REPRESENTATION

Marcel Campen
New York University

REPRESENTATION

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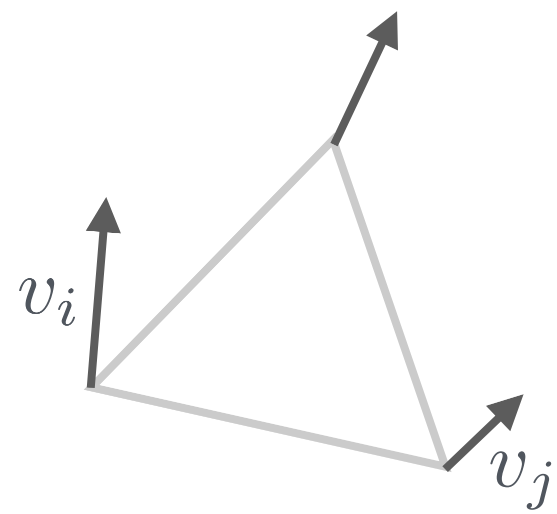
Model problem:

$|\nabla v|^2 \to \min$



v_i "$-$" v_j

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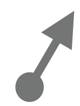
- 1 directional 

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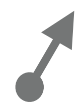
- 1 directional 
    - Cartesian

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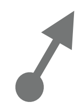
- 1 directional 
  - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

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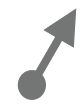
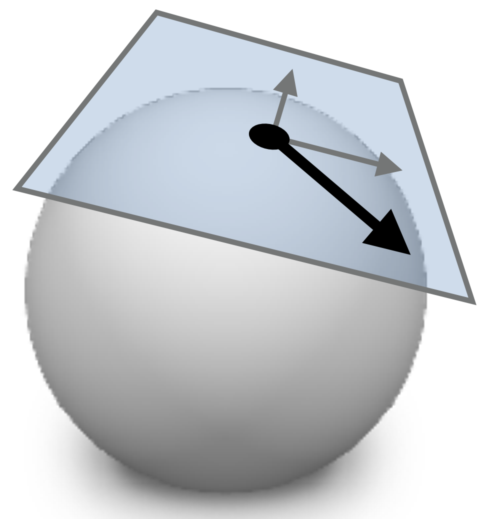
- 1 directional 
    - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

REPRESENTATION

- 1 directional
  - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



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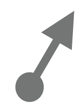
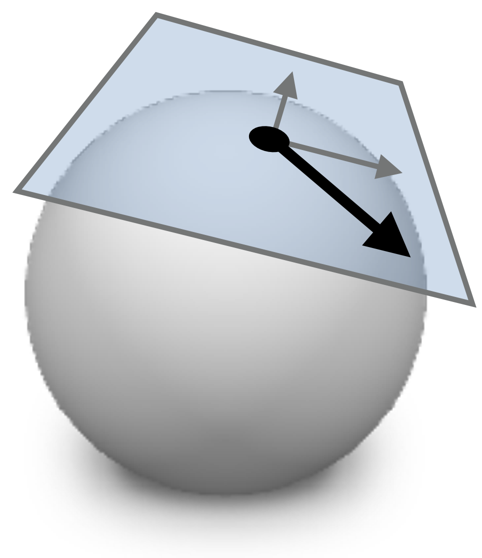
- 1 directional 
    - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

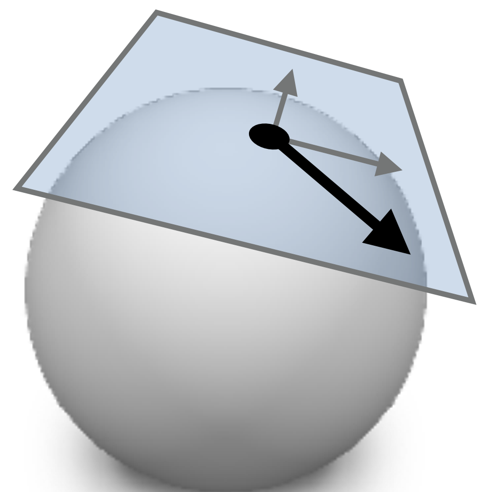


$$\begin{pmatrix} b_{1,p} & b_{2,p} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

REPRESENTATION

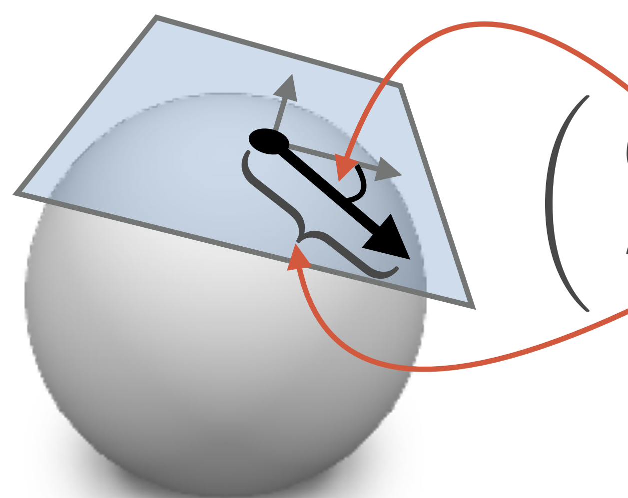
- 1 directional
  - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$$\begin{pmatrix} b_{1,p} & b_{2,p} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

  - Polar



$$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

REPRESENTATION

- 1 directional

direction field

  - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

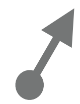
$$x^2 + y^2 = 1$$

  - Polar

$$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

$$\left(\theta \right)$$

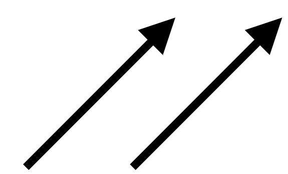
REPRESENTATION

- 1 directional 

direction field

  - Cartesian

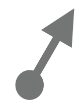
$$\begin{pmatrix} x \\ y \end{pmatrix} \qquad x^2 + y^2 = 1 \qquad \begin{pmatrix} 0.7 \\ 0.7 \end{pmatrix} - \begin{pmatrix} 0.7 \\ 0.7 \end{pmatrix} = 0$$



  - Polar

$$\begin{pmatrix} \theta \\ r \end{pmatrix} \qquad \begin{pmatrix} \theta \end{pmatrix} \qquad 45^\circ - 405^\circ \neq 0$$

REPRESENTATION

- 1 directional 
  - Cartesian

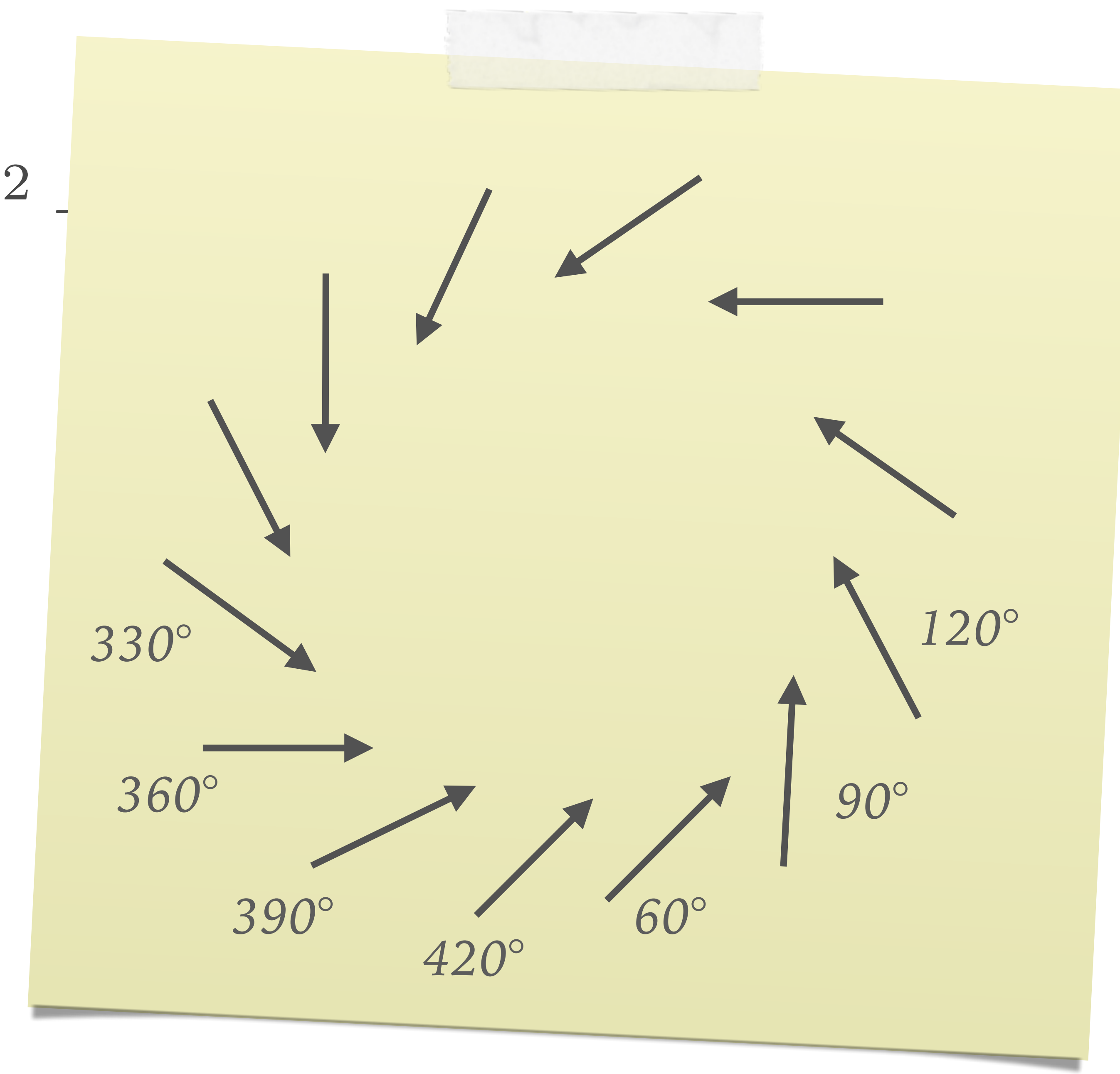
$$\begin{pmatrix} x \\ y \end{pmatrix}$$

