Hw1

Saturday, September 6, 2025 1:30 PM

Q1:

From:

Sviatov, Kirill, Nadejda Yarushkina, Daniil Kanin, Ivan Rubtcov, Roman Jitkov, Vladislav Mikhailov, and Pavel Kanin. 2021. "Functional Model of a Self-Driving Car Control System" *Technologies* 9, no. 4: 100. https://doi.org/10.3390/technologies9040100

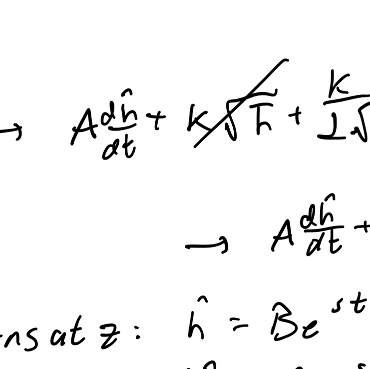
Sensing Mechanisms:
- Lidar Ouster OS-1, 16-channel.
- Stereo Camera Stereolabs ZED.
- GPS sensor U-blox GY-GPSV3-NEO-M8N.
- Eight ultrasonic parking sensors.
- Limit switches.

Control Mechanism (Actuator):
- For gear shifting—2 Linak LA15 linear actuators, self-developed mechanical parts for connecting actuators.
- Linak LA36 actuators for brake and clutch pedals.
- To rotate the steering wheel—electric power steering from the Lada Kalina car.
- An additional rotation sensor was installed on the steering shaft with the transmission of the shaft rotation angle without the need for synchronization.

Computational Element:
- A single-board computer Nvidia Jetson Xavier is used for control, and STM32 family controllers are used for motors controlling.

Feedback in this system includes constantly calculating the acceleration of the vehicle based upon the input from the sensing block and changing the response of the car in order for it to follow the real time scenario going on in the road.

Q2



$A\frac{dh}{dt} = q_{in} - q_{out}$, $q_{out} = k\sqrt{h}$

a. $A\dot{h} = q_{in} - k\sqrt{h}$

$\boxed{A\dot{h} + k\sqrt{h} = q_{in}}$

b. let $h = \bar{h} + \hat{h}$ and $q_{in} = \bar{q}_{in} + \hat{q}_{in}$

Taylor expansion:

$\sqrt{h} = \sqrt{\bar{h}} + \frac{1}{2\sqrt{\bar{h}}}(h - \bar{h})$

$= \sqrt{\bar{h}} + \frac{1}{2\sqrt{\bar{h}}}\hat{h}$

$\rightarrow A\frac{d\hat{h}}{dt} + \cancel{k\sqrt{\bar{h}}} + \frac{k}{2\sqrt{\bar{h}}}\hat{h} = \cancel{\bar{q}_{in}} + \hat{q}_{in}$ at steady state: $k\sqrt{\bar{h}} = \bar{q}_{in}$

$\rightarrow A\frac{d\hat{h}}{dt} + \frac{k}{2\sqrt{\bar{h}}}\hat{h} = \hat{q}_{in}$

Ansatz: $\hat{h} = Be^{st}$ $\quad A\,Bse^{st} + \frac{k}{2\sqrt{\bar{h}}}Be^{st} = 0$

$\frac{d\hat{h}}{dt} = Bse^{st}$ $\quad As + \frac{k}{2\sqrt{\bar{h}}} = 0$

$B = \hat{h}(0)$ $\quad s = -\frac{k}{2A\sqrt{\bar{h}}}$

$\rightarrow \boxed{\hat{h}(t) = \hat{h}(0)e^{-\frac{kt}{2A\sqrt{\bar{h}}}}}$

c. Givens $A = 205\,cm^2$, $h(0) = 22\,cm$, $\bar{h} = 15.3\,cm$, $\bar{q}_{in} = 23.7\,cm^3/s$ at $\hat{q}_{in} = 0$

$h(0) = \bar{h}(0) + \hat{h}(0) \rightarrow \hat{h}(0) = 22\,cm - 15.3\,cm = 6.7\,cm$

$\bar{q}_{in} = k\sqrt{\bar{h}} \rightarrow k = \frac{\bar{q}_{in}}{\sqrt{\bar{h}}} = \frac{23.7\,cm^3/s}{\sqrt{15.3\,cm}} = 6.06\ cm^{5/2}/s$

