Note: The conversion process is of particularity, so we have not placed it in the main text. Specifically, this conversion can only be used when the c_{k2}=0 and c_{k1} of each generating unit in a case are all equal. (Photovoltaic units are excluded, because the power output of photovoltaic units is a fixed value, that is, $P^{min}=P=P^{max}$). When the above conditions are met, minimizing the cost of generating units is equivalent to minimizing the active power loss cost. In the series of cases set in this paper, these conditions are all satisfied, so the cases used in this paper can use this conversion.

** The validity of this conversion has been rigorously tested using MATLAB code **

Here we only present the conversion process, and the specific proof can be referred to Reference [Zero Duality Gap in Optimal Power Flow Problem] (doi: 10.1109/TPWRS.2011.2160974). The conversion process is as follows:

First, define the following variables:

1) Define $\underline{\lambda_k}$ and $\overline{\lambda_k}$ as the Lagrange multipliers associated with the lower inequality (29) and the upper inequality (29), respectively.

2) Define $\underline{\gamma_k}$ and $\overline{\gamma_k}$ as the Lagrange multipliers associated with the lower inequality (30) and the upper inequality (30), respectively.

3) Define $\underline{\mu_k}$ and $\overline{\mu_k}$ as the Lagrange multipliers associated with the lower inequality (31) and the upper inequality (31), respectively.

4) Define λ_{lm} as the Lagrange multipliers associated with the upper inequality (33), respectively.

5) Define the symmetric matrix of Lagrange multipliers associated with the constraint (35):

$$\begin{bmatrix} r_{lm}^1 & r_{lm}^2 & r_{lm}^3 \\ r_{lm}^2 & r_{lm}^4 & r_{lm}^5 \\ r_{lm}^3 & r_{lm}^5 & r_{lm}^6 \end{bmatrix}$$

6) Define the symmetric matrix of Lagrange multipliers associated with the constraint (36) and (37):

$$\begin{bmatrix} r_k^1 & r_k^2 & r_k^3 \\ r_k^2 & r_k^4 & r_k^5 \\ r_k^3 & r_k^5 & r_k^6 \end{bmatrix}$$

Where, $k \in \{N_{\text{SOP}}^{\text{set}} \bigcup N_{\text{RPFC}}^{\text{set}}\}$, $N_{\text{SOP}}^{\text{set}}$ is the set of SOP numbers, $N_{\text{RPFC}}^{\text{set}}$ is the set of RPFC numbers, and satisfies $N_{\text{SOP}}^{\text{set}} = \{1,2,3,...,i\}$, $N_{\text{RPFC}}^{\text{set}} = \{i+1, i+2,...\}$

7) Define the symmetric matrix of Lagrange multipliers associated with the constraint (34):

$$\begin{bmatrix} 1 & (r_k^1)' \\ (r_k^1)' & (r_k^2)' \end{bmatrix}$$

Let:

$$\lambda_k = \begin{cases} -\underline{\lambda_k} + \overline{\lambda_k} + c_{k1} + 2\sqrt{c_{k2}}(r_k^1)', & \text{if } k \in G \\ -\underline{\lambda_k} + \overline{\lambda_k}, & \text{else} \end{cases}$$

$$\gamma_k = -\underline{\gamma_k} + \overline{\gamma_k}$$

$$\mu_k = -\underline{\mu_k} + \overline{\mu_k}$$

Then the dual OPF-CE can be written as:

$$\begin{aligned} \max\ h = & \sum_{k \in N} [\underline{\lambda_k} P_k^{\min} - \overline{\lambda_k} P_k^{\max} + \lambda_k P_k^D + \underline{\gamma_k} Q_k^{\min} - \overline{\gamma_k} Q_k^{\max} + \gamma_k Q_k^D + \underline{\mu_k}(V_k^{\min})^2 - \overline{\mu_k}(V_k^{\max})^2] + \\ & \sum_{k \in G} [c_{k0} - (r_k^2)'] - \sum_{(l,m) \in L} [\lambda_{lm} P_{lm}^{\max} + (S_{lm}^{\max})^2 r_{lm}^1 + r_{lm}^4 + r_{lm}^6] - \\ & \sum_{k \in N_{\text{SOP}}^{\text{set}}} [(C_k^{\text{sop}})^2 r_k^1 + r_k^4 + r_k^6] - \sum_{k \in N_{\text{RPFC}}^{\text{set}}} [(C_k^{\text{rpfc}})^2 r_k^1 + r_k^4 + r_k^6] \end{aligned}$$

s.t.

$$A = \sum_{k \in N} [\lambda_k \mathbf{Y}_k + \gamma_k \overline{\mathbf{Y}_k} + \mu_k \mathbf{M}_k] + \sum_{(l,m) \in L} [(2r_{lm}^2 + \lambda_{lm}) \mathbf{Y}_{lm} + 2r_{lm}^3 \overline{\mathbf{Y}_{lm}}] \succ 0$$

$$\overline{\lambda_k} \geq 0,\ \underline{\lambda_k} \geq 0,\ \overline{\gamma_k} \geq 0,\ \underline{\gamma_k} \geq 0,\ \overline{\mu_k} \geq 0,\ \underline{\mu_k} \geq 0,\ \lambda_{lm} \geq 0$$

$$\begin{bmatrix} r_{lm}^1 & r_{lm}^2 & r_{lm}^3 \\ r_{lm}^2 & r_{lm}^4 & r_{lm}^5 \\ r_{lm}^3 & r_{lm}^5 & r_{lm}^6 \end{bmatrix} \succ 0$$

$$\begin{bmatrix} r_k^1 & r_k^2 & r_k^3 \\ r_k^2 & r_k^4 & r_k^5 \\ r_k^3 & r_k^5 & r_k^6 \end{bmatrix} \succ 0$$

$$\begin{bmatrix} 1 & (r_k^1)' \\ (r_k^1)' & (r_k^2)' \end{bmatrix} \succ 0$$

$$-\underline{\lambda_k} + \overline{\lambda_k} + \underline{\lambda_j} - \overline{\lambda_j} + 2 \times (r_k^2 + r_j^2) = 0$$

$$\begin{cases} \text{for sop:} \begin{cases} -\underline{\gamma_k} + \overline{\gamma_k} + 2 \times r_k^3 = 0 \\ -\underline{\gamma_j} + \overline{\gamma_j} + 2 \times r_j^3 = 0 \end{cases} \\ \text{for rpfc: } -\underline{\gamma_k} + \overline{\gamma_k} + \underline{\gamma_j} - \overline{\gamma_j} + 2 \times (r_k^3 + r_j^3) = 0 \end{cases}$$

In the last two sets of constraints, k and j represent the buses directly connected to the SOP or

RPFC, respectively.

Finally, the power loss cost and equipment investment cost can be added.

After the above conversion, we no longer need to solve for the huge **W** matrix (with $2n \times 2n$ variables), and only need to solve for the variables defined above.