  - Polar

$$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

direction field

x^2 - … 0



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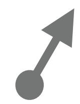
- 1 directional 
  - Cartesian

    $$\begin{pmatrix} x \\ y \end{pmatrix}$$

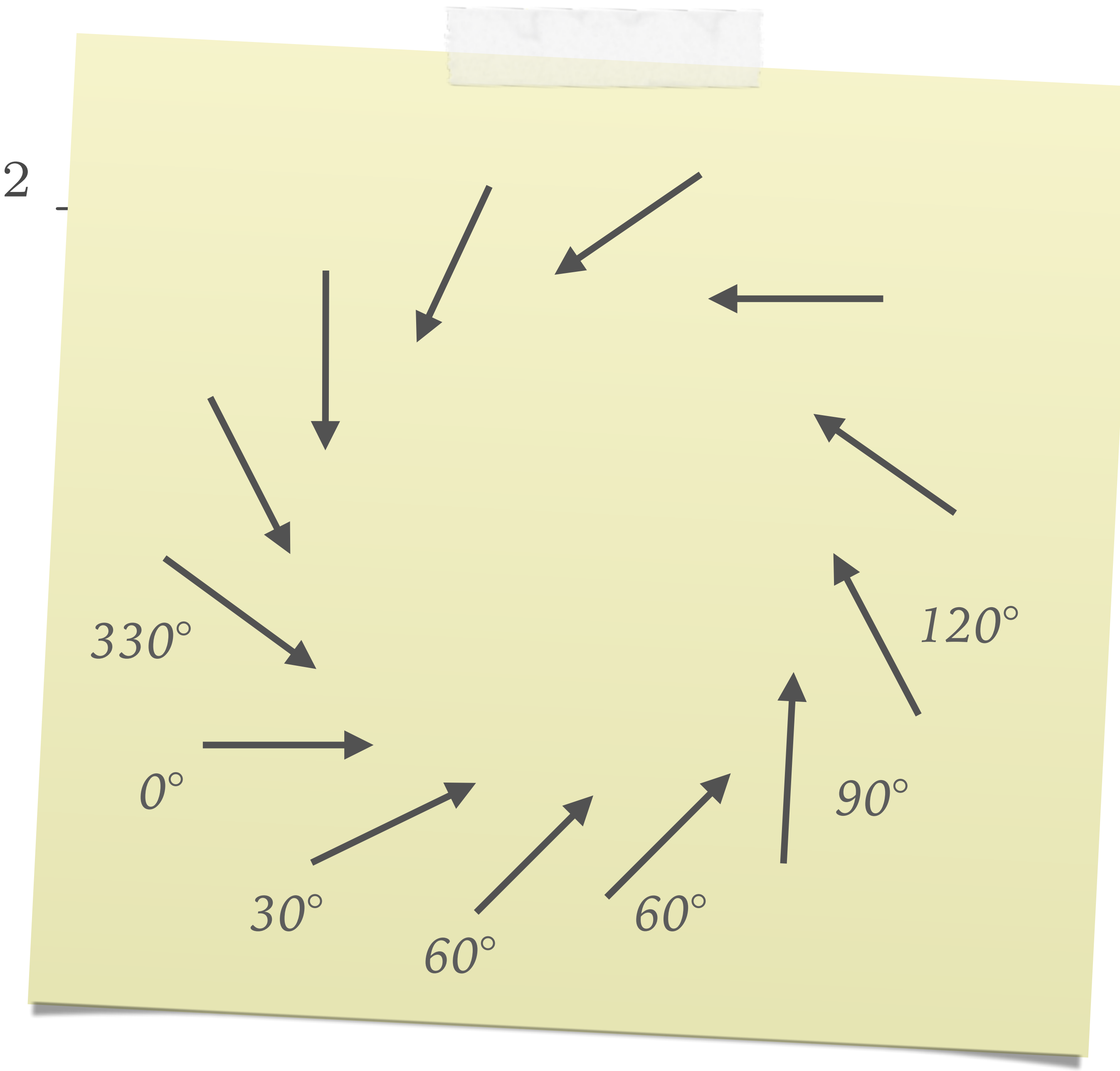
  - Polar

    $$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

direction field

x^2 -

0



REPRESENTATION

- 1 directional

	direction **field**	2π-invariance
• Cartesian		
$\begin{pmatrix} x \\ y \end{pmatrix}$	$x^2 + y^2 = 1$	built-in
• Polar		
$\begin{pmatrix} \theta \\ r \end{pmatrix}$	$(\ \theta\ )$	$\theta_i - \theta_j \mod 2\pi$

REPRESENTATION

- 1 directional

	direction field	2π-invariance

  - Cartesian

$\begin{pmatrix} x \\ y \end{pmatrix}$	$x^2 + y^2 = 1$	built-in

  - Polar

$\begin{pmatrix} \theta \\ r \end{pmatrix}$	$\begin{pmatrix} \theta \end{pmatrix}$	$\min_{k \in \mathbb{Z}} \ \theta_i - \theta_j + k2\pi$

REPRESENTATION

- 1 directional

direction field | 2π-invariance

- Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

$$x^2 + y^2 = 1$$

built-in

- Polar

$$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

$$(\ \theta \ )$$

$$\theta_i - \theta_j + k2\pi$$

k const.

explicit choice of period

$\Rightarrow$ control over topology

[Li et al. 2006]

REPRESENTATION

- 1 directional
  - Cartesian

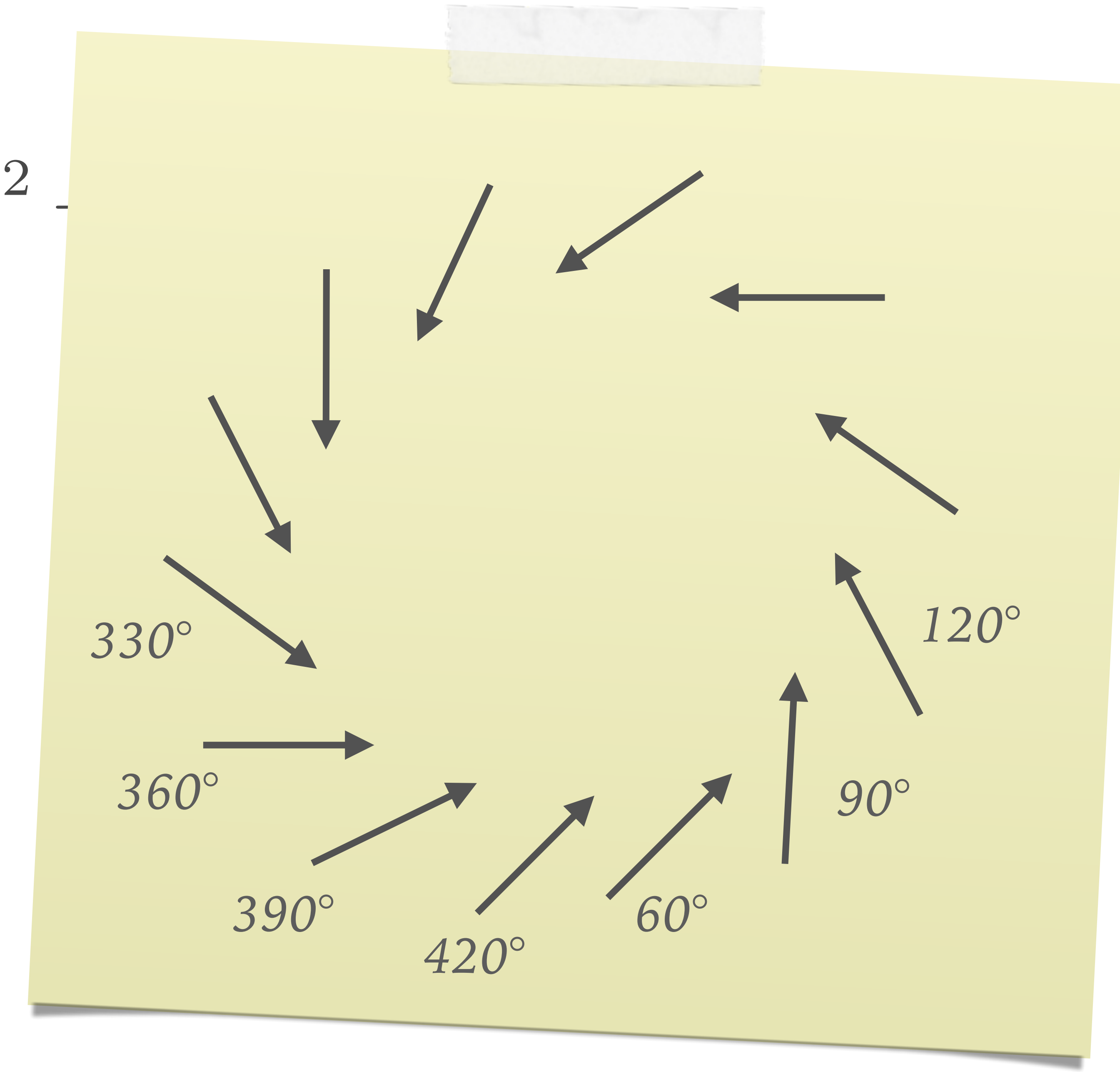
    $$\begin{pmatrix} x \\ y \end{pmatrix}$$

