

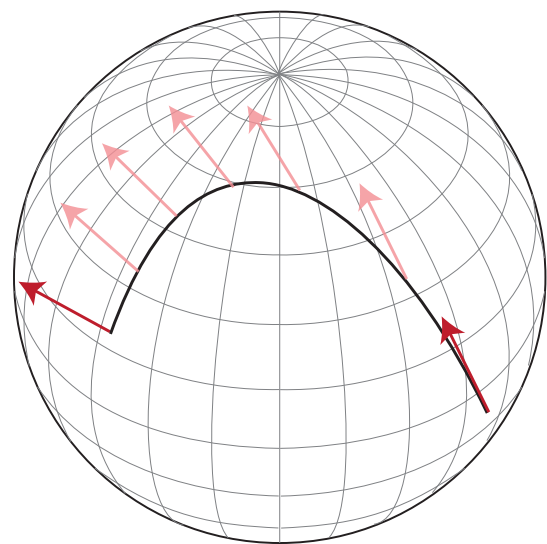
DISCRETIZATION

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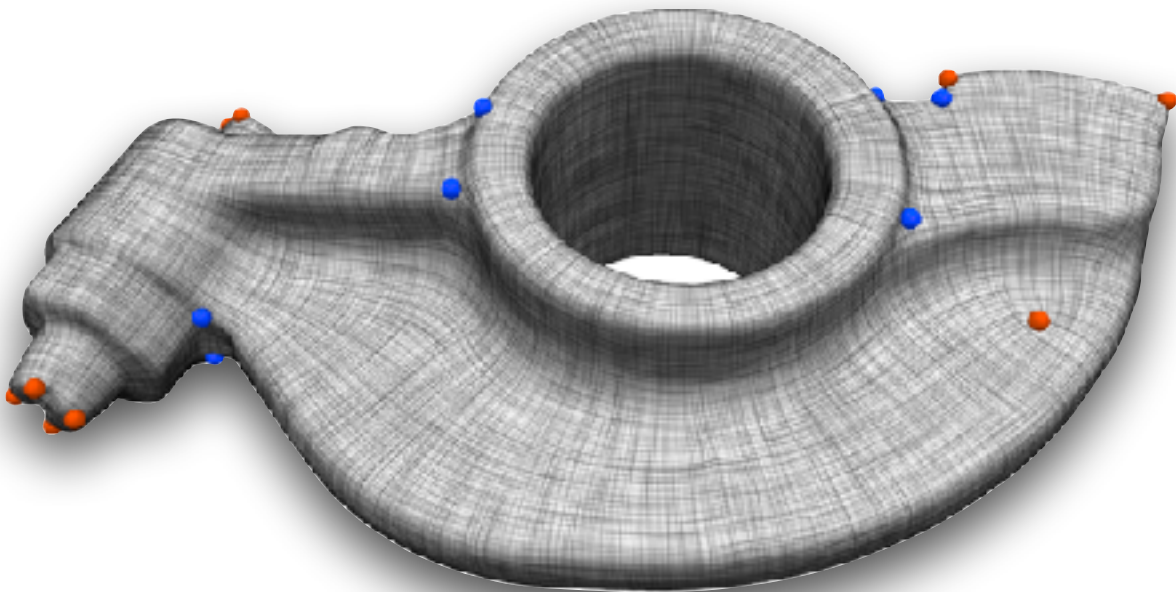
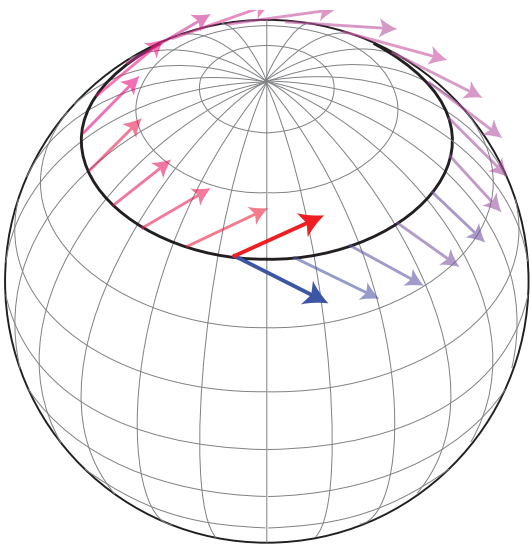
ALL IS WELL KNOWN IN THE CONTINUUM

.....