$\tau = \frac{1}{s} = \frac{2A\sqrt{\bar{h}}}{k} = 264.68\,s$

$\rightarrow \hat{h}(t) = 6.7e^{-t/264.68}$ cm

$\rightarrow \boxed{h(t) = 15.3 + 6.7e^{-t/264.68}\ cm}$

d. $A\frac{d\hat{h}}{dt} + \frac{k}{2\sqrt{\bar{h}}}\hat{h} = 0$

$\Downarrow \mathcal{L}$

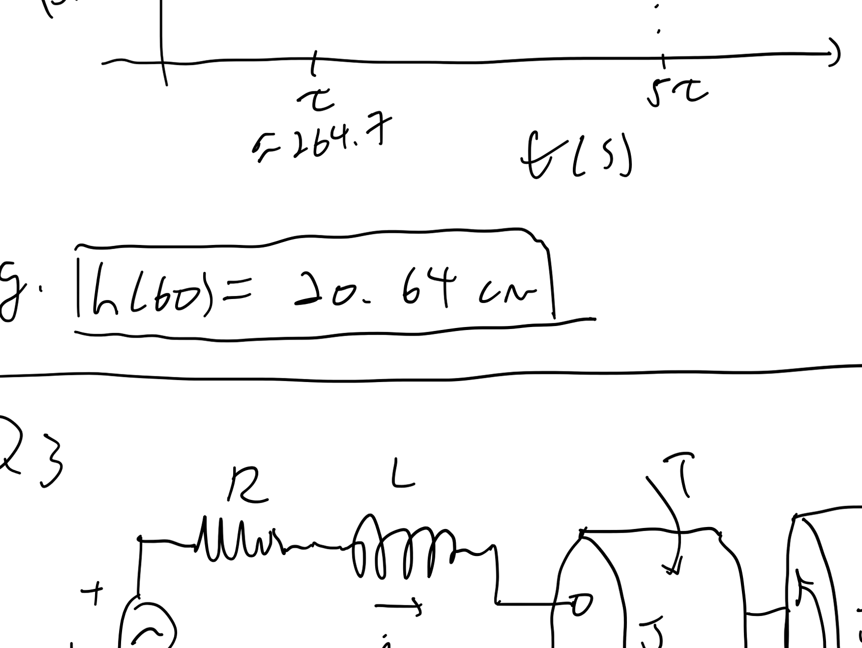
$As\hat{H}(s) - A\hat{h}(0) + \frac{k}{2\sqrt{\bar{h}}}\hat{H}(s) = 0$

$\hat{H}(s) = \frac{A\hat{h}(0)}{As + \frac{k}{2\sqrt{\bar{h}}}} \Rightarrow \boxed{\hat{h}(t) = \hat{h}(0)e^{-\frac{kt}{2A\sqrt{\bar{h}}}}}$ ✓

$\mathcal{L}^{-1}\left\{\frac{1}{As + \frac{k}{2\sqrt{\bar{h}}}}\right\} = \frac{1}{A}e^{-\frac{k}{2A\sqrt{\bar{h}}}t}$

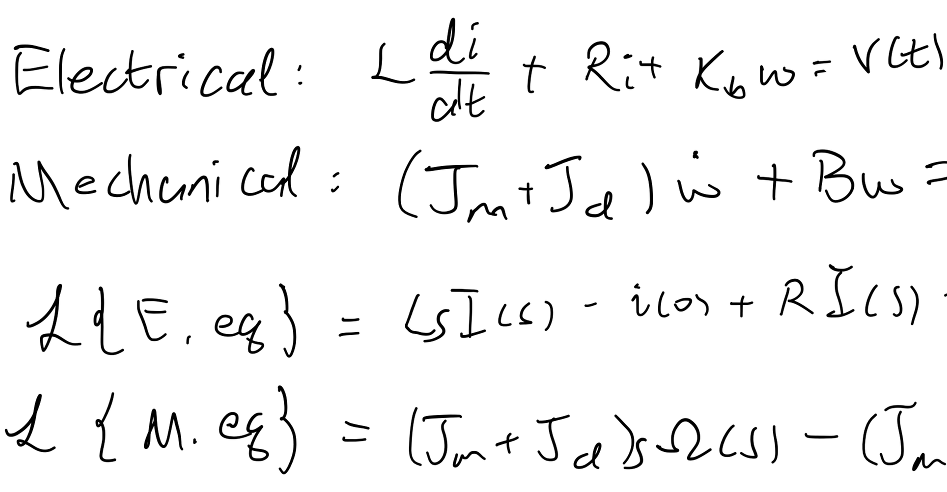
e. $\boxed{\tau = 264.7\,s}$

f.



