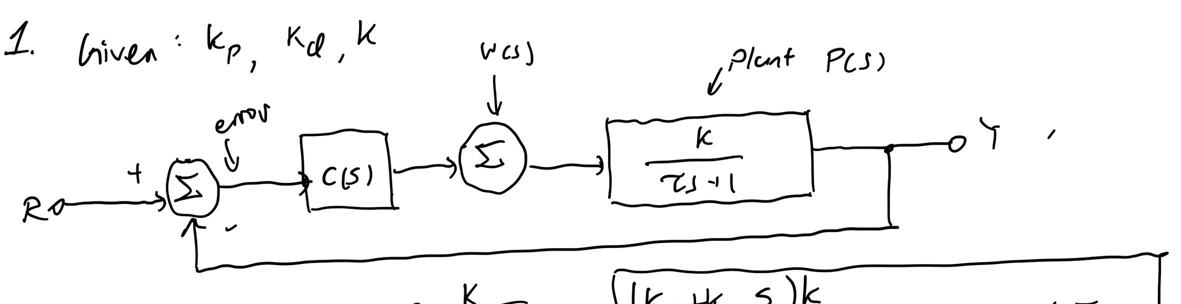
HW2

Sunday, September 21, 2025 12:22 PM

1. Given : k_p, K_d, K



a. $G(s) = \frac{Y(s)}{R(s)} = \frac{C\frac{K}{\tau s+1}}{1+\frac{CK}{\tau s+1}} = \boxed{\frac{(K_p+K_d s)K}{\tau s+1+(K_p+K_d s)K} = \text{CLTF}}$

$C = K_p + K_d s$

FVT:

b. $\lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{(K_p+K_d s)K}{\tau s+1+(K_p+K_d s)K} = \boxed{\frac{K_p K}{1+K_p K}}$

c. $R(s) = \frac{1}{s}$ $\quad Y = CPE$ $\quad e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{R}{1+CP}$

$E = R - Y = R - CPE$

$E(1+CP) = R$

$E = \frac{R}{1+CP}$

$= \frac{1}{1+C(0)P(0)}$

$\boxed{e_{ss} = \frac{1}{1+K_p K}}$

2. let $C(s) = K_p + \frac{K_i}{s}$, $R=0$

b $Y(s) = P(s)(C(s)E(s) + W(s)) = P(-CY+W)$

$E(s) = \cancel{R(s)} - Y(s) = -Y(s) \Rightarrow Y(1+PC) = PW$

$Y = \frac{PW}{1+PC}$

$(1+PC)E = PW$

$\frac{E}{W} = \frac{-P}{1+PC} = \frac{\frac{-K}{\tau s+1}}{1+\frac{K(K_p+\frac{K_i}{s})}{\tau s+1}} = \frac{-K}{\tau s+1+KK_p+\frac{K_i K}{s}}$

$\boxed{\frac{E}{W} = \frac{-Ks}{\tau s^2+s+KK_p s+KK_i}}$

b. $W = \frac{1}{s}$

$e_{ss} = \lim_{s\to 0} \cancel{s}\frac{1}{\cancel{s}} \frac{-Ks}{\tau s^2+s+KK_p s+KK_i} = \boxed{0}$

3. Given $p(t) = \dot{y}(t) + 2y(t) = 2u(t)$ where

a. $u(t) = K_p e(t) + K_i\int^t e(t)dt$

Assume 0 init conds,

$\mathcal{L}\{P(t)\} = sY(s) + 2Y(s) = 2U(s)$

$Y(s+2) = 2U$

$P = \frac{Y}{U} = \frac{2}{s+2}$

$\mathcal{L}\{u(t)\} = K_p + K_i\frac{1}{s} \equiv C(s)$

CLTF: $G(s) = \frac{CP}{1+CP} = \frac{\frac{2}{s+2}\cdot\frac{K_p s+K_i}{s}}{1+\frac{2}{s+2}\cdot\frac{K_p s+K_i}{s}}$

$\boxed{G(s) = \frac{2(K_p s+K_i)}{s(s+2)+2(K_p s+K_i)}}$

b. $r = 1(t) \overset{\mathcal{L}\{r\}}{\Rightarrow} R(s) = \frac{1}{s}$

$y_{ss} = \lim_{s\to 0} sG(s)R(s) = \lim_{s\to 0} \cancel{s}\frac{1}{\cancel{s}} \frac{2(\cancel{K_p s}+K_i)}{\cancel{s(s+2)}+2(\cancel{K_p s}+K_i)} = \frac{2K_i}{2K_i}$

$\rightarrow \boxed{y_{ss} = 1}$

c. $d = 1(t) \overset{\mathcal{L}\{d\}}{\Rightarrow} D(s) = \frac{1}{s}$

$Y(s) = \frac{P(s)D(s)}{1+CP}$

$y_{ss} = \lim_{s\to 0} \cancel{s}\cdot\frac{1}{\cancel{s}} \frac{\frac{2}{s+2}}{1+\left(\frac{2}{s+2}\cdot\frac{K_p s+K_i}{s}\right)}$

$= \lim_{s\to 0} \frac{2s}{s(s+2)+2(K_p s+K_i)} = \boxed{0}$

d. The integral forces the steady-state error to zero for step responses and step disturbances.