  - Polar

    $$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

direction field

$x^2 -$ 0



REPRESENTATION

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    - Cartesian

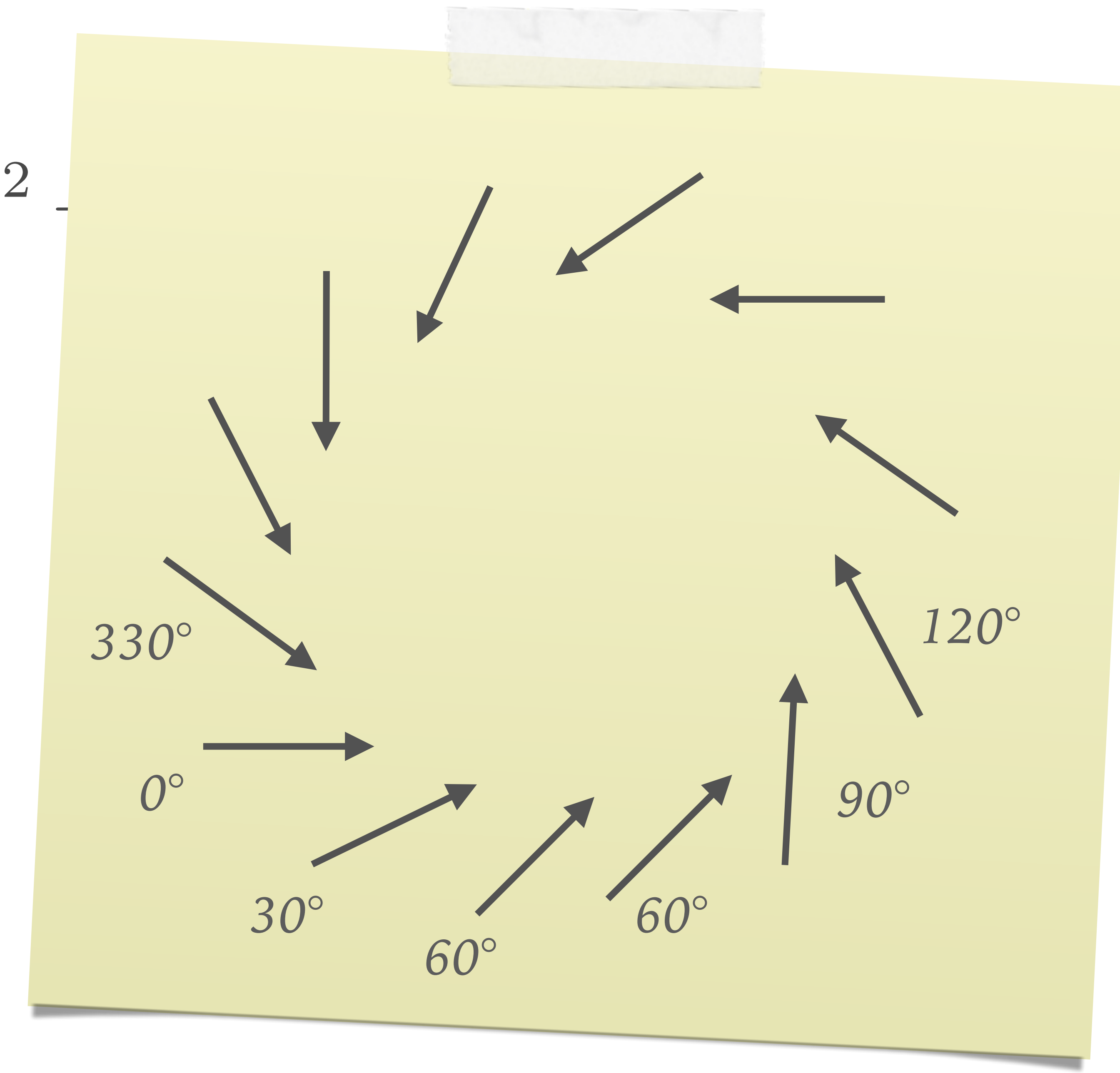
$$\begin{pmatrix} x \\ y \end{pmatrix}$$

    - Polar

$$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

direction field

x^2 - 0



REPRESENTATION

- 1 directional
  - Cartesian

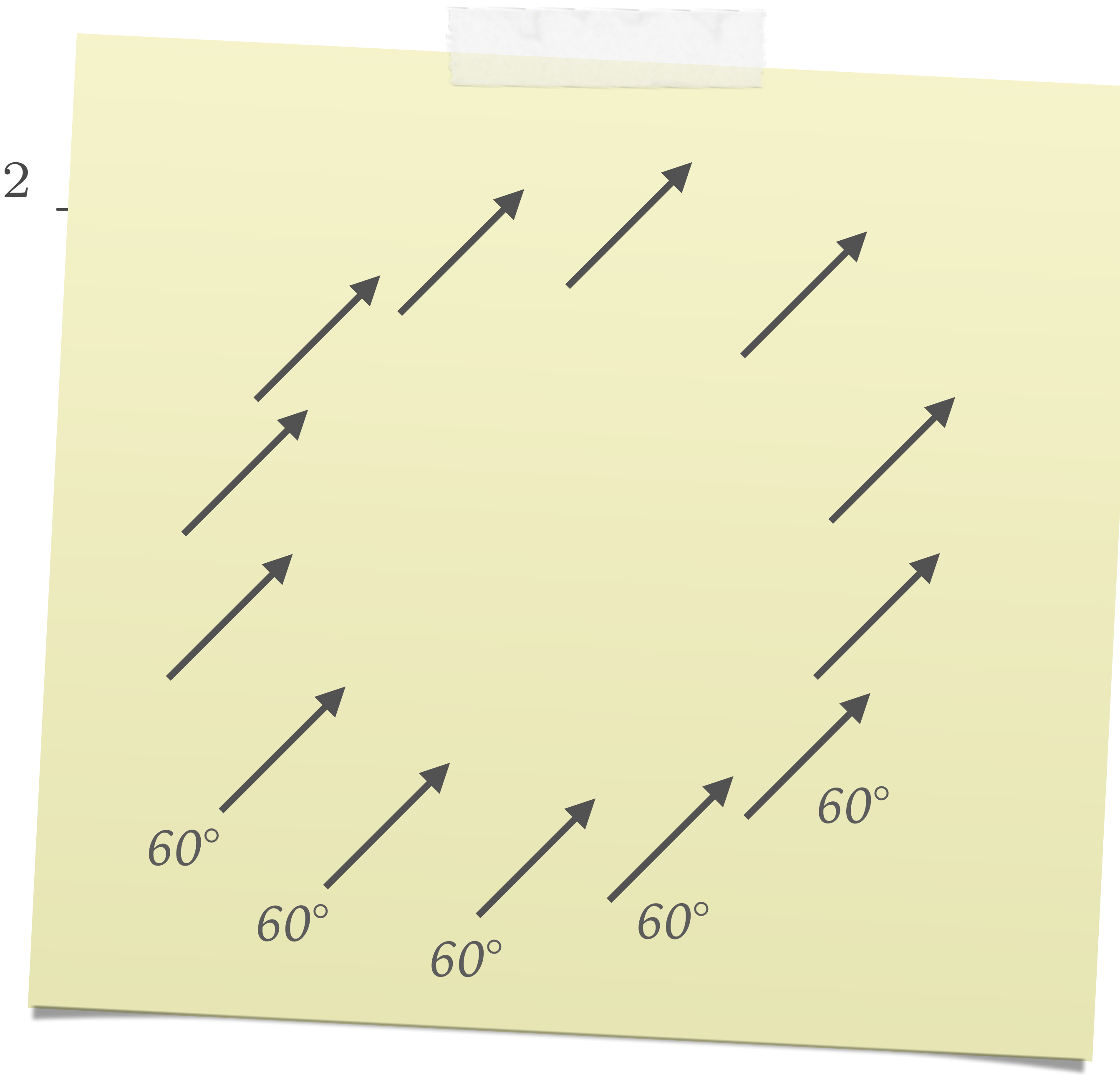
    $$\begin{pmatrix} x \\ y \end{pmatrix}$$