g. $\boxed{h(60) = 20.64\ cm}$

Q3



Given: K_b and K_m

a. Electrical: $L\frac{di}{dt} + Ri + K_b w = V(t)$

Mechanical: $(J_m + J_d)\dot{w} + Bw = K_m i$

$\mathcal{L}\{E.eq\} = LsI(s) - i(0) + RI(s) + K_b\Omega(s) = V(s)$ 1)

$\mathcal{L}\{M.eq\} = (J_m + J_d)s\Omega(s) - (J_m + J_d)\ w(0) + B\Omega(s) = K_m I(s)$ 2)

Let $J = J_m + J_d$ and set $i(0) = w(0) = 0$ i.e. machine starts at rest

solve for $I(s)$ and then equalize 1) and 2):

$\frac{Js\Omega(s) + B\Omega(s)}{K_m} = \frac{V(s) - K_b\Omega(s)}{Ls + R}$

$(Ls+R)Js\Omega(s) + (Ls+R)B\Omega(s) = K_m V(s) - K_m K_b\Omega(s)$

$\Omega(s)\ (Ls+R)(Js+B) + K_m K_b\Omega(s) = K_m V(s)$

$\Omega(s)\left[(Ls+R)(Js+B) + K_m K_b\right] = K_m V(s)$

$\boxed{G(s) = \frac{K_m}{(Ls+R)(Js+B) + K_m K_b}}$

b. $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} sG(s)$

$W_{ss} = \lim_{s\to 0} s\cdot\frac{1}{s}\frac{K_m}{(Ls+R)(Js+B) + K_m K_b}$

$\boxed{w_{ss} = \frac{K_m}{RB + K_m K_b}\ rad/s}$

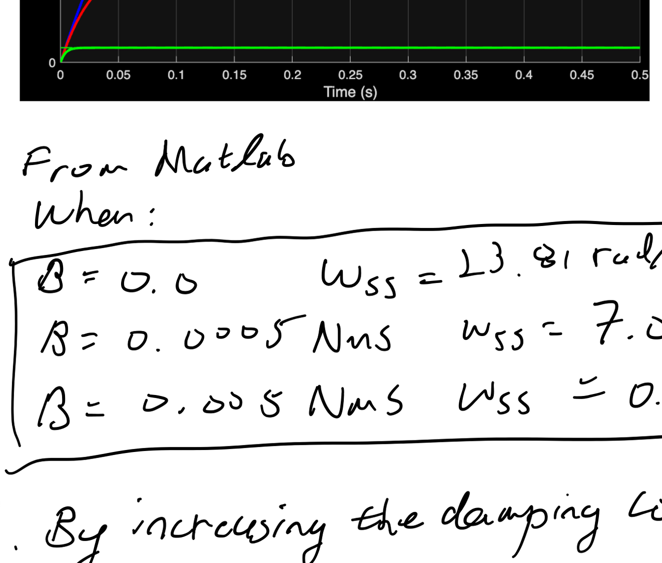
c. Let $L = B = 0$: $\quad G(s) = \frac{K_m}{RJs + K_m K_b}$

DC Gain: $\lim_{s\to 0}\frac{1}{s}s[G(s)] = \lim_{s\to 0}\frac{1}{s}\frac{sK_m}{RJs + K_m K_b}$

$= \lim_{s\to 0}\frac{K_m/(K_m K_b)}{RJs/(K_m K_b) + 1}$

dc gain $\boxed{= \frac{K_m}{K_m K_b}}$ $\quad \boxed{\tau = \frac{RJ}{K_m K_b}}$

d.



e. From Matlab

When:

$B = 0.0$ $\quad W_{ss} = 23.81$ rad/s

$B = 0.0005$ Nms $\quad W_{ss} = 7.04$ rad/s

$B = 0.005$ Nms $\quad W_{ss} = 0.96$ rad/s

They are the same

f. By increasing the damping coefficient significantly reduces steady-state angular velocity.

Without damping the system reached max speed limited only by the back EMF which is higher than the other results with damping.