HW3

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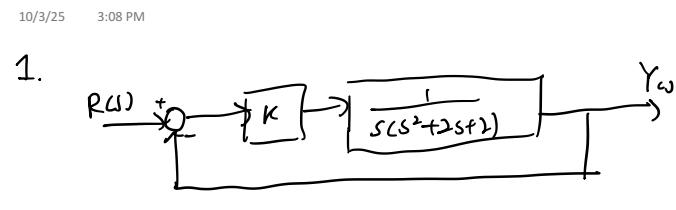
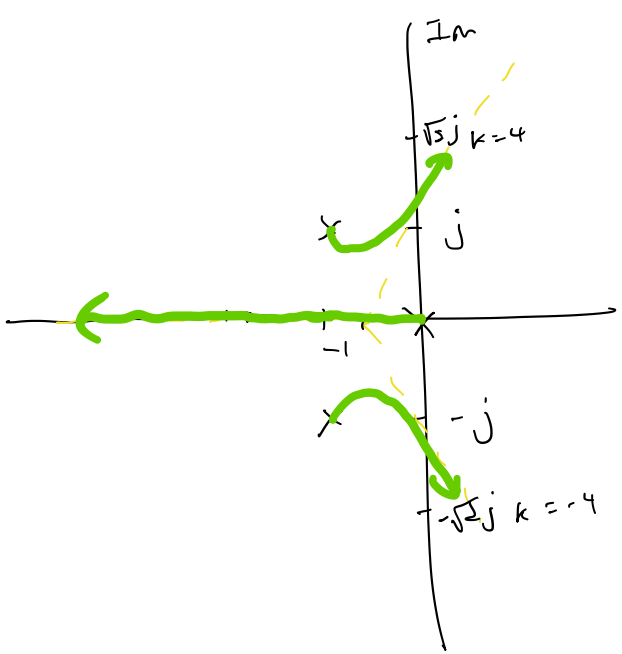
1.



a)



i. poles: $s=0$

$s=-1\pm j$

ii. Zeroes: None

$n=3$

$m=0$

asymptotes:

$\theta = \pm\frac{180(2k+1)}{n-m}$ or $\theta = \frac{180^\circ + 360^\circ(l-1)}{n-m}$

$\theta = \pm 60^\circ,\ k=0$; $\theta = 60^\circ$ $l=1$

$\theta = 180^\circ,\ k=1$; $\theta = 180^\circ$ $l=2$

$\theta = 300^\circ$ $l=3$ $\to \theta = -60^\circ$

$\alpha = \frac{0 + -1 \cancel{+j} + -1 \cancel{-j}}{3} = -\frac{2}{3}$

iii. $\frac{d}{ds}\left(\frac{1}{s(s^2+2s+2)}\right) = 0$

$\frac{d}{ds}\left(\frac{1}{s^3+2s^2+2s}\right) = 0$

$3s^2+4s+2=0$ take only the top of fraction

$s_{1,2} = \frac{-4 \pm \sqrt{16+24}}{6} = -\frac{2}{3} \pm j\frac{\sqrt{2}}{3}$ $s=-1.73$ and $s=\cancel{0.38}$ not on locus

plug into $1+K\frac{1}{s(s^2+2s+2)} = 0$ and solve for k

$K = \cancel{-1.13}$ and $\underline{2.61}$ want $K>0$ for $s=-1.72$

iv. $\sum\theta_p - \cancel{\sum\theta_z}^{\,0} = 180^\circ$

$\angle(s_0 - s_{p_1}) + \angle(s_0 - s_{p_2}) + \angle(s_0 - s_{p_3}) = 180^\circ$

$\theta_{p_1} + 90^\circ + 135^\circ = 180^\circ$

$\theta_{p_1} = -45^\circ$

$\theta_{p_2} = +45^\circ$

v. $s=j\omega$

$s(s^2+2s+2)+K=0$

$s^3+2s^2+2s+K=0$

$(j\omega)^3+2(j\omega)^2+2(j\omega)+K=0$

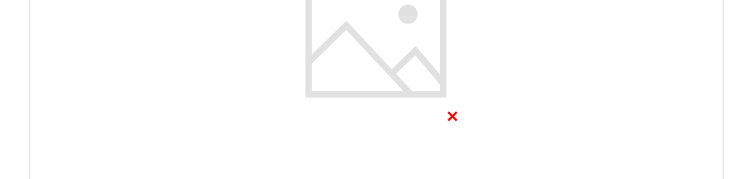
$-j\omega^3 - 2\omega^2 + 2j\omega + K = 0 + 0j$