  - Polar

    $$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

direction field

x^2 - 0



REPRESENTATION

- 1 directional

  direction field — 2π-invariance

  - Cartesian

    $$\begin{pmatrix} x \\ y \end{pmatrix}$$

    $$x^2 + y^2 = 1$$

    built-in

  - Polar

    $$\begin{pmatrix} \theta \\ r \end{pmatrix} \quad r\mathrm{e}^{\mathrm{i}\theta}$$

    $$\begin{pmatrix} \theta \end{pmatrix}$$

    $$\theta_i - \theta_j + k2\pi$$

    k const.

    explicit choice of period
    $\Rightarrow$ control over topology

    [Li et al. 2006]

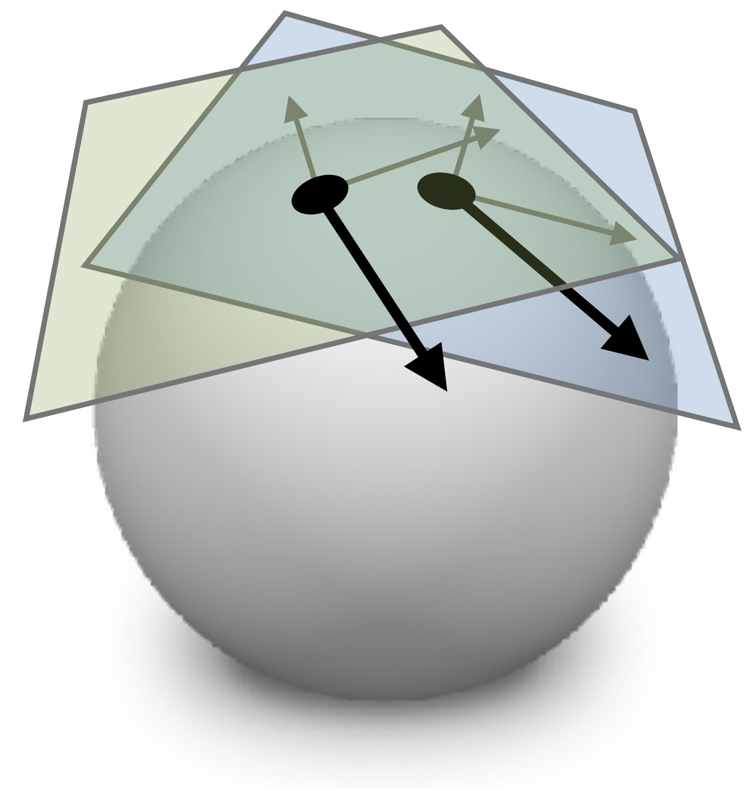
REPRESENTATION

- Differences between tangent vectors?
  - Cartesian

    $$\begin{pmatrix} x \\ y \end{pmatrix}$$

  - Angle

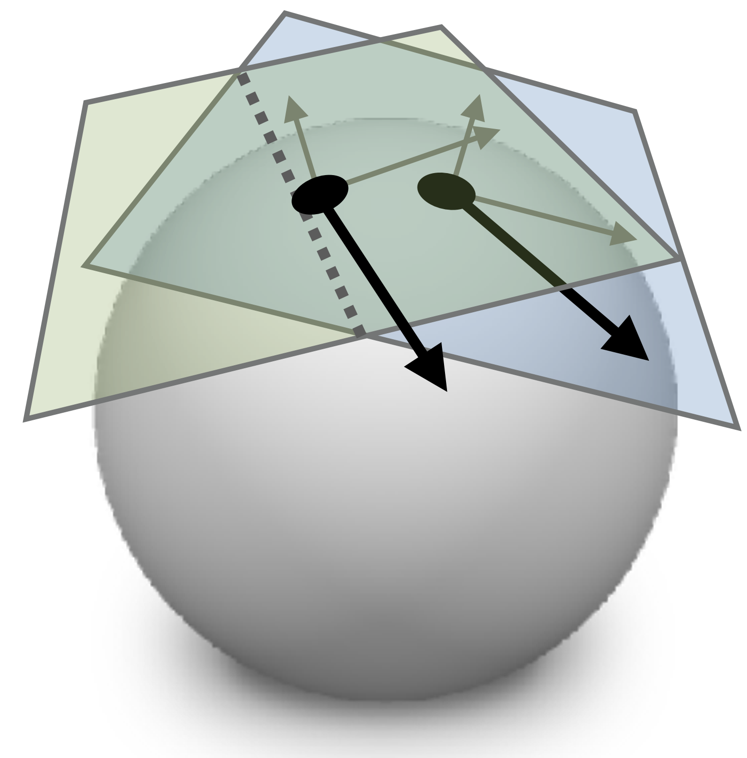
    $$\theta$$



REPRESENTATION

- Differences between tangent vectors?
  - Cartesian

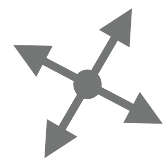
$$\begin{pmatrix} x \\ y \end{pmatrix}_j - \begin{pmatrix} \cos X_{ij} & -\sin X_{ij} \\ \sin X_{ij} & \cos X_{ij} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}_i$$



  - Angle

$$\theta_j - \theta_i + X_{ij} + p_{ij} 2\pi$$

REPRESENTATION

- N directionals 

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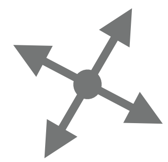
- N directionals 
  - simply use multiple $\begin{pmatrix} x \\ y \end{pmatrix}$ or $\begin{pmatrix} \theta \\ r \end{pmatrix}$ per location?

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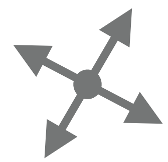
- N directionals 
  - simply use multiple $\begin{pmatrix} x \\ y \end{pmatrix}$ or $\begin{pmatrix} \theta \\ r \end{pmatrix}$ per location?
    - perhaps okay for mere representation,
      but problematic for synthesis, optimization, …

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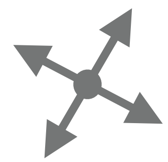
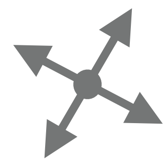
- N directionals 
  - simply use multiple $\begin{pmatrix} x \\ y \end{pmatrix}$ or $\begin{pmatrix} \theta \\ r \end{pmatrix}$ per location?
    - perhaps okay for mere representation,
      but problematic for synthesis, optimization, …
      - symmetries $\Rightarrow$ additional constraints

REPRESENTATION

- *N* directionals 
    - symmetric
        - just use one representative

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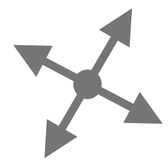
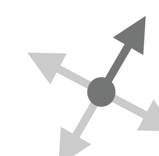
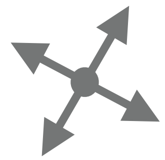
- N directionals 
  - symmetric
    - just use one representative
      - others implied by rotation by $k2\pi/N$



[Palacios & Zhang 2007]

[Ray et al. 2008]

REPRESENTATION

- N directionals 
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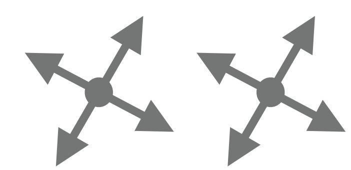