4. $P(s) = \frac{1}{s-2}$

a. $\mathcal{L}^{-1}\{P\} = e^{2t}$

pole: $s=2>0$ positive real pole, unstable

$s=2 \quad B=1/2$

$A(s-2)+Bs=1 \quad s=0 \quad A=-1/2$

$Y(s) = P(s)R(s) = \frac{1}{s-2}\cdot\frac{1}{s} = \frac{1}{s(s-2)}$

$Y(s) = -\frac{1}{2s} + \frac{1}{2(s-2)}$

$\downarrow\mathcal{L}$

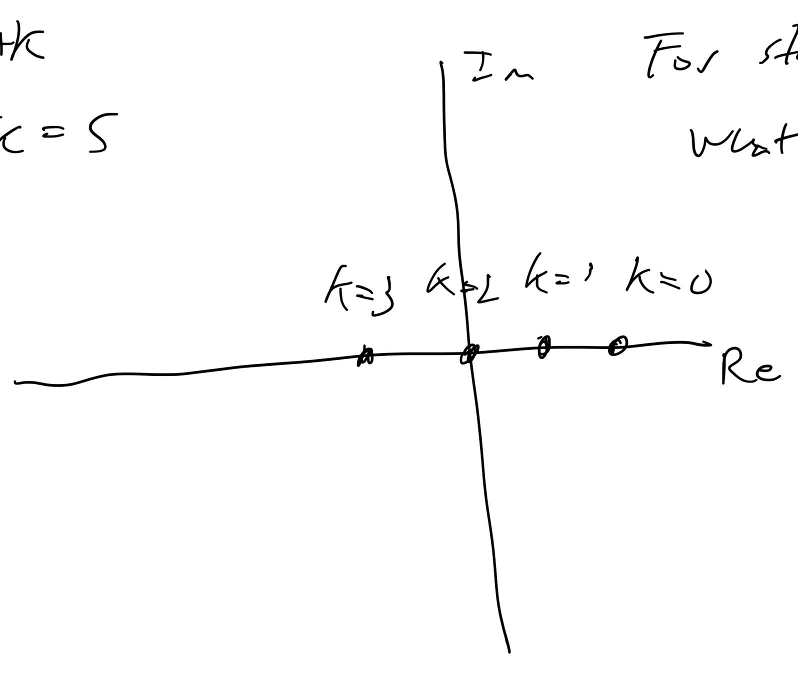
$\boxed{y(t) = \left(-\frac{1}{2}+\frac{1}{2}e^{2t}\right)u(t)}$

b. $\text{CLTF} = \frac{CP}{1+CP} = \frac{KP}{1+KP} = \frac{K\frac{1}{s-2}}{1+\frac{K}{s-2}} = \boxed{\frac{K}{s-2+K}}$

c. char. eq $= s-2+K$

roots: $2-K = s$

K	s
0	2
1	1
2	0
3	-1



For stability want $\boxed{K>2}$

d. Increase K moves system leftward with $K>2$ required for stability.