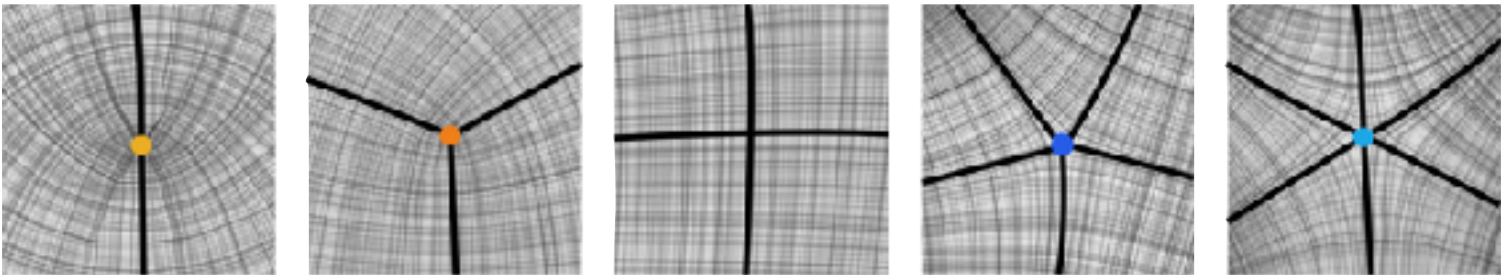
Parallel Transport

Holonomy



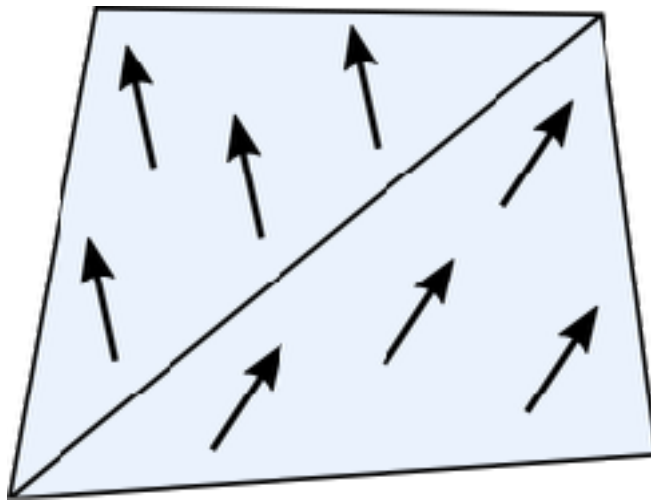
Poincare-Hopf Theorem

Singularities

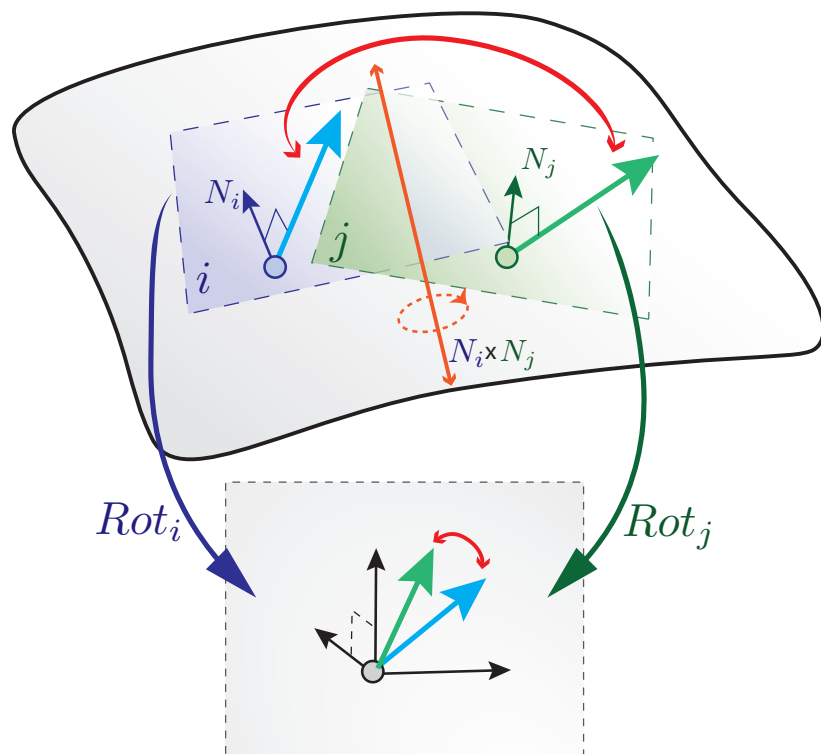


CHALLENGES IN THE DISCRETE SETTING