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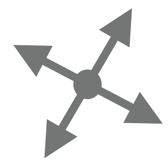
- N directionals 
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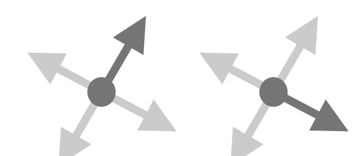
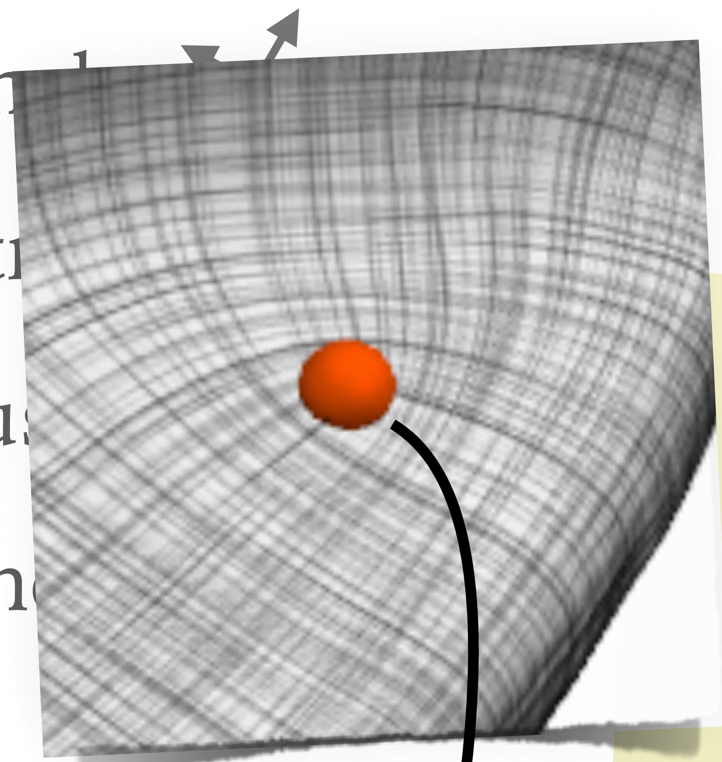
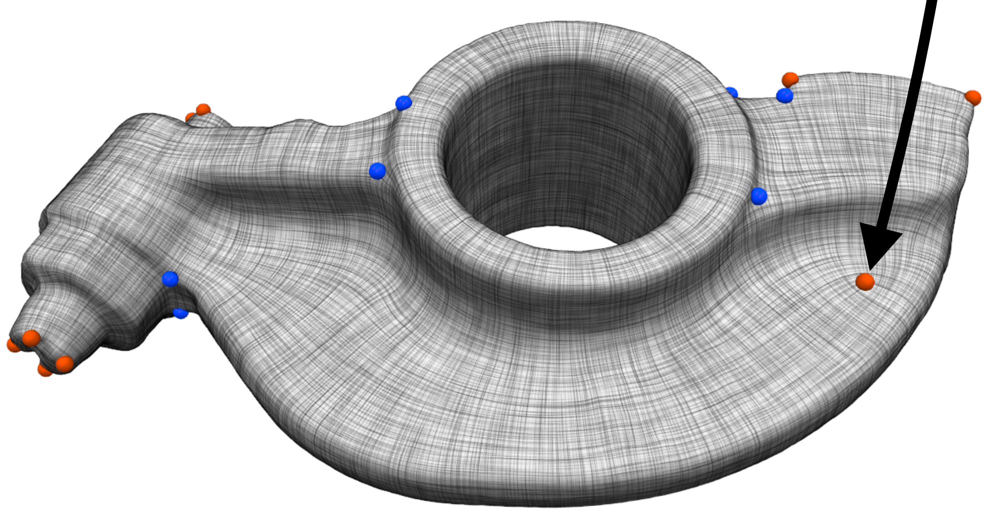
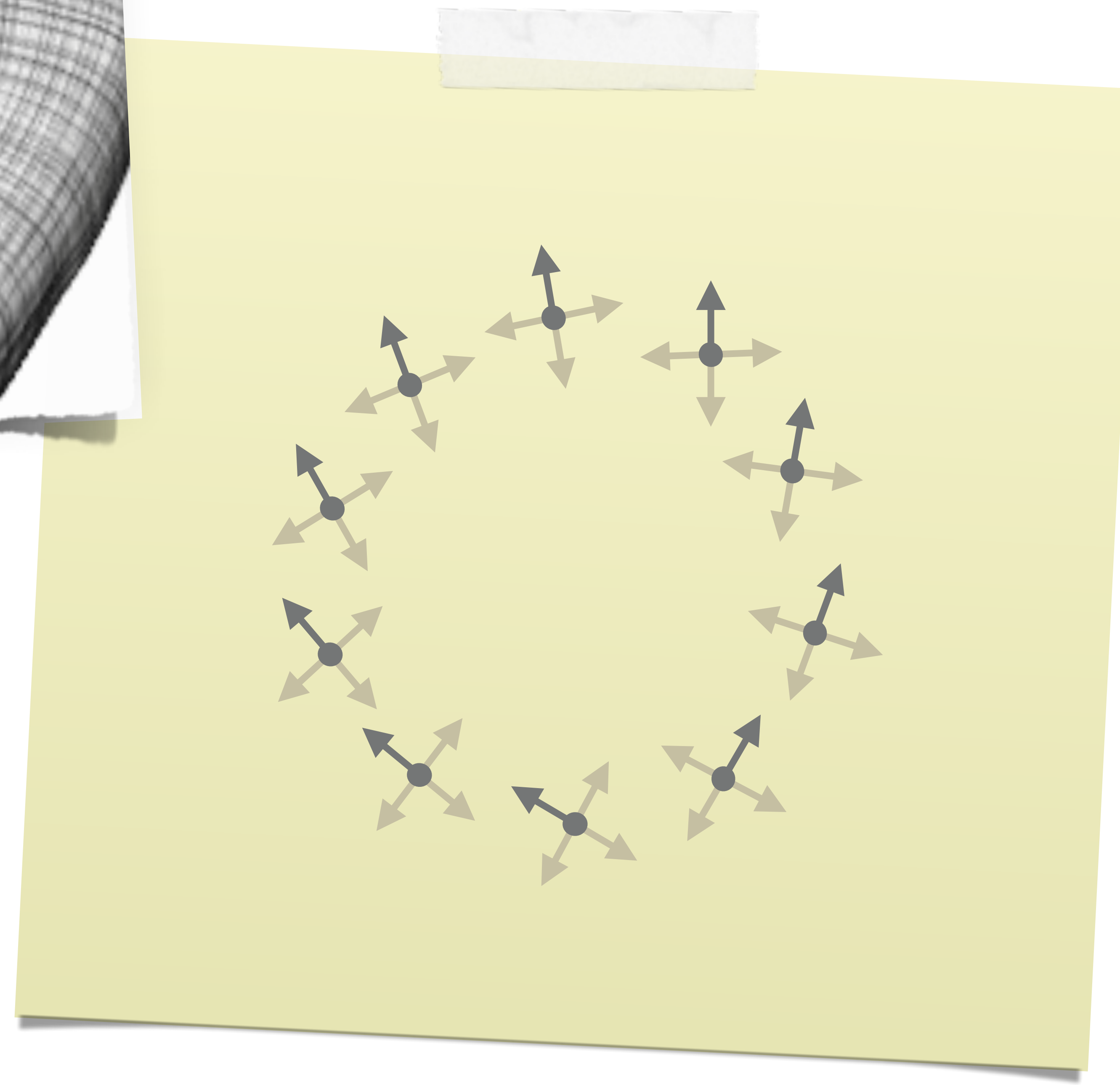


REPRESENTATION

- *N* direction
  - symmetr
    - just us
      - oth







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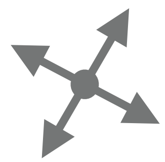
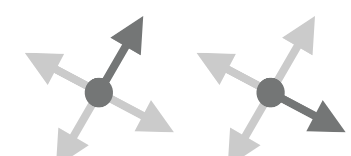
- N directionals 
  - symmetric
    - just use one representative
      - others implied by rotation by $k2\pi/N$

$$\theta_j - \theta_i + X_{ij} + p_{ij} 2\pi/N$$



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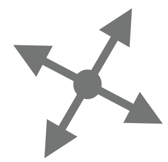
- N directionals 
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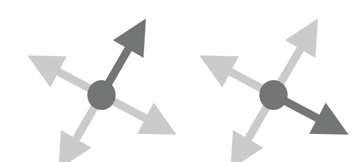
$$\theta_j - \theta_i + X_{ij} + p_{ij}2\pi/N$$



- Cartesian:

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

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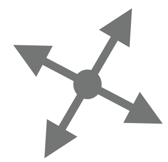
- N directionals 
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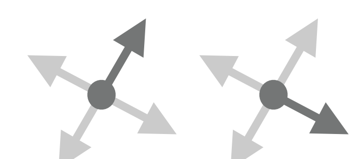
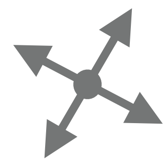


- Cartesian:

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \theta = \mathrm{atan2}(y, x)$$

REPRESENTATION

- N directionals 
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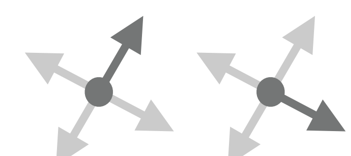
$$\theta_j - \theta_i + X_{ij} + p_{ij} 2\pi/N$$



- Cartesian:

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \theta = \operatorname{atan2}(y, x) \rightarrow \begin{pmatrix} \sin N\theta \\ \cos N\theta \end{pmatrix}$$

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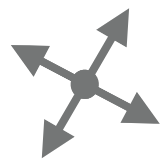
- N directionals 
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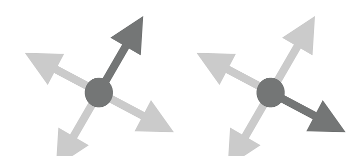
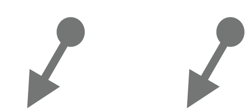
$$\theta_j - \theta_i + X_{ij} + p_{ij} 2\pi/N$$





- Cartesian:

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \theta = \operatorname{atan2}(y, x) \rightarrow \begin{pmatrix} \sin N\theta \\ \cos N\theta \end{pmatrix}$$

"representation vector"

[Hertzmann & Zorin 2000]