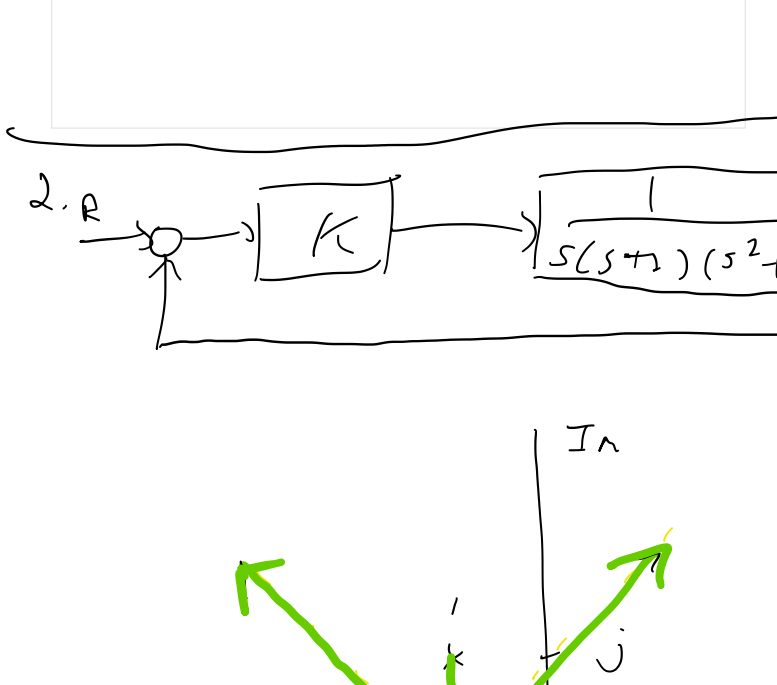
$\begin{cases} -\omega^3 + 2\omega = 0 \to \omega(-\omega^2+2)=0 & \omega = 0, \pm\sqrt{2} \\ -2\omega^2 + K = 0 \to K=4 & \to \text{not on the root loci} \end{cases}$

b. Since the system is type 1. The error at SS would be finite for a $\boxed{\text{ramp input}}$

c. To decrease ts we must increase σ and move the pole further left by decreasing the gain K we can move the poles more left based on the middle branch of the root locus graph.

2.





is the graph

breakin: s(-1,0)

i. poles: $0, -2, -1\pm j$

Zeros: None

$n=4$

$m=0$

ii. $\theta = \pm\frac{180(2k+1)}{n-m}$

$k=0$ $\theta = \pm 45^\circ$

$k=1$ $\theta = \pm 135^\circ$

$\alpha = \frac{0+(-2)+(-1+j)+(-1-j)}{4}$

$= -1$

iii. $\frac{d}{ds}\left(\frac{1}{s(s+2)(s^2+2s+2)}\right) = 0$

$4s^3+12s^2+12s+4=0$ take the top

$s^3+3s^2+3s+1=0$

$s=-1$

$1+K\frac{1}{s(s+2)(s^2+2s+2)} = 0 \to k=1$ ✓

iv. $\sum\theta_p - \cancel{\sum\theta_z}^{\,0} = 180^\circ$

$\theta_1 + 90^\circ + 135^\circ + 45^\circ = 180^\circ$

$\theta_1 = -90^\circ$

$\theta_2 = +90^\circ$

$\theta_3 + 180^\circ + 135^\circ + 45^\circ = 180^\circ$

$\theta_3 = -180^\circ$

$\theta_4 = +180^\circ$

b. Routh Array $s(s+2)(s^2+2s+2) = s^4+4s^3+6s^2+4s+k$

s^4	1	6	k
s^3	4	4	0
s^2	5	k	0
s^1	$4-\frac{4}{5}k$	0	
s^0	k		

$4-\frac{4}{5}k > 0$

$4 > \frac{4}{5}k$

$20 > 4k$

$5 > k$

$\boxed{\begin{array}{l} k<5 \text{ For stability} \\ k>5 \text{ For unstable} \end{array}}$ $\omega = \pm j$

