

Problem

Define a number is legal if

- ① it is a natural number (≥ 0 , integer)
- ② it does not contain the digit anywhere in its base 10 representation
- ③ it is not divisible by 9

Find how many number in $[F, L]$ is legal.

And F, L do not contain a 9 digit.

F, L is not divisible by 9

Solution

① Define a function f . $f(x)$ means how many number in $[0, x]$ is legal.

The answer to this problem is $f(L) - f(F) + 1$.

② How to calculate $f(x)$?

a) Turn x to $nums[]$

example: $x = 2018$

$nums = [2, 0, 1, 8]$

Then $x = \sum_{i=0}^{len(nums)-1} nums[i] \times 10^{len(nums)-1-i}$

③ The number of numbers in $[0, x]$ and do not contain digit 9 is.

$$\begin{aligned}
 & \sum_{i=0}^{len(nums)-1} nums[i] \times 9^{len(nums)-1-i} \\
 = & \sum_{i=0}^{len(nums)-2} \left(\sum_{j=0}^{len(nums)-1-i} 9^j \right) \times nums[i] \times 9^{len(nums)-1-i} \\
 & + nums[len(nums)-1] \times 9^{len(nums)-1}
 \end{aligned}$$

For this part, the last digit will be 0, 1, 2, ..., 8. We need to multiply $\frac{8}{9}$ to it.

For this part, we will check the numbers individually.

Example: 2018

$$(2 \times 9^3 + 0 \times 9^2 + 1 \times 9) \times \frac{8}{9} + 8 \times 9^0$$

Test 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018
Individually

Reminder

python: `2**3`

c++: `#include <stdio.h>`

`#include <math.h>`

~~`pow(2,3)`~~

Don't use this function.

Disable the scientific notation:

`cout.setf(cios::fixed, ios::floatfield);`