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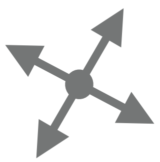
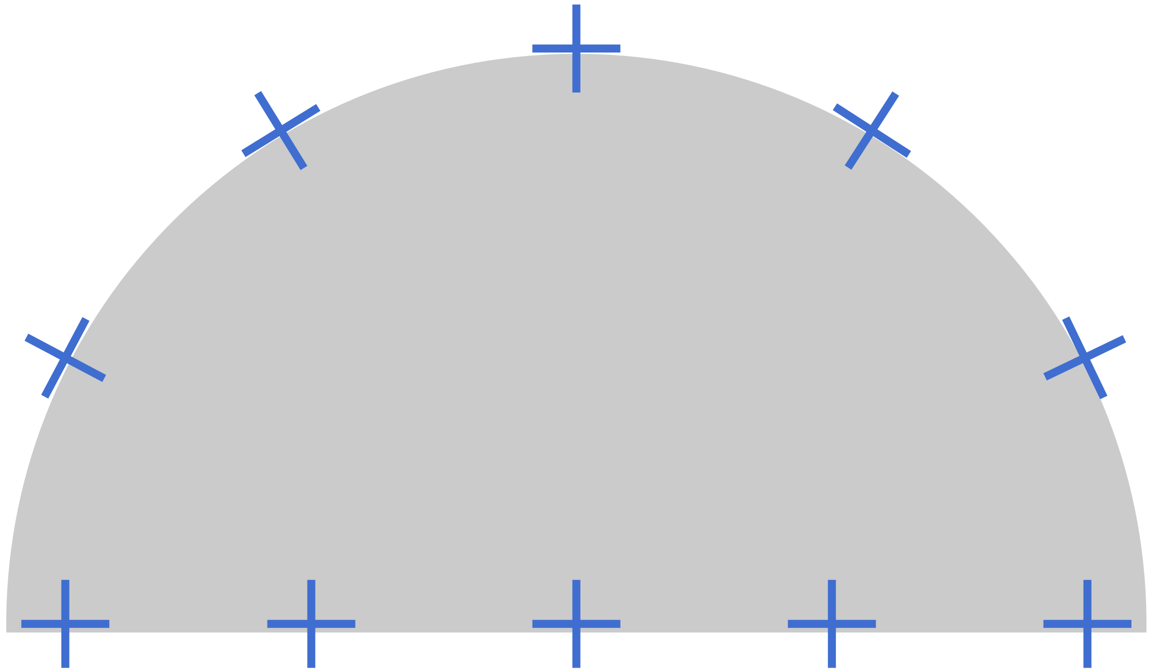
- N directionals 
    - symmetric
        - just use one representative
            - others implied by rotation by $k2\pi/N$



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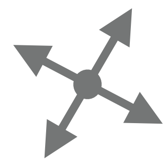
- N directionals 
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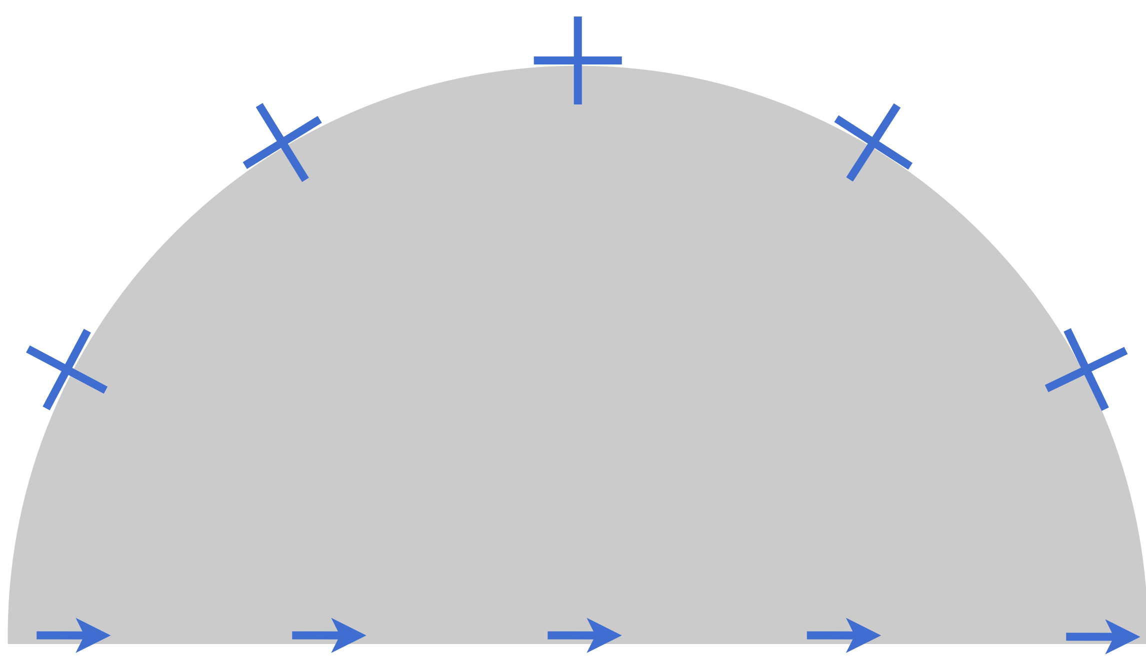

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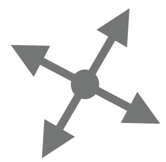
- N directionals 
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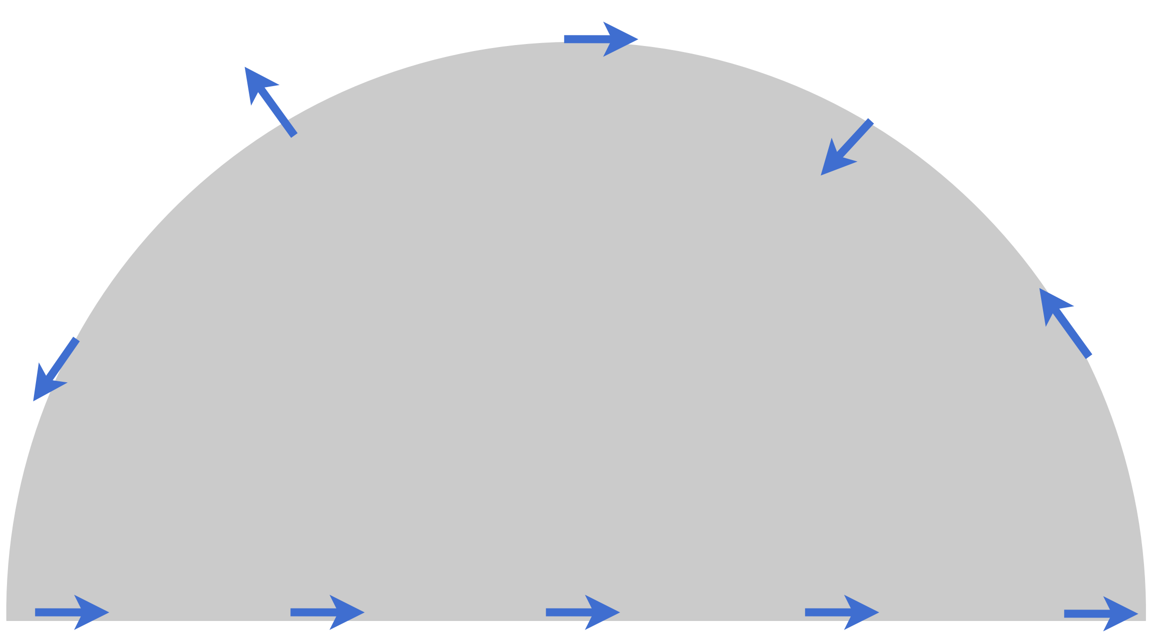
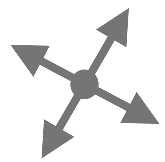
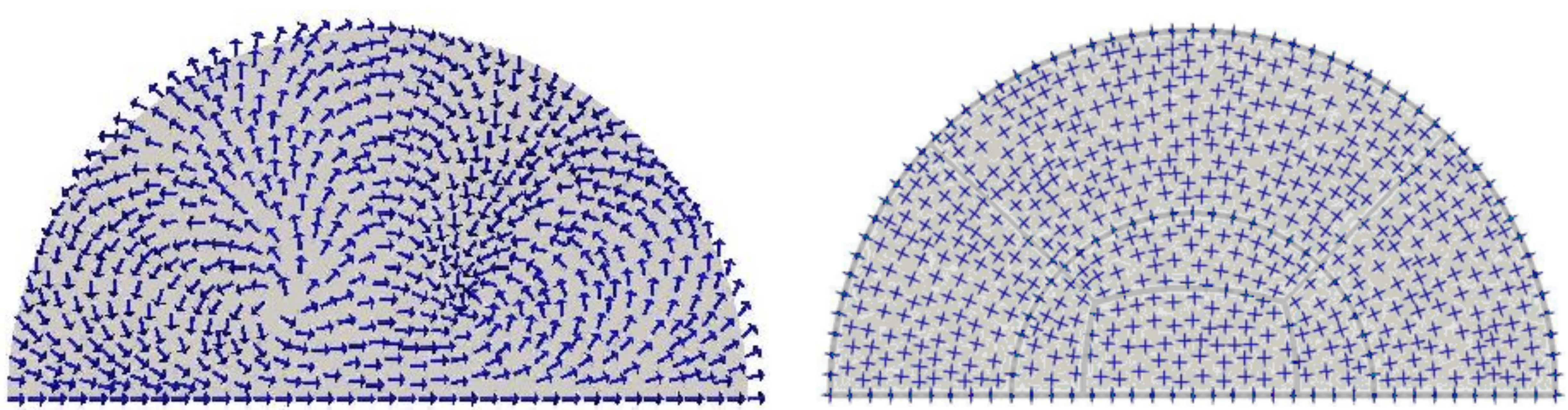


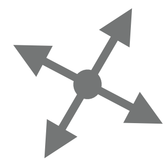
REPRESENTATION

- N directionals 
  - symmetric
    - just use one representative
      - others implied by rotation by $k2\pi/N$