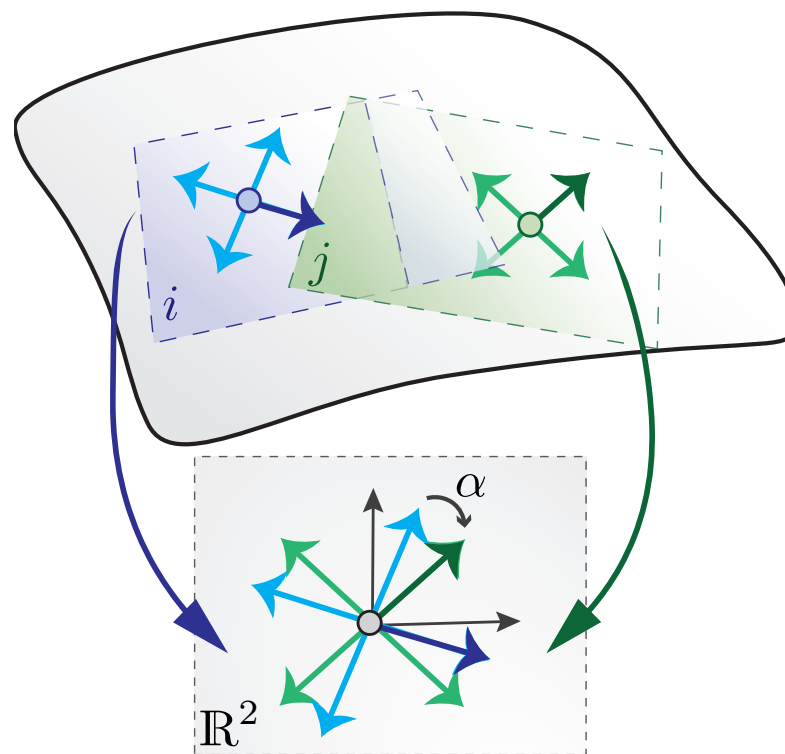
Discontinuity



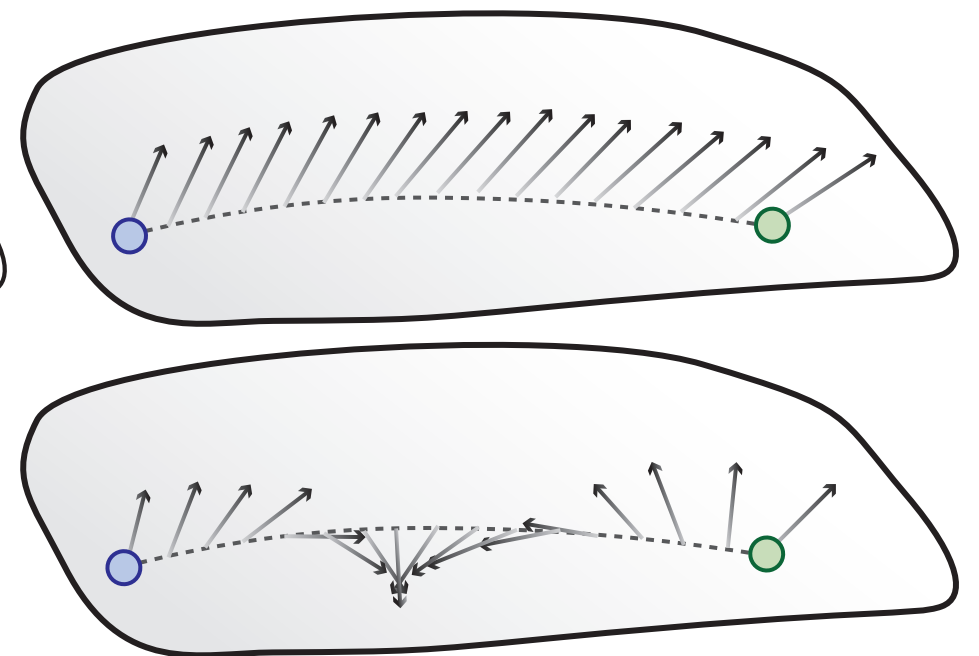
Connection



Matching

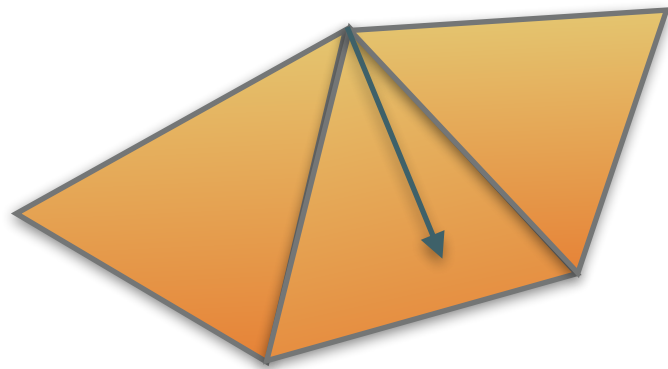


Interpolation



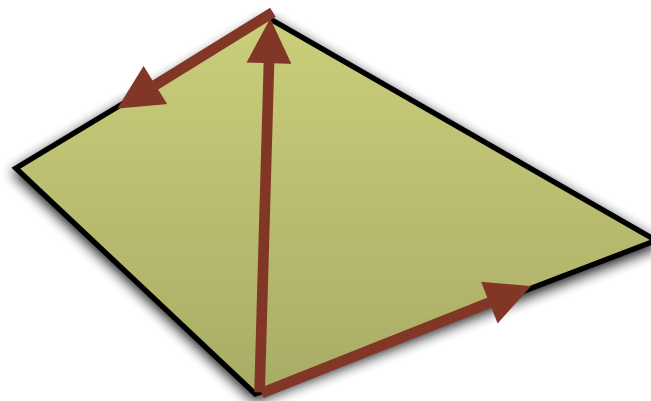