0 sign changes = 0 RHP poles

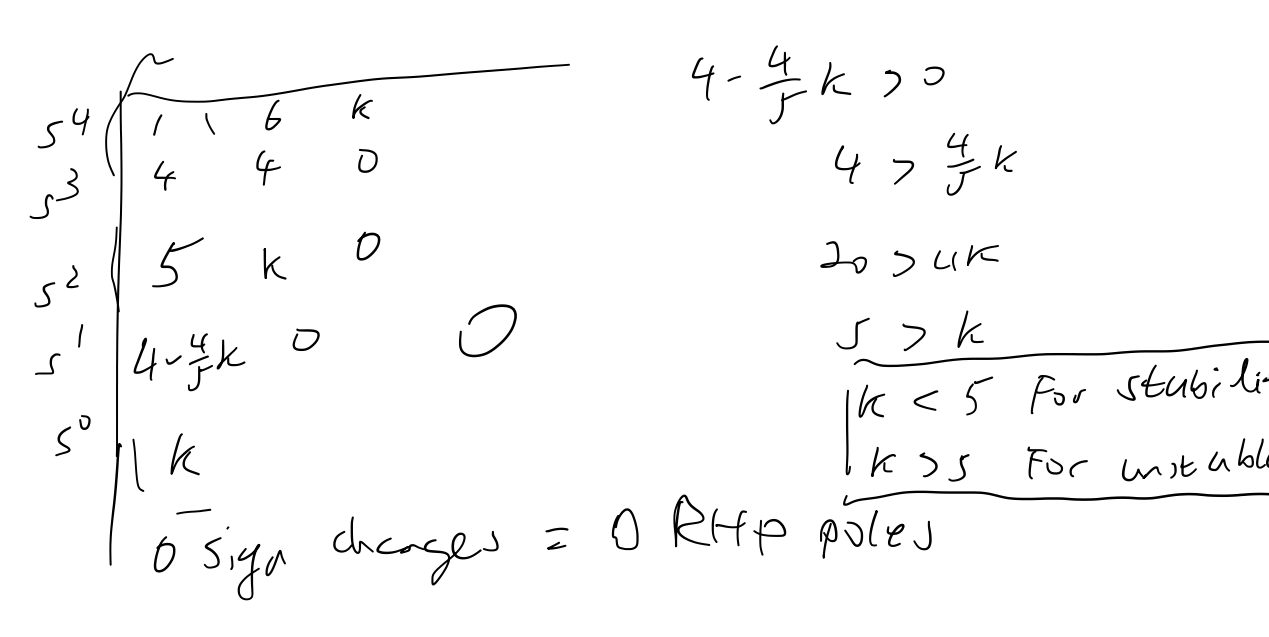
c. $t_s = \frac{4}{\sigma}$ want t_s small so σ bigger

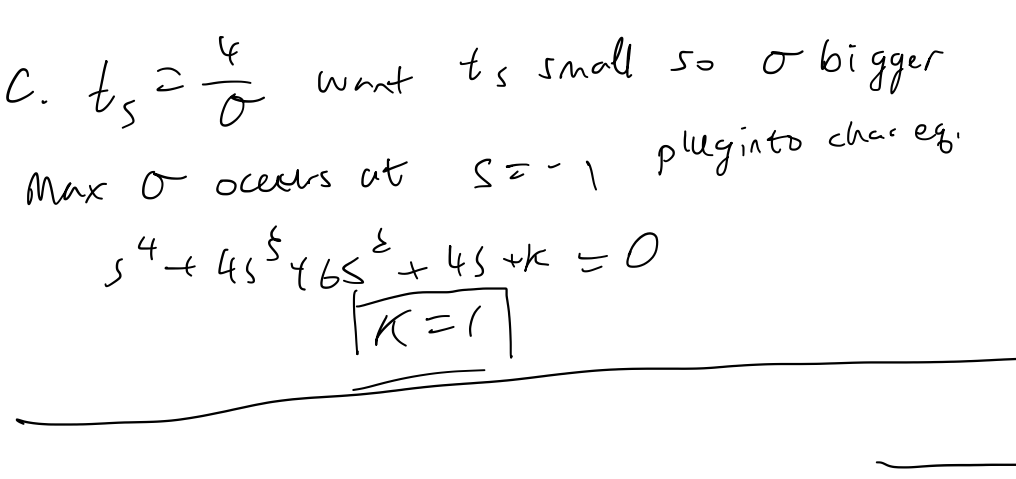
Max σ occurs at $s=-1$ plug into char eq.