e. $Y(s) = \frac{1}{s}\frac{K}{s-2+K} = \frac{1}{s}\frac{K}{s-(2-K)}$

$A(s-a)+Bs = K$

$a = 2-K \qquad B = \frac{K}{a} = \frac{K}{2-K}$

$A = \frac{-K}{2-K}$

$\rightarrow Y(s) = \frac{K}{(2-K)s} - \frac{K}{(2-K)(s-(2-K))}$

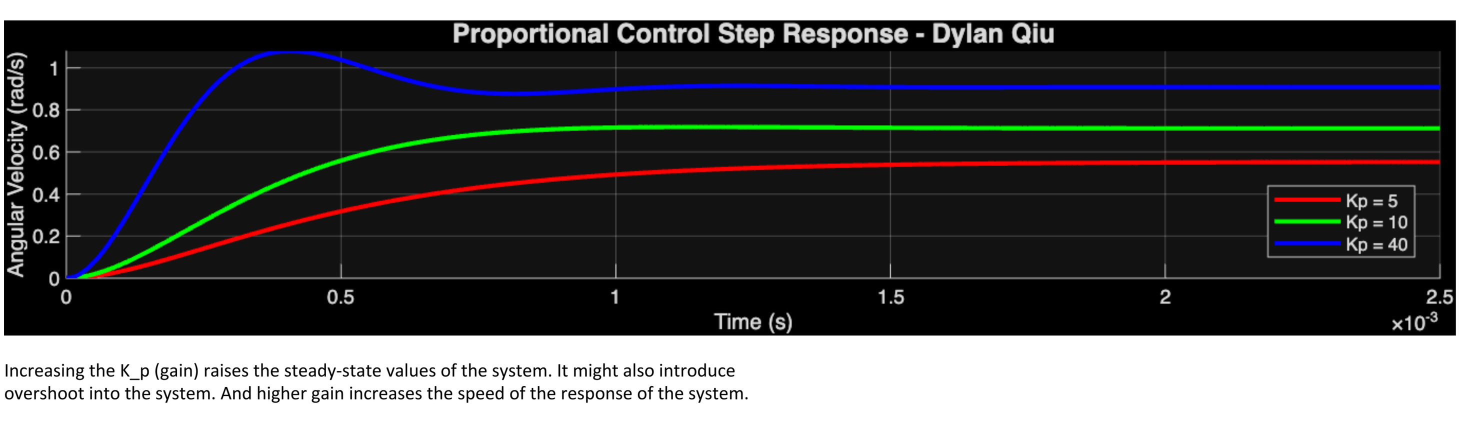
$\downarrow\mathcal{L}$

$y(t) = \frac{K}{2-K} - \frac{K}{2-K}e^{(2-K)t}$

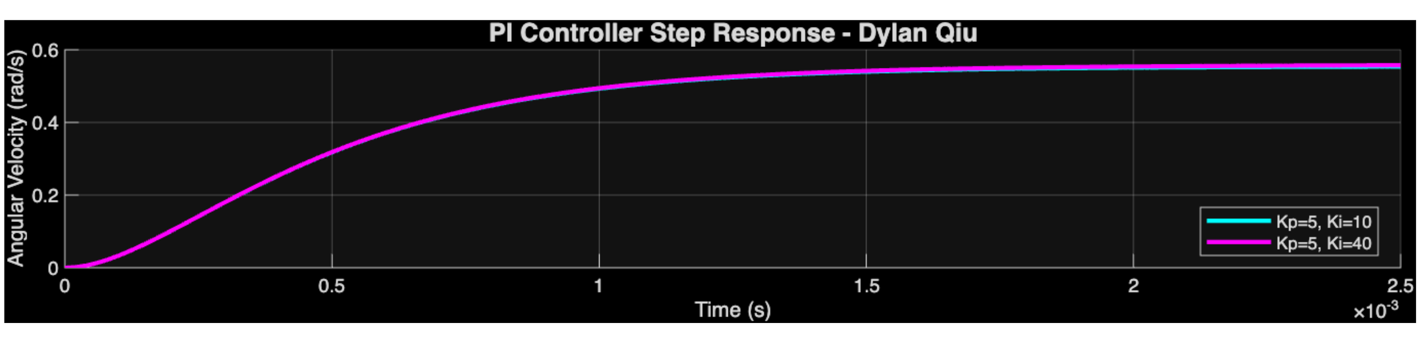
$\boxed{y(t) = \frac{K}{2-K}\left(1-e^{(2-K)t}\right)}$

$K>2$ is stable

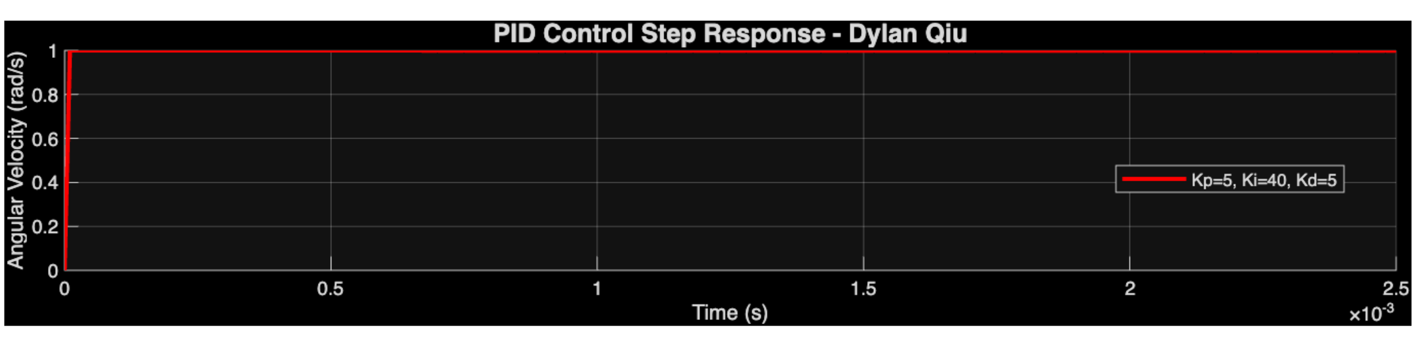
5.