TANGENT SPACES

Vertex based



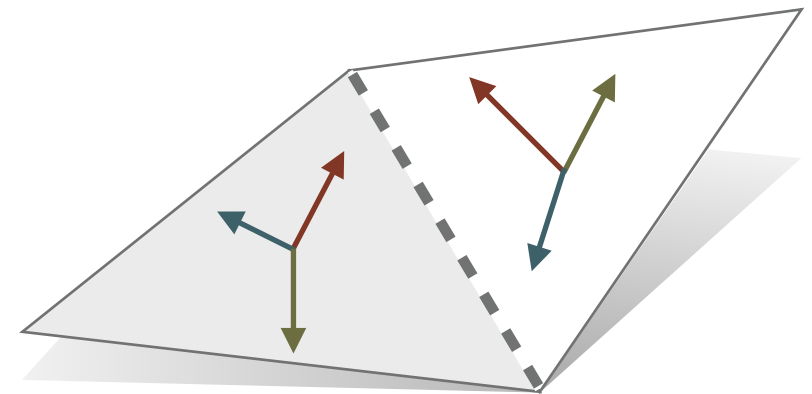
[Polthier and Schmies 98]
[Zhang *et al.* 2006]
[Knöppel *et al.* 2013]

Edge based



[Desbrun *et al.* 2005]
[Fisher *et al.* 2007]
[Ben-Chen *et al.* 2010]