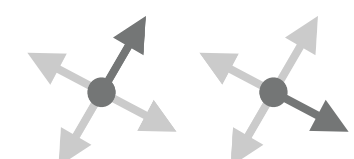


[Kowalski et al. 2013]

REPRESENTATION

- N directionals 
  - symmetric
    - just use one representative
      - others implied by rotation by $k2\pi/N$

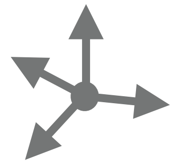
$$\theta_j - \theta_i + X_{ij} + p_{ij} 2\pi/N$$



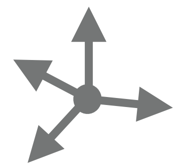
- Cartesian:

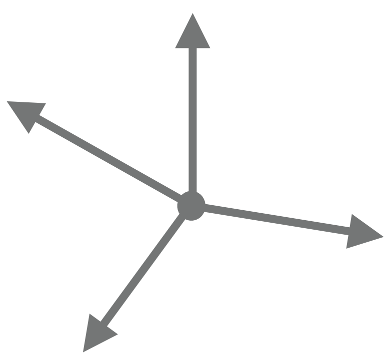
$$\begin{pmatrix} x \\ y \end{pmatrix} \to \theta = \mathrm{atan2}(y, x) \to \begin{pmatrix} \sin N\theta \\ \cos N\theta \end{pmatrix}$$

REPRESENTATION

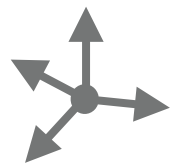
- N directionals 
    - non-symmetric

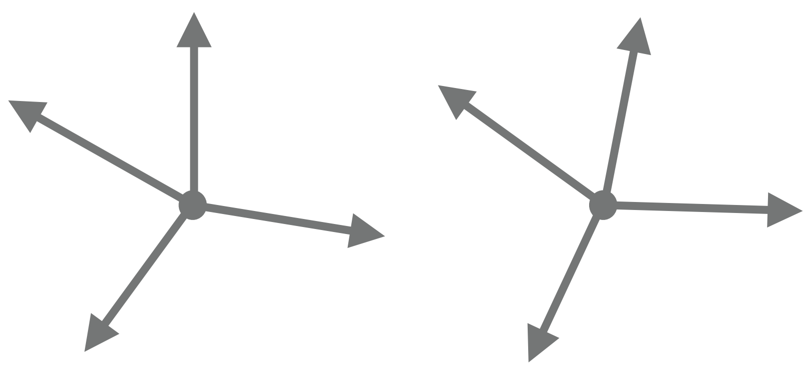
REPRESENTATION

- N directionals 
  - non-symmetric

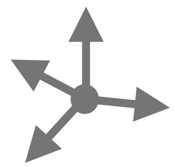


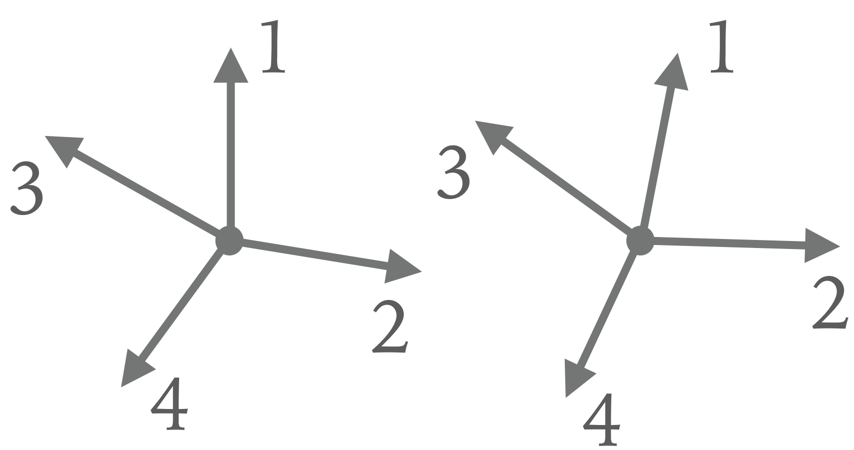
REPRESENTATION

- N directionals 
  - non-symmetric

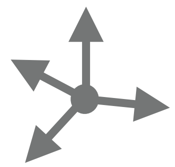


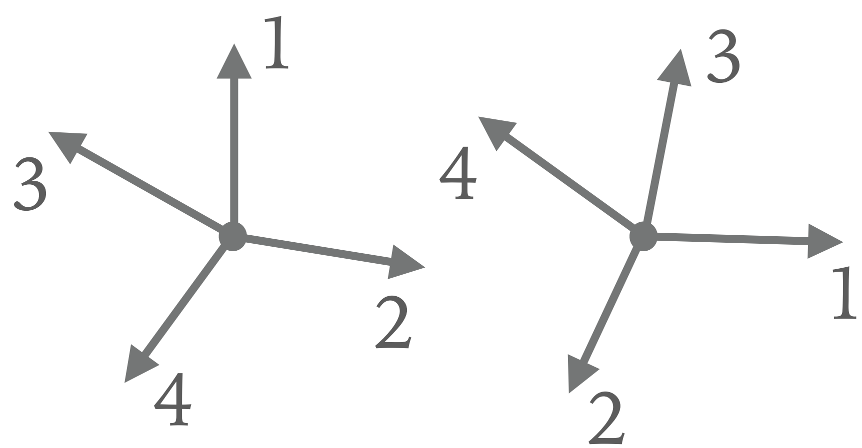
REPRESENTATION

- N directionals 
    - non-symmetric

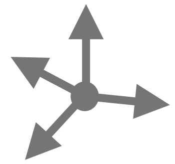


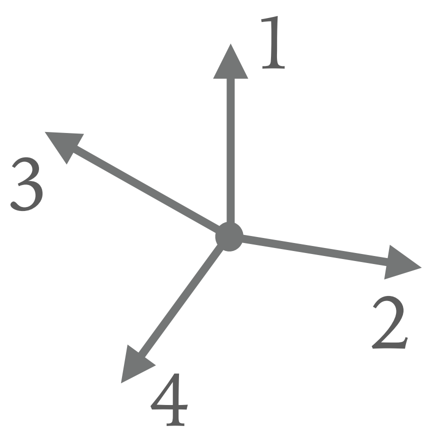
REPRESENTATION

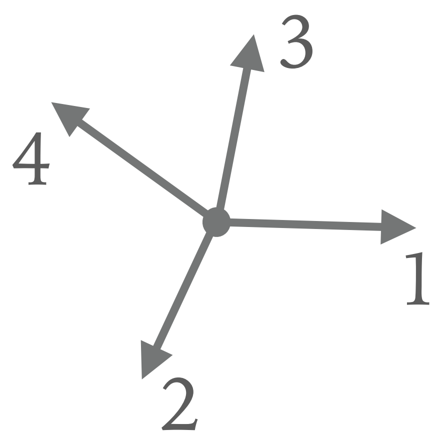
- *N* directionals 
  - non-symmetric



REPRESENTATION

- *N* directionals 
  - non-symmetric



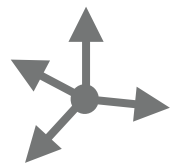


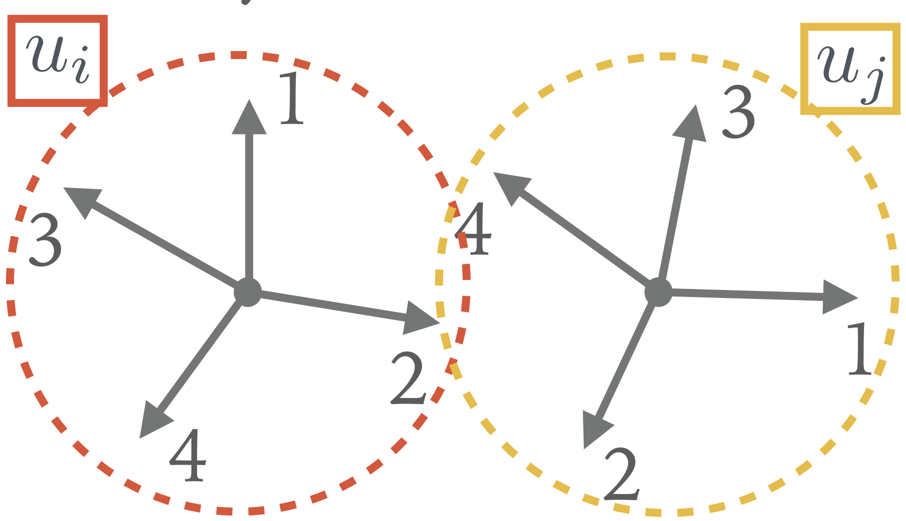
$$F(v_1, v_2, v_3, v_4) = u$$

F symmetric

$$F(v_1, v_2, v_3, v_4) = F(v_3, v_2, v_4, v_1)$$

REPRESENTATION

- N directionals 
  - non-symmetric



$F(v_1, v_2, v_3, v_4) = u$

F symmetric

F invertible (up to symmetry)

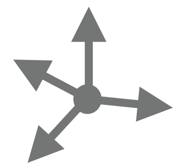
$$(x-a)(x-b)(x-c)$$
$$= x^3 + (a+b+c)x^2 + (ab+ac+bc)x + (abc)$$
$$\to (a+b+c \mid ab+ac+bc \mid abc)$$
$$(\quad u \quad , \quad v \quad , \quad w \quad) =: F(a,b,c)$$

