**Appendix C**

We describe here a simple but efficient algorithm to extract the relaxation rate  from magnetization measurements at different spin-lock time samples. The assumed dynamics is either  (two parameters to estimate) or  (three parameters to estimate). The idea being very similar in both cases, we will detail it only in the three-parameter case.

Estimating , , and  from the data  acquired at several  samples can be done by minimizing the mean square error, , between the actual data and the model for , , and , where



The minimization problem over  and  alone is easy to solve (a linear system of equations) and we find

 and 

where the functions , ,  and  are defined by



是平均值

What’s the size of x and y? 🡪 [1,5] [102400, 5]

Using these estimates allows to express the MSE as a function of  only: . It turns out that this function has only one global minimum, in cases of practical interest. To find this minimum, we resort to a dichotomic strategy whereby the estimation interval of  is shrank by a factor two at each iteration (see D.J. Wilde, Optimum seeking methods, Prentice-Hall (1964)). Starting from an interval 

1. We evaluate the derivative of the criterion  at .
2. If the derivative is positive (i.e.,  is increasing at that point), this means that the minimum of  is located in the interval  and so, update .
3. If the derivative is negative (i.e.,  is decreasing at that point), this means that the minimum of  is located in the interval  and so, update .
4. Repeat 1 until the  is smaller than a pre-requisite accuracy; typically, after 10 iterations, the length of the original interval is reduced by a factor 1000.

This algorithm is implemented in vectorized form in Matlab, making it possible to estimate  at every pixel of sequence of images in one shot (no loops over pixels), resulting in a very short computation time.