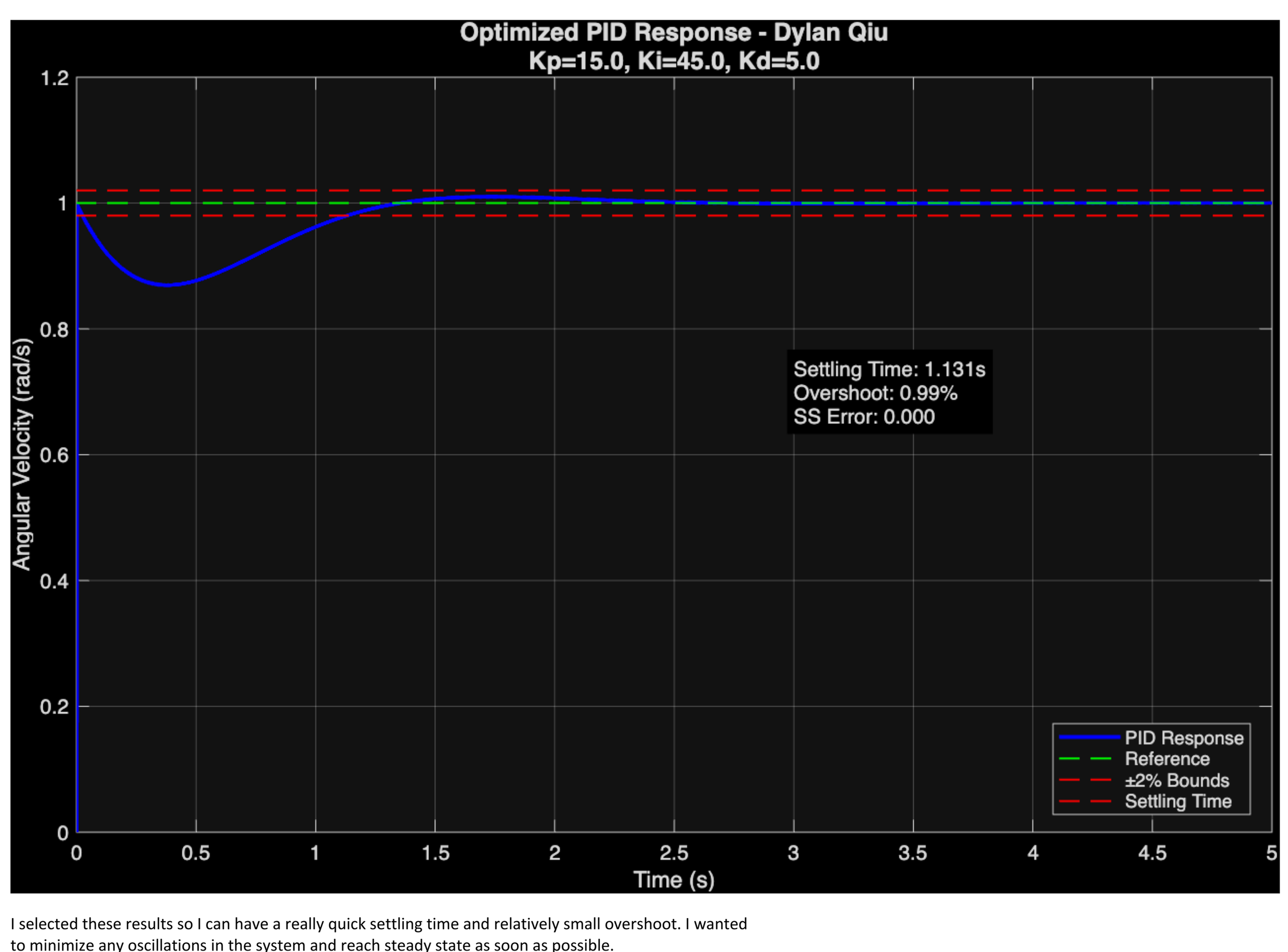
Increasing the K_p (gain) raises the steady-state values of the system. It might also introduce overshoot into the system. And higher gain increases the speed of the response of the system.



Changing the K_i has almost zero effect on the system. The higher K_I only results in a very slightly higher steady state error than the lower value.



Adding the derivative term made the rise time really fast and it stays at steady state without any error.



I selected these results so I can have a really quick settling time and relatively small overshoot. I wanted to minimize any oscillations in the system and reach steady state as soon as possible.