$s^4+4s^3+6s^2+4s+k=0$

$\boxed{K=1}$

3.





i. poles: $s=0, -1\pm j$

zeros: $s=-2$

$n=3$

$m=1$

ii. $\theta = \frac{180^\circ(2k+1)}{n-m}$

$k=0$ $\theta = 90^\circ$

$K=1$ $\theta = 270^\circ$

$\alpha = \frac{0 + (-1\cancel{+j}) + (-1\cancel{-j}) - (-2)}{2}$

$\alpha = 0$

iii. $\frac{d}{ds}\left(\frac{1}{s(s^2+2s+2)}\right) = 0$

$\frac{d}{ds}\left(\frac{1}{s^3+2s^2+2s}\right) = 0$

$\to 3s^2+4s+2=0$

$s = -\frac{2}{3} \pm \frac{\sqrt{2}}{3}$

iv. $\sum\theta_p - \sum\theta_z = 180^\circ$

$\theta_1 + 90^\circ + 135^\circ - 45^\circ = 180^\circ$

$\theta_1 = 0$

$\theta_2 = 0$

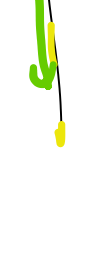
$\theta_3 + 135^\circ + 45^\circ - 180^\circ = 180^\circ$

$\theta_3 = 180^\circ$

v. No Im-Axis intersections

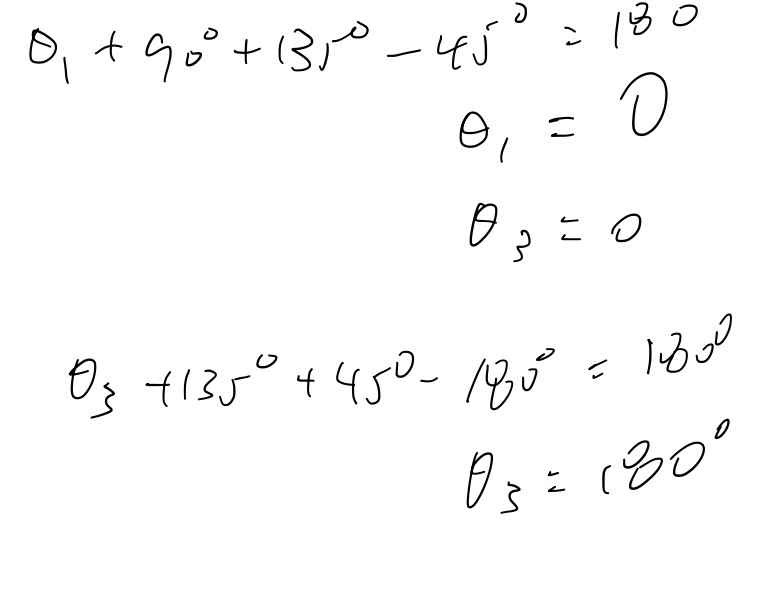
b. The (s+2) added an extra zero where the other 2 root loci plots don't.
The system also became always stable no matter what value of K it is with everything in the LHP

4.



$P(s) = \frac{1}{Js^2}$ $J = 1\ \text{kgm}^2$ $1+\frac{K_p}{s^2}$ s^2+K

a. $C(s) = K_p$



i. poles: $s=0$

Zeroes: None

$n=2$

$m=0$

ii. $\frac{180^\circ+360^\circ(k-1)}{n-m} = 180^\circ$

$\alpha = 0$

iii. $\frac{d}{ds}\left(\frac{1}{s^2}\right) = 0$

$-\frac{2}{s^3} = 0$

No breakaway pts.