Face based



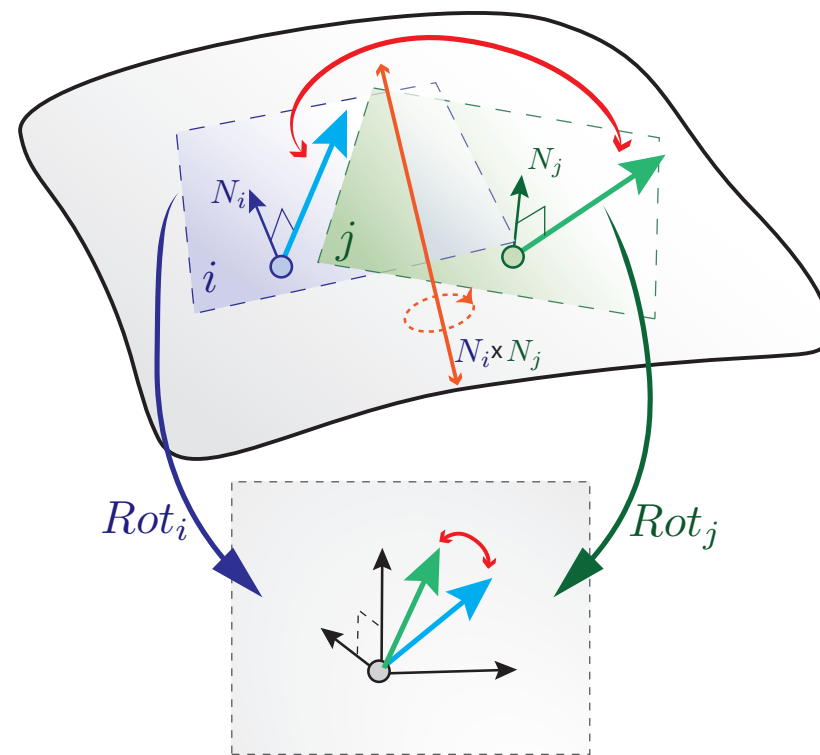
[Bommes *et al.* 2009]
[Crane *et al.* 2010]
[Diamanti *et al.* 2014]

*Choice of tangent space and differential implications: see [deGoes *et al.* 2016].*

DISCRETE CONNECTION

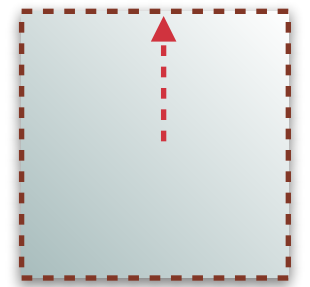
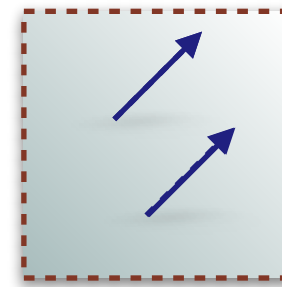
- Bijective linear map between adjacent tangent spaces.
- **Popular choice:** flattening + single axis system.

[Ray *et al.* 2008]
[Crane *et al.* 2010]
[Knöppel and Pinkall 2015]



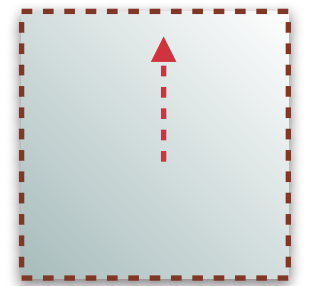
DISCRETE TOPOLOGY: ROTATION

- What happens in between?
- Valid **rotation** choices:



$$\delta_{ij} = \frac{\pi}{4} + 2\pi k, k \in \mathbb{Z}$$

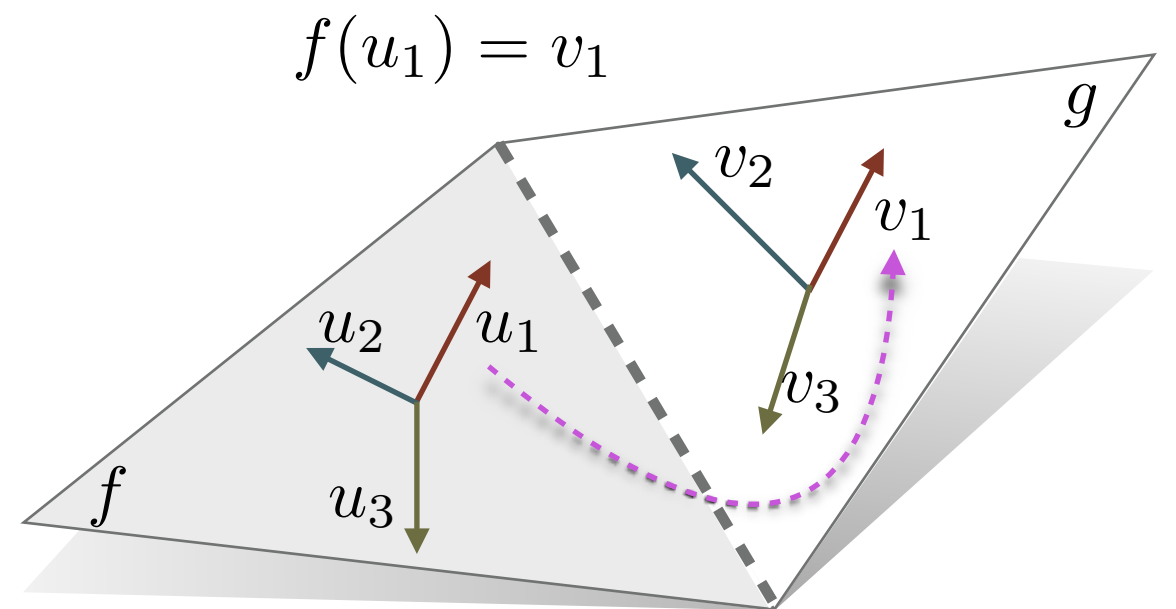
Principal Rotation *Period Jump*
[Li et al. 2006]



- **Implicit:** can only assume principal.
- **Explicit:** period given.

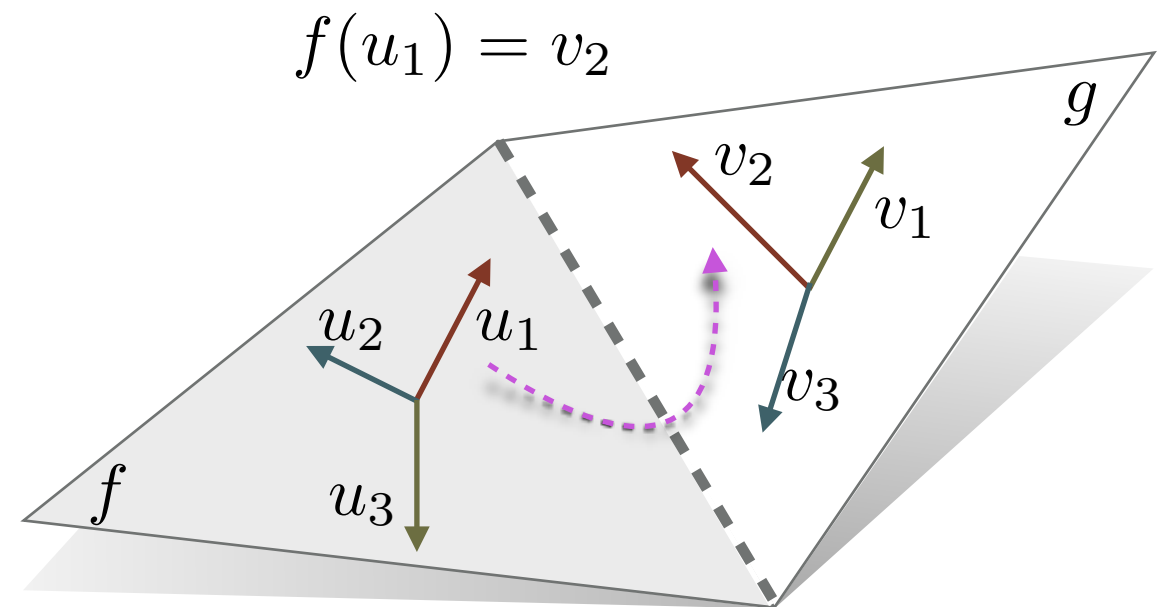
MATCHING

- Which direction to which other?
- **Reduction**: order-preserving. [Diamanti *et al.* 2014]
- **N -directional**: N possible choices.
 - How best to choose?



