trust? In particular, how do player's utilities in the equilibria when beliefs diverge (and known to all) compare with their utility if the general public is relatively homogeneous?

Sorting effects dominate in a heterogeneous society.

5. A research agenda

Decentralized opinion exchange may have a mixed effect. In a homogeneous society, for instance, an individual is extremely unlikely to receive a rebellious message from her peer, which disciplines biased agents. But in the unlikely event that she does, such an extreme event will stir up her skepticism toward the rumor that social trust is undermined and distort the information a neutral agent is trying to convey. I thus suggest the global game approach, where "players' beliefs about other players' beliefs" [4]is concerned, to study the problem in detail.

[4] Stephen Morris and Hyun Song Shin(2001): *Global Games: Theory and Applications*