iv. $\sum\theta_p - \sum\theta_z = 180^\circ$

$\theta_p = 180^\circ$

v. $Js^2+k=0$ $s=j\omega$

$-J\omega^2+k=0$

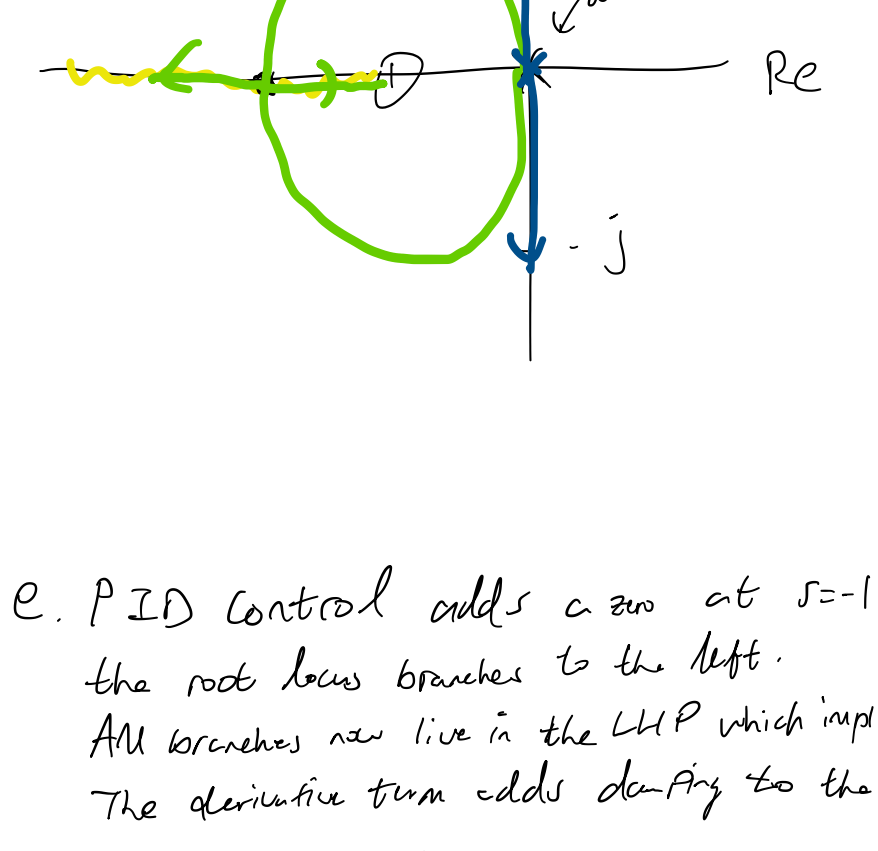
$\omega = \pm\sqrt{\frac{K}{J}}$ entire Im-axis

$s=0$

b. As you vary K_p you have marginal stability over the control of the system which will oscillate indefinitely as K_p increases, the frequency of oscillation also increases.

c. $C(s) = k_p + k_d s$ $K = k_p$ and $K_p/K_D = 1$

char eq: $1+CP = 1+\frac{K_p}{s^2}+\frac{K_d}{s} \Rightarrow \boxed{1+\frac{K(s+1)}{Js^2} = 0}$



i. poles: 0, 0,

zero: -1

$n=2$

$m=1$

ii. $\theta = \frac{180^\circ+360^\circ(k-1)}{n-m}$

$k=1$ $\theta = \pm 90^\circ$

$\alpha = \frac{1}{1} = 1$

iii. $\frac{d}{ds}\left(\frac{s+1}{s^2}\right) = 0$

$\frac{-s-2}{s^3} = 0$

$s = -2$

iv. No complex poles so no angles of departure/arrival

e. PID control adds a zero at $s=-1$ which pulls the root locus branches to the left.
All branches now live in the LHP which implies the system is now stable.
The derivative term adds damping to the system.

f. $C(s) = K\frac{(s+z)}{(s+p)}$

char eq:

$1+CP = 1+K\frac{s+z}{s+p}\cdot\frac{1}{s^2} = 1+\frac{K(s+z)}{s^2(s+p)}$

root locus: $\boxed{s^2(s+p)+K(s+z)=0}$

g. $\frac{s+1}{s^3+ps^2} = 0$ $z=1$

i. $p=20$ $n=3$ $m=1$

$\theta = \frac{180(2k-1)}{n-m}$

$k=1$ $\theta = 90^\circ$

$k=2$ $\theta = 270^\circ$

$\alpha = \frac{-(20-1)}{2} = \boxed{-9.5}$

ii. $\theta = 90^\circ$, $\theta = 270^\circ$ same steps

$\alpha = \frac{-(9-1)}{2} = \boxed{-4}$

iii. $\boxed{\begin{array}{l}\theta = 90^\circ \\ \theta = 270^\circ\end{array}}$ same steps