MATCHING

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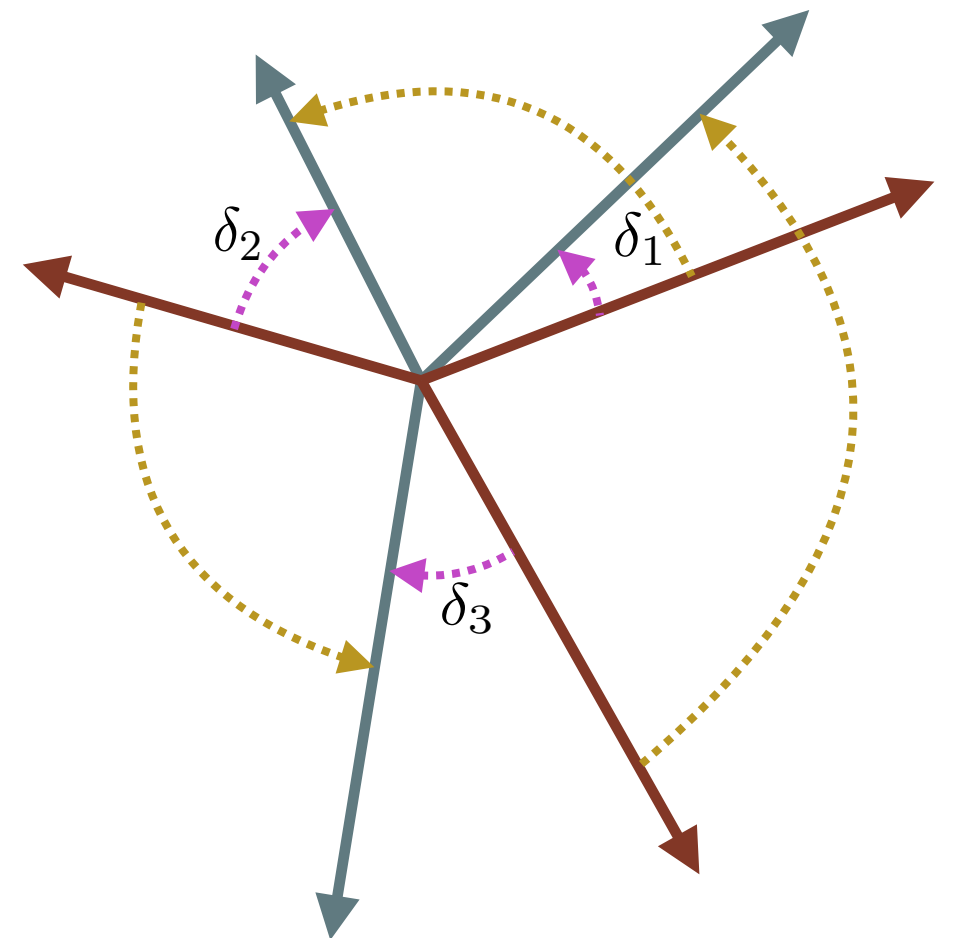


EFFORT

- Sum of matching rotations.
 - **Intuition:** generalize “closest angle” to “minimum effort”.
- All order-preserving matchings differ by $2\pi k, k \in \mathbb{Z}$
- **Principal matching:** the matching with $Y = [-\pi, \pi)$