N-PolyVector

[Diamanti et al. 2014]

$$(a+b+c+d \mid ab+ac+ad+bc+bd+cd \mid abc+abd+acd+bcd \mid abcd)$$

REPRESENTATION

- N directionals 
  - use complex numbers, complex polynomials

    $$a = x + iy$$
    $$a = re^{i\theta}$$

  - non-symmetric

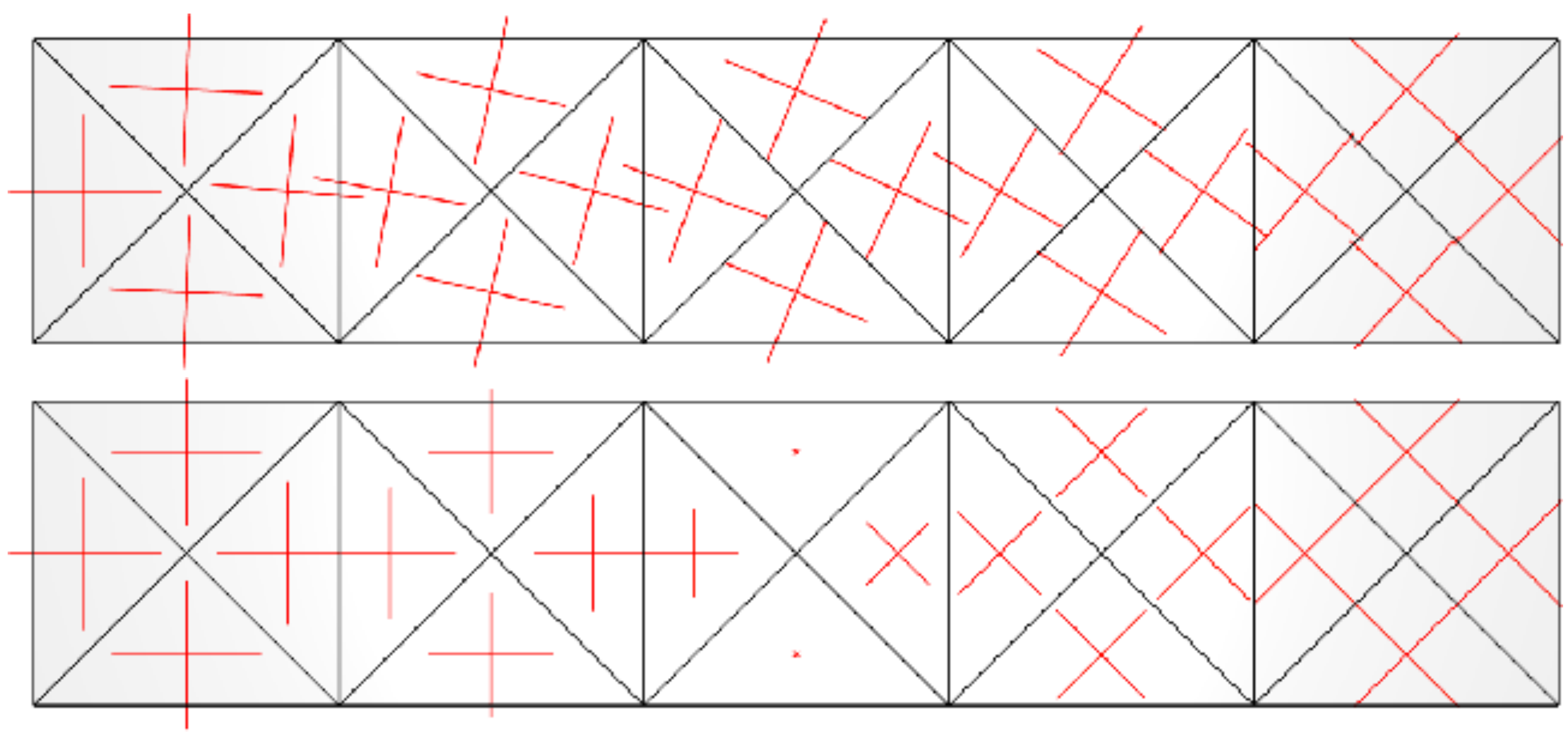
$$(a + b + c + d \mid ab + ac + ad + bc + bd + cd \mid abc + abd + acd + bcd \mid abcd)$$

  - symmetric

$$(\underbrace{a + b + c + d}_{0} \mid \underbrace{ab + ac + ad + bc + bd + cd}_{0} \mid \underbrace{abc + abd + acd + bcd}_{0} \mid \underbrace{abcd}_{\substack{-a^4 \\ =-re^{i4\theta}}})$$

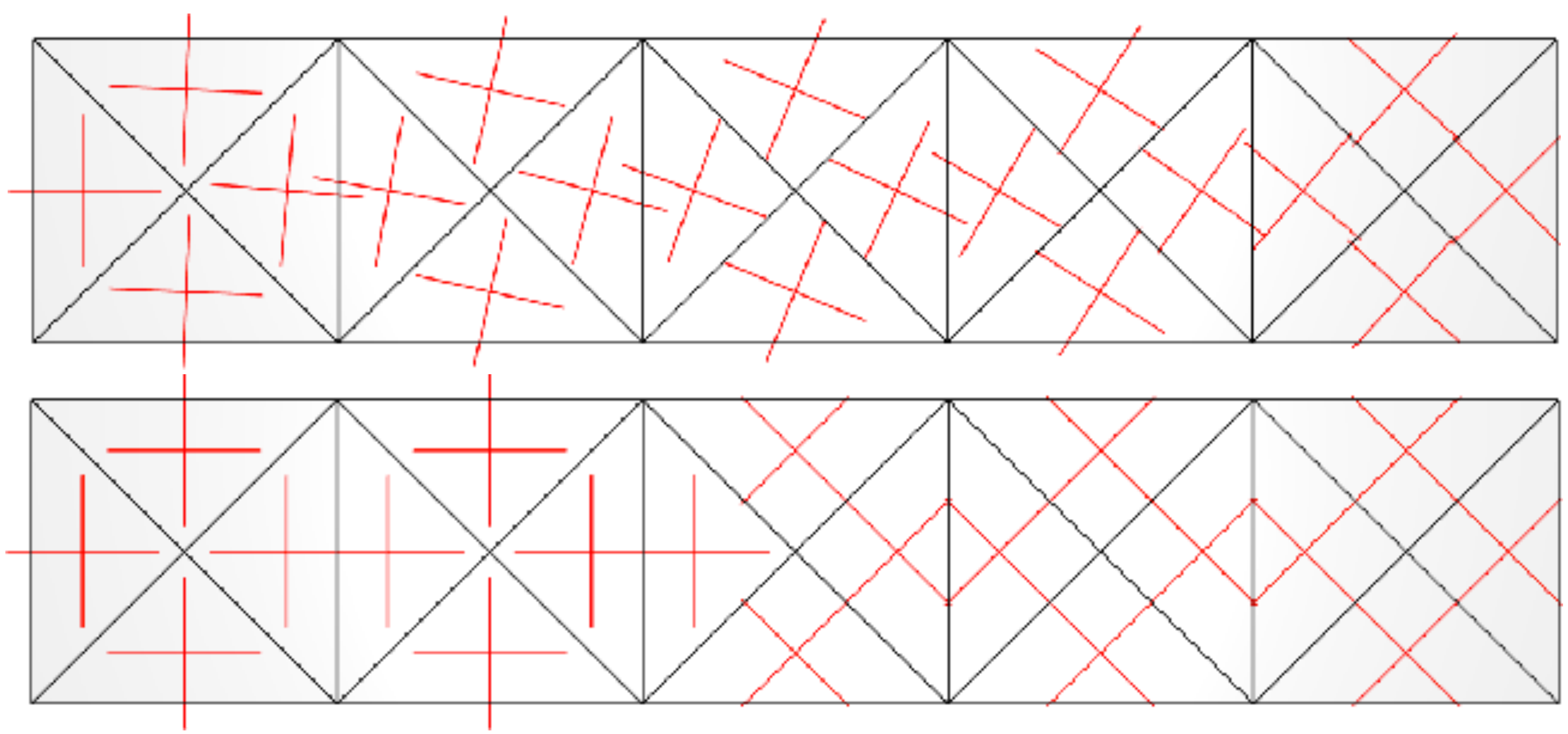
REPRESENTATION

	fixed topology	free topology
smoothest	**angle** linear	**angle** linear, mixed-integer **Cartesian** linear, non-linear constraints Eigenvalue problem



REPRESENTATION

	fixed topology	free topology
smoothest	angle linear	angle linear, mixed-integer Cartesian linear, non-linear constraints Eigenvalue problem



REPRESENTATION

	fixed topology	free topology
smoothest	**angle** linear	**angle** linear, mixed-integer **Cartesian** linear, non-linear constraints Eigenvalue problem
+ directional constraints	**angle** linear, (mixed integer) **1-form** linear, linear constraints	**angle** linear, mixed-integer **Cartesian** linear, non-linear constraints linear, linear constraints

REPRESENTATION

- Alternatives
  - Extrema of periodic circular functions (SH)
  - Eigenvectors of tensors (spd matrices)
  - Functional representation
  - …