$\alpha = \frac{-(3-1)}{2} = \boxed{-1}$

$\frac{d}{ds}\left(\frac{s+1}{s^3+ps^2}\right) = 0$ Taking the top only

$s^3+ps^2-(3s^2+2ps)(s+1)=0$

$s^3+ps^2-3s^3-3s^2-2ps^2-2ps=0$

$-2s^3-3s^2-ps^2-2ps=0 \to 2s^2+3s+ps+2p=0$

$s(2s^2+(3+p)s+2p)=0$

at $p=20$:

$\boxed{s=0}$ break away

$s=-2.1$

$\boxed{s=-9.4}$ breakaway

at $p=9$

$\boxed{s=0}$ breakaway

$s=2.16$

$\boxed{s=-3}$ breakaway

at $p=3$

$\boxed{s=0}$ breakaway

+ complex solution for s that is not on root locus

h. moving the poles to the right increases the settling time of the system but the branches become more damped.

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5. $M_p(\zeta) = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

solve for ζ in terms of Mp:

$\ln(M_p) = -\frac{\zeta\pi}{\sqrt{1-\zeta^2}}$

$\sqrt{1-\zeta^2}\ln(M_p) = -\zeta\pi$

$(1-\zeta^2)(\ln(M_p))^2 = (\zeta\pi)^2$

$(\ln(M_p))^2 - \zeta^2(\ln(M_p))^2 = \zeta^2\pi^2$

$(\ln(M_p))^2 = \zeta^2(\pi^2+(\ln(M_p))^2)$

$\zeta = \left[\frac{(\ln(M_p))^2}{(\pi^2+(\ln(M_p))^2)}\right]^{1/2} = \boxed{0.69}$

$t_{r_{10-90\%}} \cong \frac{1.8}{\omega_n}$

$\omega_n = \frac{1.8}{t_r} = \boxed{3600}$

$G(s) = \frac{K_m}{(Ls+R)(Js+B)+K_mK_b}$

b. $\beta = \cos^{-1}(\zeta) = \cos^{-1}(0.69) \cong 46.37^\circ$

break in: $\frac{-8196}{2} = -4098$ from evaluating the open loop TF

at $s=-4098$, and $\beta = 46.37^\circ$

$\tan(46.37^\circ) = \frac{\pm\omega_d}{s}$

$\pm j\omega_d = 4299$

$\omega_d = \pm 4299j$

at 5% overshoot select pole: $\boxed{s=-4098\pm 4299j}$

at that point $\boxed{K=16.4}$ (from Matlab)

c.

K=16.4

$G(s) = \frac{K_m}{(Ls+R)(Js+B)+K_mK_b}$

$e_{ss} = \lim_{s\to 0}\frac{1}{s}KG(s) = K\left(\frac{\frac{K_m}{JL}}{\frac{K_mK_b}{JL}}\right) = 0.247K$

d. Increase the gain increases the overshoot And it won't meet the specifications in a)

K=25

E. The effect of the lag compensator will reduce steady state error by increasing the gain at low frequencies.

It also adds more places on the root locus where the gain could be that will still satisfy a). It added an zero for the poles to go around.

f. $C = K_c\frac{(s+z)}{s+p}$

DC gain of system (G_{ss})

R_g $K_b = 0.247$ also from matlab

$e_{ss} = \lim_{s\to 0} s\frac{E(s)}{R(s)}\frac{1}{s}$

where $\frac{E(s)}{R(s)} = \frac{1}{1+CP} = \frac{1}{1+K_c\frac{(s+z)}{(s+p)}G(s)} \xrightarrow{\lim_{s\to 0}} \frac{1}{1+K_c\frac{z}{p}G_{ss}}$

so as $\lim_{s\to 0} C = K_c\frac{z}{p}$ and $\lim_{s\to 0} G = G_{ss}$ where G_{ss} is DC gain of system

$e_{ss} = \lim_{s\to 0}\frac{E(s)}{R(s)}$

$0.02 = \frac{1}{1+K_c\frac{z}{p}\cdot G_{ss}}$

$0.02 + (C\,G_{ss})0.02 = 1$

$C = \frac{(1-0.02)}{0.02\,G_{ss}} = \boxed{198.4}$

g.

$K_c\frac{z}{p} = \frac{1}{\text{Gain}}\frac{(1-0.02)}{0.02} = 198.4$

Set Kc = 25 from part d

$\frac{z}{p} = \frac{198.4}{25} = 8$ ← pole and zero ratio

Z = -0.008

P = -0.001

Kc = 198.4