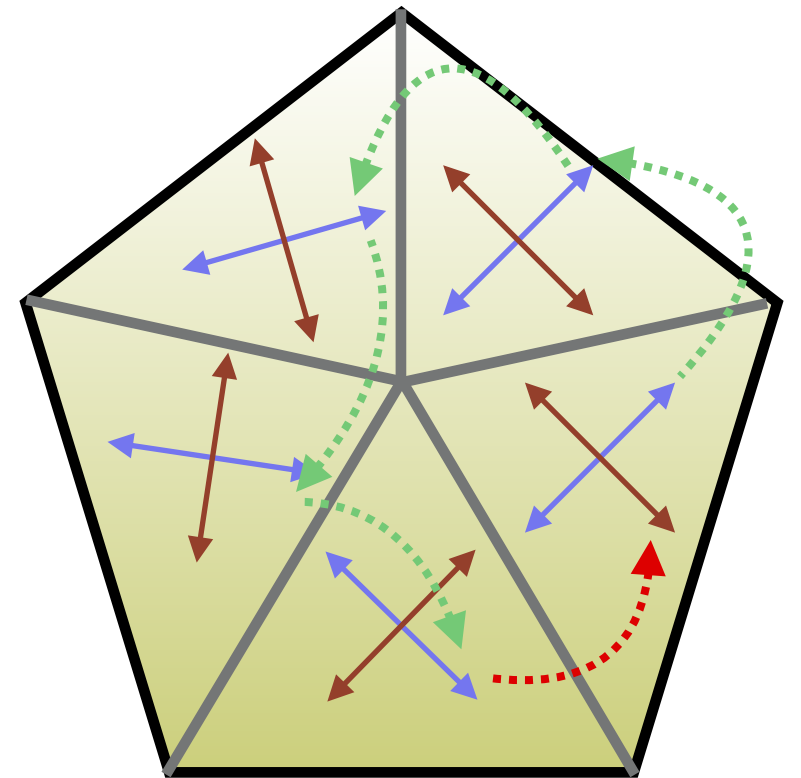
$$Y_1 = \delta_1 + \delta_2 + \delta_3$$

$$Y_2 = \delta_1 + \delta_2 + \delta_3 + 2\pi$$



SINGULARITIES

- Around a matching cycle, directional returns to itself.
- Up to a different matching!
- Directional field as **trivial connections**. [Crane *et al.* 2010]
 - Induced Curvature: $\frac{2\pi}{k}$
- Regular cycles: index 0
- Sum of indices: $2 - 2g$

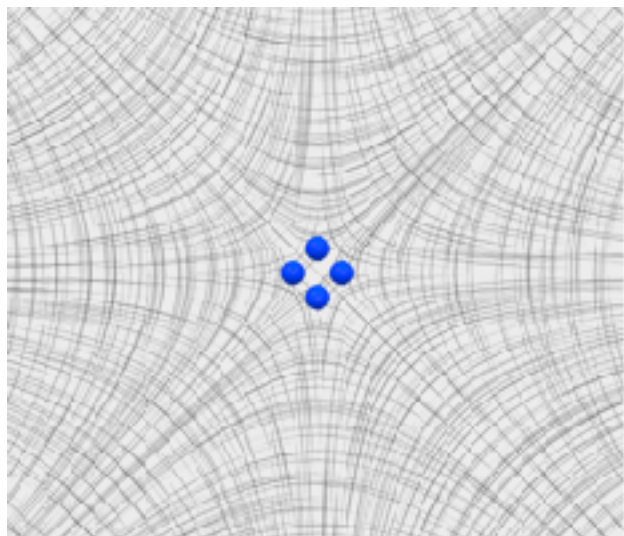


Index of singularity: $\frac{1}{k}$

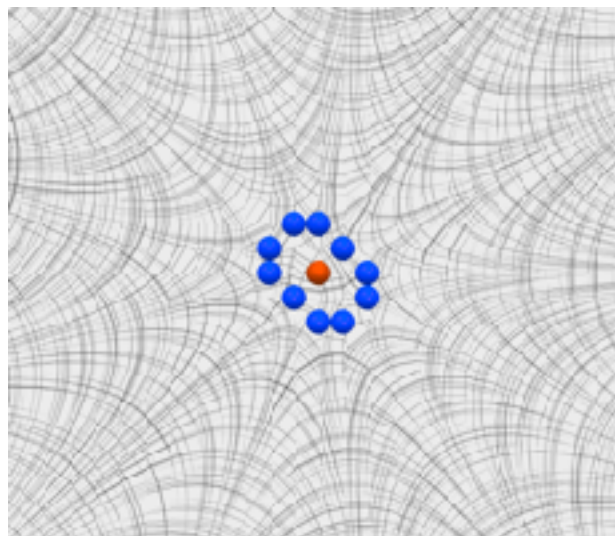
SAMPLING PROBLEM

- **Implicit field**: principal matching assumed.
- Low valence cycles: limited rotation sums.
- Higher order singularities cannot be represented!
- In practice: promoting low-degree singularity cycles (“singularity party”).

$$-\frac{4}{4}$$



$$-\frac{9}{4}$$



$$-\frac{21}{4}$$

